UK monetary and fiscal policy since the Great Recession- an evaluation

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Abstract

This paper explores the economic impacts of the Bank of England’s quantitative easing policy, implemented as a response to the global financial crisis. Using an open economy Dynamic Stochastic General Equilibrium (DSGE) model, we demonstrate that monetary policy can remain effective even when nominal interest rates have reached the zero lower bound. We estimate and test the model using the indirect inference method, and our simulations indicate that a nominal GDP targeting rule implemented through money supply could be the most effective monetary policy regime. Additionally, our analysis suggests that a robust, active fiscal policy regime with nominal GDP targeting could significantly enhance economic stabilization efforts.

Keywords: Quantitative easing, Financial friction, SOE-DSGE, Indirect inference, Zero bound

JEL classification: E44, E52, E58, C51

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1 Introduction

The onset of the Global Financial Crisis (GFC) in August 2007, which was further exacerbated by the collapse of Lehman Brothers in September 2008, triggered a severe recession in several major economies, including the United Kingdom. This paper aims to identify the drivers of the UK’s response to the GFC and draw policy implications, encompassing monetary, regulatory, and fiscal measures. To this end, we employ a Dynamic Stochastic General Equilibrium (DSGE) model of the UK economy, which we estimate and test using an indirect inference method against empirical behavior observed over the relevant period. This method confers robustness on the model’s small sample performance, as it is fully identified by structural constraints.

In the aftermath of the GFC, central banks implemented unconventional monetary policies, such as Quantitative Easing (QE), which involved injecting liquidity into the economy through large-scale asset purchases. The UK experienced a sharp contraction in output of around 20% on an annualized basis in the first quarter of 2009, coupled with an unprecedented increase in the unemployment rate to 7.6%. In response, the Bank of England (BoE) pursued expansionary monetary policies to boost demand, aggressively lowering the interest rate to its effective lower bound. Specifically, the Monetary Policy Committee of the BoE reduced the interest rate from 5% to 0.5% in 2009, followed by a further cut to 0.25% by the autumn of 2016, when the short-term rate became constrained by the zero lower bound. Successively, three rounds of QE were then announced by the BoE.

In our model, we place the banking sector at the heart of monetary transmission, embodying the key financial friction in the economy. We do so because it seems clear that this crisis primarily arose in the banking system, often being referred to as a banking crisis, and that both monetary policy and bank regulation had major effects on the behavior of bank credit. Our model adopts the financial accelerator approach of Bernanke et al. (1999) and embeds in it the effects of banking regulation and also adds a collateral element; QE, through the additional liquidity it injects, acts to reduce the cost of this collateral and lower the cost of credit. In adopting this DSGE model for the UK, we follow Le et al. (2016) who applied this model to the US over this episode within a closed economy context, and successfully matched US data behaviour. It seemed likely to us that, given the similarity of the structure of the UK and US economies, the same model would also match the UK experience, after allowing for the UK’s high openness.

Previous research by Lyu et al. (2023) used a similar DSGE structure to demonstrate that a monetary regime using a counteractive M0 rule can stabilize the UK economy. However, their study did not explore the potential benefits of alternative monetary policies, nor did it examine the interactions between fiscal and monetary policy. Our study aims to provide a more comprehensive analysis by incorporating both fiscal and monetary policy factors to better understand their combined effects on economic stability.

Joyce et al. (2011) specifies five potential transmission channels for the impact of QE: policy signalling, portfolio rebalancing, liquidity, broad money, and confidence. In our model, these channels are all present in one way or another. Though there is no direct portfolio rebalancing, since our household sector has no ‘preferred habitat’, expectations of future policy determine longer term interest rates in a similar way and also affect ‘confidence’, while policy signalling such as ‘forward guidance’ works similarly. However, these channels are all motivated via the principal banking transmission process.

There is by now an extensive literature, both theoretical and empirical, on the crisis episode and the effects of QE within it. Bhattarai and Neely (2020) provides a comprehensive survey of these studies.
Here we examine some key findings of this literature.

First, we review SVAR studies for the US and UK. Walentin (2014) finds that exogenous shocks to
US mortgage spreads have big effects on house prices, residential investment, consumption and GDP. Thus
unconventional monetary policy in the form of asset purchases in mortgage markets can affect the mortgage
spread, and it also has big effects on house prices and GDP. He finds that if QE1 had a 150 basis points effect
on the mortgage spread (as in Hancock and Passmore, 2011), then it would have raised GDP at the peak
by 3.8% and house prices by 5.1%. Using a time-varying parameter SVAR Baumeister and Benati (2013)
found that the US and UK QE1 reduced big drops in GDP and prices by narrowing term spreads. Using
the estimate of a 60 basis points drop in the US term spreads from Gagnon et al. (2011) they find that US
QE1 lowered the ten-year Treasury yield by 58 basis points, stopped inflation from reaching a low of -1%
and output from reaching a trough of -10%. With an assumption of a 50 basis points drop in term spreads
in the UK, QE1 would have prevented a fall in inflation from reaching -4% and output from a trough of
-12%. Applying various time-varying VARs to the UK data, Kapetanios et al. (2012) found that the peak
effects of QE1 were +1.5% on real GDP and +1.25% on CPI inflation.

Next we review a variety of theoretical models investigating some of the different transmission channels
mentioned above between QE and the economy. The first group derives QE’s effectiveness via an assumption
of exogenous participation constraints in financial markets, so QE can have real effects via its impact on the
yield curve. Following the theoretical work by Vayanos and Vila (2009), and other empirical investigations
following it (e.g. Hamilton and Wu, 2012), Chen et al. (2012) set up a medium size DSGE model with
segmented asset markets, where some agents trade in both long-term and short-term bonds, subject to a
transaction cost, which is a diminishing function of the ratio of central bank holdings of long-term to short-
term bonds, while other agents can only trade in long-term bonds. The transaction cost ensures there is a
term premium in the no-arbitrage asset pricing condition; central bank purchases of long-term bonds would
reduce this term premium to produce real macroeconomic effects. They estimate the model with Bayesian
methods for US postwar data. They find that US QE2, a $600 billion purchase of long-term government
bonds, causes GDP to rise by 0.13%.

A second channel assumes that the effects work through the binding leverage constraints on intermedi-
banks who intermediate between households and non-financial firms can abscond with funds. To eliminate
this incentive, there is a binding incentive constraint, which also means a binding leverage constraint. Limits
to arbitrage create a wedge/premium between the expected return on capital and risk-free debt, which is
inversely related to banks’ net worth. In these models, when leverage constraints bind and financial shocks
raise the external finance premium, government credit policy can act countercyclically by influencing net
worth. In Gertler and Karadi (2011), QE acts as direct financial intermediation by the central bank to
offset disruption to private financial intermediation. Although the central bank is less efficient than private
intermediaries, the central bank is not subject to the leverage constraint and can issue risk free debt to raise
funds elastically from households to fund non-financial firms. When the constraint on private intermediation
tightens during a crisis, QE can act countercyclically. Gertler and Karadi (2013) aim to develop a unified
framework to analyse all large scale asset purchases by extending Gertler and Karadi (2011) with an additional
assumption that banks can intermediate the funding of long-term government bonds as well as the
funding of non-financial firms, thus the interest spread depends on the long-term bonds fraction of intermedi-
aries’ assets. Also, the central bank can conduct monetary policy by either adjusting the short-term interest
rate or by purchasing long-term government bonds and private securities. They find that US QE2 would have raised GDP by around 1%. Sims and Wu (2019) extend Gertler and Karadi (2013) to include the central bank’s balance sheet with interest-bearing reserves, which financial intermediaries are required to hold and cannot be stolen. The central bank can hold either private or government bonds, which are financed via the creation of reserves. The model assumes that firms issue long-term bonds to finance investments and financial intermediaries hold long-term bonds and reserves. Market clearing requires that bonds issued by firms and debt issued by the government must be held by the central bank or financial intermediaries. When intermediaries are constrained via the costly enforcement problem, then QE, as central bank purchases of long-term bonds via creation of interest-bearing reserves, does not crowd out intermediary bond purchases so that the total demand for long-term bonds increases, causing higher bond prices, easing the constraint and elevating aggregate demand. They find that US QE can account for 2/3 of the observed decline in the shadow Federal Funds rate.

Studies of these transmission channels of QE like Chen et al. (2012) and Gertler and Karadi (2013) also find that a credible central bank commitment to hold short-term interest rates at zero generates most of the GDP effects. This commitment works via a signalling channel, i.e. by accumulating a large balance sheet through long-term bond purchases, it signals lower short-term policy rates in the future and thereby reduces current long rates. Bhattarai et al. (2019) used a model without financial frictions or participation constraints to study time-consistent discretionary policy and how the maturity composition of government debt affects optimal future interest rates. Their calibration matches Krishnamurthy and Vissing-Jorgensen (2011)’s event-study estimates of the signalling effects of US QE2 on long-term yields, expected inflation, inflation and output during the Great Recession. They find that QE2 increases output by 1.6% and inflation by 1.4% on impact.

Although this is not an exhaustive list of available studies, we observe that most of the DSGE studies in the literature focus on US experience with QE and many quantitative analyses are based on calibration or Bayesian estimation. These studies produce a wide range of implied results for QE policy. However, none was tested and statistically evaluated. Our model is closely related to the strand of the literature based on the leverage constraint on intermediaries. Its idea is similar to Gertler and Kiyotaki (2011) and Gertler and Karadi (2013) - there is a leverage constraint, but instead of QE working to relax the intermediaries’ constraint, we assume that QE eases firms’ constraints. We test it against UK data for the post-crisis period to establish empirical conclusions about the effects of monetary policy and regulation on a major open economy.

The rest of the paper is structured as follows. Section 2 sets out the model. Section 3 introduces the indirect inference method and estimates and tests the model. In Section 4, we set out the empirical findings and discuss the causes of the crisis. Section 5 considers the implications for policy; we simulate the model with alternative monetary, regulative and fiscal policies and analyse how they affect economic stability and welfare. Section 6 concludes.

2 Model set-up

This section will briefly describe the DSGE model in a small open economy. It follows Le et al. (2016), which in turn builds on Smets and Wouters (2007) and Bernanke et al. (1999).
2.1 Household sector

The representative household in the model maximizes its non-separable utility function, which determines their consumption and labor choices in each period. Merola (2014) notes that the non-separable nature of the utility function implies that expected employment growth will also influence consumption decisions. Specifically, the utility function is defined as follows:

$$ U = \max_{E_0} \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{1 - \sigma_c} (C_t - hC_{t-1})^{1-\sigma_c} \exp \left( \frac{\sigma_c - 1}{1 + \sigma_l} L_t^{1+\sigma_l} \right) \right] \right\} $$

(2.1)

The inter-temporal budget constraint for the household is defined:

$$ P_t C_t + B_t + D_t + S_t B^f_t \leq R_{t-1} B_{t-1} + R^f_{t-1} S_{t-1} B^f_{t-1} + R_{t-1} D_{t-1} + W_t L_t $$

(2.2)

where $W_t$ is the nominal wage offered by the entrepreneur, and gross nominal interest rate $R_t$ is equal to $(1 + r_t)$. Similarly, the Foreign gross nominal interest rate is $R^f_t$. $B_t$ and $B^f_t$ represent domestic and foreign bond, respectively. The disposable income can be put into the bank as a deposit $D_t$. $Q_t$ is real exchange rate and defined as $Q_t = \frac{P^f_t}{P_t} S_t$, which is treated as import price related to domestic price level. $S_t$ is the nominal exchange rate defined as the domestic currency’s value on one unit of foreign currency. $P^f_t$ represents the consumption goods price from a foreign country. According to Minford (2014), they assumed the foreign bond price to be the cost at what the foreign consumption basket would cost. $P^*_t$ is the general price level of the home country. $P^*_f$ is the foreign country’s general price level, and related to domestic currency will be $P^*_t S_t$. Here we assume exports goods from the domestic country have a little impact on the rest of the world, so $P^*_t \approx P^f_t$.

The household’s utility (equation 2.1) is maximized subject to budget constraints (equation 2.2) with respect to the variables $C_t$, $B_t$, $B^f_t$, and $L_t$. Subsequently, the consumption Euler equation is generated using the results of the first-order condition.

$$ \frac{[C_t - hC_{t-1}]^{-\sigma_c} \left( \frac{\sigma_c - 1}{1 + \sigma_l} L^t_{t+1} \right)}{[C_{t+1} - hC_t]^{-\sigma_c} \left( \frac{\sigma_c - 1}{1 + \sigma_l} L^t_{t+1} \right)} = E_t \left( \frac{P_t}{P^f_t} \beta R_t \right) $$

(2.3)

$$ \frac{1 + R^f_t}{(1 + R_t) S_t} = \frac{1}{E_t S_{t+1}} $$

(2.4)

By substiting $E_t \pi_{t+1} = \frac{E_t [P^f_{t+1}]}{P^f_t}$ and $E_t \pi^f_{t+1} = \frac{E_t [P^f_{t+1}]}{P^f_t}$, leads to the Real Uncovered Interest Rate Parity (RUIP) which generate the motion of real exchange rate:

$$ \frac{E_t q_{t+1}}{q_t} = \frac{r_t}{r^f_t} \frac{E_t \pi^f_{t+1}}{E_t \pi_{t+1}} $$

(2.5)

According to a single-industry version of Armington’s (1969) model, the total consumption for each household $C_t$ will be differentiated by produced places. Specifically, we distinguish the domestically produced products and imported goods as $C^d_t$ and $C^f_t$. The utility function for aggregated consumption can be represented via
the constant elasticity of substitution (CES) index.

\[ C_t = [\omega(C_d^t)^{-\rho} + (1 - \omega)\zeta_t]^{-\frac{1}{\rho}} \]  

(2.6)

We assume that the domestic consumers have a fixed preference bias towards the domestic products, and it is measured by \( \omega; 0 < \omega < 1 \). \( \rho \) is related to the elasticity of marginal substitution between domestic and foreign goods' variety, which is constant at \( \sigma = \frac{1}{1+\rho} \). \( \zeta \) is the preference error of demand for imported goods. The total expenditure of consumption is defined as \( P_tC_t = P_tC_d^t + Q_tC_f^t \). \( P_t^d \) is the ratio of domestic price relative to the general price level \( P_t \), defined as \( p_t^d = P_t^d/P_t \). By setting up the Lagrangian function, we can generate the demand of import:

\[ C_f^t = ((1 - \omega)\zeta_t)\sigma(Q_t)^{-\sigma}C_t \]  

(2.7)

Symmetrically, export equation is:

\[ (C_f^*) = ((1 - \omega f)\zeta_f)^{\sigma_f}(Q_f^*)^{-\sigma_f}C_t^* \]  

(2.8)

where \((C_d)^*\) and \((C_f)^*\) are foreign demand for their products and imported goods. Similarly, \( \omega_f \) is a foreign consumer’s home bias. \( C_t^* \) is total consumption. \( \sigma_f \) is a foreign country elasticity of marginal substitution between domestic and imported goods. \( \zeta^* \) represents the foreign random preference error to the demand for import.

The evolution of net foreign bonds follows the principle that current account surplus and capital account deficit sum to zero. Current account surplus is the real net exports plus the income flow from foreign bond investment, defined as \((EX_t - Q_tIM_t) + r_f^f b_f^t Q_t\). Capital account deficit captures the decrease in net foreign asset, measured by \((b_{t+1}^f - b_t^f)Q_t\). Thus the evolution of net foreign bond can be expressed as follows with the real term:

\[ \Delta b_{t+1}^f = \left( \frac{EX_t}{Q_t} - IM_t \right) + r_f^f b_t^f \]  

(2.9)

2.2 Labor union and labor packers with hybrid wage setting

Smet and Wouters (2007) describe labor markets as comprising of two key actors: labor unions and labor packers. Households supply homogeneous labor to the labor union, which then allocates and differentiates labor services before selling them to labor packers. The labor packers then aggregate the labor services from the labor union using an aggregator proposed by Kimball (1995) and provide them to intermediate goods producers for production.

\[ L_t = \left( \int_0^t L_t(i)^{\frac{1}{1+\lambda_{w,t}}} di \right)^{1+\lambda_{w,t}} \]  

(2.10)

\( L_t \) and \( L_t(i) \) represent the composite labor and differentiated labor services respectively. \( \lambda_{w,t} \) measures the shocks to aggregator function, which causes the changes in demand then mark-up, and it is following the
AR(1) process as $\ln(\lambda_{w,t}) = \rho_w \ln(\lambda_{w,t-1}) + \eta_w$. The profit function for labor packer is:

$$L_t W_t - \int_0^1 L(i) W(i) dt$$

(2.11)

$W_t$ and $W_{t(i)}$ are the wage of composite and intermediate labor respectively. Then subject to:

$$L_t = \left( \int_0^t L_t(i) \frac{1}{1+\lambda_{w,t}} \right)^{1+\lambda_{w,t}}$$

(2.12)

By FOC, the optimal demand of labor from labor unions is:

$$L_t(i) = \left( \frac{W_t(i)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t$$

(2.13)

The labor unions work as an intermediate between the household and the labor packer. Under the Calvo pricing indexation, part of labor unions can adjust their price based on the following optimization problem:

$$\max E_t \sum_{s=0}^{\infty} \beta^s \xi_w \frac{\Xi_{t+s}}{\Xi_t} P_t L_{t+s}(i) \left[ \tilde{W}_t(i)(\Pi^w_{t,t+s}) - W_t^h \right]$$

(2.14)

where $\Pi^w_{t,t+s} = \Pi_{k=1}^{\infty} \left( \frac{\pi_{t+k-1}}{\pi_t} \right)^{\xi_w}$. Subject to the labor demand function 3.47, the optimal wage will satisfy the following condition:

$$\sum_{s=0}^{\infty} \beta^s \xi_w \frac{\Xi_{t+s}}{\Xi_t} P_t \left[ (1 - \omega^w) L_{t+s} W_t(i) - \omega^w W_t^{\omega^w} \right. + \omega^w L_{t+s} W_t^h W_t(i) - \omega^w W_t^h = 0$$

(2.15)

where $\omega^w = -\frac{1+\lambda_{w,t}}{\lambda_{w,t}}$. Then the law of motion of the aggregate wage is:

$$W_t = \left[ \xi_w(W(i)_{t-1} \left( \frac{\pi_{t-1}}{\pi_t} \right)^{\xi_w} + (1 - \xi_w)(\tilde{W}_t(i)) \frac{1}{\lambda_{w,t}} \right]^{\lambda_{w,t}}$$

(2.15)

Smet and Wouters (2007) adopt a New Keynesian (NK) model in which prices and wages are sticky due to Calvo-type pricing in both goods and labor markets. However, there is significant disagreement over the extent of nominal rigidity, which is crucial as it determines the short-run non-neutrality of monetary policy and the effectiveness of monetary interventions for stabilizing the economy. Le et al. (2011) test both the NK and a New Classical (NC) version of the model using indirect inference on US postwar data, and find that both models are strongly rejected as the NK model generates too much nominal rigidity while the NC model generates too little. The authors also considered a weighted model that allowed for nominal rigidities in some parts of the economy but not others, finding that it provided the appropriate amount of nominal rigidity for the US economy. We follow Le et al. (2012) to build a hybrid wage model. We assume a fixed fraction ($\nu^w$) of labor is from imperfect competitive market and the remaining ($1 - \nu^w$) is from competitive market. If the wage is perfectly flexible and mark up equals zero, the real wage would equal the marginal substitution rate between consumption and leisure. The hybrid wage which will be passed to labor packers
is then defined as:

\[ W_t^{Hybrid} = \nu_w W_t + (1 - \nu_w) W_t^{NC} \]  

(2.16)

2.3 Final goods producer and hybrid price setting

In an environment of perfect competition, producers of final goods are price takers. They evaluate intermediate goods, and we assume, following Le et al. (2012), that the final output consists of intermediate goods from a monopoly market (\(\nu_p\)) and perfectly competitive markets (1-\(\nu_p\)). In a competitive market with zero price markup, intermediate goods are priced at their marginal cost. Thus, the final goods equation for this hybrid market is as follows:

\[ P_t^{NC} = MC \]  

(2.17)

\[ P_t^{Hybrid} = \nu_p P_t + (1 - \nu_p) P_t^{NC} \]  

(2.18)

2.4 Intermediate goods producer

To incorporate the concept of financial friction, we modify the DSGE framework in Smets and Wouters (2007) by drawing on the BGG model. In our modified framework, entrepreneurs act as the intermediate goods producers. They hire labor and purchase installed capital using constant return to scale technology to produce intermediate goods (\(Y_t(i)\)). Additionally, entrepreneurs purchase capital from capital producers using externally financed funds and net worth. The intermediate goods are produced using the following production function, with capital as an input: services \(K_s^t(i)\) and labour input \(L_t(i)\):

\[ Y_t(i) = K_s^t(i)^{\alpha} \left[ \gamma_t L_t(i) \right]^{1-\alpha} - \gamma_t \Phi \]  

(2.19)

The optimal capital utilization rate is solved by solving the maximizing problem:

\[ \max R_t^{rental} Z_t(i) K_{t-1}(i) - a(Z_t(i)) K_{t-1}(i) \]  

(2.20)

\[ \partial z_t : R_t^{rental} = a'(z_t) \]  

(2.21)

Capital services is specified as \(K_s^t(i) = Z_t(i) K_{t-1}(i)\) where \(Z_t(i)\) is real capital utilization rate. The income of renting capital services is \(R_t^{rental} Z_t(i) K_{t-1}(i)\). The cost of changing capital utilisation is \(a(Z_t(i)) K_{t-1}(i) \) . The profit function for entrepreneurs is:

\[ P_t(i) Y_t(i) - W_t L_t(i) - R_t^{rental} K_s^t(i) \]  

(2.22)

\[ ^2 \text{At steady state, } a(1) = 0, \text{ and } z = 1. \]
To attain an optimal allocation of resources, it is necessary to optimize the profit function while taking into account the constraints imposed by the production function with regard to labor and capital input:

\[
\partial L_t(i) : MC_t \gamma (1-\alpha)^t (1-\alpha) \varepsilon_t^a \left( \frac{K^a_t(i)}{L_t(i)} \right)^{\alpha} = W_t \tag{2.23}
\]

\[
\partial K^a_t(i) : MC_t \gamma (1-\alpha)^t \alpha \varepsilon_t^a \left( \frac{K^a_t(i)}{L_t(i)} \right)^{\alpha-1} = R_t^{rental} \tag{2.24}
\]

\(MC_t\) is the marginal cost. Then we generate a labor demand equation related to the capital:

\[
K^a_t = \frac{\alpha}{1-\alpha} \frac{W_t}{R_t^{rental}} L_t \tag{2.25}
\]

The marginal cost can be derived as:

\[
MC_t = \frac{(R_t^{rental}) \alpha(W_t)^{1-\alpha}}{\varepsilon_t^a \alpha^\alpha (1-\alpha)^{1-\alpha}} \tag{2.26}
\]

According to the Calvo (1983) contract, a certain proportion \(\xi^s\) of entrepreneurs are able to adjust their prices in each period. The objective of the entrepreneurs is to maximize their profits while taking into account the constraints imposed by the demand for intermediate goods. In other words, the problem at hand involves finding the optimal price setting that maximizes profits while ensuring that intermediate goods are demanded in accordance with market conditions.

\[
\text{Max} \sum_{s=0}^{\infty} \beta^s \xi^s \Xi_t^s \Pi^{t+s} P_t(i) \tilde{Y}^{t+s}(i)(\Pi_{i,t}^{t+s}) - MC_{t+s} \tag{2.27}
\]

s.t. intermediate goods demand function

\[
Y_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-1 + \lambda^{t+s}} Y_t \tag{2.28}
\]

where \(\xi\) is used to measure the fraction of intermediate good producer, that will adjust their price level. \(\Xi_t^s P_t^{\beta^s} / \Xi_t^s P_t^{i+s}\) is the firm nominal discount factor. \(\Pi^{t+s} = \prod_{k=1}^{t+s} (\frac{\pi_t^{t+s}}{\pi_t^{t+s-1}})^{\lambda^t} \). \(P_t(i)\) is the chosen optimal price level. \(MC\) is marginal cost of intermediate goods production:

\[
\text{Max} \sum_{s=0}^{\infty} \beta^s \xi^s \Xi_t^s P_t(i) \tilde{P}^{t+s}_t(i) \left( \frac{P_t(i)}{P_t} \right)^{-1 + \lambda^{t+s}} Y_t \tilde{P}^{t+s}_t(i)(\Pi_{i,t}^{t+s}) - MC_{t+s} \tag{2.29}
\]

Finally, the optimal choice of price can be generated by first order condition with respect to \(P_t(i)\):

\[
\sum_{s=0}^{\infty} \beta^s \xi^s \Xi_t^s P_t(i) \left[(1-\omega)Y_{t+s} P_t(i)^{-\omega} + \omega Y_{t+s} MC_{t+s} P_t(i)^{-\omega-1} P_t(i) \right] = 0 \tag{2.30}
\]

\(^3\) According to SW2007, the nominal discount factor here is equals the discount factor for the households.
Then the optimal price level chosen by intermediate goods producers is:

\[
\tilde{P}(i) = \frac{\sum_{s=0}^{\infty} \beta^s \varepsilon_p s \pi_i ^{s+1} P_{t+s} Y_{t+s} MC_{t+s} P_t^{-\omega} P_t^{\omega} - \omega}{\sum_{s=0}^{\infty} \beta^s \varepsilon_p s \pi_i ^{s} P_{t+s} Y_{t+s} P_t^{-\omega} P_t^{\omega}} (\omega - 1)
\]

(2.31)

Given that each firm updates its prices using the same mechanism, the aggregate price index for the intermediate goods in the imperfectly competitive market can be derived as follows:

\[
P_t = [\varepsilon_p (P(i)_{t-1} (\frac{\pi_{t-1}}{\pi_t})_{t-1}^{1 \beta}] + (1 - \varepsilon_p) (P_t(i)_{t}^{1 \omega})^{1 \beta}_{p,t}
\]

(2.32)

2.5 Capital producer

In this subsection, we will discuss the behaviour of capital producers. Refer to SW07, and capital producer takes prices as given in a competitive market. Each period, they purchase the capital left from the last period with intermediate goods producer, then combine with the newly invested resources. With every unit of investment, they will produce \(1 - S \left( \frac{I_t}{I_{t-1}} \right) I_t \) capital. Then the capital evolution equation is:

\[
K_t = (1 - \delta) K_{t-1} + \varepsilon_t \left[ 1 - S \left( \frac{I_t}{I_{t-1}} \right) \right] I_t
\]

(2.33)

Capital producers are subject to quadratic investment adjustment costs which is specified as \(S \left( \frac{I_t}{I_{t-1}} \right)\), with steady state = 0, \(S' > 0\), \(I_t\) is investment, and \(\delta\) is depreciated rate of capital. \(\varepsilon_t\) denotes the random investment shock following AR(1) Progress, specified as: \(ln \varepsilon_t = \rho_{\varepsilon} \varepsilon_{t-1} + \eta_t, \eta_t \sim N(0, \sigma_t)\). The objective function is profit function of capital producer:

\[
Max E_t \sum_{t=0}^{\infty} \beta^t [P_t K_t - P_t^k (1 - \delta) K_{t-1} - I_t]
\]

(2.34)

Then through the first order condition with respect to \(I_t\), we generate the investment Euler equation:

\[
1 = \varepsilon_t P_t^k (1 - S \left( \frac{I_t}{I_{t-1}} \right) - S' \left( \frac{I_t}{I_{t-1}} \right) \frac{I_t}{I_{t-1}}) \frac{I_t}{I_{t-1}})
\]

\[
- \beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} P_t + 1^{k} \varepsilon_t^{k+1} S' \left( \frac{I_t}{I_{t-1}} \right) \left( \frac{I_t}{I_{t-1}} \right)^{2} \right]
\]

(2.35)

2.6 External finance premium

Due to the existence of asymmetric information between borrowers and lenders, there is a cost associated with external finance premium charged by banks for credit, which affects the costs of intermediate goods producers, the group of entrepreneurs borrowing to buy capital in excess of their net worth. Specifically, there exists an equation for the external finance premium that is charged to these intermediate goods producers. In each period, they purchase capital \(k_t\) from a capital producer at a price \(P^k_t\). In the subsequent period, they can resell the depreciated capital back to the capital producer at a price \(P_{t+1}^k\). Similar to the assumptions outlined in BGG (2009), \(P(i)_t\) represents the relative price of intermediate goods, while \(\frac{\lambda Y_{t+1}}{K_{t+1}}\) is the marginal product of capital. Under these conditions, the expected rate of return of capital for the entrepreneur can
be expressed as:

\[ E_t \left[ R^k_{t+1} \right] = E_t \left[ \frac{P(i)_{t+1} \alpha_{Yt+1} + P^k_{t+1}(1 - \delta)}{P^k_t} \right] \]  

(2.36)

The expression \( P(i)_{t+1} \alpha_{Yt+1} + P^k_{t+1}(1 - \delta) \) denotes the value of the marginal product of capital, which is also referred to as the rental rate of capital. However, according to BGG (2009), default risk resulting from the information asymmetry between entrepreneurs and financial intermediaries can increase the cost of external finance relative to internal funds. From the perspective of a financial intermediary, if an entrepreneur defaults, they will be required to pay the auditing cost and keep whatever they find. Conversely, if the entrepreneur is able to fully repay the loan, the intermediary will receive a return. For the intermediary, the return must not be less than the opportunity cost, which can be represented as \( R_t B_t \), where \( B_t \) is the amount of borrowing by the entrepreneur, and can be calculated as \( (Q_t K_{t+1} - N_t) \).

With the above state-contingent constraints from a financial intermediary, the entrepreneur will maximize the profit by choosing the optimal amount of capital. Referring to the BGG (2009), the optimal capital purchases should be proportional to the net worth and determined by the expected discounted rate of return of capital

\[ s_t = E \{ R_k \} \]

(2.37)

Equivalently, we can re-write the above equation as:

\[ E[R^k_{t+1}] = \varepsilon^{pr}_t s(\cdot) \left( \frac{N_{t+1}}{P^k_{t+1} K_{t+1}} \right) R_{t+1}, s(\cdot) < 0 \]

(2.38)

The function \( s(\cdot) \) represents the cost of external finance, which is dependent on the leverage ratio. The equation above states that, in equilibrium, the discounted rate of return to capital for each entrepreneur who is not self-financed must be equal to the external finance premium. The variable \( \varepsilon^{pr}_t \) represents the finance premium shock, which follows an AR(1) process given by \( \ln \varepsilon^*_t = \rho \varepsilon^*_t-1 + \eta^*_t, \eta^*_t \sim N(0, \sigma) \). This exogenous premium shock can be viewed as a shock to the supply of credit or as a shock that can modify the premium. Equation 2.38 captures the idea that, for the partly self-financed entrepreneur, the return on their capital should be equivalent to the marginal cost of external finance. Additionally, the finance premium is inversely related to the net worth-to-investment ratio. The difference between the gross return on capital and the external finance cost reflects the equity of the entrepreneur who has survived from the previous period.

\[ R^k_t P^k_{t-1} K_t - E_{t-1}[R^k_{t+1} (P^k_{t-1} K_t - NW_{t-1})] \]

(2.39)

The evolution of net worth can be rewritten as:

\[ NW_{t+1} = \varepsilon^n_{t} \theta V_t = \varepsilon^n_{t} \theta [R^k_{t+1} P^k_{t-1} K_t - E_{t-1}[R^k_{t+1} (P^k_{t-1} K_t - NW_{t-1})]] \]

(2.40)

We assume that entrepreneurs who exit the market will consume their equity. Therefore, we define entrepreneur consumption as:

\[ C^n_t = (1 - \theta) V_t \]

(2.41)
Extension with QE and ZLB

The big financial crisis resulted in a liquidity crisis rather than an insolvency problem, caused by a sharp decline in demand and cash flow. In the aftermath of the crisis, financial intermediaries became more risk-averse and reluctant to lend to firms due to the higher risk of bankruptcy. According to Le et al. (2016), money is the cheapest form of collateral due to its lack of disposal costs. Thus, quantitative easing (QE) increased liquidity in the private sector by providing collateral that lowered default risk, thereby reducing the external finance premium required by financial intermediaries. Importantly, to mitigate the risk of bankruptcy, financial intermediaries are required to hold counterpart funds for the assets on their balance sheets. Additionally, macro-prudential measures based on Basel Agreements 1 and 2 are treated as exogenous $I(1)$ time-series process shocks that affect the cost of credit, according to Le et al. (2016).

Table 1 describes how QE will affect each sector’s balance sheet. The household will receive deposits from bond sales, and uses this to buy non-monetary collateral (such as property) from firms, who place the resulting deposits in the banks where they act as collateral. The banks then make loans to entrepreneurs, helped by this collateral. With the cheaper collateral, the external finance premium required by the bank will fall, which leads to higher investment and higher household savings (the value of the firms’ net worth) in capital market equilibrium. Notice that the firm sector by creating capital also creates savings deposits, so that there is a bank deposit credit-multiplier process.

### Table 1: Balance sheets of each sector in the economy

<table>
<thead>
<tr>
<th>Asset</th>
<th>Liability</th>
<th>Asset</th>
<th>Liability</th>
<th>Asset</th>
<th>Liability</th>
<th>Asset</th>
<th>Liability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coll$_{NonM}$(-)</td>
<td>Net worth</td>
<td>Credit(+)</td>
<td>Deposit(+)</td>
<td>Deposit(+)</td>
<td>Savings(+)</td>
<td>Borrowing</td>
<td>GB (-)</td>
</tr>
<tr>
<td>Coll$_M$(+)</td>
<td>Credit(+)</td>
<td>M0(+)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>M0 (+)</td>
</tr>
<tr>
<td>K(+</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Resources from Le et al (2014). Coll$_{NonM}$ is the collateral in non-monetary form; Coll$_M$ is the collateral in monetary form; GB is the government bonds. And + and - are used to describe how the balances change with the quantitative easing.

Our model incorporates occasional zero lower bound (ZLB) constraints, allowing us to consider two states for the economy: a normal scenario where the Bank of England sets interest rates using the Taylor rule, and a "crisis model" where quantitative easing (QE) is implemented, and interest rates are fixed at the ZLB (0.025% quarterly). We introduce two new variables: $\xi$ and $m_t$, which are associated with QE. $\psi$ measures how M0 impacts the premium, and given a specific level of leverage $(k-n)$, a firm facing bankruptcy can cover more with a lower bankruptcy threshold and return rate on assets. $\xi_t$ is a macro-prudential instrument that we incorporate into the error term, as Le et al. (2016) note. We consider $\xi_t$ as the increase in costs for banks resulting from regulatory constraints, such as holding surplus equity. As a result, the credit premium equation in log-linearized form becomes:

$$E_{t+1}^k - (r_t - \pi_{t+1}) = \chi(qq_t + k_t - n_t) - \psi m_t + \xi_t + epr_t \quad (2.42)$$

The supply function of M0 differs across the two states of the economy. In normal times, M0 supports the supply of money (whose log is $m_t$) via the discount window; in the ZLB crisis model, M0 targets the credit premium, aiming to bring it back to normal. These functions are as follows:

$$\text{When } r_t > 0.0625\%, \quad m_{0t} = m_{0t-1} + \psi_1(m_t - m_{t-1}) + e_t^{m0}$$
When $r_t = 0.0625\%, m_{0t} = m_{0t-1} + \psi_2(cy_t - cy^*) + \epsilon_{t, z_{lb}}$;
where $\psi_1, \psi_2 \in (0,1)$ are used to measure the elasticity of response. The money supply shocks follow AR(1) processes.

Following Le et al. (2016), the supply of money is assumed to be equal to the (deposits(=credit) + M0)\(^4\). Then use firms’ account balance\(^5\) to express the credit with (K + COLL − NW). Therefore, total money supply $M_t$ is defined as:

$$M_t = \text{Deposits}_t + \text{M0}_t = \text{Credit}_t + \text{M0}_t = (K_t + \text{COLL}_t - \text{NW}_t) + \text{M0}_t$$

where Deposits\(_t\) and Credit\(_t\) represent the level of deposits and credit at time \(t\), respectively, and \(\text{M0}_t\) represents the level of monetary base at time \(t\). \(K_t\), \(\text{COLL}_t\), and \(\text{NW}_t\) represent the levels of capital stock, non-monetary collateral, and net worth, respectively, at time \(t\).

$$M = K + \text{COLL} − \text{NW} + \text{M0} \quad (2.43)$$

In the log-linearized form equation is written as:

$$M_t = (1 + v - c - \mu)K_t + \mu m_{0t} - \nu n_t \quad (2.44)$$

Notably, the collateral is treated as a fixed proportion of the money. \(\mu\), \(\nu\), and \(c\) are the steady state ratios of net worth to money, \(\text{M0}\) to money and collateral to money, respectively.

### 2.7 Monetary and fiscal policy

In this section, we introduce monetary policy during normal times, in which the central bank follows a Taylor rule. The nominal interest rate reflects deviations of output and inflation from their targeted values.

$$r_t = \rho r_{t-1} + (1 - \rho)(r_p \pi_t + r_y y_t) + r_{\delta y}(y_t - y_{t-1}) + e r_t \quad (2.45)$$

In this equation, parameter \(r_p\) measures the response of inflation to changes in the nominal interest rate, while \(r_y\) and \(r_{\delta y}\) determine the response of output and changes in output, respectively. The parameter \(\rho\) represents the degree of interest rate smoothing. The exogenous monetary policy shock, denoted by \(er_t\), follows an AR(1) process. Additionally, we consider an exogenous spending process \(G_t\) financed by lump-sum taxes, resulting in the government budget constraint.

$$P_t G_t + R_{t-1} B_t = T_t + B_{t+1} \quad (2.46)$$

---

\(^4\)Since M2 data captures bank money such as deposit and we assume the deposit move one to one with the bond purchases, the QE’s effect would also be detected by the M2

\(^5\)The balance sheet for the firm will be presented in the next section with the balance sheets of other sectors
2.8 Market clearing condition

The overall resource constraint on the whole economy can be integrated by combining domestic budget constraints with the evolution of net foreign assets:

\[ Y_t = C_t + I_t + a(Z_t)K_{t-1} + C^e_t + EX_t - IM_t + \varepsilon^g_t \] (2.47)

where \( \varepsilon^g_t \) is government spending shock and follow AR(1) process, \( \ln \varepsilon^g_t = \rho_g \ln \varepsilon^g_{t-1} + \eta^g_t \).

The full model is listed in Appendix part A.

3 Indirect Inference

We use the method of Indirect Inference, which was first proposed by Smith (1990) and then extended by Gourieroux et al. (1993), Gourieroux and Monfort (1995) and Canova (2007). Indirect inference uses an auxiliary model as a descriptor of the data, which is entirely independent of the theoretical model. The target of the method is to find a set of parameters which makes the behaviour of the auxiliary model from simulated data closest to the one based on the actual data; this amounts to minimising the model Wald statistic which reflects the probability of the data-based auxiliary model. The indirect inference method applied in this work is that proposed by Meenagh et al. (2009) and refined by Le et al. (2012) with Monte Carlo experiments. They compared the power of the indirect inference test with that from Maximum Likelihood and found that the power of indirect inference is much higher in small samples.

3.1 Indirect Inference estimation

The model has two sets of parameters: steady-state values, such as the investment-output ratio and the capital-output ratio, which are fixed based on observable data; and parameters related to agents’ behavior, which are estimated. We use the Simulated Annealing (SA) algorithm to find the optimal values of the latter group of parameters. SA starts from an initial point and searches the parameter space by considering neighboring states and choosing to move the system to other states or stay in the current state.

In this paper we are using the unfiltered non-stationary data from 1986 to 2016. Due to the non-stationarity of the data, we use a VECM as the auxiliary model descriptor of data. In practice this is a VARX(1) with a deterministic trend and the non-stationary productivity residual as an exogenous variable, which equates to a cointegrating relationship.

Table 2 shows the estimation results, which indicate that the Wald statistic of 41.67 for a VAR model of the three central variables, output, interest rate, and inflation \((y, r, \pi)\), is not rejected, with the maximum p-value at 0.253 (>0.05). This result suggests that the model passes the Wald test with ease. In addition, Table 4 demonstrates that the individual VARX (1) parameters are within the 95% bounds based on simulated data, indicating a good fit of the model to the data.

---

\( ^6 \)We used the ADF and KPSS tests to check shock stationarity. All shocks, except for productivity, were stationary or trend stationary. As productivity shock was integrated of order one, we modeled it as an ARMR(1,1,0) after differencing.
Table 2: Structural Parameter Estimates and test results

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbols</th>
<th>Calibration</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Household sector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Discount factor</td>
<td>$\beta$</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Elasticity of consumption</td>
<td>$\sigma_c$</td>
<td>1.39</td>
<td>1.26</td>
</tr>
<tr>
<td>Elasticity of labor supply</td>
<td>$\sigma_l$</td>
<td>2.83</td>
<td>2.70</td>
</tr>
<tr>
<td>External habit formation</td>
<td>$h$</td>
<td>0.70</td>
<td>0.79</td>
</tr>
<tr>
<td>Degree of wage stickiness</td>
<td>$\xi_w$</td>
<td>0.70</td>
<td>0.83</td>
</tr>
<tr>
<td>Degree of Wage indexation</td>
<td>$l_{w}$</td>
<td>0.58</td>
<td>0.68</td>
</tr>
<tr>
<td>Proportion of sticky wage</td>
<td>$w^w$</td>
<td>0.10</td>
<td>0.08</td>
</tr>
<tr>
<td>Preference bias in consumption</td>
<td>$\omega$</td>
<td>0.70</td>
<td>0.70</td>
</tr>
<tr>
<td>Firm sector</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Degree of price stickness</td>
<td>$\xi_p$</td>
<td>0.67</td>
<td>0.85</td>
</tr>
<tr>
<td>Degree of price indexation</td>
<td>$l_p$</td>
<td>0.43</td>
<td>0.26</td>
</tr>
<tr>
<td>Proportion of sticky price</td>
<td>$w^p$</td>
<td>0.40</td>
<td>0.38</td>
</tr>
<tr>
<td>Entrepreneur Survival rate</td>
<td>$\theta$</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>Capital share in production</td>
<td>$\alpha$</td>
<td>0.30</td>
<td>0.15</td>
</tr>
<tr>
<td>Capital depreciation rate</td>
<td>$\delta$</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>Fixed cost in production</td>
<td>$\phi$</td>
<td>1.50</td>
<td>1.54</td>
</tr>
<tr>
<td>Elasticity of capital adjustment</td>
<td>$\varphi$</td>
<td>5.74</td>
<td>8.02</td>
</tr>
<tr>
<td>Elasticity of capital utilisation</td>
<td>$\psi$</td>
<td>0.05</td>
<td>0.13</td>
</tr>
<tr>
<td>Monetary policy</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taylor rule response to inflation</td>
<td>$r_p$</td>
<td>2.30</td>
<td>2.55</td>
</tr>
<tr>
<td>Interest rate smoothing</td>
<td>$\rho$</td>
<td>0.74</td>
<td>0.63</td>
</tr>
<tr>
<td>Taylor rule response to output</td>
<td>$r_y$</td>
<td>0.03</td>
<td>0.02</td>
</tr>
<tr>
<td>Taylor rule response to output change</td>
<td>$r_{sy}$</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>M0 response to M2</td>
<td>$\psi_1$</td>
<td>0.05</td>
<td>0.01</td>
</tr>
<tr>
<td>Money response to credit growth</td>
<td>$\psi_2$</td>
<td>0.04</td>
<td>0.13</td>
</tr>
<tr>
<td>Financial friction</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elasticity of premium to leverage</td>
<td>$\chi$</td>
<td>0.04</td>
<td>0.06</td>
</tr>
<tr>
<td>Elasticity of premium to money</td>
<td>$\psi$</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td><strong>Test results</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wald value</td>
<td>41.674</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P-value</td>
<td>0.253</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3: Statistical properties of shock

<table>
<thead>
<tr>
<th>Description</th>
<th>Symbols</th>
<th>AR(1) coef</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government spending shock</td>
<td>$\rho_g$</td>
<td>0.8034</td>
</tr>
<tr>
<td>Preference shock</td>
<td>$\rho_c$</td>
<td>0.4526</td>
</tr>
<tr>
<td>Investment shock</td>
<td>$\rho_I$</td>
<td>0.5198</td>
</tr>
<tr>
<td>Taylor rule shock</td>
<td>$\rho_r$</td>
<td>0.1877</td>
</tr>
<tr>
<td>Productivity shock</td>
<td>$\rho_p$</td>
<td>0.6521</td>
</tr>
<tr>
<td>Price mark-up shock</td>
<td>$\rho_p$</td>
<td>0.6008</td>
</tr>
<tr>
<td>NK wage mark-up shock</td>
<td>$\rho_{wnk}$</td>
<td>0.4533</td>
</tr>
<tr>
<td>NC wage mark-up shock</td>
<td>$\rho_{wnk}$</td>
<td>0.7689</td>
</tr>
<tr>
<td>Risk premium shock</td>
<td>$\rho_{pm}$</td>
<td>0.2390</td>
</tr>
<tr>
<td>Net worth shock</td>
<td>$\rho_n$</td>
<td>3.4532</td>
</tr>
<tr>
<td>Quantitative easing shock</td>
<td>$\rho_{q}$</td>
<td>0.0912</td>
</tr>
<tr>
<td>Export demand shock</td>
<td>$\rho_c\times$</td>
<td>1.2476</td>
</tr>
<tr>
<td>Import demand shock</td>
<td>$\rho_{im}$</td>
<td>1.9351</td>
</tr>
<tr>
<td>Foreign interest rate shock</td>
<td>$\rho_{f}$</td>
<td>0.4786</td>
</tr>
<tr>
<td>Foreign consumption shock</td>
<td>$\rho_{f}$</td>
<td>0.6139</td>
</tr>
</tbody>
</table>
Table 4: VECM parameters and Bootstrap Bounds for $y \pi r$ with estimated parameter

<table>
<thead>
<tr>
<th>$y \pi r$</th>
<th>actual VAR coefs</th>
<th>lower bound</th>
<th>upper bound</th>
<th>In/Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{yy}$</td>
<td>0.9463</td>
<td>0.1568</td>
<td>1.3762</td>
<td>In</td>
</tr>
<tr>
<td>$b_{y\pi}$</td>
<td>-0.0231</td>
<td>-1.3833</td>
<td>2.320</td>
<td>In</td>
</tr>
<tr>
<td>$b_{yr}$</td>
<td>-0.3387</td>
<td>-1.219</td>
<td>1.087</td>
<td>In</td>
</tr>
<tr>
<td>$b_{\pi\pi}$</td>
<td>0.3129</td>
<td>0.1335</td>
<td>0.4382</td>
<td>In</td>
</tr>
<tr>
<td>$b_{\pi y}$</td>
<td>0.0712</td>
<td>-0.2248</td>
<td>0.2850</td>
<td>In</td>
</tr>
<tr>
<td>$b_{\pi r}$</td>
<td>0.029</td>
<td>-0.008</td>
<td>0.1733</td>
<td>In</td>
</tr>
<tr>
<td>$b_{rr}$</td>
<td>0.8810</td>
<td>0.4195</td>
<td>0.9332</td>
<td>In</td>
</tr>
<tr>
<td>$b_{ry}$</td>
<td>0.0427</td>
<td>-0.011</td>
<td>0.1782</td>
<td>In</td>
</tr>
<tr>
<td>$b_{r\pi}$</td>
<td>0.0210</td>
<td>-1.763</td>
<td>0.5482</td>
<td>In</td>
</tr>
</tbody>
</table>

3.2 Robustness check

In order to assess the robustness of our estimation procedure, we conducted a Monte Carlo experiment. We assumed that the estimated model was the true model and then created a false model by altering the estimated parameters by a certain percentage. Specifically, we increased or decreased the estimated parameters by $+x\%$ or $-x\%$ and then tested whether the resulting model was still consistent with the data.

Table 5 shows the rejection rate for different levels of parameter falseness, ranging from 1% to 10%. The rejection rate increases sharply with increasing falseness, indicating that our test has strong power. For example, when we falsified the model by 10%, the model was 100% rejected, meaning that our estimated coefficients cannot deviate from the true coefficients by more than 10% for the model to be valid.

Table 5: Monte Carlo Power test

<table>
<thead>
<tr>
<th>Parameter Falseness</th>
<th>True</th>
<th>1%</th>
<th>5%</th>
<th>7%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rejection rate</td>
<td>5%</td>
<td>12.1%</td>
<td>55.86%</td>
<td>78.4 %</td>
<td>100 %</td>
</tr>
</tbody>
</table>

3.3 The Variance decomposition of shocks

Table 6 provides insights into the various factors that impact key variables’ variation. Productivity is the dominant contributor to output variation, accounting for over a third of the variation. Among financial shocks, the external finance premium explains approximately 10% of the variation. Export and import demand shocks, as foreign shocks, each contribute around 5% to the variation. Monetary policy exerts a total impact of 10%, which is evenly split between the Taylor rule and QE shocks. The influence of other shocks on output variation is relatively minor. The patterns of consumption variation are similar to those of output variation.

Regarding financial variables, interest rate movements are mainly driven by shocks to productivity, monetary policy, and the bank premium. These same shocks are the primary drivers of exchange rate variation as well. Additionally, trade shocks significantly impact exchange rates.
Table 6: Variance decomposition of shocks: 2006Q1 to 2016Q4

<table>
<thead>
<tr>
<th>Shocks</th>
<th>Exchange rate</th>
<th>Output</th>
<th>Consumption</th>
<th>Interest rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>government spending shock</td>
<td>3.432334</td>
<td>5.008639</td>
<td>1.3332652</td>
<td>2.1071515</td>
</tr>
<tr>
<td>preferences shock</td>
<td>1.042222</td>
<td>1.108837</td>
<td>12.910514</td>
<td>0.0016258</td>
</tr>
<tr>
<td>Investment shock</td>
<td>3.809868</td>
<td>8.071381</td>
<td>2.0001068</td>
<td>4.0051501</td>
</tr>
<tr>
<td>Monetary policy shock</td>
<td>4.67901</td>
<td>5.812723</td>
<td>4.971046</td>
<td>35.217756</td>
</tr>
<tr>
<td>Productivity shock</td>
<td>32.98789</td>
<td>36.37000</td>
<td>22.009242</td>
<td>19.161725</td>
</tr>
<tr>
<td>Price mark-up shock</td>
<td>4.876478</td>
<td>10.862621</td>
<td>9.547019</td>
<td>5.5119655</td>
</tr>
<tr>
<td>Wage mark up</td>
<td>1.46E-05</td>
<td>0.000024</td>
<td>2.763E-05</td>
<td>4.439E-07</td>
</tr>
<tr>
<td>Labour supply</td>
<td>4.013433</td>
<td>5.033975</td>
<td>2.0003234</td>
<td>0.0061056</td>
</tr>
<tr>
<td>Premium shock</td>
<td>6.876755</td>
<td>9.674527</td>
<td>22.0093845</td>
<td>15.003631</td>
</tr>
<tr>
<td>Net worth shock</td>
<td>3.798575</td>
<td>1.085220</td>
<td>10.000833</td>
<td>3.0006949</td>
</tr>
<tr>
<td>Quantitative easing shock</td>
<td>0.876487</td>
<td>5.000011</td>
<td>4.3610951</td>
<td>9.6402881</td>
</tr>
<tr>
<td>Import shock</td>
<td>18.70903</td>
<td>5.000301</td>
<td>4.4197725</td>
<td>2.1404486</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

3.4 Dynamic response from the model

- Taylor rule shock

Figure 1 depicts the IRFs to a positive Taylor rule shock, which increases the nominal interest rate of the baseline model. Notably, when the economy gets into the ZLB crisis, the Taylor rule will be suspended, so the monetary policy shock only applies to the model without a ZLB crisis. A standard Taylor rule transmission mechanism suggests that a monetary policy contraction usually discourages borrowing, investment, and consumption and reduces output. Then the downward pressures on the demand-side are gradually fed through the changes in the output. Meanwhile, the output gap can lead to a lower inflation level. The demand for labour also falls with reduced aggregate demand in the labour market. In terms of the financial sector, the falls in the capital price lower the net worth of the entrepreneur. Consequently, the external finance premium is pushed up, then works counter-cyclically with an amplified impact that further reduces lending and investment.

In the foreign sector, deflation and higher nominal interest rate appreciate the British pound with a higher real interest rate, which also reduces the real exchange rate, then makes exports less competitive with a higher demand of imports. Net foreign bond position decreases overall and gets back after around ten quarters.
- **Quantitative easing shock**

Figure 2 illustrates the impulse responses to a QE shock obtained from two models - one depicted by the red dotted line, which represents an economy constrained by the zero lower bound (ZLB), and the other depicted by the blue solid line, which represents the model without ZLB. These impulse responses provide valuable insights into the impact of QE on a ZLB-constrained economy. Our analysis reveals that QE, which involves injecting money through significant asset purchases, mitigates default risk and external finance premiums, resulting in a positive impact on demand-side variables. We find that, compared to the model without ZLB constraints, consumption responses are stronger, and inflation is bolstered, leading to a rapid reduction in the real interest rate. The lower real interest rate subsequently triggers a depreciation of the British Pound in the foreign sector, leading to a more competitive export market. Additionally, we observe that import
responses are higher compared to the baseline environment. This is because the traditional monetary policy rule does not have a contractionary effect on the increased money supply.

Our results suggest that monetary policy encompasses more than just adjusting policy rates. These findings are consistent with previous studies, such as Le et al. (2016), which conclude that QE can play a pivotal role in reviving an economy during a recession, including the ZLB crisis.

Figure 2: Quantitative easing shock to both Non-crisis and Crisis models
• Fiscal policy shock

Figure 3 illustrates the impact of an expansionary fiscal policy, which is captured through a fiscal multiplier. Our results show that higher government spending leads to higher aggregate demand, pushing up inflation and the price level. The nominal interest rate also rises via the response of the Taylor rule. The increase in government spending also leads to a direct increase in output, which in turn induces higher expectations for income among consumers, resulting in higher consumption. In the labour market, firms provide higher wages to attract labour, leading to an increase in both investment and capital price for the accelerated production process. The higher value of capital indicates that the net worth on entrepreneur balance also increases, resulting in a lower external finance premium required by the bank. Additionally, the counter-cyclical effect of premium further increases the net worth of the entrepreneur, leading to more lending and investment.

In the foreign sector, the increase in domestic demand leads to an increase in imports to satisfy this demand, which could result in an appreciation of the real exchange rate (a decrease in $Q$) and an appreciation of the British pound. However, this weakens the competitiveness of domestic goods in the foreign market, leading to a drop in export, and consequently, a decrease in the accumulated net foreign asset.

When the model is constrained by the ZLB, we observe a slightly higher response from the demand side, including consumption, capital, and investment, due to a lack of contraction from the monetary policy, which could give downward pressure on the demand side. In the foreign sector, there is a devaluation of the domestic currency due to a lower real interest rate, making exports more competitive. Import is also slightly increased to satisfy excess domestic demands. Finally, we observe an increase in the accumulated net foreign asset.

3.5 Causes of the crisis

Using our sample analysis, we explore further what a crisis is, what causes it, and what it means. A crisis is defined as an interruption of output for at least three years, while a financial crisis constitutes a crisis with a binding ZLB on nominal interest rates. In practice, we bootstrap the shocks based on UK observations from 1985Q1 to 2007Q4 to develop a sample of "standard shock scenarios", during which there are no major financial shocks. In parallel, shock samples for 1985Q1 through 2016Q4 are gathered as "crisis-inclusive scenarios". We simulate the model with the two shock sets and compare their effects. Based on the bootstrap simulation examples in figure 6 and 7, the following results were found.

1 Shocks without financial shocks do cause real crisis but not financial crisis. Crisis is a normal part of the UK economy. Under the "standard shock scenarios", without the financial shocks, there are several significant drops in output while the ZLB was not hit, as shown in figure 4.

2 Figure 5 shows that financial crisis and big recession were more likely to be caused when financial shocks are included. The Great Recession was therefore a crisis and a financial crisis triggered by both real and financial shocks.

3 Financial shocks alone are not sufficient to create a significant economic crisis. In our bootstrap scenarios using only financial shocks, there was no economic crisis.
4 Policy Implications

The choice of an optimal monetary target has been a longstanding issue in monetary economics, but empirical evidence on the matter is scarce. Two alternatives that have received significant attention are price-level targeting (PLT) and nominal GDP targeting (NGDPT). In the next section, we explore the implications of the UK model that we have estimated for the potential effects of transitioning to PLT or NGDPT as a monetary rule.

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Sweden experimented with price-level targeting in the 1930s for a brief period of around two years, but this did not yield much insight due to the short duration of adoption. For more details, see Jonung (1979) and Berg and Jonung (1999)
4.1 Inflation targeting implications of the model

Inflation targeting is a traditional monetary policy where the central bank sets a specific inflation rate as its target and stabilizes the economy by keeping inflation anchored to its long-run target. It was introduced in the 1990s to help reduce inflation expectations and avoid high inflation. Inflation targeting has been widely employed by developed countries, such as the US, UK, and EU countries, since then. It is the monetary rule
estimated in our model as:

\[ r_t = \rho r_{t-1} + (1 - \rho) (r_p \pi_t + r_y y_t) + r_\delta (y_t - y_{t-1}) + e_r \tag{4.1} \]

where \( \rho \) measures the interest rate smoothing, and \((1 - \rho)\) captures the short-run feedback from inflation and output gap.

The global crisis in 2008 exposed critical flaws in inflation targeting and fuelled recent interest in optimal monetary policy. In the aftermath of the financial crisis, when interest rates hit the ZLB, so reducing monetary policy potency, some researchers have proposed setting a higher inflation target under inflation-targeted rules in an attempt to solve the ZLB crisis— for example, Krugman (1998), Woodford and Eggertsson (2003), and Bernanke (2017). Alternatively, Hatcher and Minford (2014) suggest that price-level targeting could be a valuable mechanism for helping the economy recover from deflationary shocks.

To compare the ability of each monetary regime to stabilise the economy and avoid financial crisis, we perform bootstrap simulations with a large sample size to measure the frequency of crises and welfare costs of different regimes. As shown in subsection 4.3, under the baseline inflation targeting regime, there would be 99.9 crises per 1000 years, and the welfare cost for "Output" and "Inflation" are 2.84 and 0.025, respectively.

### 4.2 Alternative monetary policy

- **Price-level targeting**

  Price-level targeting (PLT) is a monetary policy strategy in which the central bank sets a specific path for the price level and commits to correcting deviations from that path within a given period. Unlike inflation targeting, which aims to stabilize inflation, PLT provides more guidance to the economy. As noted by Svensson (1999), using PLT can help solve the time-inconsistency problem. With rational expectations, PLT can lead to lower inflation and output variability, essentially providing a free lunch.

  PLT has two key advantages over inflation targeting:

  First, under inflation targeting, past deviations from the target are effectively ignored, whereas PLT takes history into account and corrects past deviations. This approach affects the expectations of the forward-looking public, making PLT a more effective tool for maintaining price stability. For instance, if inflation unexpectedly rises from 2% to 3%, inflation targeting would simply allow the deviation to continue, gradually approaching the target. In contrast, PLT would require a below-average inflation rate and maintain a specific price-level path, promoting stability over time. Although PLT may result in longer reactions to inflation deviations, it ensures price and inflation stability in the long run, making it a valuable monetary policy tool.

  Secondly, when the nominal interest rate is constrained by the zero lower bound, an unexpected change in aggregate demand can cause real interest rate to rise under an inflation targeting scheme. This can result in a decrease in inflation expectations and an increase in recession risk. However, under price-level targeting with an inflation target of 2%, people expect inflation to exceed 2% since past deviations are not ignored, and the central bank can make up for any shortfall. This expectation stimulates aggregate demand and increases the price level. Eggertsson and Woodford (2003) confirmed this intuition, finding that price-level targeting reduces welfare losses during financial and zero-bound crisis periods, as compared to inflation targeting in New Keynesian models. Additionally, Coibion et al. (2012) found that price-level targeting
can reduce the frequency and severity of zero-bound episodes. Other studies, such as Coletti and Woodford (1999), Dittmar, Gavin, and Kydland (1999), and Dittmar and Gavin (2000), provide further evidence in support of price-level targeting.

To perform the empirical study, we specify the price-level targeting as follows:

\[ r_t = \rho_1 r_{t-1} + (1 - \rho_1) \{ \rho_p (p_t - p^*) + \rho_y (y_t - y^*) \\
+ \rho_\delta y \{(y_t - y^*) + (y_{t-1} - y^*)\}\} + \epsilon r_t \] (4.2)

where the steady-state of price level \( p^* \) is assumed constant and normalized to 0, practically, we choose the average value of output from actual data as the steady-state value of output as \( y^* \). \( \rho_1 \) is the interest rate smoothing rate, and \( \rho_p \) is the value of Taylor rule response to price level, and \( \rho_y \) and \( \rho_\delta y \) are Taylor rule response to output and output change respectively. We estimate the parameters in the above equation by minimizing the crisis times. We search for the values which can allow the model to stabilize the economy most by simulation:

\[ r_t = 0.545 * r_{t-1} + (1 - 0.545) \{ 1.745 * (p_t - p^*) + 0.02 * (y_t - y^*) \\
+ 0.03 * [(y_t - y_{t-1})] + \epsilon r_t \] (4.3)

From table 8, with a single price-level targeting adopted, there has been a significant decrease in the frequency of economic and financial crises compared with the results generated by inflation targeting. Within the expected 1000 years, the frequency of both crises comes down to 87. The total welfare cost drops from 2.87 to 0.724 with a significant contribution from output variance, which drops down to 0.698.

- **Nominal GDP targeting**

This section will discuss another desirable strategy for monetary policy, nominal GDP targeting or nominal income targeting, which strives to get a certain level of nominal GDP growth. The most attractive feature of the nominal GDP targeting is closely related to output and prices, which are the variables the central bank cares about most. Frankel (2012) concluded that the central bank under nominal GDP makes decisions regarding the importance of inflation and real output rather than the breakdown between the two.

Additionally, superior to inflation targeting, it can respond effectively to demand and supply shock. For example, facing a negative supply shock, there will be a decline in output and a rise in inflation. Under inflation targeting, the central bank would choose to carry out the contractionary monetary policy to maintain a lower inflation rate, but at the cost of further exacerbating the recession. In contrast, nominal GDP targeting can avoid a worse situation by an expansionary monetary policy and return the nominal GDP to target. Though the inflation rate will be temporarily above the potential, it can decrease unemployment by letting inflation rise, particularly during the recession.

The NGDP targeting can avoid default and create more financial stability on the front of financial friction. Koenig (2013) and Sheedy (2014) remarked that if the aggregate income can keep close to the steady growth path by nominal GDP targeting, it would not fall as much during the recession, allowing people to repay their loans, then avoid default and bankruptcy.

Then we bootstrap our model with nominal GDP targeting, and the rule is defined as follows.
\[ r_t = \rho_1 r_{t-1} + \rho_y (y_t + p_t - \bar{y} - \bar{p}) + \epsilon r_t \]  

(4.4)

where \( y_t + p_t - \bar{y} - \bar{p} \) indicates the deviation of the nominal GDP from targeted value. \( \bar{p} = 0 \) and \( \bar{y} \) follow the real output data. \( \rho_y \) is treated as the partial elasticity of interest rate responding to the nominal GDP deviation. The parameters that minimized crisis were:

\[ r_t = 0.625 \times r_{t-1} + 2.21 \times (y_t + p_t - 8.71) + \epsilon r_t \]  

(4.5)

Nominal GDP targeting has the potential to significantly reduce economic volatility. Table 7 presents the results of simulations, which show that the financial crisis can be reduced from 87 to 66 when compared to the price-level targeting case, indicating that nominal GDP targeting is more effective in lowering zero-bound episodes than the other two rules. In terms of welfare cost, the output variance is the lowest at 0.690, and the inflation variance is similar to that of the inflation-targeting rule at 0.025. Overall, the simulated results suggest that nominal GDP targeting and price-level targeting are more effective than the traditional inflation-targeting rule in stabilizing the economy.

4.3 Combining the monetary rules with the use of QE in the ZLB- "Monetary reform"

We also consider these different regimes in combination with the use of QE when the ZLB is triggered, which we term "monetary reform". Table 8 brings all these results together. It shows that crises are reduced further under either combination than the single rule adopted; NGDPT+Monetary reform outperforms the other two types of monetary regimes in the standard welfare cost measure with the lowest value of 1.147.

Table 7: Frequency of Crisis under Different Types of Monetary Regimes

<table>
<thead>
<tr>
<th>Economic Crisis</th>
<th>Inflation targeting</th>
<th>Inflation targeting+Monetary reform</th>
<th>Price level targeting</th>
<th>NGDPT</th>
<th>PLT+Monetary reform</th>
<th>NGDPT+Monetary reform</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration between two crisis</td>
<td>10.01</td>
<td>10.84</td>
<td>11.41</td>
<td>10.58</td>
<td>18.11</td>
<td>17.24</td>
</tr>
<tr>
<td>Frequency of crisis (expected economic crisis per 1000 years)</td>
<td>99.90</td>
<td>91.91</td>
<td>87.64</td>
<td>94.51</td>
<td>55.21</td>
<td>58.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Financial Crisis</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration between two crises</td>
<td>10.12</td>
<td>10.98</td>
<td>11.42</td>
<td>14.94</td>
<td>19.97</td>
<td>20.04</td>
</tr>
<tr>
<td>Frequency of crisis (expected economic crisis per 1000 years)</td>
<td>98.78</td>
<td>91.07</td>
<td>87.62</td>
<td>66.93</td>
<td>50.08</td>
<td>49.90</td>
</tr>
<tr>
<td>Welfare cost</td>
<td>Var(Output)</td>
<td>2.840</td>
<td>1.668</td>
<td>0.698</td>
<td>0.690</td>
<td>0.683</td>
</tr>
<tr>
<td></td>
<td>Var(Consumption)</td>
<td>1.458</td>
<td>0.896</td>
<td>0.910</td>
<td>1.003</td>
<td>0.458</td>
</tr>
<tr>
<td></td>
<td>Var(Inflation)</td>
<td>0.025</td>
<td>0.043</td>
<td>0.026</td>
<td>0.025</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>4.323</td>
<td>2.857</td>
<td>1.634</td>
<td>1.718</td>
<td>1.166</td>
</tr>
</tbody>
</table>

Total: 4.323

To further present how different monetary regimes behave in stabilizing the economy, in Figures 6 and 7, we plot the graphs for simulated "output" from randomly drawn examples. It shows that alternative

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8By applying identical shocks to both models, any observed differences are solely attributed to the varying monetary policy regimes.
monetary regimes can better stabilize the economy, particularly after combining with monetary reform. It is easy to detect several significant fluctuations with inflation targeting (solid blue line). However, there is more stability created by other regimes. For instance, the big ups and downs under inflation targeting are squashed with alternative regimes at periods 20 to 30. The price-level targeting and nominal GDP targeting can perform better, especially when the crisis collapses. Around the period 90, there was a significant slump under the inflation targeting (solid blue line). While under the PLT+monetary reform (solid red line) and the NGDPT+monetary reform (solid grey line), the big crisis is stabilized into a moderate drop or small swings and lasts for a shorter period.

![Figure 6: Simulated output under different rules (Example1)](image)

![Figure 7: Simulated output under different rules (Example2)](image)

4.4 The role of fiscal policy

A further important policy issue is the role of fiscal policy in this context where monetary policy is also being optimized and is using unconventional methods, notably QE, at the ZLB. It could well be the case that fiscal policy can reduce the challenges faced by monetary policy, in particular by dealing more effectively with the ZLB, so reducing the pressure to use QE. In this section we investigate this important issue by considering two possible fiscal policies- one in which fiscal policy follows a strong counter-cyclical rule and another in which it reacts rapidly to any threat of reaching the ZLB, by pushing fiscal intervention to whatever is needed to prevent interest rates reaching it. We do this while maintaining monetary policy at their optimized rules as set out above. The following are three different fiscal policy regimes:

1 Baseline policy regime
\[ g_t = \rho g_{t-1} + \rho_a e^a_t + e^g_t \]

Where \( g_t \) is the government expenditure shock; \( e^a_t \) is the productivity i.i.d innovation; \( e^g_t \) is the government expenditure i.i.d innovation.

2 Supressing fiscal policy regime

\[ g_t = \rho g_{t-1} + \rho_a e^a_t + e^g_t + f_t \]

where \( f_t \) is a fiscal shock pushing interest rate out of the ZLB.

3 Strong fiscal feedback policy regime

\[ g_t = \rho g_{t-1} + \rho_a e^a_t + e^g_t - \theta (y_t - \bar{y}_t) \]

where \( \bar{y}_t \) is the base run output; \( (y_t - \bar{y}_t) \) is the output gap; \( \theta = 1 \)

Table 8 shows the results of adding these two fiscal rules to the policy mix. What it reveals is that fiscal policy has a major contribution to make in stabilizing the economy, both output and inflation. It can make a substantial inroad into instability by suppressing the ZLB when it threatens to occur. However, it makes an even bigger inroad when it reacts forcefully in a counter-cyclical way at all times; this also largely prevents the emergence of the ZLB, by preventing the violent lurches in output and inflation that trigger the large interest rate changes that hit the ZLB. Simulations of this policy combination imply the ZLB is only hit 3% of the time, effectively sidelining it; meanwhile the fiscal stabiliser greatly lowers output variance while keeping inflation and interest rate variance low. The welfare cost is minimised in this policy combination, revealing the importance of having a fiscal component in the policy mix. The figure below shows the simulated output paths for a typical simulation under each policy regime. It can be seen clearly how output is stabilized compared with the optimal monetary regime alone.

<table>
<thead>
<tr>
<th>Variance</th>
<th>Baseline NGDP targeting</th>
<th>ZLB-suppressing fiscal shock</th>
<th>Strong fiscal feedback</th>
</tr>
</thead>
<tbody>
<tr>
<td>Var(output)</td>
<td>0.0108</td>
<td>0.0067</td>
<td>0.0034</td>
</tr>
<tr>
<td>Var(inflation)</td>
<td>0.0371</td>
<td>0.0282</td>
<td>0.0251</td>
</tr>
<tr>
<td>Welfare loss</td>
<td>0.0425</td>
<td>0.0350</td>
<td>0.0284</td>
</tr>
<tr>
<td>Var(interest rate)</td>
<td>0.0186</td>
<td>0.0306</td>
<td>0.0227</td>
</tr>
<tr>
<td>Utility</td>
<td>-52.38</td>
<td>-51.03</td>
<td>-51.97</td>
</tr>
</tbody>
</table>
5 Conclusion

This work is based on the ongoing challenge for monetary policy during the aftermath of the financial crisis in 2008. When conventional monetary policy was limited by the zero lower bound, the central banks turned to unconventional monetary policy, such as quantitative easing. In light of the impressive developments in monetary policy, we would like to better understand the unconventional monetary tool by studying its transmission mechanism through the financial intermediary, for example.

According to the transmission method proposed by Le et al. (2016), we analyzed quantitative easing through the bank lending channel to capture the dynamic response to unconventional monetary policy. Ultimately, the model shows the importance of unconventional monetary policy, showing that the money supply needs to be controlled to ensure economic stability. Besides, both monetary policy and expansionary fiscal policy have proven beneficial to the economy, including the zero lower bound period. The model's estimation results indicate that it can explain the observations well and replicate the fluctuations of the key
endogenous variables: output, inflation, and interest rate, for which we are primarily concerned. Notably, we employed the indirect inference method to investigate whether the model could explain the data behaviour in the UK.

Simulation results show that monetary policy and monetary reform significantly improve monetary regime behavior. In particular, monetary reform can help squash the enormous crisis and stabilize the economy with fewer significant fluctuations from simulated output under various schemes. Based on our study, nominal GDP targeting and monetary reform have the lowest welfare cost and crisis frequency. Thus, we argue that the single Taylor rule will not be enough to combat financial friction. A better-performing monetary regime, such as nominal GDP targeting combined with monetary reform, could be considered.

In terms of fiscal policy, we first examined a ZLB-suppressing fiscal policy that consistently employs fiscal expansion to prevent the rate from falling into the ZLB. Additionally, we proposed a strong active fiscal policy that responds aggressively to the output gap. Our analysis indicates that fiscal policy plays a crucial role in stabilizing the economy by suppressing the ZLB and reacting counter-cyclically, thereby preventing violent fluctuations in output and inflation that may trigger large interest rate changes at the ZLB. While our study has made substantial progress in exploring the challenges faced by monetary and fiscal policies, further exploration of the model is still necessary. We hope that our study’s findings will contribute to ongoing efforts to enhance economic stability and inform policymaking.
Bibliography


[29] Neely, C.J., 2010. The large scale asset purchases had large international effects (pp. 1-45). Federal Reserve Bank of St. Louis, Research Division.


Appendices

A  DSGE model list (log-linearised)

In this part, to describe the whole framework, we list all the model equations in the log-linearized form. Each equation is normalised with one endogenous variable. And all the variables are in natural logarithm format, apart from variables that are already in the form of percentages and ratios.

Consumption Euler equation

\[ C_t = C_1 C_{t-1} + C_2 E_t C_{t+1} + C_3 (L_t - E_t L_{t+1}) - C_4 (r_t - E_t \pi_{t+1}) + \epsilon_t \]

\[ C_1 = \frac{\frac{1}{1+\frac{h}{\theta}}}{1+\frac{h}{\theta}} C_{t-1} C_2 = \frac{1}{1+\frac{h}{\theta}} C_3 = \frac{\frac{1}{1+\frac{h}{\theta}}}{(1+\frac{h}{\theta})\sigma_c} C_4 = \frac{1}{1+\frac{h}{\theta}} \]

Real Unconverted Interest Rate Parity

\[ E_t^{\pi}_{t+1} - q_t = (r_t - E_t \pi_{t+1}) - (r_t^{f} - E_t^{f} \pi_{t+1}) \]

Labor Demand Equation

\[ l_t = -w_t + (1 + \frac{1-\psi}{\psi}) r k_t + k_{t-1} \]

External Finance Premium Equation without the QE

\[ E_t c y_{t+1} - (r_t - E_t \pi_{t+1}) = \chi (q q_t + k_t - n_t) + \xi_t + e p r_t \]

External Finance Premium Equation with the QE

\[ E_t c y_{t+1} - (r_t - E_t \pi_{t+1}) = \chi (q q_t + k_t - n_t) - \psi m_t + \xi_t + e p r_t \]

Net Worth Evolution Equation

\[ n_t = \frac{\lambda}{\psi} (c y_t - E_{t-1} c y_t) + E_{t-1} c y_t + \theta n_{t-1} + e n w_t \]

Capital Services Equation

\[ k_t^{s} = k_{t-1} + z_t \]

Capital Utilisation Equation

\[ z_t = \frac{1-\psi}{\psi} r k_t \]

Hybrid Wage Equation

\[ w_t^{NK} = \frac{\beta^\gamma \frac{1-\sigma}{1+\beta^\gamma \frac{1-\sigma}{1+\beta^\gamma \frac{1-\sigma}{\sigma_c}}}}{1+\beta^\gamma \frac{1-\sigma}{1+\beta^\gamma \frac{1-\sigma}{1+\beta^\gamma \frac{1-\sigma}{\sigma_c}}}} w_{t-1} + \frac{\beta^\gamma \frac{1-\sigma}{1+\beta^\gamma \frac{1-\sigma}{1+\beta^\gamma \frac{1-\sigma}{\sigma_c}}}}{1+\beta^\gamma \frac{1-\sigma}{1+\beta^\gamma \frac{1-\sigma}{1+\beta^\gamma \frac{1-\sigma}{\sigma_c}}}} E_t \pi_{t+1} - \frac{1}{1+\beta^\gamma \frac{1-\sigma}{1+\beta^\gamma \frac{1-\sigma}{\sigma_c}}} \pi_t \]

\[ - \frac{\lambda}{1+\beta^\gamma \frac{1-\sigma}{\sigma_c}} \pi_{t-1} - \frac{\lambda}{1+\beta^\gamma \frac{1-\sigma}{\sigma_c}} \left( \frac{(1-\beta^\gamma \frac{1-\sigma}{\sigma_c}) \xi_w (1-\xi_w)}{(1-\beta^\gamma \frac{1-\sigma}{\sigma_c}) \xi_w} \right) (w_t - \sigma l_t - (\frac{1}{1-\psi}) (c_t - \psi c_{t-1})) + e w_t \]

\[ w_t^{NC} = \sigma l_t - (\frac{1}{1-\psi}) (c_t - \psi c_{t-1}) - (\pi_t - E_{t-1} \pi_t) + e w_t \]
\[ w^\text{hybrid}_t = w^w w^w_{t}^{NK} + (1 - w^w)w^w_{t}^{NC} \]

**Hybrid Keynesian Phillips Curve**

\[ \pi^{NK}_t = \frac{1}{1 + \beta \gamma^{1-\sigma_c}} E_t \pi_{t+1} + \frac{r_p}{1 + \beta \gamma^{1-\sigma_c} \phi_p} (1 - \frac{\gamma_c}{\phi_p} (1 - \gamma_c)) (\rho \pi_{t-1}^2 + (1 - \alpha) w_t) - e p_t \]

\[ \pi^{NC}_t = (1 - \alpha) w_t + \alpha t^k \]

\[ \pi^\text{hybrid}_t = w^w \pi^{NK}_t + (1 - w^w) \pi^{NC}_t \]

**Tobin Q Equation**

\[ q_{t} = \frac{1 - \sigma}{1 + \sigma + R_c^{2}} E_t q_{t-1} + \frac{R^k}{1 - \sigma + R_c^{2}} E_t r k_{t+1} - E_t c_1 y_{t+1} \]

**Investment Euler Equation**

\[ I_t = \frac{1}{1 + \beta \gamma^{1-\sigma_c}} I_{t-1} + \frac{\beta^{1-\sigma_c}}{1 + \beta \gamma^{1-\sigma_c} \phi_p} E_t I_{t+1} + \frac{1}{1 + \beta \gamma^{1-\sigma_c} \phi_p} q_{t} + e i_t \]

**Production Function**

\[ y_t = \phi[^3 + (1 - \alpha)^3 t + ea] \]

**Taylor Rule Equation**

\[ r_t = \rho r_{t-1} + (1 - \rho) (r_p \pi_t + r_y y_t) + r_y (y_t - y_{t-1}) + e r_t \]

**Quantitative Easing with ZLB crisis**

\[ m_t = m_{t-1} + \psi_2 (c y_t - c y^*) + err m_{t, zlb} \]

**Money supply equation without the QE**

\[ m_t = m_{t-1} + \psi_1 (M_t - M_{t-1}) + err m_{t, r_t} \]

**M2 Equation**

\[ M_t = \beta (1 + \nu - \mu) k_t + \mu m_t - \nu m_t \]

**Foreign Bond Equation**

\[ b^f_t = (1 + r^f_t) d^f_t - 1 + \frac{\rho^p}{\rho^y} p^p_t + p^d_t + \frac{\rho^d}{\rho^y} - \frac{1}{\rho^m} m_t \]

**Export Equation**

\[ x_t = c^e_t + \frac{1}{2} \sigma x_t + e x_t \]

**Import Equation**

\[ m_t = c_t - \sigma q_t + e m_t \]

**Resource Constraint**

\[ y_t = \gamma c_t + \gamma l_t + \frac{k}{y} R^k z_t + \epsilon c^e_t + \gamma x_t - \frac{m}{m} m_t + e g_t \]

**Stochastic Shock Process**

To determine the dynamics of the model, we set up 15 shocks including two exogenous variable, foreign consumption \( C^f_t \) and foreign interest rate \( r^f_t \). The shock process is listed as following:

**Government spending shock (market clearing equation)**

\[ e g_t = \rho_1 e g_{t-1} + \rho_2 \eta^3_t + \eta^1_t \]

**Preference shock (consumption euler equation)**

\[ e b_t = \rho_2 e b_{t-1} + \eta^2_t \]

**Productivity shock (production function)**

\[ (e a_t - e a_{t-1}) = \rho_3 (e a_{t-1} - e a_{t-2}) + \eta^3_t \]

**Investment shock (Investment euler equation)**

\[ e i_t = \rho_4 e i_{t-1} + \eta^4_t \]

**Monetary policy shock (Taylor rule equation)**

\[ e r_t = \rho_5 e r_{t-1} + \eta^5_t \]
Price mark-up shock (Hybrid inflation rate equation)
\[ ep_t = \rho_6 ep_{t-1} + \eta_t^6 \]

Wage mark-up shock (Hybrid wage equation for NK)
\[ ew_t = \rho_7 ew_{t-1} + \eta_t^7 \]

External finance premium shock (External finance premium equation)
\[ epr_t = \rho_9 epr_{t-1} + \eta_t^9 \]

Net worth shock (Net Worth equation)
\[ enw_t = \rho_{10} enw_{t-1} + \eta_t^{10} \]

Money supply shock (M0 equation with crisis)
\[ errm_t = \rho_{11} errm_{t-1} + \eta_t^{11} \]

Money supply shock (M0 equation without crisis)
\[ errm_t = \rho_{12} errm_{t-1} + \eta_t^{12} \]

Export demand shock (Export demand equation)
\[ eex_t = \rho_{13} eex_{t-1} + \eta_t^{13} \]

Import demand shock (Import demand equation)
\[ eim_t = \rho_{14} eim_{t-1} + \eta_t^{14} \]

Exogenous foreign consumption process
\[ c_t^f = \rho_{15} c_t^{f-1} + \eta_t^{15} \]

Exogenous foreign interest rate process
\[ r_t^f = \rho_{16} r_t^{f-1} + \eta_t^{16} \]

B Data and Resources
<table>
<thead>
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<th>Symbol</th>
<th>Variable</th>
<th>Definition, Description</th>
<th>Sources</th>
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<td>R</td>
<td>Nominal interest rate</td>
<td>3 month average sterling T-bill</td>
<td>BoE</td>
</tr>
<tr>
<td>I</td>
<td>Investment</td>
<td>Gross fixed capital formation + Changes in inventions</td>
<td>ONS</td>
</tr>
<tr>
<td>( p_k )</td>
<td>Price of capital</td>
<td>Calculated from model equation</td>
<td>N/A</td>
</tr>
<tr>
<td>K</td>
<td>Capital</td>
<td>Calculated from model equation</td>
<td>N/A</td>
</tr>
<tr>
<td>( \pi )</td>
<td>Inflation</td>
<td>Quarterly percentage change in price GDP deflator</td>
<td>ONS</td>
</tr>
<tr>
<td>W</td>
<td>Wage</td>
<td>Average wage and earning / Total actual working hours, divided by GDP deflator</td>
<td>ONS</td>
</tr>
<tr>
<td>C</td>
<td>Consumption</td>
<td>Household final consumption expenditure</td>
<td>ONS</td>
</tr>
<tr>
<td>Y</td>
<td>Output</td>
<td>Gross domestic product</td>
<td>ONS</td>
</tr>
<tr>
<td>L</td>
<td>Labour</td>
<td>employment / total actual hour worked</td>
<td>ONS</td>
</tr>
<tr>
<td>( r^k )</td>
<td>Rental rate of capital</td>
<td>Calculated from equation</td>
<td>N/A</td>
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<tr>
<td>S</td>
<td>External finance premium</td>
<td>Difference of bank lending rate and risk-free rate</td>
<td>BoE</td>
</tr>
<tr>
<td>N</td>
<td>Net worth</td>
<td>FTSE all share index , divided by GDP deflator</td>
<td>Data Stream</td>
</tr>
<tr>
<td>M0</td>
<td>Quantitative easing</td>
<td>M0 Stock in UK</td>
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<td>M2</td>
<td>Total money supply</td>
<td>M2 money stock in UK</td>
<td>Federal Reserve Economic Data</td>
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<tr>
<td>EX</td>
<td>Export</td>
<td>Total UK export</td>
<td>ONS</td>
</tr>
<tr>
<td>IM</td>
<td>Import</td>
<td>Total UK import</td>
<td>ONS</td>
</tr>
<tr>
<td>Q</td>
<td>Real exchange rate</td>
<td>Inverse of quarterly average sterling effective exchange rate</td>
<td>ONS</td>
</tr>
<tr>
<td>P</td>
<td>General price level</td>
<td>Consumer Price Index of All items in the UK</td>
<td>Federal Reserve Economic Data</td>
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### C Stationarity of shocks

<table>
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<th>KPSS Statistic</th>
<th>AR(1)Parameters</th>
<th>Process</th>
</tr>
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<tbody>
<tr>
<td>Government spending</td>
<td>0.0324**</td>
<td>0.1285</td>
<td>0.9022</td>
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<tr>
<td>Preferences shock</td>
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<td>0.3833</td>
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<tr>
<td>Investment shock</td>
<td>0.0038***</td>
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</tr>
<tr>
<td>Monetary policy shock</td>
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<td>0.3027</td>
<td>-0.0560</td>
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</tr>
<tr>
<td>Productivity shock</td>
<td>0.7327</td>
<td>1.0547***</td>
<td>-0.0905</td>
<td>Non-Stationary</td>
</tr>
<tr>
<td>Price mark-up shock</td>
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</tr>
<tr>
<td>Wage mark-up shock</td>
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<td>-0.2713</td>
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</tr>
<tr>
<td>Premium shock</td>
<td>0.070*</td>
<td>0.2275</td>
<td>0.7961</td>
<td>Trend Stationary</td>
</tr>
<tr>
<td>Networth shock</td>
<td>0.0541*</td>
<td>0.1510</td>
<td>0.5939</td>
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<tr>
<td>Mzero shock (M0 eq)</td>
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<td>0.4072</td>
<td>0.0267</td>
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<tr>
<td>Mzero shock (crisis)</td>
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<td>0.0275</td>
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<tr>
<td>Export shock</td>
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<td>0.3262</td>
<td>0.8083</td>
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<tr>
<td>Import shock</td>
<td>0.002***</td>
<td>0.2636</td>
<td>0.9553</td>
<td>Trend Stationary</td>
</tr>
</tbody>
</table>

Note: a denotes the Augmented Dickey-Fuller (ADF) test, ***, **,* indicate reject the null hypothesis (with unit root) at 10 % 5% and 1% significant level respectively.

b denotes the Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test, ***, **,* indicate reject the null hypothesis (stationary) at 10 % 5% and 1% significant level respectively.