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A heterogeneous-agent model of growth and inequality for the UK - what has caused the UK's growth collapse since 2008?

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June 2, 2026

Abstract

This paper builds on Yang et al. (2021). This analysed the effect on UK economic growth of wealth inequality and tax or regulative disincentives to entrepreneurs, over recent decades with a heterogeneous-agent growth model where agents can enhance individual productivity growth by undertaking entrepreneurship. In this further note we update the study with the latest data and also examine how far recent policies have contributed to the collapse in UK growth since 2008. The model is estimated and tested by indirect inference, with data updated to 2024. The model explains the UK rise in growth after the 1970s and its recent collapse in terms of changing disincentives to entrepreneurs.

Keywords: Heterogeneous-agent Model, Entrepreneurship, Growth, Inequality, Indirect Inference, planning, infrastructure

JEL codes: E10, O30, O40

We thank Xialiang Yang for the data and model codes used in Yang et al. (2021).

1 Introduction

Theories of growth focus on a wide range of factors, as is apparent in the huge literature on factors causing or preventing growth. In this paper we build on Yang et al. (2021) who develop a theory of growth and inequality for the UK and test it on a long episode of UK history when policies towards the economy changed radically, with strong effects on economic growth and general macroeconomic behaviour. They used Indirect Inference to test the match between the facts of inequality and growth and the predictions of their DSGE model of growth and inequality in the UK economy. Here we both bring the data set up to date, as far as 2024,[1] and we also extend the model to include other factors that have been argued to contribute to growth- including public spending on infrastructure and housing planning permission, both of which have been hindered in the UK by government policies that have given power to special interest groups opposing development on environmental grounds, see Zenghelis et al. (2024). These factors have also been stressed by the current Labour government, which in its first budget raised tax rates sharply but has since emphasised the need to reduce obstacles to infrastructure and housing investment, see speech by Chancellor Rachel Reeves on growth, January 22, 2024[2]. We also include in our data the effects of the latest tax and regulative policy changes introduced by the current government.

For the UK Yang et al. (2021) found that the DSGE model was not rejected. Their model emphasises the key role of tax and regulation in driving growth via entrepreneurial action. Inequality both results

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[1] It therefore updates Minford and Zhu (2025) by including the latest policy changes introduced by the current Labour government, with their computed effect on growth since 2008.

[2] https://www.gov.uk/government/speeches/chancellor-vows-to-go-further-and-faster-to-kickstart-economic-growth

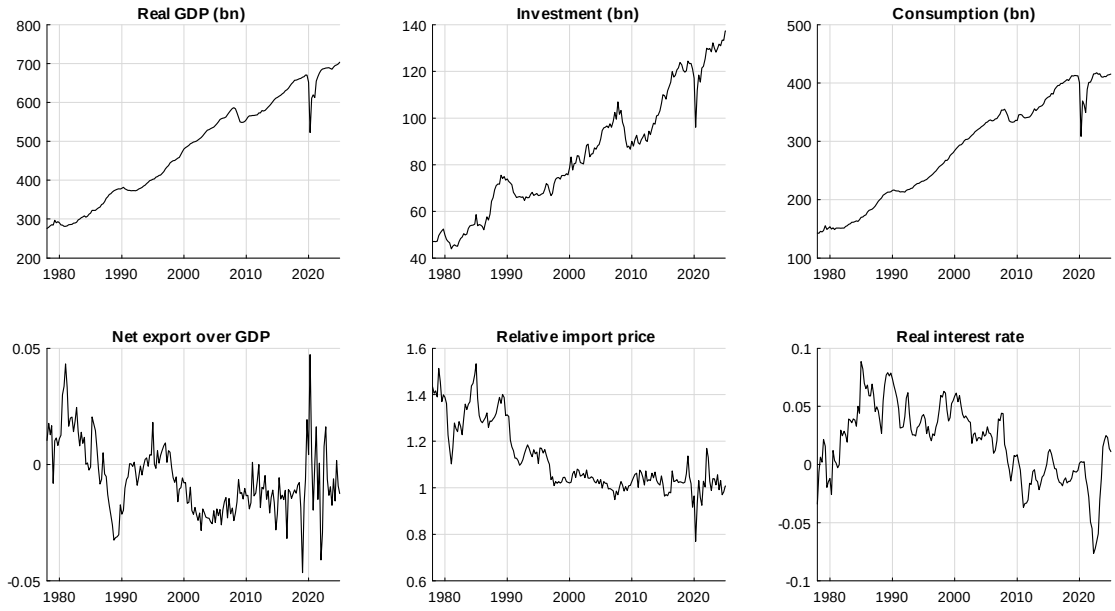
from growth and reinforces growth via a link between wealth and lower exposure to the costs of taking entrepreneurial risks - the wealthy are both less risk-averse and have a lower marginal disutility of costs. Our aim in this paper is to extend the model to include the potential effects of infrastructure and housing investment, as well as the latest tax and regulative policy changes and to test this extended model against the same data behaviour as the original model but updated to 2024. What we find in this paper's extension is that the introduction of housing and infrastructure worsen the model's explanatory power; and that if they alone are included as explanations of growth, the model is strongly rejected. Thus we find that these are not factors that explain growth. Furthermore, we find that it is the introduction of higher tax rates on entrepreneurs and more intrusive labour market regulation since the 1990s that is responsible for the collapse of UK growth in recent decades.

In the rest of this paper we explain our new data and results. The reader is referred to Yang et al. (2021) for our review of the literature on growth and inequality, and on heterogeneous agent models of the type used here. For convenience, we include the model's set up and specification in a Model Appendix and an account of our Indirect Inference methodology in a second appendix.

In the remainder of this paper, the first section reviews the data we use in the various models we test. The second sets out our empirical results. The last concludes.

2 UK Data used in this study

Figure 1: Facts of the postwar economy in the UK



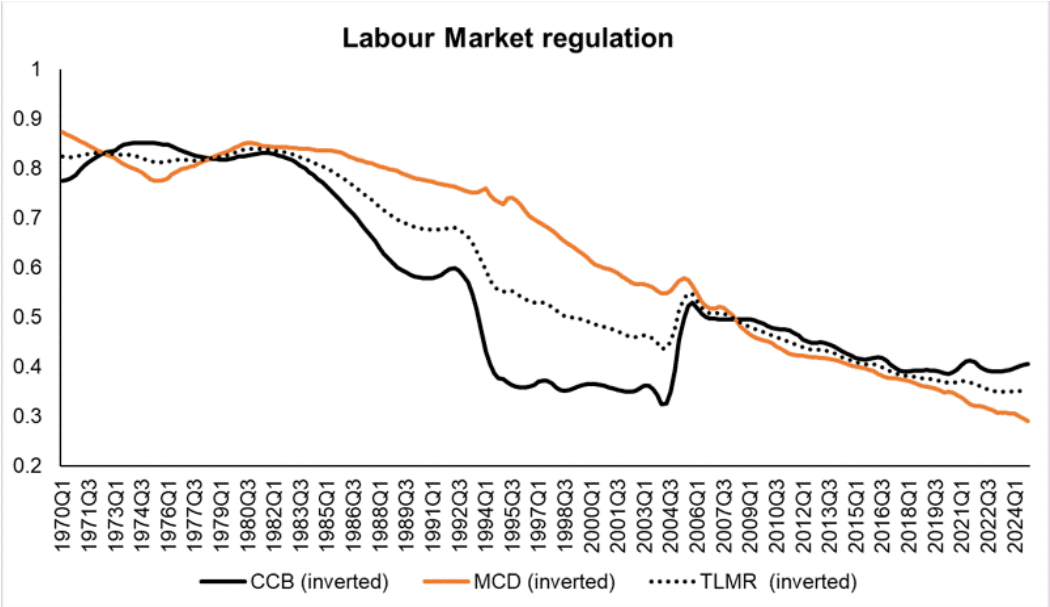
The facts of inequality also show substantial movement during this period, as indicated in the charts following for the shares of the top 10% of the population in income, wealth and consumption. For example, the income share of the top 10% rose from 1980 to the mid-1990s before levelling off, while their wealth share rose to a peak at the end of the 1990s before falling back almost to its starting point. Their consumption share fluctuated around a slightly rising trend. Notice how income and consumption inequality rise much less than capital inequality, due to the operation of the benefits system which was boosted during this period.

Figure 2: Inequality Indicators



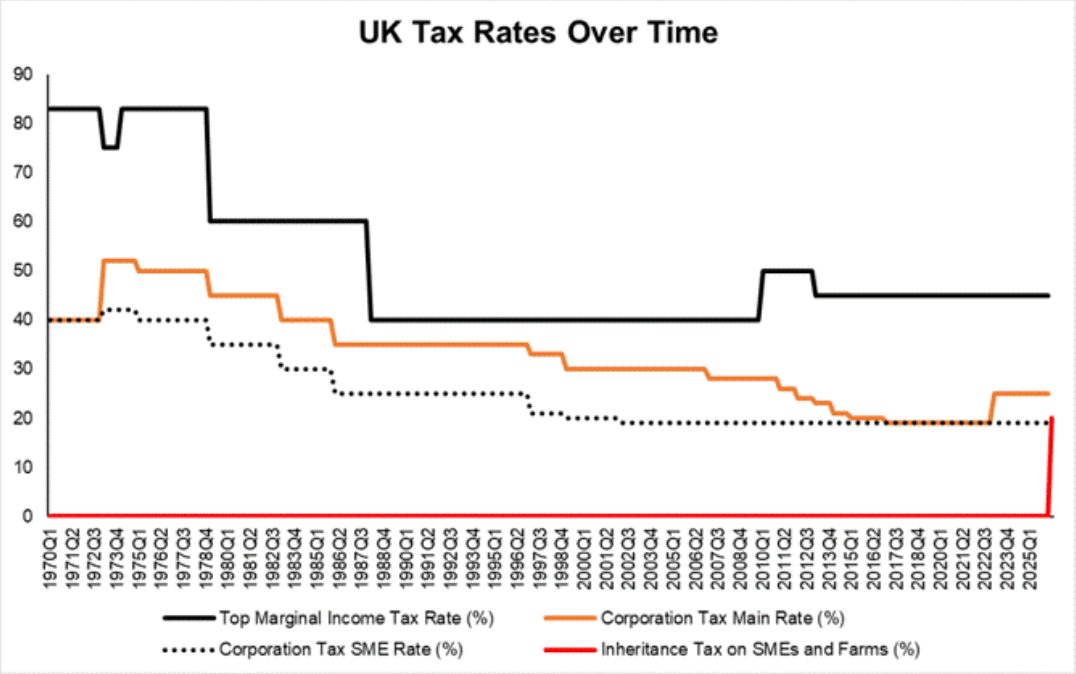
Following Yang et al. (2021), the Labour Market Regulation (LMR) index is constructed as the arithmetic mean of two sub-indicators: Centralised Collective Bargaining (CCB) and the Mandated Cost of Worker Dismissal (MCD), obtained from the Fraser Institute's Economic Freedom of the World database. Both indicators are originally measured on a 0–10 scale, where higher values indicate greater economic freedom and lower regulatory intensity. To ensure consistency with the notion of regulatory cost, each component is first inverted (10 minus the original index), and the Transformed Labour Market Regulation (TLMR) is then defined as the average of the transformed components. A higher TLMR therefore reflects greater labour market rigidity and higher regulatory costs.

Figure 3: Labour Market Regulation



We replace the corporation tax rate (CTR) with the top marginal income tax rate (TMITR), obtained from the OECD Database. Figure 4 compares the TMITR with alternative corporate tax measures, including the main corporation tax rate and the SME corporation tax rate.

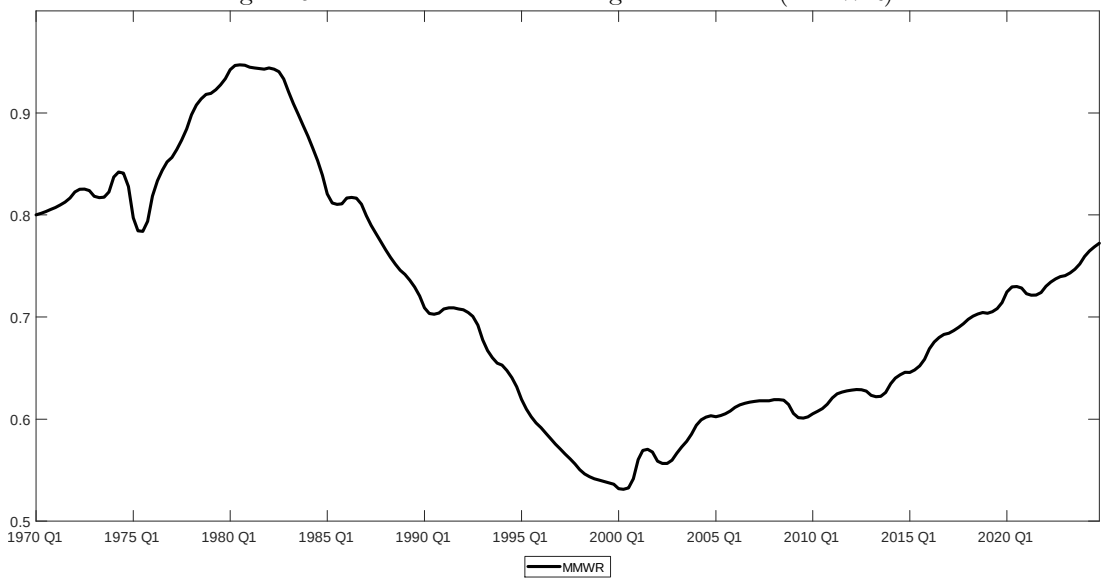
Figure 4: UK Tax Rates Over Time



Note: Inheritance tax on SMEs and farms is set at 20% on value above the £2.5 million threshold from April 2026.

For the minimum-to-median wage ratio (MMWR), the data are available only from 1999 onwards and are obtained from OECD statistics. For earlier periods, it is proxied by the unionisation rate from UK labour market statistics. This allows the combined MMWR series to be extended back to 1978.

Figure 5: Minimum-to-median wage ratio index (MMWR)



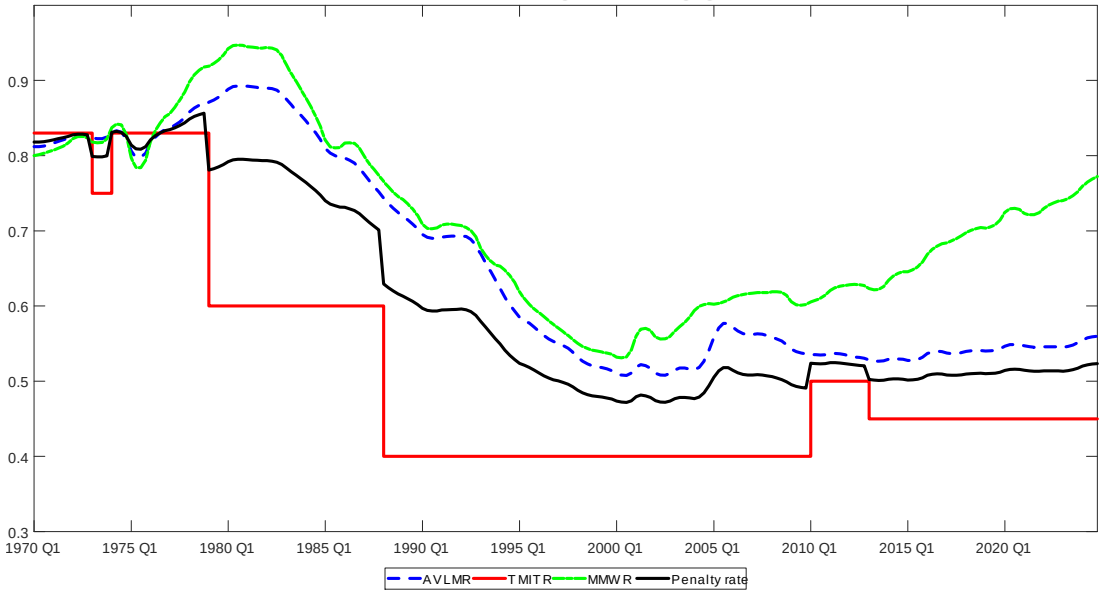
In Yang et al. (2021), the entrepreneurship penalty rate is constructed as the average of a transformed labour market regulation index (TLMR) and the corporation tax rate (CTR), we extend and refine this

measure by defining the penalty rate as an equally weighted average of the transformed labour market regulation index (TLMR), the top marginal income tax rate, and the combined MMWR. Formally,

$$\pi_t = \frac{1}{3}TLMR_t + \frac{1}{3}TMITR_t + \frac{1}{3}MMWR_t \tag{1}$$

Therefore a higher value of π_t indicates a greater regulatory, tax, and labour market burden on entrepreneurial activity.

Figure 6: Generated entrepreneurship penalty rate



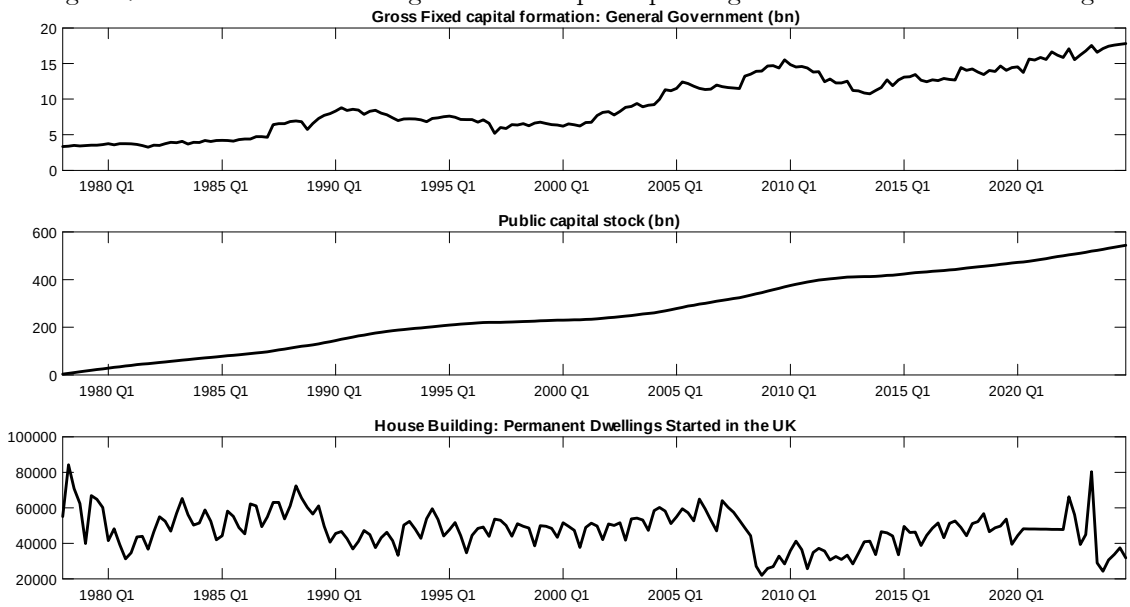
In this paper, we extend Yang et al. (2021) to include data on planning restrictions for housing and the stock of government capital spending on infrastructure. Here, we proxy infrastructure investment using gross fixed capital formation (GFCF) for general government, which reflects public spending on long-term assets. To obtain the public capital stock k_t we accumulate the GFCF and apply an annual depreciation rate δ of 2.5% by the perpetual inventory method as follows:

$$k_t = (1 - \delta)k_{t-1} + I_t \tag{2}$$

In relation to planning constraints in the housing sector, we use quarterly data on permanent dwellings started in the UK which serves as a practical proxy.

It can be seen that the public stock of infrastructure grew more rapidly in the 2010 decade, otherwise accumulating at a quite steady rate. Planning constraints tightened in that same decade, as evidenced by the fall-off in housing starts.

Figure 7: Data for the stock of government capital spending on infrastructure and housing



3 Empirical results:

Testing the model for the role of planning and infrastructure. We follow Yang et al. (2021) in their test of the model on UK data from 1978Q1 but now extend the sample period to 2024Q4. Our aim is to see whether adding planning and infrastructure to the productivity equation and updating the data for the latest policy changes affects the model's fit to the data behaviour.

We first report here the results for testing our basic model of endogenous growth, featuring the tax on entrepreneurs and inequality. For this model the p-value was 6.3%, above the 5% confidence threshold, a somewhat closer fit than in Yang et al. (2021). Next we add the two variables suggested in the LSE work: planning permissions (proxied by house starts) and infrastructure (proxied by the publicly created capital stock). These when added, the p-value is reduced to 0.4%, indicating the model is strongly rejected. If we include them but exclude the tax and inequality terms (model 2) the model is strongly rejected with a p-value of zero. A general model (model 3) of exogenous productivity with no identified causal theory, is also strongly rejected with a p-value of zero. What this all means is that planning and infrastructure do not explain growth but rather worsen its explanation. What explains growth is the entrepreneurial penalty and the extent of inequality in capital ownership. These results are more decisively unfavourable to the models that are modified from the Yang et al. (2021) set-up than we found in Minford and Zhu (2025) before we updated the sample.

Table 1: Fixed Parameters

Fixed parameters		Value
Share of capital in Cobb-Douglas production	α	0.3000
Utility discount rate	β	0.9975
Capital depreciation rate	δ	0.0034
Share of consumption preference in CRRA utility	ϕ	0.5000
Elasticity of consumption in CRRA utility	ψ_1	1.0000
Elasticity of labour in CRRA utility	ψ_2	1.0000
Drift in individual entrepreneurship penalty equations	ρ_0^{π}	0.3690
Steady-state consumption share by top 10% income decile	$\varpi_{C,1}$	0.2000
Steady-state aggregate output/consumption ratio	Y/C	1.7100
Steady-state aggregate capital/consumption ratio	K/C	33.535

Table 2: Estimated Parameters

Estimated parameters		Baseline	Model1*	Model2**	Model3***
Entrepreneurship time effect on individual productivity growth	θ_2	0.4356	0.5238	0.4982	0.5352
Capital on individual entrepreneurship penalty rate	ρ_2^{π}	-0.0009	-0.0011	-0.0013	-0.0083
Penalty rate on productivity growth for the rich	$\phi_{2,1}$	-0.6754	-0.5758		
Penalty rate on productivity growth for the poor	$\phi_{2,2}$	-0.2657	-0.3214		
Infrastructure investment on productivity growth for the rich	$\kappa_{1,1}$		0.4372	0.5421	
Infrastructure investment on productivity growth for the poor	$\kappa_{1,2}$		0.2547	0.3617	
House planning on productivity growth for the rich	$\kappa_{2,1}$		0.5538	0.6125	
house planning on productivity growth for the poor	$\kappa_{2,2}$		0.1684	0.1876	
T-stats		1.52	2.67	5.75	3.53
P-value		6.3%	0.4%	0.0%	0.0%

*Model 1 includes infrastructure investment and house planning effects in the productivity
**Model 2 excludes entrepreneurship tax effects on productivity
**Model 3 excludes entrepreneurship tax, infrastructure investment and house planning effects on productivity

We can see from various outputs from the model how the key policy changes have caused the sharp slowdown in the UK's productivity since 2008 after the resurgence in the 1980s and 1990s. The model explains the slowdown in growth as due to the rise in marginal tax rates and in labour market regulation. Figure 8 shows the model predictions for productivity of the top 10% and the bottom 90%, from 1970. Figure 9 shows the shock decomposition of average productivity from the model for the same period. What one can see here is that the effects of the tax/regulation disincentives worsen in the past two decades, offsetting the expansionary effects from the trend and shock terms, and so causing overall stagnation of productivity after 2008. Figure 10 shows the UK's performance relative to France and Germany in GDP per capita, which mirrors what the model is predicting from the productivity slowdown.

Figure 8: Productivity: Top 10% vs Bottom 90%

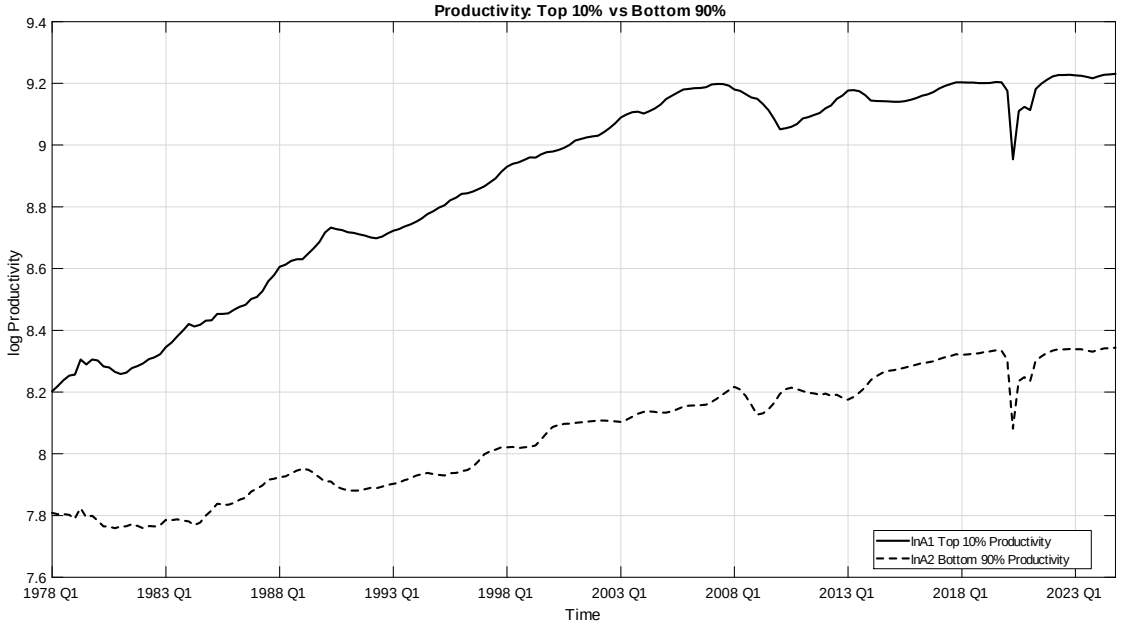


Figure 9: UK productivity timeline - shock decomposition

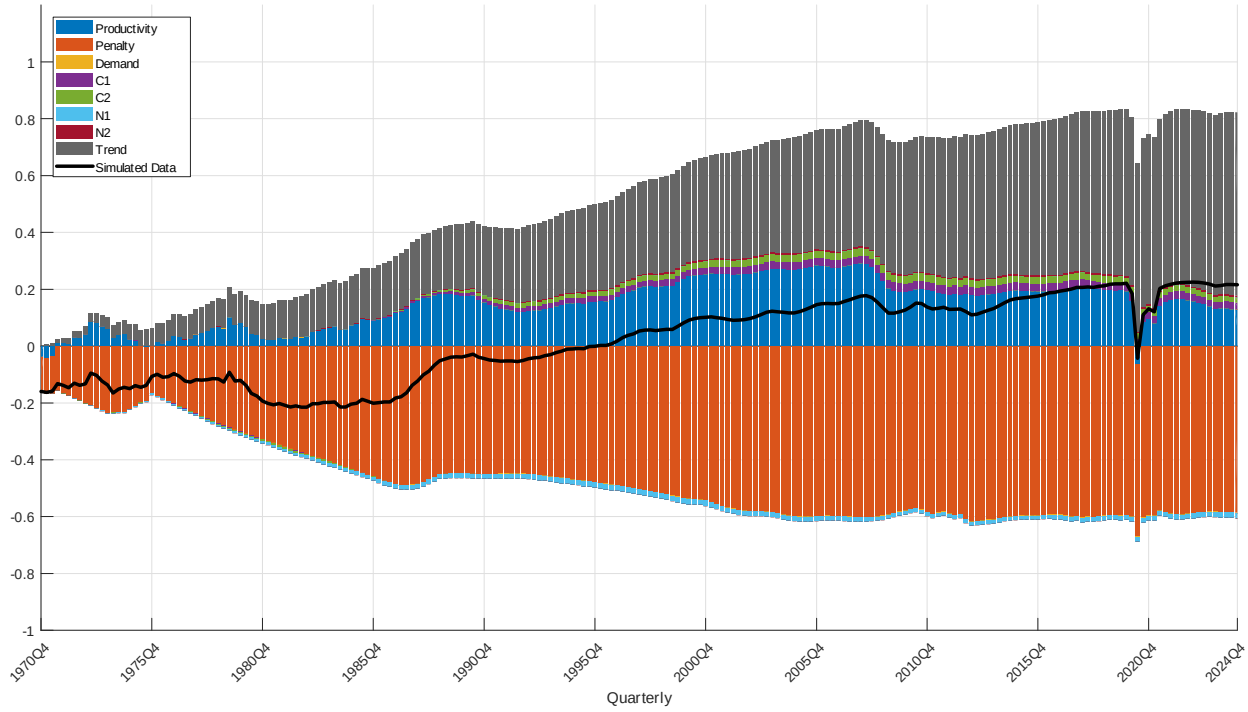
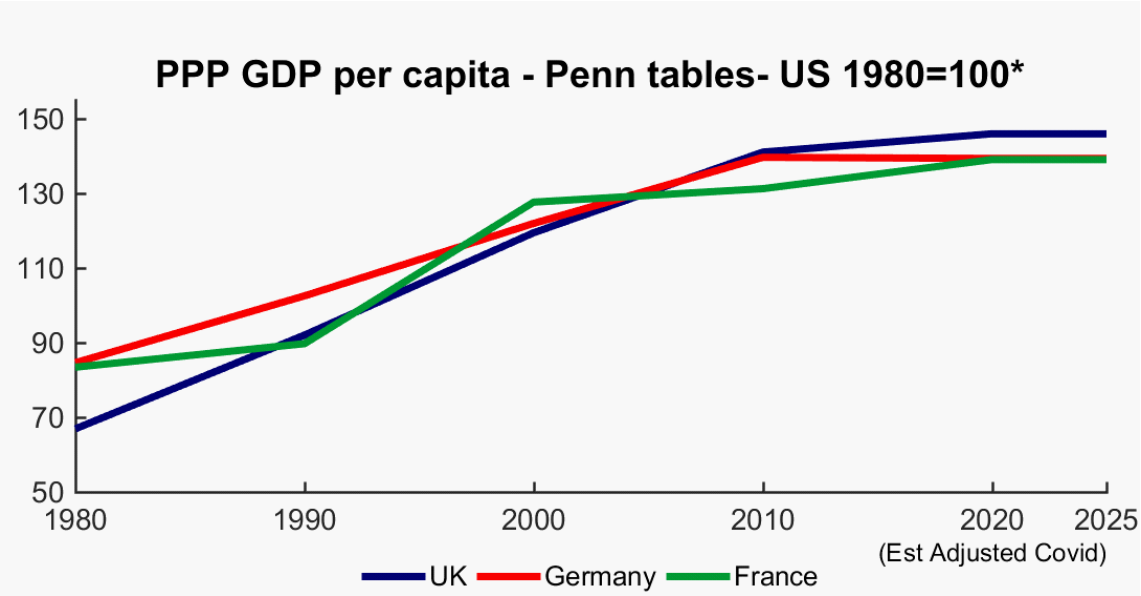


Figure 10: GDP per capita, UK vs France and Germany since 1980



4 Conclusion

We have followed Yang et al. (2021) in setting out a model of endogenous growth by entrepreneurial innovation in which the major influences are the tax/costs levied on firms' innovation and the extent of inequality (indicating the proportion of entrepreneurs who are rich). These last two factors stimulate innovation; this in turn creates growth which increases inequality as much of it accrues to rich entrepreneurs. We tested this model by indirect inference and found that it passed our test at 5% confidence.

Our aim in this paper was to add to this model an explanatory role for planning permission and infrastructure in the promotion of productivity growth, as argued by Zenghelis et al. (2024). At the same time we extended the data sample to the latest date, 2024 Q4, for which it is available. What we found was that these extra factors worsen the model's explanatory power. When put into the model as the sole explanatory factors (i.e.without the tax and inequality terms), the model was strongly rejected. Hence we find these factors make a negative explanatory contribution, while the tax and inequality factors on their own produce satisfactory explanatory power over the whole sample period. Their evolution also accounts both for the UK's big growth improvement after the 1970s and its collapse in the recent two decades.

We conclude that the UK data support the original model of Yang et al. (2021) based on entrepreneurs' incentives to innovate. The model suggests that both the UK's improvement in growth after the 1970s and its worsening since the 1990s were due to changes in these incentives.

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5 MODEL APPENDIX: A model of growth based on entrepreneurial incentives with an extension to planning and infrastructure

Following Lucas (1988) we examine representative households operating in competitive markets who own a Cobb-Douglas production function with constant returns to scale dividends and wages add up to total GDP. We assume that each household can work or may undertake some activity to innovate its methods, so raising its productivity. This is the model's mechanism of growth. We assume that it is 'entrepreneurship'. This is affected by the 'business tax rate'. The economy is closed. This economy can be summarised in the equations that follow.

The representative household's utility function seen from period 0 is:

$$U_t = E_0 \sum_{t=0} \beta^t (\ln c_t + \alpha_t l \ln x_t) \tag{3}$$

where α_t is a stationary preference error process,
subject to

$$(1+\tau_t)c_t + k_t - (1-\delta)k_{t-1} + b_t = y_t + (1+r_{t-1})b_{t-1} + \Gamma_t - \pi_t z_t \tag{4}$$

where:

τ is the tax rate on consumption — this is assumed to be the sole general tax (so that dividends and wages are taxed indirectly through consumption);

π is tax levied on entrepreneurial activity;

consumption (c), capital stock (k), foreign bonds (b), leisure (x), productivity-enhancing activity (z) and government transfers (Γ) are all expressed per capita;

δ is depreciation and r is the real rate of interest on foreign bonds. Goods are bought by some system of organised barter and so we ignore the role of money in this economy;

$y_t = A_t k_t^{\gamma} X_t^{\zeta} (1 - x_t - z_t)^{1-\gamma-\zeta} + e_t$ is the Cobb-Douglas production function of the household (and firm combined). X represents exogenous other production factors — such as 'land'/natural resources — assumed to be owned by households; e_t represents firm-level idiosyncratic risk, with mean zero and variance $vare$.

The household's first order conditions familiarly yield

$$c_t = \frac{1}{\beta(1+\tau_t)} E_t \frac{c_{t+1}(1+\tau_{t+1})}{1+r_{t+1}} \tag{5}$$

the condition relating the marginal product of capital (which we also denote by the shadow real dividend rate, d_t) to world real interest rates plus depreciation:

$$r_t + \delta = \frac{\gamma y_t}{k_t} = d_t \tag{6}$$

and the condition relating labour supply to the marginal product of labour (which we also denote by the shadow real wage,w_t):

$$a) w_t = \frac{(1-\gamma-\zeta)y_t}{(1-x_t-z_t)};\ b) x_t = \frac{\alpha_t l c_t (1+\tau_t)}{w_t} \tag{7}$$

Using the marginal productivity of capital condition, we can replace capital in the production function by terms in the shadow dividend (determined in 5).

$$y_t = A_t^{\frac{1}{1-\gamma}} \left(\frac{\gamma}{d_t}\right)^{\frac{\gamma}{1-\gamma}} X_t^{\frac{\zeta}{1-\gamma}} (1 - x_t - z_t)^{\frac{1-\gamma-\zeta}{1-\gamma}} \tag{8}$$

We now turn to the determination of productivity growth and the marginal condition determining z, the productivity-enhancing activity of workers.

We write the growth of productivity as:

$$\frac{A_{t+1}}{A_t} = a_0 + a_1 z_t + u_t(1) \tag{9}$$

where u_t is an error process, which includes any effect of planning and infrastructure; and the parameter a_1 is the effect of the entrepreneurial activity on productivity growth. This equation embodies our key hypothesis, that time spent in entrepreneurial activity, z_t, seeking improved processes and market opportunities, instead of working in an existing firm activity, yields innovation and so productivity gains.

Going back therefore to the household's optimising decision, its first order condition for z_t at time 0 is given by

$$0 = E_0 \sum_{t=1}^{\infty} a_1 \frac{A_0}{A_1} \beta^t \frac{y_t}{(1+\tau_t)c_t} - [Vare/c_t] - \lambda_0(w_0 + \pi_0) \tag{10}$$

Where the first term is the expected net return in terms of marginal utility from the extra productivity growth and the second term is the utility value of risk, the expected value of $(e_t/c_t)^2$, that is of the square of the marginal utility at t of the error, converted to equivalent marginal utility of permanent consumption, so becoming$[Vare/c_t]$[3].

$$\frac{A_1}{A_0} = E_0 \sum_{t=1}^{\infty} a_1 \beta^t \frac{y_t}{(1+\tau_t)c_t} / [\lambda_0(w_0 + \pi_0) + [Vare/c_t]\}] \tag{11}$$

Hence from (1) we have the choice of z_0 that delivers this productivity growth:

$$z_0 = E_0 \sum_{t=1}^{\infty} \beta^t \frac{y_t}{(1+\tau_t)c_t} / \{\lambda_0(w_0 + \pi_0) + [Vare/c_t]\} - [a_0 + u_0]/a_1(1) \tag{12}$$

Now we note that $E_0 \frac{y_t+i}{c_{t+i}} = \frac{y_0}{c_0}$- this is because $c_t = \overline{y_t}$ which is permanent income (for a fuller formal derivation see Minford et al., 2007, Appendix A) so that $\frac{y_t}{c_t} = \frac{y_t}{\overline{y_t}} = \epsilon_t$, a random error which is temporary and we assume for convenience to last for only one period; similarly we assume conveniently that the current tax rate is expected to be fixed into the future. Applying these assumptions and using the solutions above for w_t yields:

$$\begin{aligned} \frac{A_1}{A_0} &= \left\{ \frac{a_1\beta}{1-\beta} \frac{1}{(1+\tau_0)} \frac{y_0}{c_0} \right\} / \left\{ \frac{1}{c_0(1+\tau_0)} \left(\frac{[\alpha_0 + u_0] l c_0 (1+\tau_0)}{x_0} + \pi_0 \right) + [Vare/c_t] \right\} \\ &= \left\{ \frac{a_1\beta}{1-\beta} \frac{y_0}{c_0} \frac{x_0}{\alpha_0 l} \right\} / \{(1+\tau_0)[1 + \pi_0/w_0] + [Vare/c_t]\} \end{aligned} \tag{13}$$

We now linearise this as:

$$\frac{A_1}{A_0} = \phi_0 - \phi_1(\tau_0 + \pi_0') - \phi_2[Vare/c_t] + error_0 \tag{14}$$

where $\pi_0' = \pi_0/w_0$ is the tax/subsidy on the productivity-enhancing activity as a fraction of the real wage; and since $\frac{A_{t+1}}{A_t} = \Delta \ln A_{t+1} + 1$, gathering constants, including $-\phi_2[Vare/c_t]$, as ϕ_0' and letting $u_t' = error_t$ we obtain

$$\Delta \ln A_{t+1} = \phi_0' - \phi_1(\tau_t + \pi_t') + u_t' \tag{15}$$

What we see is that the tax rate on the activity z consists of both the general tax rate and the particular imposts (or subsidies) levied on the activity as such. Notice that the error includes any effect of planning and infrastructure; this enters through their effect on z_0 and hence on wages.

[3] We derive this as follows:
Utility $u = \ln c; U_t = E_t \sum \beta^t u_t = \sum \beta^t [E_t \ln c_t +$
$E_t 1/2 \sum_{i=0}^{i=\infty} (\epsilon_{t+i}/(c_t - \epsilon_{t+i}) - \epsilon_{t+i}/(c_t + \epsilon_{t+i}) =$
$\sum \beta^t [E_t \ln c_t + E_t 1/2 \sum_{i=0}^{i=\infty} (\epsilon_{t+i}(c_t + \epsilon_{t+i}) - \epsilon_{t+i}(c_t - \epsilon_{t+i})]/(c_t + \epsilon_{t+i})(c_t - \epsilon_{t+i}) \approx \sum \beta^t [E_t \ln c_t + E1/2 \sum_{i=0}^{i=\infty} \{\epsilon_{t+i}(c_t + \epsilon_{t+i}) - \epsilon_{t+i}(c_t - \epsilon_{t+i})\} c_t^2 = \sum \beta^t [E_t \ln c_t - (Vare]/c_t^2$where ϵ_{t+i} is the absolute value of e_{t+i}; thus there are twice as many values in the distribution as for e_{t+i}, since both positive and negative values share the same absolute value- however only half are counted. To obtain the certainty equivalent of the risk element note that:
$dU_t = \frac{\overline{c}}{(1-\beta)c_t} d\overline{c} - \frac{1}{1-\beta} dVare/c_t^2$;so that $\frac{\overline{c}}{c_t} d\overline{c} = dVare/c_t^2$ and so $d\overline{c} = c_t dVare$
Hence the cost of risk in terms of certainty equivalent consumption is $Vare/c_t$.

The model is completed with the government budget constraint and goods market clearing- which determines the real interest rate.

We now introduce the role of inequality. We divide our households into two groups, the rich (the top 10% by holdings of capital) and the rest. All households have the same utility and production functions; they only differ in capital holdings and hence in income. But this difference means that the marginal utility of consumption differs between them: the rich have a lower marginal utility of consumption and so they value both the costs of entrepreneurship and the value the risks less in marginal utility terms than the rest. Hence both the costs, $\phi_1(\tau_t + \pi'_t)$, and the risks, ϕ'_0, depend on the household relative capital wealth. For simplicity we redefine π'_t to include all tax and risk costs, which in turn are affected by relative wealth, so we write the determinants of it as:

$$\ln \pi'_{i,t} = \rho_0^{\pi} + \rho_1^{\pi} \ln \pi'_{i,t-1} - \rho_2^{\pi} \cdot Q(K_{i,t-2}/K_{t-2}) + \ln \pi'_t \tag{16}$$

where $\rho_1^{\pi} \geq 0$; $\rho_0^{\pi}, \rho_2^{\pi} > 0$. We assume that our function (effectively the inverse of consumption marginal utility, rising with income and so capital) takes the form $Q(K_{i,t}/K_t) = (\mu_i/\omega_{Y,i})\,(K_{i,t}/K_t)^2$ where capital and income are normalised at unity if they are equal to the average. $K_{i,t}/K_t = 1$ represents perfect equality while either greater or less than 1 implies inequality. Appendix B of Yang et al. (2021) shows that $Q(K_{i,t}/K_t)$ has a minimum at perfect equality where $K_{i,t}/K_t = \mu_i/\omega_{Y,i} = 1$, and penalty rates are identical across agents.

We now apply this model to the UK data after 1970, which includes the Thatcher reform period where the tax/regulation costs on entrepreneurs were reduced sharply. We are particularly interested in this extension paper in whether including planning restrictions on housing and spending on infrastructure adds to the explanation of the UK data behaviour in this period. These are included in the extended model by including them explicitly in the productivity equation above as additional factors, with the error now collecting the effects of other omitted factors.

5.1 Appendix on Indirect Inference

In this appendix, we set out and explain our methodology of model testing and parameter estimation; Indirect Inference (II), developed by Le et al. (2011).[4] II is based on the idea that if the structural model is true in terms of both specification and parameters, the properties of the actual data should come from the distribution of the properties of the simulated data with some critical minimum probability. The data properties can be captured by a simple auxiliary model such as a VAR, Impulse response functions or moments. Define the parameters of the structural model and the auxiliary model as θ and β respectively. We first use the actual data to estimate the auxiliary parameters, say $\widehat{\beta}$. Given the null hypothesis $H_0 : \theta = \theta_0$, we simulate S samples using the structural model and estimate the auxiliary parameters using each simulated sample to obtain estimators $\widetilde{\beta}_s(\theta_0); s = 1, \cdots, S$. To evaluate whether $\widehat{\beta}$ comes from the distribution of $\left\{\widetilde{\beta}_s(\theta_0)\right\}$, we compute the Wald statistic

$$Wald_a = \left[\widehat{\beta} - \overline{\widetilde{\beta}_s(\theta_0)}\right]' \sum(\theta_0)^{-1} \left[\widehat{\beta} - \overline{\widetilde{\beta}_s(\theta_0)}\right] \tag{17}$$

which asymptotically follows a $\chi^2(k)$ distribution where k is the number of elements in β and $\sum(\theta_0)$ is the variance-covariance matrix of $\widehat{\beta} - \widetilde{\beta}_s(\theta_0)$.[5] We can check the allocation of $Wald_a$ in the distribution of simulated $Wald_s; s = 1, \cdots, S$ where $Wald_s$ is computed when using the s^{th} simulated sample to estimate $\widehat{\beta}$. If $Wald_a$ is less than the c^{th} percentile value of $\{Wald_s\}$ sorted from smallest to largest, H_0 cannot be rejected in a $c\%$ confidence interval; otherwise the model is false.[6] An alternative way is to compute the

[4] II is different from indirect estimating. See details of the latter in Smith (1993), Gourieroux et al. (1993) and Davidson and MacKinnon (2004, 556-44).

[5] Because $\sqrt{S}\left[\widehat{\beta} - \widetilde{\beta}_s(\theta_0)\right]$ asymptotically follows a normal distribution, proved by Gourieroux et al. (1993).

[6] We do not directly compare $Wald_a$ with a certain critical value of $\chi^2(k)$ because of the possible simulated sample-variant issue.

Table 3: Power test against numerical falsity of parameters

Parameter Falseness	True	1%	3%	5%	7%	9%
Rejection Rate with 0.95 Confidence	0.05	0.134	0.27	0.672	0.82	1.00

transformed Mahalanobis Distance (TMD) and compare it with the critical value of t distribution on the $c\%$ confidence interval.[7]

$$Z = T_c \left[\frac{\sqrt{2Wald_a} - \sqrt{2k-1}}{\sqrt{2Wald_c} - \sqrt{2k-1}} \right] \tag{18}$$

where T_c is the critical value of a one-tail t distribution on the $c\%$ confidence interval.

Generally, a (linearised) DSGE model can be represented as a VARMA or a VAR(∞) which can be further represented to a VAR(p) with a finite order or even a VAR(1) (Dave and De Jong 2007, 9-9; Giacomini 2013; Wickens 2014). However, the long-run solution of our model can only be approximated as a VARX with non-stationary lagged endogenous variables X due to nonstationary productivities.[8] Both Le et al. (2011) and Le et al. (2015) conduct Monte Carlo experiments to find the power of test as the variables included and the order of the VAR vary. They find that a VAR(1) in 3 endogenous variables typically has good power, while raising the order or the variable number further can boost the power too far for any hope of finding a tractable model that can pass the test. Hence, we use a VARX(1) with 3 variables, aggregate output, aggregate capital and capital inequality, combined with the lagged individual productivities as the "X".[9] The auxiliary parameter vector β contains 9 VAR coefficients and 3 variances of the VAR residuals.

Given the null hypothesis that the structural model is true, one can back out the structural errors from the model and the actual data and then bootstrap these structural errors to obtain simulated samples. II is also used to estimate the parameters by searching for the parameter values such that the relevant Wald or TMD is smallest.

Le et al. (2012) and Le et al. (2016) conduct Monte Carlo power tests on three testing methods on different models: II, the Likelihood ratio test; and the "unrestricted Wald" test (in which the reduced form VAR on the data sample rather than the VAR from the structural model is bootstrapped). II is found to give far more potential power than the other classical testing methods.

To evaluate the power of II on our model here, we use Monte Carlo experiments to compute the power of the test both against parameter mis-estimation, and more importantly, against model mis-specification, including models with different causal sequencing and capable apparently of providing 'observationally equivalent' data.

We first generate 500 samples from the true model and the actual data. Then treating each simulated sample from the true model as the observation, we test the false model by II and calculate the rejection rate out of the 500 Monte Carlo experiments. Table 3 shows the result of our power test against the false models with mis-estimation where both structural parameters and the AR coefficients of the errors are steadily falsified by +/- x each time. The probability of rejecting the false models rises sharply with an increase in the falsity of parameters.

We then test the power of II against a mis-specified model in which the basic mechanism of wealth inequality on entrepreneurship is turned off: the equations of the penalty rate (C15)-(C16) in Appendix C are replaced by a simple AR(1) process. Wealth inequality in this false model continues to be generated by randomness but does not in turn generate more innovation. We keep the parameters the same as the

[7] $\sqrt{2Wald_a}$asymptotically follows a normal distribution $N(\sqrt{2k-1}, 1)$.

[8] See Meenagh et al. (2012) for more details of the VARX approximation.

[9] Appendix B has shown that the aggregate growth is determined by aggregate output, aggregate capital and capital inequality while the aggregate growth is the difference in logarithm of aggregate output and thus in the VARX(1), series of aggregate output can implicitly describe growth.

fully-estimated values in the benchmark model. Growth therefore continues to occur as a result of shocks to the aggregate penalty rate. But there is no longer any mechanism linking growth to inequality. On the same 500 samples the rejection rate of this mis-specified model at 95% confidence is no less than 99.4% and hence we can conclude that II provides huge power against mis-specified models which attempt to mimic the results from the true model.