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Macroeconomic Shocks and Volatility Spillovers Between Stock, Bond, Gold and Crude Oil Markets

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Abstract

This paper introduces a novel model to analyse the impact of macroeconomic shocks on volatility spillovers within key financial markets, such as Stock, Bond, Gold and Crude Oil. By treating macroeconomic variables as external factors to financial market volatility, our study distinguishes between internal financial volatility spillovers and external shocks arising from macroeconomic changes. Our analysis reveals that without macroeconomic shocks, the Stock market predominantly acts as the main source of volatility spillovers, with Crude Oil being the principal spillover recipient. However, the Stock market's role in driving volatility spillover, especially towards the Crude Oil market, changes markedly in the context of macroeconomic shocks. These shocks exert a more substantial impact on Crude Oil compared to other markets. In contrast, the Bond and Gold markets exhibit a lower level of volatility transmission and are less influenced by macroeconomic shocks, thereby reinforcing their roles as stabilizers within the financial system.

Key Words: Volatility spillover, Macroeconomic shocks, Multiplicative Error Model, Realized volatility, Financial markets

JEL Classification Codes: C32, C52, G14.

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1. Introduction

The spillover of risks, which is commonly known as the volatility spillover effects, characterize how shocks and risks propagate and spread among different markets (**??**). Numerous studies have highlighted the significant effects of changes in macroeconomic conditions on the volatilities of financial markets. Given their importance for risk valuation and portfolio diversification strategies (**?**), there is a need for precise quantification of the impact of macroeconomic shocks on these volatility spillovers.

The exploration of volatility spillover effects in financial markets has been extensively covered, with contributions from **?????**. Typically, these studies are anchored in the Vector Auto-regression (VAR) models or the multivariate GARCH model, often integrating the volatility spillover index as highlighted by **?**. More recently, **?** introduced the Multiplicative Error Model (MEM), which addresses some limitations of the VAR model (e.g., **???**) and the multivariate GARCH model (e.g., **??**). For instance, unlike the VAR model, MEM is not prone to the issue of zero and non-negative predictions of volatility. Compared to the multivariate GARCH model, it avoids the 'curse of dimensionality' problem, as noted by **?**.

Macroeconomic shocks often stem from changes in broader macroeconomic conditions that impact asset markets. These risks can manifest in various forms, including monetary policy risks (**?**), interest rate risks, inflation risks, economic policy uncertainty (**?**), and geopolitical risks (**?**). The influence of policy-induced uncertainty on commodities, currencies, and Crude Oil has been well-established (**??**). For instance, **?** identified a causal effect of U.S. economic policy uncertainty on the interconnectedness between Crude Oil and currency markets in both emerging and developed economies. Similarly, inflationary pressures and shifts in interest rates have also been found to significantly affect financial markets. For example, **?** discovered that term and credit spreads have strong predictive power for total return spillovers and total volatility spillovers in financial markets. Lastly, financial markets, particularly equities and Bonds, are also subject to climate and geopolitical risks. Research by **?**, **?**, and **?** has examined the responses of equity and Bond mar-

kets to geopolitical and environmental risks, with **?** noting that geopolitical risks can positively influence the performance of green Bonds and equity.

Despite extensive research on how macroeconomic shocks affect individual market's return and volatility, there is a noticeable research gap in understanding their impact on spillovers and contagion between financial markets. Our study addresses this gap by examining how macroeconomic shocks influence the volatility spillovers in Stocks, Gold, Bonds, and Crude Oil markets. Existing models, mainly VAR models, assume a two-way influence between financial volatilities and macroeconomic factors. However, this mutual influence is not well-supported by evidence. In fact, macroeconomic changes, driven by economic policies and market conditions, tend to have a greater effect on market volatilities. This is supported by findings that show financial markets are more reactive to macroeconomic shifts than vice versa (**??**). Recent studies provide further evidence supporting the dominant influence of macroeconomic factors on financial market volatilities. For example, **?** show significant information transmission from equity-related uncertainty to oil prices, but not vice versa. Similarly, **?** find that macroeconomic fundamentals explain the increased spillover between global equity markets and the Dow Jones Industrial Average (DJI) during financial crises. **?** demonstrate a strong sensitivity of German and US implied volatility indices to macroeconomic announcements. **?** argue that heightened macroeconomic uncertainty impacts the volatility of the US stock market, which is particularly evident in the aftermath of the 2007 US Great Recession. **?** finds a significant role of geopolitical events on oil price volatility and, to a lesser extent, on stock market volatility. Furthermore, **?** demonstrate that the US default spread, the TED (Treasury Eurodollar) spread, US stock volatility, and the Risk Aversion Index (RAI) contribute to volatility spillovers at various levels of volatility states.

Inspired by these insights, our first contribution extends **?** vector Multiplicative Error Model (vMEM) by incorporating macroeconomic variables as additional exogenous factors. This extension, known as the vMEM-X model, is designed to investigate the influence of macroeconomic shocks on volatility spillovers across financial markets. In the vMEM-X framework, macroeco-

nomic variables are treated as external factors that impact market volatilities independently. These variables are integrated into the model using a VAR framework, under the assumption that financial market volatility does not reciprocally affect macroeconomic conditions. The vMEM-X model distinguishes between two types of shocks: those originating within financial markets and those resulting from macroeconomic changes. To assess the impact of these shocks, we also extend and develop new formulas for the calculation of volatility spillover balance and impulse response functions proposed by **?**. These enhancements enable a thorough examination of market responses to both types of shocks, providing valuable insights into the dynamics of volatility transmission across financial markets.

We apply the vMEM-X model to the Stock, Bond, Gold and Crude Oil markets, and explore their volatility spillovers using key macroeconomic variables such as term spread, short-term interest rates, inflation rates, US real economic activity, economic policy uncertainty, and geopolitical risk [2]. Our findings reveal the followings. Firstly, our analysis identifies the Stock market as the primary provider of volatility spillovers, with the Crude Oil market predominantly acting as the recipient of these spillovers. This pattern holds true whether or not macroeconomic variables are included in the model. Secondly, we observe that these macroeconomic shocks impact Crude Oil and Stock volatility more than they do other markets. Thirdly, in scenarios excluding macroeconomic variables, the apparent spillover effects from Stocks to Crude Oil can be misinterpreted. Our analysis show that the substantial volatility transmitted from the Stock market to the Crude Oil market, is not merely a result of the Stock market's inherent traits. Instead, it mirrors the increased sensitivity of both the Stock and Crude Oil markets to macroeconomic shocks. Lastly, the Bond and Gold markets, typically regarded as safe havens, demonstrate lower levels of volatility spillover and are less affected by macroeconomic shocks, confirming their roles as stabilizers (**???**) in the financial markets amidst economic uncertainty.

[2]We chose the six macroeconomic variables based on existing literature and data availability from 2003 onwards. Detailed references regarding the use of these variables can be found in the Section 4

One of the most notable findings is the nuanced role of the Stock market in volatility transmission. The literature has consistently identified the Stock market as a major volatility spillover provider (**???**). Initially, our empirical analysis suggests that the Stock market is the primary force driving volatility. However, a deeper examination reveals a more intricate interaction. While the Stock market indeed transmits significant volatility to other markets, especially Crude Oil, this does not solely indicate its inherent propensity to generate spillovers. Instead, this pattern reflects the acute sensitivity of both the Stock and Crude Oil markets to macroeconomic shocks. These findings suggest that changes in macroeconomic conditions, rather than inherent financial market shocks, primarily dictate the observed volatility transmission from Stocks to other markets.

The rest of the paper is organized as follows. Section 2 introduces multiplicative error model for the volatility with exogenous variables. Section 3 derive the volatility spillover balance for this model. Section 4 presents the dataset and Section 5 contains the empirical results and their interpretations. Finally, Section 6 concludes with policy implications and suggestions.

2. The Methodology Framework

? introduced a natural estimator for the quadratic variation of a process, known as the realized variance (RV), defined as the sum of frequently sampled squared returns. To simplify, let us assume that prices $p_0, \ldots, p_n$ are observed at $n+1$ intervals, evenly distributed over the interval $[0, t]$. Using these returns, the n-sample realized variance, RV, can be defined as follows:

$$RV = \sum_{j=1}^{n} r_j^2 \tag{1}$$

where $r_j = p_j - p_{j-1}$.the realized variance (RV) converges in probability to the quadratic variation of log prices as the number of intraday observations increases, i.e., as $n \to \infty$.

2.1. Multiplicative Error Models

Since the RV is non-negatively valued and highly persistent over time, we follow the work of **?**, **?**, **?**, and **?** to use the MEM for modeling the dynamics of RV. The MEM was initially proposed

by **?** and has been widely used for modeling the dynamics of non-negative, highly persistent financial time series, such as absolute return, daily range, realized volatility, trading duration, trading volume, and bid-ask spread.

Given the information set I_{t-1}, the realized volatility in market i, denoted as $RV_{i,t}$, is modeled as follows:

$$RV_{i,t}|I_{t-1} = \mu_{i,t}\epsilon_{i,t}, \quad i = 1, 2, ..., k, \tag{2}$$

where k represents the number of assets/markets studied in the system, which in our case is 4. The innovation term $\epsilon_{i,t}$ is a unit mean random variable, such that $\epsilon_{i,t}|I_{t-1} \sim \text{i.i.d}(1, \sigma_i)$. Consequently, $\mu_{i,t} = E(RV_{i,t}|I_{t-1})$, which can be specified as a basic MEM(1,1):

$$\mu_{i,t} = \omega_i + \alpha_{ii} RV_{i,t-1} + \beta_i \mu_{i,t-1} \tag{3}$$

where ω_i, α_{ii}, and β_i are defined according to a standard MEM model, as described in Engle (2012).

Furthermore, the heterogeneous autoregressive (HAR) model of **?** has emerged as a simple and powerful way to include the long-memory feature of realized volatilities. Adding HAR terms to the realized semi-variance equations, results in the richer dynamic equations

$$\mu_{i,t} = \omega_i + \alpha_{ii} RV_{i,t-1} + \beta_i \mu_{i,t-1} + \alpha_{ii}^w {RV^w}_{i,t-1} + \alpha_{ii}^m {RV^m}_{i,t-1}, \tag{4}$$

where $RV_{i,t}^w = \frac{1}{5}\sum_{l=1}^{5} RV_{i,t-l}$, $RV_{i,t}^m = \frac{1}{22}\sum_{l=1}^{22} RV_{i,t-l}$.

To study the volatility spillover effects, we include the lagged daily volatility observed in other markets to the specification. The general volatility spillover model is then:

$$\mu_{i,t} = \omega_i + \alpha_{ii} RV_{i,t-1} + \beta_i \mu_{i,t-1} + \alpha_{ii}^w {RV^w}_{i,t-1} + \alpha_{ii}^m {RV^m}_{i,t-1} + \sum_{j \neq i}^{k} \alpha_{ij} RV_{j,t-1} \tag{5}$$

Next, we add macro condition variables to the volatility volatility spillover model. Let $Z_t =$

$(z_{1t}, z_{2t}, ...z_{mt})'$ be m macro economic variables, then

$$\begin{aligned}\mu_{i,t} = \omega_i + \alpha_{ii} RV_{i,t-1} + \beta_i \mu_{i,t-1} + \alpha_{ii}^w {RV^w}_{i,t-1} + \alpha_{ii}^m {RV^m}_{i,t-1} \\ + \sum_{j \neq i}^{k} \alpha_{ij} RV_{j,t-1} + \sum_{l=1}^{m} c_{il} z_{l,t-1},\end{aligned} \tag{6}$$

Following **?** and **?**, the volatility models in (6) can be estimated using quasi-maximum likelihood estimation. This is under the assumption that the innovation terms $\epsilon_{i,t}|I_{t-1}$ and follow exponential distributions.

Now let us write (6) in a compact matrix form. Let $\mathbf{x}_t = (RV_{1,t}, RV_{2,t}, \cdots, RV_{k,t})'$, $\boldsymbol{\mu}_t = (\mu_{1,t}, \mu_{2,t}, \cdots, \mu_{k,t})'$, $\mathbf{x}_t^w = (RV_{1,t}^w, RV_{2,t}^w, \cdots, RV_{k,t}^w)'$, $\mathbf{x}_t^m = (RV_{1,t}^m, RV_{2,t}^m, \cdots, RV_{k,t}^m)'$ and $\boldsymbol{\epsilon}_t = (\epsilon_{1,t}, \epsilon_{2,t}, \cdots, \epsilon_{k,t})'$. Denote $Z_t = (z_{1t}, z_{2t}, ...z_{mt})'$, conditional on the information available at time t, (6) can be stacked in a compact matrix form as

$$\begin{aligned}&\mathbf{x}_t = \boldsymbol{\mu}_t \odot \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim \mathrm{D}(\mathbf{1}, \boldsymbol{\Sigma}), \\ &\boldsymbol{\mu}_t = \boldsymbol{\omega} + \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}\boldsymbol{\mu}_{t-1} + \mathbf{A}^w \mathbf{x}_{t-1}^w + \mathbf{A}^m \mathbf{x}_{t-1}^m + C Z_{t-1}.\end{aligned} \tag{7}$$

where $\odot$ denotes the Hadmard (element by element) product. The innovation vector $\boldsymbol{\epsilon}_t$ has support over $[0, +\infty)$, with a unit mean vector $\mathbf{1}$ and general variance-covariance matrix $\boldsymbol{\Sigma}$. This is a vMEM with exogenous variable, the model is lablled as vMEM-X model.

The first two moment conditions of the vMEM are given by $\mathrm{E}(\mathbf{x}_t|\Omega_t) = \boldsymbol{\mu}_t$ and $\mathrm{var}(\mathbf{x}_t|\Omega_t) = \boldsymbol{\mu}_t\boldsymbol{\mu}_t' \odot \boldsymbol{\Sigma}$, with the latter a positive definite matrix by construction. Processes such as those defined by (7) can be written as VARMA-X(1,1) by defining appropriate error terms (see Appendix A for the derivations). Given this representation, the covariance stationarity condition requires that the largest eigenvalue of $\mathbf{A} + \mathbf{B} + \mathbf{A}^w + \mathbf{A}^m$ be less than unity. Consequently, the unconditional first moment can be obtained as $E(\mathbf{x_t}) = (I_k - \mathbf{A} + \mathbf{B} + \mathbf{A}^w + \mathbf{A}^m)^{-1} \boldsymbol{\omega}$.

3. Spillover Analysis

? and **?** propose a quantitative measure for the volatility spillover effects across multiple markets, premised on the measure of spillovers as responses to shocks. Following their methodology, we derive analogous measures for our volatility models.

Next, we derive a multiple-step ahead forecasting $\mathbf{x}_{t+\tau}$ (where $\tau > 0$) computed at date t, which is not known and needs to be substituted with its corresponding conditional expectation $\boldsymbol{\mu}_{t+\tau|t}$, hence

$$\boldsymbol{\mu}_{t+1|t} = \boldsymbol{\omega} + \mathbf{A}\mathbf{x}_t + \mathbf{B}\boldsymbol{\mu}_t + \mathbf{A}^w\mathbf{x}_t^w + \mathbf{A}^m\mathbf{x}_t^m + CZ_t, \tag{8}$$

and for $2 \leq \tau < 22$,

$$\boldsymbol{\mu}_{t+\tau|t} = \boldsymbol{\omega} + (\mathbf{A}+\mathbf{B})\boldsymbol{\mu}_{t+\tau-1|t} + \mathbf{A}^w\mathbf{x}_{t+\tau-1|t}^w + \mathbf{A}^m\mathbf{x}_{t+\tau-1|t}^m + C\hat{Z}_{t+\tau-1|t}, \tag{9}$$

where $x_{t+\tau-1|t}^w = \frac{1}{5}\sum_{l=1}^{5} x_{t+\tau-l|t}$, $x_{t+\tau-1|t}^m = \frac{1}{22}\sum_{l=1}^{22} x_{t+\tau-l|t}$ and $x_{t+\tau-l|t} = \mu_{t+\tau-l|t}$ if $\tau > l$. And then, for any $\tau \geq 22$,

$$\boldsymbol{\mu}_{t+\tau|t} = \boldsymbol{\omega} + (\mathbf{A}+\mathbf{B}+\mathbf{A}^w+\mathbf{A}^m)\boldsymbol{\mu}_{t+\tau-1|t} + C\hat{Z}_{t+\tau-1|t}, \tag{10}$$

As long as we know $\hat{Z}_{t+\tau-1}$, the multiple-step ahead forecasting can be solved recursively for any horizon τ.

The variables Z_t primarily represent macroeconomic condition variables. In our empirical analysis, we select various proxies for macroeconomic variables, including term spread, short term interest rate, inflation rate, US real economic activity, Economic policy uncertainty, geopolitical risk, among others. It is reasonable to assume that these macro variables are exogenous to the volatility of the financial market. However, changes in macroeconomic conditions significantly affect the volatility of the financial market. Following the standard approach in macroeconomic analysis, we employ a reduced form Vector Autoregression (VAR) model for the macro variables, as it is well-acknowledged that many structural macroeconomic models, for instance, the renowned

Dynamic Stochastic General Equilibrium (DSGE) model by **?**, possess a reduced form VAR representation.

$$Z_t = PZ_{t-1} + \eta_t, \tag{11}$$

where $\eta_t \sim i.i.d(0, \Sigma_\eta)$ is the shocks of macroeconomic conditions. The VAR coefficient P can be estimated by OLS. The multiple-step ahead forecasts of $\hat{Z}_{t+\tau-1}$ is given by

$$\hat{Z}_{t+\tau} = P^\tau Z_t \tag{12}$$

3.1. Volatility spillover from the financial market shocks

Firstly, let us consider the shocks are from financial markets. we can derive a spillover balance index. Let us recall that the vMEM-X in a system,

$$\mathbf{x}_t = \boldsymbol{\mu}_t \odot \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim \mathrm{D}(\mathbf{1}, \boldsymbol{\Sigma}). \tag{13}$$

The innovation vector $\boldsymbol{\epsilon}_t$ has a mean vector $\mathbf{1}$ with all components unity and general variance-covariance matrix $\boldsymbol{\Sigma}$. We can interpret $\boldsymbol{\mu}_{t+\tau} = E(\mathbf{x}_{t+\tau}|I_t, \boldsymbol{\epsilon}_t) = \mathbf{1}$, that is, the expectation of $\mathbf{x}_{t+\tau}$ conditional on $\boldsymbol{\epsilon}_t$ being equal to the unit vector $\mathbf{1}$: this is on the basis of the dynamic forecast obtained before. Let us now derive a different dynamic solution, $\boldsymbol{\mu}^{(i)}_{t+\tau} = E(\mathbf{x}_{t+\tau}|I_t, \boldsymbol{\epsilon}_t = \mathbf{1} + \mathbf{s}^{(i)})$, for a generic ith element $\mathbf{s}^{(i)}$, where $i = 1, 2, ..., k$. The ith element equal to the unconditional standard deviation of $\epsilon_{i,t}$ and the other terms $j \neq i$ equal to the linear projection $E(\epsilon_{j,t}|\epsilon_{i,t} = 1 + \sigma_i) = 1 + \sigma_i \frac{\sigma_{i,j}}{\sigma_i^2}$. The element-by-element division ($\oslash$) of the two vectors,

$$\rho^{(i)}_{t,\tau} = \boldsymbol{\mu}^{(i)}_{t+\tau} \oslash \boldsymbol{\mu}_{t+\tau} - 1. \tag{14}$$

Given the multiplicative nature of the model, $\rho^{(i)}_{t,\tau}$ gives us the set of responses (relative changes) in the forecast profile starting at time t for a horizon τ brought about a 1 standard deviation shock in the ith market. The cumulated impact of the shock from market i to market j is:

$$\Phi^{j,i}_t = \sum_{\tau=1}^{K} \rho^{j,i}_{t,\tau}. \tag{15}$$

where K is the forecast horizon. The total spillover effect (TSI) as:

$$TSI = \sum_{i \neq j} \sum_{t=1}^{T} \Phi_t^{j,i} \tag{16}$$

which measures the overall contribution of volatility spillover shocks across markets.

This is also a way to assess the total change induced by the shock of different markets. Following **?**, we express the spillover balance as the ratio of the average responses "to" to the average response "from" (excluding one's own) :

$$Balance_i = \frac{\sum_{j \neq i} \sum_{t=1}^{T} \Phi_t^{j,i}}{\sum_{j \neq i} \sum_{t=1}^{T} \Phi_t^{i,j}}. \tag{17}$$

where $Balance_i$ denotes volatility spillover balance. A $Balance$ value greater than 1 signals that the market is a net creator of volatility spillover, while a $Balance$ value smaller than 1 signals that the market is a net accepter of volatility spillover.

3.2. Volatility spillover from macroeconomic shocks

Now, let us consider the volatility spillover from the macroeconomic shocks.

The vMEM-X model innovation vector $\boldsymbol{\epsilon}_t$ has a mean vector $\mathbf{1}$. Let us consider that macroeconomic VAR model innovation vector η_t has a mean vector not equal to 0, but $(0 + \sigma^l)$, where $l = 1, 2, ..., m$. The lth element σ^l equal to the unconditional standard deviation of $\eta_{l,t}$, keeping other macroeconomic shocks unchanged. Let us derive a different dynamic solution, $\boldsymbol{\mu}_{t+\tau}^{(l)} = E(\mathbf{x}_{t+\tau}|I_t, \boldsymbol{\eta}_t = \sigma^l)$, for a generic lth element $\sigma^{(l)}$. The multiple-step ahead forecasts of $\hat{Z}_{t+\tau}$ is given by

$$\hat{Z}_{t+\tau}^l = P^\tau Z_t^l \tag{18}$$

Replacing $\hat{Z}_{t+\tau}$ in the multiple step ahead forecasting equations in (9) and (10), we get $\boldsymbol{\mu}_{t+\tau}^{(l)} = E(\mathbf{x}_{t+\tau}|I_t, \boldsymbol{\eta}_t = \sigma^l)$. The element-by-element division ($\oslash$) of the two vectors is

$$\rho_{t,\tau}^{(l)} = \boldsymbol{\mu}_{t+\tau}^{(l)} \oslash \boldsymbol{\mu}_{t+\tau} - 1. \tag{19}$$

Given the multiplicative nature of the model, $\rho_{t,\tau}^{(l)}$ gives us the set of responses (relative changes) in the forecast profile starting at time t for a horizon τ brought about a 1 standard deviation shock in the lth macroeconomic condition. The cumulated impact of the shock from macroeconomic shock l to financial market j is expressed as:

$$\Phi_t^{j,l} = \sum_{\tau=1}^{K} \rho_{t,\tau}^{(l)}, \tag{20}$$

and the total spillover effect (TSI) as:

$$TSI = \sum_{j,l} \sum_{t=1}^{T} \Phi_t^{j,l} \tag{21}$$

which measures the overall contribution of volatility spillover shocks across different macroeconomic condition variables.

The total spillover to financial market from macroeconomic shock l is given by $\sum_{j=1}^{k} \sum_{t=1}^{T} \Phi_t^l$ nd the total spillover from all the macroeconomic shocks to financial market j is given by $\sum_{l=1}^{L} \sum_{t=1}^{T} \Phi_t^j$.

4. Data

We follow **??** and choose the four futures contracts: S&P 500 futures (ES: CME GROUP), Treasury Bond futures (US: CCBOT/CME GROUP), Gold futures (GC: COMEX/CME GROUP), and Crude Oil futures (CL: NYMEX/CME GROUP) to represent the Stock, Bond, Gold, and Crude Oil markets, respectively. The data are obtained from TickData, Inc. The sample period is July 1, 2003 to August 5, 2022, with a total of 4864 trading days. We choose July 1, 2003 as starting date, as trading occurs both in the daytime and in the evening (e.g., from 7:20 to 16:00 and from 17:00 to 7:20 for Bond) from that day, ensuring that the realized variance of our dataset closely approximates whole-day variance.

There are two benefits to using future contract rather than using the spot price of the four markets. First, the four contracts are traded for 23 hours during the sample periods, which closely approximates whole-day variance, enhancing the accuracy of the realized variances. Second, the

four futures are traded on the same exchange, which eliminates the need to adjust for time zones. This consistency simplifies the analysis and allows for more accurate comparisons across the different futures contracts. Detailed information regarding data cleaning and the construction of the realized variance process can be found in **?** and **?**.

We select the following six variables to assess the macroeconomic conditions of the US, based on the availability of daily data from 2003 onwards.

- Term spread (TSD): TSD represents the difference between the yield of 10-year constant maturity Treasury Bonds and that of 3-month Treasury bills. As highlighted by **?** and **?**, the Term Spread is a significant predictor of future Stock returns and volatility. Notable studies employing TSD as a measure of macroeconomic conditions include , **?**, **?**, **? ?**, and **?**.

  Source: $https://fred.stlouisfed.org$

- Effective Federal Funds Rate (FFR): The FFR denotes the overnight interest rate at which US banks and credit unions lend excess reserves to each other. This very short-term rate serves as one of the primary monetary policy tools utilized by the Federal Reserve to either stimulate or decelerate overall economic activity. Notable studies that have employed the FFR as a metric for macroeconomic conditions include **?**, **?**, **?**, **?**, and **?**.

  Source: $https://fred.stlouisfed.org$

- Inflation (INF): INF is a forward-looking metric that reflects the expectations of economic agents (consumers, investors, businesses) regarding the trajectory of US inflation rates over the ensuing five years. This market-based measure is derived from Treasury spreads. Notable studies utilizing INF as a macroeconomic condition indicator include **?**, **?**, **?**, **?**, **?**, and **?**.

  Source: $https://fred.stlouisfed.org$

- The Aruoba-Diebold-Scotti (ADS) Business Conditions Index: The ADS Index, as proposed by **?**, serves as a real-time gauge for the overall economic activity in the US. Given that the

average value of the ADS index is zero, positive (negative) values of the index signify better-(worse-) than-average economic conditions. Notable studies employing the ADS Index as a macroeconomic condition indicator include **?**, Giovannelli et al. (2021), **?**,**?**, and **?**.

Source: $https://www.philadelphiafed.org$

- US Economic Policy Uncertainty (EPU): The EPU index, developed by **?**, serves as a proxy for the overall uncertainty pertaining to economic policy in the US. The construction of the index is predicated on three core components: (i) policy-related economic uncertainty gleaned from news reports; (ii) uncertainty regarding prospective tax legislation, as garnered from Congressional Budget Office reports; and (iii) disparate forecasts among economists concerning public expenditure and future inflation rates. Notable studies utilizing the EPU as an indicator for macroeconomic conditions include **?**, **?**, **?**, **?**, and **?**.

  Source: $https://www.policyuncertainty.com$

- Geopolitical Risk (GPR): The Geopolitical Risk index, developed by **?**, encapsulates risks stemming from interactions between countries. These interactions encompass trade relationships, security partnerships, alliances, multinational climate initiatives, supply chains, and territorial disputes. Notable studies employing the GPR index as a macroeconomic condition indicator include **?**, **?**, **?**, **?**, and **?**.

  Source: $https://www.matteoiacoviello.com/gpr.htm$

The descriptive statistics for realized volatility and macroeconomic condition variables are summarized in Table 1, with their time series evolution depicted in Figures 1 and 2.

In the realized volatility, Crude Oil stands out with the highest volatility, suggesting it is riskier compared to other markets. Conversely, Bonds exhibit the lowest mean of realized variance, reflecting their status as a safe-haven asset with less credit risk and more predictable payments, consistent with **?**. The Ljung Box statistics indicate strong serial autocorrelations in realized variance. Figure 1 shows a marked increase in volatility of the four markets during the global financial

crisis, followed by significant declines with occasional jumps. The volatility surged at the onset of the COVID-19 pandemic, although it did not persist as during the financial crisis. The persistent nature of realized variances over time suggests that MEM-type models are well-suited for modeling these dynamics.

Insert Table 1 here

Insert Figure 1 and 2 here

Turning to macroeconomic condition variables, the mean values for TSD, FFR, and INF (1.63, 1.22, and 2.18 respectively) generally align with the economic policy objectives in the US, indicating a stable economy during the sample period. However, the negative mean for ADS reflects below-average real business conditions, likely influenced by the COVID-19 pandemic's adverse impacts, as seen in Figure 2. The negative skewness in TSD, FFR, INF, and ADS mirrors the significant downturns experienced during crises, including the 2008 global financial crisis and the COVID-19 pandemic. The EPU and GPR values, at 1.1 and 1.07 respectively, slightly exceed the normalized value of 1.

Figure 2 highlights two sharp declines in TSD coinciding with the global financial crisis and COVID-19 pandemic periods. These dips in TSD during crisis periods are indicative of market stress and monetary policy responses. The Inflation (INF) rate, on the other hand, displays relative stability throughout the sample period. FFR shows a prolonged period of near-zero values between 2009 and 2016, and again from 2020 to early 2022, reflecting the Federal Reserve's response to economic downturns with lower interest rates. EPU spiked significantly during the financial crisis and even more so during the COVID-19 pandemic. This aligns with the heightened economic uncertainty during major global events. GPR also shows a significant increase in 2022, likely a reflection of the Russian-Ukrainian crisis, as noted by **?**.

By comparing Figure 1 and 2, it seems that the volatilities in markets such as Crude Oil, Stocks, and Gold seems highly responsive to economic policy uncertainties, and major shifts in monetary

policy. In contrast, the Bond market typically exhibits stability during these periods.

5. Empirical results

5.1. Estimates

Based on the equation-by-equation estimation results, we proceed to select a more parsimonious specification, based on the significance of zero restrictions. The large number of coefficients in the general specification in (7) yields inefficient parameter estimates and, therefore, less precise spillover forecasts analysis (**?**). We report only the coefficients estimates that are significant at 5 percent level in Table 2. The model diagnostics are summarized in the lower panel of Table 2. where the values of the log-likelihood functions, Bayesian Information Criteria (BIC) and Ljung box (LB) statistics for residuals are reported.

Insert Table 2 here

Columns 2 to 5 of Table 2 present the primary estimated results from the vMEM-X model with macroeconomic shocks incorporated as additional exogenous variables. Firstly, we observe that the estimated α is relatively large, while the estimated β is relatively small, indicating persistence and slow mean revision in volatility. Secondly, the two HAR parameters, α_w and α_m, are significant in all eight cases, indicating a high level of persistence in the variance. Thirdly, lagged Stock volatility has a positive effect on the other three markets. This suggests a strong contagion effect where shocks in the Stock market are transmitted to other markets, influencing their volatility. The lagged Bond volatility has negative impact on the other three market. It aligns with the stabilizing role of Bonds, suggesting that increased volatility in Bonds could potentially lead to reduced volatility in other markets. The lagged Gold volatility does not affect the other three markets, nor does the lagged Crude Oil volatility. This can be attributed to the safe-haven property of Gold and Crude Oil markets, where investors use them to hedge and diversify their portfolios to mitigate risk exposures. Lastly, TSD has a positive impact on Stock, Bond and Gold volatilities, implying that an increase in term spread increases the volatility in these markets. The negative coefficients for INF across

all markets, especially oil, indicate that higher inflation expectations reduce market volatilities, possibly due to the anticipation of central bank interventions, such as interest rate hikes, which might stabilize market fluctuations. ADS negatively affects Stock and oil volatilities, suggesting that better economic conditions might lead to lower volatility in these markets. EPU, GPR, and FFR only impact stock market volatility and not the volatilities of the other three markets. This may be attributed to the perception of Bonds, Gold, and Crude Oil as safe-haven assets, especially during periods of economic uncertainty or geopolitical tensions.

For comparative purposes, we also estimates the vMEM model of **?**, which excludes the macroeconomic variables. These results are displayed in columns 6 to 9 of Table 2. It is evident that incorporating macroeconomic shocks in the vMEM model does not alter the sign and significance of other parameters. However, with the inclusion of macroeconomic shocks, the vMEM-X model exhibits higher log-likelihood values and smaller BIC values, suggesting an overall better fit. The LB statistics are small and insignificant, indicating that both models successfully capture the dynamics of the semivariance processes, while the vMEM-X model demonstrates a slightly better capture of the dynamics, as evidenced by smaller LB values.

5.2. Volatility Spillovers from financial market shocks

From Table 2, it is evident that the four markets are interdependent. The key questions we address are: Which market plays the most significant role in spreading volatility shocks across these four markets? And, do changes in macroeconomic variables influence the direction and strength of volatility spillover effects among them? To answer these questions, we employ the volatility spillover formulas developed in Section 3. The spillover balance index, particularly when its value exceeds 1, indicates that a market is a net contributor to volatility spillovers.

Firstly, we investigate the volatility spillovers originating from the financial market. The results are reported in Table 3 and 4. Table 3 shows the outcomes without incorporating macroeconomic shocks, using the vMEM model and volatility spillover index developed by **?**. The findings unveil significant volatility spillovers among the four asset markets, particularly towards the oil market

from Stock market. The Stock market exhibits a spillover balance index of 2.39, illustrating its role as a major volatility spillover provider. Conversely, the Bond and Gold markets display spillover balance indices of 1.18 and 1.06 respectively, indicating a relatively balanced stance in terms of volatility spillover. The oil market, with a spillover balance index of 0.27, significantly below 1, emerges as the principal volatility spillover recipient.

Insert Table 3 and 4 here

In Table 4, which includes macroeconomic variables and represents the vMEM-X model, the results show distinct variations compared to Table 3. Although the Stock market continues to be a key source of volatility, its impact is less pronounced when macroeconomic variables are considered, as the spillover balance index drops from 2.39 to 1.72. The Bond market's spillover balance index increases from 1.18 to 1.23, indicating a heightened role in transmitting volatility. Notably, the magnitude of changes in the stock market's spillover balance is much larger compared to the changes in the Bond market's spillover balance index. This is consistent with the observations in Table 2, where all six macroeconomic variables have a significant impact on stock market volatility, whereas only two of them affect bond market volatility. Gold and oil markets transition to being recipients of spillovers, aligning with the scenario under investigation. Notably, the spillover from the Stock market to Crude Oil reduces significantly from 17.16 to 5.55 upon including macroeconomic variables, suggesting that these spillovers might be a response to macroeconomic rather than Stock market shocks. This aspect warrants further exploration in the following subsection.

5.3. Volatility Spillovers from macroeconomic shocks

Table 5 presents the outcomes of volatility spillover resulting from macroeconomic shocks. The key findings are following.

Insert Table 5 here

Firstly, the overall spillover effect on the four financial markets from inflation expectation (INF) shocks is 30.85, the highest among all macroeconomic shocks, followed by term spread (TSE)

shocks at 21.98. Other macroeconomic shocks show similar levels of overall volatility spillover to the financial markets. This indicates that inflation expectations and term structure predominantly drive financial market spillover and contagion, a conclusion supported by studies from **?**, **?**, and **?**.

Secondly, Crude Oil exhibits the highest sensitivity to macroeconomic shocks (with a "Total from" value of 53.12), highlighting its vulnerability to economic fluctuations. The Stock market also responds considerably (with a "Total from" value of 35.79), while Bond and Gold markets, known for their safe-haven attributes, are less impacted (**??**).

Thirdly, contrasting Tables 4 and 3 reveals that the Stock market's high volatility spillover balance index, mainly due to significant spillover to Crude Oil, does not solely indicate its dominant role in transmitting spillovers among these markets. Rather, it underscores the swift and substantial reaction of both Stock and Crude Oil market volatilities to macroeconomic shocks, with these shocks impacting Crude Oil more heavily than Stocks. In scenarios lacking macroeconomic variables, this differential response to macroeconomic shocks is observed as heightened spillover from Stocks to Bonds, as shown in Table 3.

5.4. Impulse response analysis

Lastly, we use the vMEM-based impulse response to show how the shocks propagate to other markets for a crises. We select March 11, 2020, the date when the World Health Organisation (WHO) declared COVID-19 a global pandemic, as starting date a shock to illustrate how the shock propagates to other markets [3].

Insert Figure 3 here

The IRF, assuming shocks originating from the financial market, shows that volatility shocks in the Stock market have a notable ripple effect on other markets. As seen in Figure 3, these shocks create an immediate and substantial response in the Stock market itself (approximately 0.7

[3] Utilizing alternative dates as starting points, such as September 16, 2008, marking the onset of the global financial crisis, reveals a similar pattern. The results are available upon request.

impact), which gradually declines over time. Notably, there is a delayed and smaller impact on the Bond, Gold, and Crude Oil markets, typically ranging between 30 percent and 40 percent. This response grows, forming a hump-shaped curve, and peaks between ten and twenty days post-shock, reflecting the contagion effect from the Stock market to others. This hump-shaped response is not observed when shocks originate in Bond, Gold, or Crude Oil markets, where other markets respond with a monotonically declining trend. These patterns show the significant influence of the Stock market on other markets. Notably, the effects of these shocks dissipate relatively quickly, becoming negligible after 150 days.

Insert Figure 4 here

When considering shocks originating from macroeconomic variables, a different pattern emerges. Figure 4 shows hump-shaped responses in all four financial markets to various macroeconomic shocks, illustrating the pervasive impact of macroeconomic conditions. Crude oil, in particular, displays the largest responses to TSE, INF, EPU, and GPR shocks, highlighting its heightened sensitivity to changes in these economic indicators. Gold responds most significantly to FFR shocks, likely reflecting its sensitivity to interest rate changes. The Stock market's relatively smaller response to most macroeconomic shocks (except ADS) suggests a lower susceptibility to these factors compared to Crude Oil and Gold. Bonds and Gold, while less impacted than Crude Oil, still show significant responses, with Gold often reacting more robustly than Bonds. These findings are consistent with Table 5, reinforcing the idea that macroeconomic shocks, especially those influencing Crude Oil, have deep and lasting effects on market volatilities. Notably, the effects of macroeconomic shocks last much longer than financial market shocks, not reaching a new equilibrium even after 200 periods. This observation underscores the long-term influence of macroeconomic factors, particularly on the volatility of the Crude Oil market.

5.5. Robustness and Extension

As a robustness check, we conducted an estimation using different forecasting horizons (10-step, 20-step, and 100-step ahead) and different orders (2nd order, 4th order, and optimal order) for the VAR model of macroeconomic shocks. Our findings are robust across these specifications. The results are available upon request.

We also expand the vMEM-X model to include a semi-variance case, where realized volatility is divided into good volatility and bad volatility. For details on constructing good and bad volatility, see **?**, **?**, **?** and **?**. Our focus is to determine whether shocks from macroeconomics impact the semi-variance in an asymmetric manner. The results of this investigation are presented in Table 6.

Insert Table 6 here

Surprisingly, our analysis reveals that macroeconomic shocks impact both good and bad volatility in a relatively symmetrical manner. For example, the total responses of bad and good volatility in Stocks to macroeconomic shocks are 19.80 and 19.55 , respectively. The difference is very small and statistically insignificant. [4]. Similar patterns are found for the other three markets as well. This finding indicates a lack of asymmetric response to macroeconomic shocks in our empirical analysis.

6. Conclusions and Policy Suggestions

This study employs a novel vMEM-X model to analyse how macroeconomic shocks influence volatility spillovers across key financial markets. The model treats macroeconomic conditions as external factors affecting market volatility, distinguishing between internal volatility spillovers within financial markets and external shocks from macroeconomic conditions. Additionally, new volatility spillover balance indices and impulse response functions are derived.

[4] **?** has defined an asymmetric volatility spillover index, which is the difference between the total responses of bad and good volatility of the four markets to different shocks, and proposed a bootstrap procedure to test if the asymmetric spillover is significant. We have also computed the asymmetric volatility spillover and conducted a similar test. However, the difference is so small that the test is insignificant.

Our application of this model to the Crude Oil, Stock, Bond, and Gold markets reveals distinct findings. In scenarios excluding macroeconomic variables, Stocks emerge as primary sources of market volatility, with Crude Oil being the main recipient. However, the role of the Stock market in driving volatility, particularly towards Crude Oil, is significantly altered when macroeconomic shocks are considered. These shocks have a more pronounced impact on Crude Oil than on Stocks, emphasizing the importance of macroeconomic factors in market behavior. The Bond and Gold markets, traditionally considered as safe havens, display a lesser degree of volatility spillover compared to Crude Oil and Stocks. This reinforces their roles as stabilizers in the financial market landscape, especially in times of economic uncertainty.

Our findings carry significant policy implications. Policymakers should prioritize stabilizing financial markets through various measures. Implementing macro-prudential regulations such as the capital adequacy ratio and liquidity coverage ratio in banks can bolster financial market stability, thus averting financial crises. Measures like Quantitative Easing (QE) and liquidity support may also diminish the risk of contagion. Additionally, managing spillover effects from interest rate decisions is crucial for effective monetary policy formulation. This strategy can help maintain a stable inflation rate, a key macroeconomic objective, thereby contributing to overall financial market stability and mitigating future financial crises and systemic risks. Other recommended policies include strengthening coordination of monetary policy and the global safety net at a global level, and enhancing supervision in the financial and banking system. Given the volatility spillovers from the stock market to Crude Oil, efforts to stabilize US and international financial markets should prioritize mitigating stock market risk spillovers to potentially prevent Crude Oil market destabilization. Policies such as employing circuit breakers with short-selling restrictions can help prevent excessive investor losses. From the perspective of investors, adjusting asset weights in portfolios appropriately to hedge against risk contagion by investing more, particularly in Gold and Crude Oil for diversification, is advisable. This strategy aligns with the role of both assets as spillover receivers, as noted by **?** and **?**.

For future research, we recommend pursuing two directions. Firstly, while we focused on six macroeconomic variables primarily from the US, it is essential to recognize the significant influence of emerging markets, particularly China, on the volatility of Crude Oil and Gold markets. Secondly, the vMEM-X approach and volatility spillover balance formula proposed in our general methods can be extended to international financial markets. For instance, exploring volatility spillover effects between financial markets in different countries and the effects of external shocks would be interesting.

Table 1 – Summary statistics

	Mean	Std	Min	Max	Skewness	Kurtosis	LB(12)
Panel A: Realized volatilities							
Stock	1.298	2.623	0.031	39.409	7.016	69.668	34164
Bond	0.450	0.487	0.002	10.262	8.191	123.104	14195
Gold	1.285	1.575	0.054	23.827	5.536	52.738	17833
Crude Oil	4.743	5.230	0.100	56.532	3.940	25.414	38341
Panel B: Macroeconomic variables							
TSD	1.630	1.110	-0.640	3.850	-0.030	2.080	46980
FFR	1.220	1.580	0.000	5.410	1.420	3.860	52389
INF	2.180	0.530	0.000	3.050	-2.310	9.950	9230
ADS	-0.260	2.140	-26.480	9.310	-7.180	85.360	50062
EPU	1.100	0.820	0.030	8.080	2.420	12.100	25672
GPR	1.070	0.450	0.090	5.430	2.250	15.110	11272

Notes: This table reports summary statistics of realized volatilities and semivariances. LB(12) is the Ljung–Box statistics for the serial correlation of order 12. The Term Spread (TSD) represents the yield difference between 10-year Treasury Bonds and 3-month Treasury bills. Effective Federal Funds Rate (FFR) denotes the overnight lending rate between banks. Inflation (INF) represents the expected inflation rates over five years. Aruoba-Diebold-Scotti (ADS) Index measures real-time economic activity. Economic Policy Uncertainty (EPU) index indicates policy-related economic uncertainty. Geopolitical Risk (GPR) index tracks risks from international interactions.

Table 2 – Estimated Results

	vMEM-X Model With macroeconomic variables				vMEM Model Without macroeconomic variables			
	Stock	Bond	Gold	Crude Oil	Stock	Bond	Gold	Crude Oil
w	0.133	0.104	0.127	0.605	0.073	0.055	0.069	0.126
α	0.659	0.194	0.379	0.419	0.635	0.195	0.368	0.399
β	0.069	0.001	0.179	0.149	0.072	0.001	0.197	0.125
α^w	0.188	0.385	0.181	0.228	0.207	0.391	0.175	0.270
α^m	0.062	0.163	0.206	0.151	0.072	0.228	0.212	0.174
Stock	0.659	0.025	0.024	0.119	0.635	0.022	0.021	0.117
Bond	-0.121	0.194	-0.098	-0.143	-0.109	0.195	-0.074	-0.211
Gold			0.379				0.368	
Crude Oil				0.419				0.399
TSD	0.018	0.016	0.019					
FFR	0.010							
INF	-0.047	-0.022	-0.034	-0.196				
ADS	-0.014			-0.057				
EPU	0.033							
GPR	-0.017							
LL	-3440	-255.6	-4867	-11060	-3447	-270.5	-4869	-11065
BIC	6982	619.0	9810	22198	6946	592.0	9797	22190
LB	15.55	9.21	9.50	8.06	16.99	9.32	9.61	10.15

LL denotes the values of the log-likelihood. BIC is Bayesian Information Criteria. LB(12) denotes the Ljung Box statistics up to order 12.

Table 3 – Volatility Spillovers between Stock, Bond, Gold and Crude Oil markets - without Macroeconomic Variables

	From Volatility				
	Stock	Bond	Gold	Oil	Total From
To Volatility					
Stock	15.93	4.96	6.11	3.49	14.55
Bond	7.84	4.44	3.59	1.87	13.30
Gold	9.74	4.64	11.59	3.35	17.74
Crude Oil	17.16	6.09	9.15	12.07	32.41
Total To	34.75	15.69	18.85	8.71	0.00
Balance	2.39	1.18	1.06	0.27	

The table presents spillovers for the full sample with a forecast horizon of $K = 200$ days.

Table 4 – Volatility Spillovers between Stock, Bond, Gold and Crude Oil markets - with Macroeconomic Variables

	From Volatility				
	Stock	Bond	Gold	Oil	Total From
To Volatility					
Stock	9.68	3.33	3.88	2.13	9.33
Bond	4.24	2.63	2.02	1.01	7.28
Gold	6.25	3.22	7.86	2.20	11.67
Crude Oil	5.55	2.42	3.21	4.18	11.18
Total To	16.05	8.98	9.11	5.34	
Balance	1.72	1.23	0.78	0.48	

The table presents spillovers for the full sample with a forecast horizon of $K = 200$ days.

Table 5 – Volatility Spillovers between Stock, Bond, Gold, and Crude Oil Markets - from Macroeconomic Shocks

	From shocks of						
	TSD	FFR	INF	ADS	EPU	GPR	Total from
To volatility of							
Stock	5.92	3.81	7.79	8.36	5.74	4.17	35.79
Bond	1.82	2.51	3.26	1.82	1.67	1.70	12.78
Gold	3.17	3.78	5.22	1.37	2.19	2.73	18.46
Crude Oil	11.06	7.85	14.58	5.00	7.11	7.51	53.12
Total to	21.98	17.94	30.85	16.56	16.72	16.11	

The table presents spillovers for the full sample with a forecast horizon of $K = 200$ days.

Table 6 – Asymmetric Volatility Spillovers between Stock, Bond, Gold and Crude Oil markets - from Macroeconomic shocks

		From shocks of						
To		TSD	FFR	INF	ADS	EPU	GPR	Total from
Bad volatility	Stock	2.12	1.79	3.30	6.86	3.79	1.94	19.80
	Bond	1.42	2.34	2.82	1.79	1.49	1.49	11.36
	Gold	0.88	1.16	1.78	1.56	1.29	1.01	7.68
	Crude Oil	8.19	5.40	10.76	4.69	5.77	5.41	40.23
Good volatility	Stock	2.10	1.68	3.23	6.80	3.82	1.92	19.55
	Bond	1.10	2.19	2.46	1.73	1.34	1.30	10.11
	Gold	0.99	1.31	2.03	1.61	1.42	1.16	8.53
	Crude Oil	7.33	4.93	9.70	4.50	5.30	4.86	36.62
	Total to	24.14	20.81	36.08	29.55	24.23	19.07	

The table presents spillovers for the full sample with a forecast horizon of $K = 200$ days.

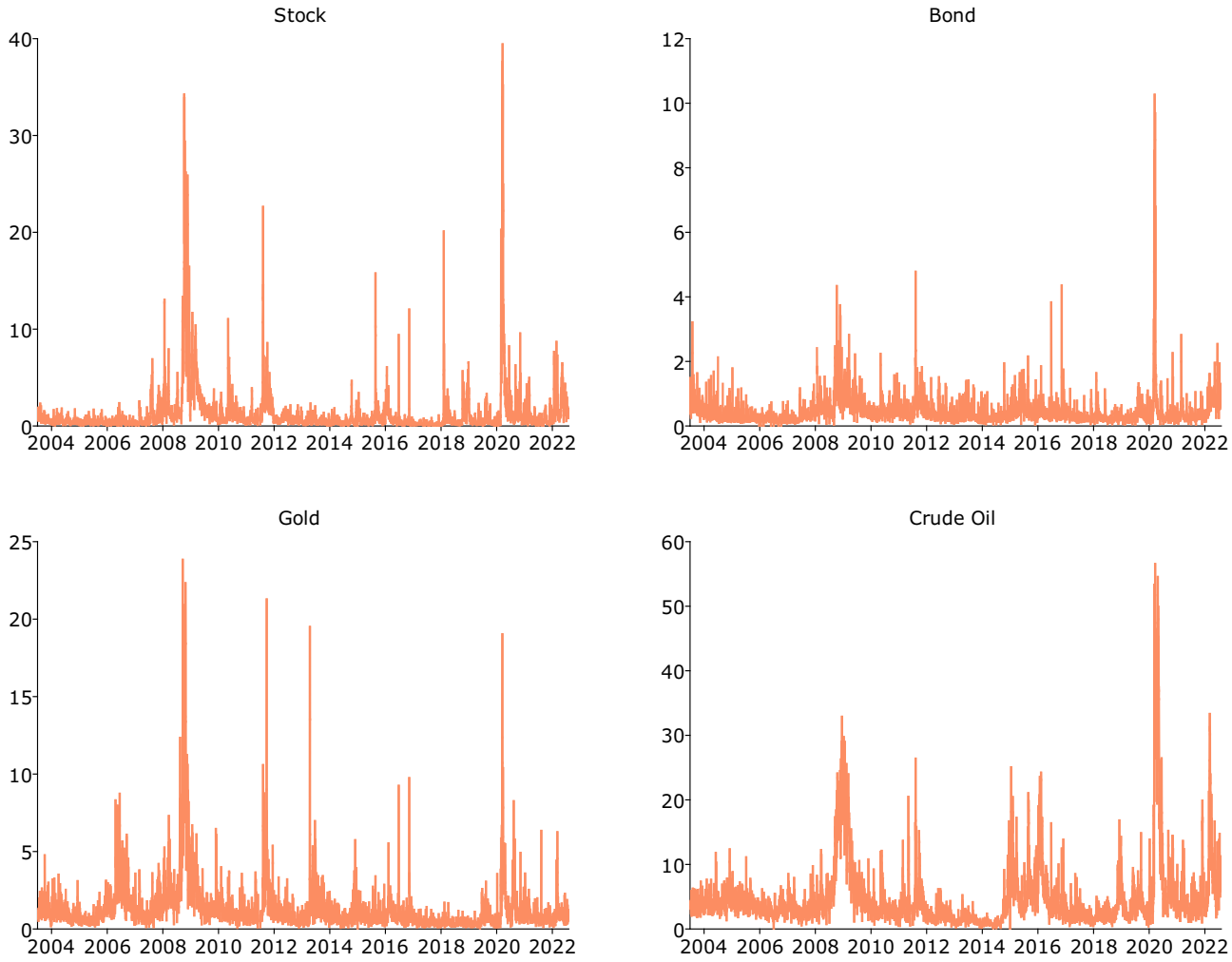


Figure 1 – The time evolution of realized volatility of Stock, Bond, Gold, and Crude Oil



Figure 2 – Time evolution of macroeconomic variables

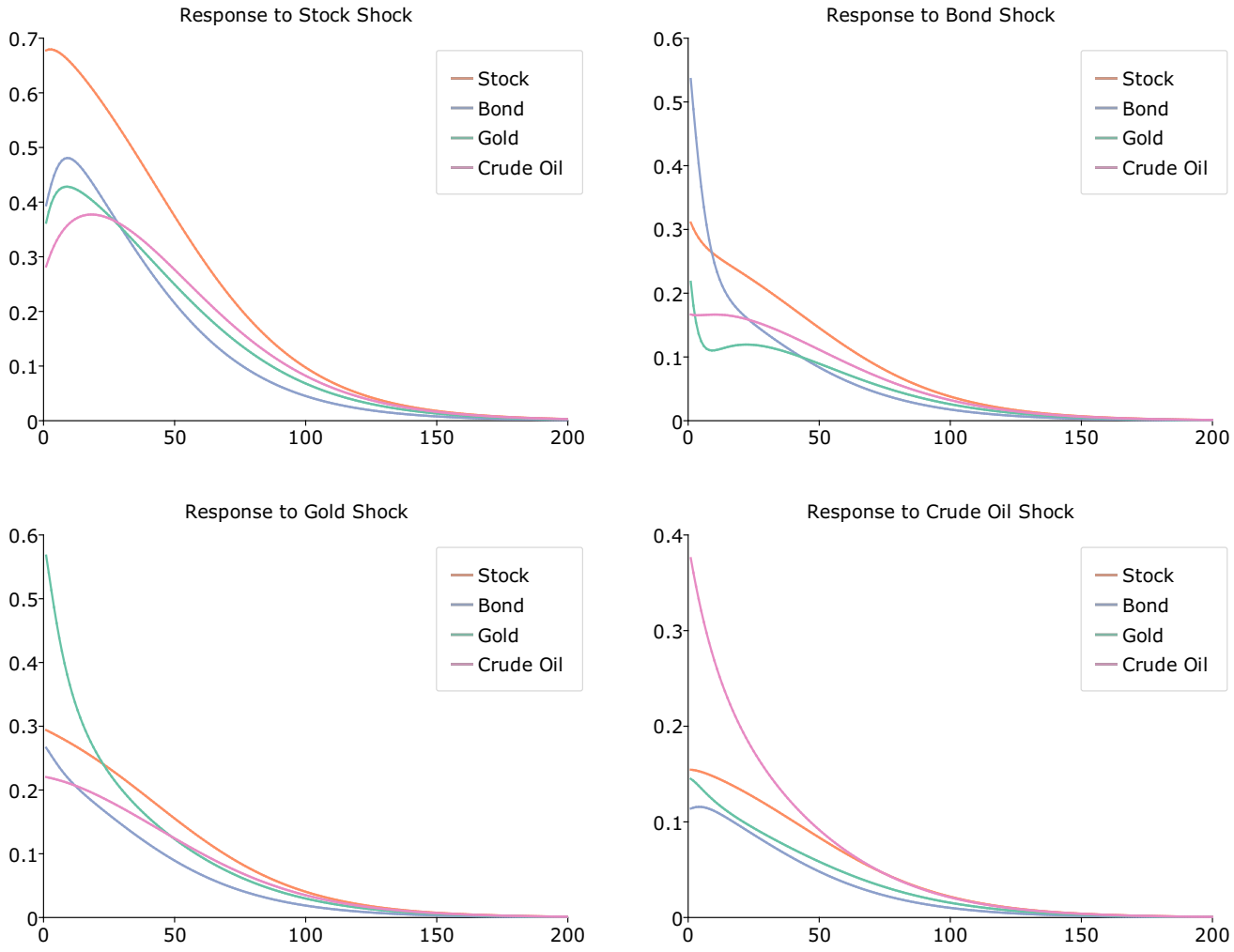


Figure 3 – Impulse response of the volatility in the four markets to realized volatility shocks

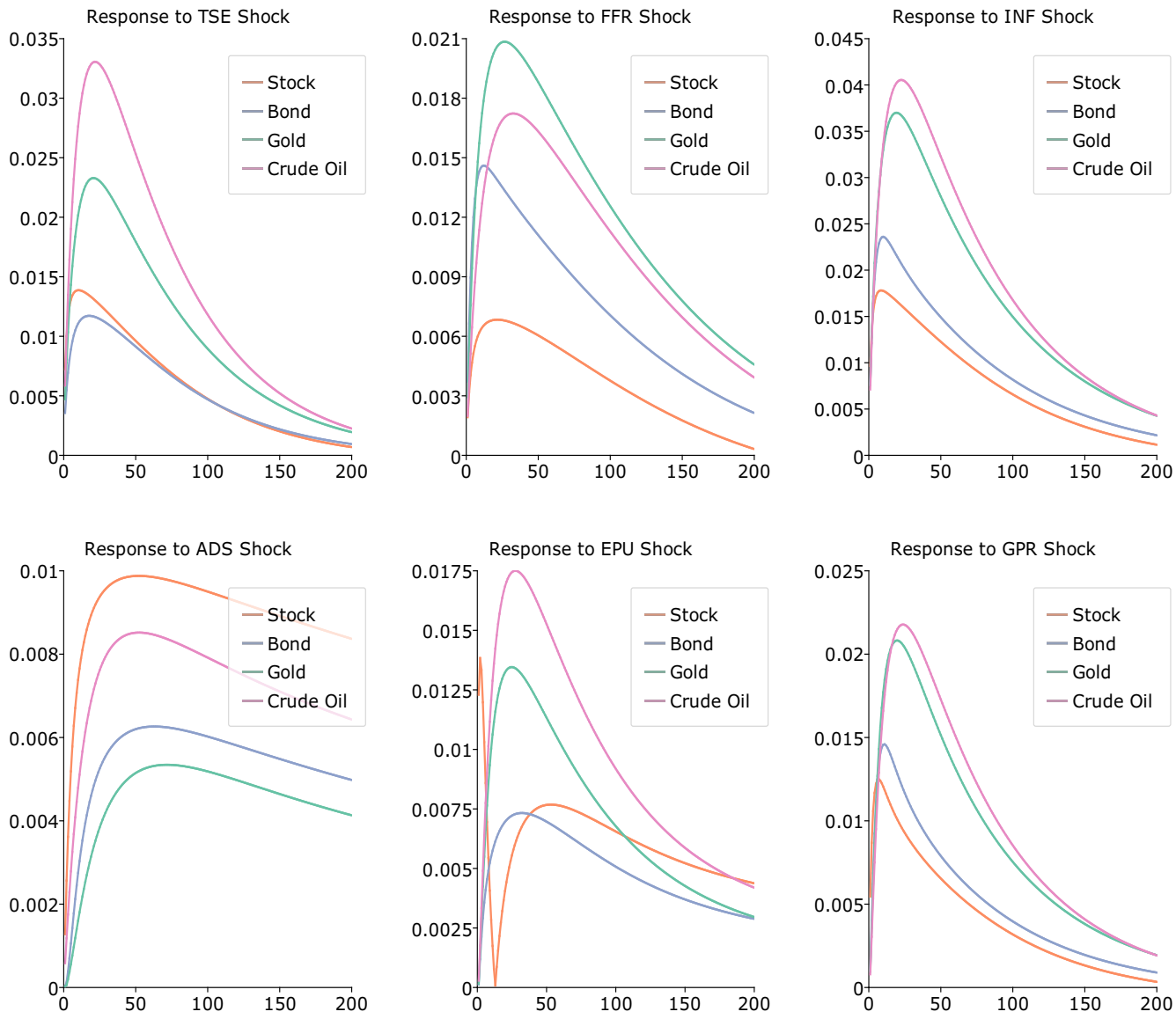


Figure 4 – Impulse response of the volatility in the four markets to markets macroeconomic shocks

Appendix A: VARMA-X Representation

? demonstrated that vMEM(p, q) can be converted to a VARMA(p, q) process. In this appendix, we follow **?** and demonstrate that the vMEM-X can be represented by a VARMA-X model.

Consider the following vMEM-X:

$$\begin{aligned}
&\mathbf{x}_t = \boldsymbol{\mu}_t \odot \boldsymbol{\epsilon}_t, \quad \boldsymbol{\epsilon}_t \sim \mathrm{D}(\mathbf{1}, \boldsymbol{\Sigma}), \\
&\boldsymbol{\mu}_t = \boldsymbol{\omega} + \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}\boldsymbol{\mu}_{t-1} + \mathbf{A}^w\mathbf{x}^w_{t-1} + \mathbf{A}^m\mathbf{x}^m_{t-1} + CZ_{t-1}.
\end{aligned} \tag{22}$$

where $\odot$ denotes the Hadmard (element by element) product. The innovation vector $\boldsymbol{\epsilon}_t$ has support over $[0, +\infty)$, with a unit mean vector $\mathbf{1}$ and general variance-covariance matrix $\boldsymbol{\Sigma}$. Taking the difference between $\mathbf{x}_t$ and $\boldsymbol{\mu}_t$, we obtain

$$\mathbf{x}_t - \boldsymbol{\mu}_t = \mathbf{e}_t, \quad \mathbf{e}_t \sim \mathrm{D}(\mathbf{0}, \boldsymbol{\Pi}). \tag{23}$$

It follows that

$$\boldsymbol{\mu}_t = \mathbf{x}_t - \mathbf{e}_t, \tag{24a}$$

$$\mathbf{B}_i\boldsymbol{\mu}_t = \mathbf{B}_i\mathbf{x}_t - \mathbf{B}_i\mathbf{e}_t. \tag{24b}$$

Substituting the expressions in (24a) and (24b) into (22) and rearranging we obtain the following representation:

$$\begin{aligned}
\mathbf{x}_t &= \boldsymbol{\omega} + \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}\mathbf{x}_{t-1} + \mathbf{A}^w\mathbf{x}^w_{t-1} + \mathbf{A}^m\mathbf{x}^m_{t-1} + CZ_{t-1} + \mathbf{e}_t - \mathbf{B}\mathbf{e}_{t-1}, \\
&= \boldsymbol{\omega} + (\mathbf{A} + \mathbf{B})\mathbf{x}_{t-1} + \mathbf{A}^w\mathbf{x}^w_{t-1} + \mathbf{A}^m\mathbf{x}^m_{t-1} + CZ_{t-1} + \mathbf{e}_t - \mathbf{B}\mathbf{e}_{t-1},
\end{aligned} \tag{25}$$

which is a HAR-VARMA-X(1,1) model. Given this representation, the covariance stationarity condition requires that the largest eigenvalue of $\mathbf{A}+\mathbf{B}+\mathbf{A}^w+\mathbf{A}^m$ be less than unity. Consequently, the unconditional first moment can be obtained as $E(\mathbf{x_t}) = (I_k - \mathbf{A} + \mathbf{B} + \mathbf{A}^w + \mathbf{A}^m)^{-1}\,\boldsymbol{\omega}$.