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Is there international risk-sharing between developed economies? New evidence from indirect inference

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Abstract

It has been an 'empirical consensus' that data from developed economies generally do not support the hypothesis of international risk-sharing, either in the form of full risk-pooling via state-contingent assets or in the form of uncovered interest parity enforced by trading non-contingent assets. We reassess these hypotheses in the context of a full DSGE model, as opposed to testing them as single regressions in previous work. We prove that the two model versions behave identically, suggesting that consumers would receive the same scope of protection against risks whether bonds are state-contingent. We further find that the model, when tested appropriately as a whole embracing risk-pooling/UIP, fits the data well and universally through the lens of indirect inference; hence, we provide new evidence of the hypotheses' empirical validity spuriously rejected by single regressions.

Keywords: consumer risk-pooling; UIP; two-country DSGE model; indirect inference test

JEL Classification: C12, E12, F41

1 Introduction

International borrowing markets are both active and deep, besides being supported by wide central bank cooperation, such as the dollar swaps extended by the US Fed. So it has seemed paradoxical that consumer risk-pooling via these markets or even a weaker version of it in the form of uncovered interest parity (UIP) has been difficult to find empirically. The empirical testing in this work has been via predictive tests on the exchange rate based on single-equation regressions, where among others one of the main difficulties in assessing this evidence has been that all the variables in these regressions are endogenous. A notable recent example is Burnside (2019) who rejects UIP for a dozen pairs of industrialised economies on single-equation tests. The paper joins an 'empirical consensus' – now barely questioned – that UIP fails to fit, which is a 'puzzle' many including Burnside attempt to solve with a variety of model features.

However, the difficulties with the single-equation tests are circumvented by Minford, Ou and Zhu (2021) (MOZ hereafter) who embed the risk-pooling hypothesis and its UIP variant in a two-country, IS/Phillips curve and Taylor rule model and test the model as a whole. By using the method of Indirect Inference, they find that neither hypothesis is rejected, with risk-pooling being more probable, on the US-EU data. They account for the discrepancy between these findings and the rejection of both hypotheses in conventional

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single-equation tests by showing, in a Monte Carlo experiment on that model, when either hypothesis was true, that certain single-equation tests would bias heavily towards the hypothesis' rejection.

In this paper, we revisit the risk-pooling hypothesis within a full DSGE model where, as we prove below, the explicit modelling of the consumption Euler equations (which are not substituted out for the forward-looking IS curves as in MOZ, by contrast) implies the formal equivalence of risk-pooling through state-contingent assets, and UIP; so under either hypothesis consumers are provided with the same scope for smoothing consumption over time and across borders. When this model is tested as a whole – for the same currency pairs considered by Burnside (2019) – by indirect inference, we find strong, universal evidence for these hypotheses.

The two-fold contribution of this paper is thus: first, we prove that full risk-pooling is always obtainable in free markets, whether or not state-contingent bonds are made available; second, we correct the spurious 'empirical consensus' that international risk-sharing is not supported by the data by showing that, once the hypothesis is tested as part of a full model that circumvents the rejection bias of single-equation tests, it exists universally.

In the rest of this paper: we demonstrate the formal equivalence of risk-pooling and UIP when the Euler equation is present in Section 2; in Section 3 we verify the numerical equivalence of the two model versions; in Section 4 we explain the method of indirect inference for testing DSGE models; in Section 5 we report our test result; Section 6 concludes.

2 The equivalence of risk-pooling and UIP – a theoretical proof

These two models of consumer behaviour in the open economy can be derived following Chari, Kehoe and McGrattan (2002) as follows:

Case A: full risk-pooling via state-contingent nominal bonds

Let the price at time $t = 0$ (when the state was s_0) of a home nominal state-contingent bond paying 1 unit of home currency in state s_t be:

$$n(s_t|s_0) = \beta f(s_t|s_0)\frac{U_c(s_t|s_0)}{P(s_t|s_0)}/\frac{U_c(s_0)}{P(s_0)} \tag{1}$$

where β is time discount factor, $f(s_t|s_0)$ is the probability of s_t occurring given s_0 has occurred, U_c is the marginal utility of consumption, P is the general price level. Now note that foreign consumers can also buy this bond freely via the foreign exchange market and its value as set by them will be:

$$n(s_t|s_0) = \beta^* f(s_t|s_0)\frac{U_c^*(s_t|s_0)}{P^*(s_t|s_0)Q(s_t|s_0)}/\frac{U_c^*(s_0)}{P^*(s_0)Q(s_0)} \tag{2}$$

where '$*$' denotes foreign variables, Q is the nominal exchange rate. Here they are equating the expected marginal utility of acquiring this bond with foreign currency, with the marginal utility of a unit of foreign currency at time 0. Plainly, (1) and (2) must be equal by arbitrage, so that:

$$\beta\frac{U_c(s_t|s_0)}{P(s_t|s_0)}/\frac{U_c(s_0)}{P(s_0)} = \beta^*\frac{U_c^*(s_t|s_0)}{P^*(s_t|s_0)Q(s_t|s_0)}/\frac{U_c^*(s_0)}{P^*(s_0)Q(s_0)} \tag{3}$$

Now we note that the terms for the period $t = 0$ are the same for all s_t so that for all t from $t = 0$ onwards:

$$\frac{U_c(s_t|s_0)}{U_c^*(s_t|s_0)} = \Theta\frac{P(s_t|s_0)}{P^*(s_t|s_0)Q(s_t|s_0)} \tag{4}$$

where $\Theta = \frac{U_c(s_0)}{P(s_0)} / \frac{U_c^*(s_0)\beta}{P^*(s_0)Q(s_0)\beta^*}$ is a constant.

Let the utility function be $U = C_t^{(1-\sigma)}\varepsilon_{j,t}/(1-\sigma)$ where σ is the inverse of the consumption elasticity and $\varepsilon_{j,t}$ is the time-preference shock, and $\hat{q}_t = -\hat{P}_t + \hat{P}_t^* + \hat{Q}_t$ is the real exchange rate (with '$\widehat{x}_t$' denoting a variable x_t in percentage deviation from its steady-state value). Equation (4) implies:

$$\sigma(\hat{c}_t - \hat{c}_t^*) = \hat{q}_t - v_t \tag{5}$$

ignoring the constant, which is the risk-pooling condition; v_t is the difference between the logs of the two countries' time-preference shocks[1].

To see that this implies the UIP relationship, use the Euler equations for consumption (e.g. for home consumers $\hat{c}_t = -\frac{1}{\sigma}\left(\frac{r_t}{1-B^{-1}} - \hat{\varepsilon}_{j,t}\right)$ where B^{-1} is the forward operator keeping the date of expectations constant). Substituting for consumption into (5) gives us UIP:

$$E_t\hat{q}_{t+1} - \hat{q}_t = r_t - r_t^* \tag{6}$$

Case B: when there are only non-contingent bonds

In this case arbitrage enforces UIP. When (6) is substituted back into the Euler equations it yields:

$$(1-B^{-1})\sigma(\hat{c}_t - \hat{c}_t^*) = (1-B^{-1})\left(\hat{q}_t - v_t\right) \tag{7}$$

Thus the risk-pooling condition occurs in expected form from where it currently is. But it can be shown to yield the same risk-pooling outcome period by period – exactly as (5) – most easily by dividing through the equation by $(1-B^{-1})^2$.

What these two cases have illustrated is that, whether there are state-contingent bonds or simple borrowing with non-contingent bonds, relative consumption is exactly correlated with the real exchange rate and time-preference shocks. Thus, *even without explicit insurance contracts, consumers can insure themselves by borrowing from foreigners, smoothing out consumption across good and bad times; we do not need explicit future contingent contracts to supplement the workings of free markets.* Indeed, these futures can be thought of as copying the market solution in advance, much like Arrow-Debreu contracts map out the future of the economy as it will respond freely to shocks. It follows that *an open economy model with optimising consumers will behave the same under risk-pooling via state-contingent contracts as under UIP; so the two models are identical*[3].

[1] An implicit simplifying assumption here is that home and foreign consumers share the same consumption elasticity, such that σ is the same for both economies. Allowing σ to take different values for the two economies does not change the implication.

[2] An alternative way to show this equivalence is to first write UIP as $\hat{q}_t = -\left(\hat{r}_t - \hat{r}_t^*\right)/(1-B^{-1})$, where in effect the real exchange rate mirrors the whole expected future path of the real interest rate; then using directly the Euler equations, in which also current consumption reflects the same whole expected path of real rates, which yields: $\hat{q}_t = \sigma(\hat{c}_t - \hat{c}_t^*) + v_t$, so $\sigma(\hat{c}_t - \hat{c}_t^*) = \hat{q}_t - v_t$.

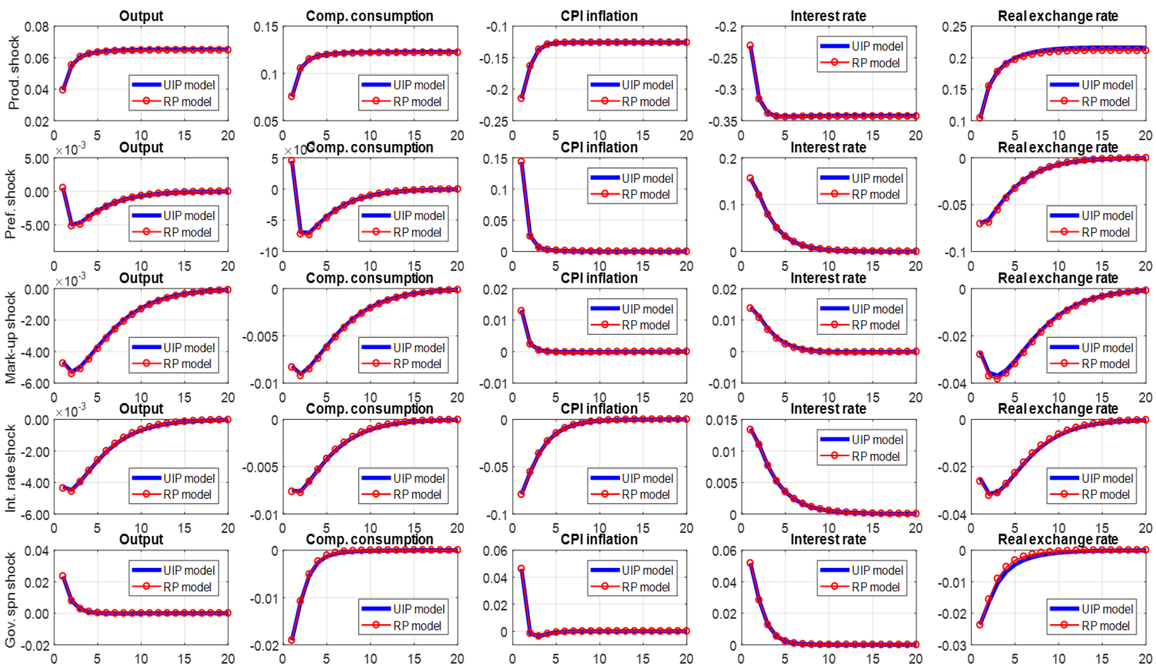
[3] Note however that if there is no explicit Euler equation in the model and instead there is a forward-looking IS curve reflecting a variety of demand shocks (as in MOZ), the IS curve implies: $\hat{y}_t - \hat{y}_t^* = -\frac{1}{\sigma}\frac{c}{y}\frac{r_t - r_t^*}{1-B^{-1}} + err_t$ (where $\frac{c}{y}$ is the steady-state consumption ratio and err_t includes the effect of v_t). If we impose UIP now we will get a relationship between relative outputs and the real exchange rate under UIP as: $\hat{y}_t - \hat{y}_t^* = \frac{1}{\sigma}\frac{c}{y}\hat{q}_t + err_t$. If instead of imposing UIP we impose risk-pooling, then $\hat{q}$ will be solved from the risk-pooling equation (5) conditional on output and market-clearing consumption. This generally will not deliver the same real exchange rate as under UIP, because the consumption derived from the market-clearing condition will not generally be the one implied by the IS curve used.

3 The equivalence of risk-pooling and UIP in a full DSGE model – a numerical verification

Here we construct a full, two-country DSGE model both in the 'standard' form without state-contingent assets (the UIP version), and in the form with them (the RP version). The two versions only 'differ' in how the real exchange rate is determined: in the UIP model it is adjusted to clear the interest gap between the two economies, as a result of international arbitrage, as shown by (6); in the RP version it is set to reflect the consumption gap between the two economies, as an equilibrium condition, as shown by (5)[4]. The rest of the model, which we detail in the appendix to save space, is canonical and identical.

Figure 1 compares a representative set of impulse response functions of the two models assigned the same parameter values. It confirms that these models – widely believed to be different in previous work (e.g., Chari, Kehoe and McGrattan, 2002) – indeed work in the same manner, as just proven.

Figure 1: Key IRFs of the UIP and RP models



4 Is there risk-pooling/UIP in the data?

4.1 Testing the model by indirect inference

The method, developed by Minford, Theodoridis and Meenagh (2008), is a formal probability test for DSGE models[5]. We choose this method because, unlike the Bayesian method which does not test models, or the Maximum Likelihood method which suffers from low test power, it provides 'Goldilocks' power such that a false model would be strongly rejected.

The idea is to ask 'how likely what we see with the data through the lens of a pure, unrestricted empirical model used to provide benchmark description of the data, is generated by the DSGE model should this DSGE

[4] We assume log utility for our model here for simplicity; so $\sigma = 1$.

[5] See also Meenagh, Minford and Wickens (2009), Le et al. (2011, 2016) and Minford, Wickens and Xu (2019).

model be true'. The first step of the test is to calculate the sample errors using the DSGE model and the data. The second step is to bootstrap these errors to generate a large number of parallel simulations using the DSGE model. The final step is to evaluate how probable the sample data could be a random realisation of the DSGE model, by comparing the data's features as described by the empirical model, to the joint distribution of them found using the same empirical model and the parallel simulations.

As in our practice of testing risk-pooling here we are mostly interested in the model's ability to explain the international business cycle and real exchange rate behaviour, so we choose as the empirical model a VARX(1) of both the home and foreign outputs, and the real exchange rate:

$$Y_t = AY_{t-1} + BX_{t-1} + e_t \tag{8}$$

where $Y_t \equiv (\hat{y}_t, \hat{y}^*_t, \hat{q}_t)'$; $X_t \equiv (\hat{\varepsilon}_{z,t}, \hat{\varepsilon}^*_{z,t}, t)'$, where the two $\hat{\varepsilon}_z$'s are the home and foreign productivity, respectively, while t is a deterministic time factor, all are assumed to be cointegrated with the two outputs; e_t is a vector of the VARX residuals.

The data features against which the DSGE model is evaluated is chosen to be the autoregressive coefficients, and the variances of the residuals, of the VARX, such that in effect the test evaluates the DSGE model's capacity in fitting the data's dynamic behaviour and volatility. The estimates found with the actual data are collected by vector Φ_{Act}; their counterparts found with the simulations are collected by vectors $\Phi_{Sim1}, \Phi_{Sim2} ... \Phi_{SimN}$ (where N is the number of parallel simulations), and $\overline{\Phi}$ and $\sum_{(\Phi\Phi)}$ are, respectively, a vector of the mean values, and the variance-covariance matrix, of the same set of these estimates.

To judge whether the DSGE model is rejected by the data, the test calculates the Wald statistic:

$$Wald_{(i)} = (\Phi_i - \overline{\Phi})' \sum\nolimits_{(\Phi\Phi)}^{-1} (\Phi_i - \overline{\Phi}) \tag{9}$$

(where $i = Act, Sim1, Sim2 ... SimN$) and evaluates the percentile of $Wald_{(Act)}$ under the distribution of $Wald_{(i)}$. The p-value of the test, under the null hypothesis that 'the DSGE model is true', is calculated by:

$$p = (100 - WP)/100 \tag{10}$$

where WP is $Wald_{(Act)}$'s percentile. The DSGE model is rejected/not rejected should the p-value be below/above the usual 1%, 5% or 10% threshold.

4.2 The test result on 10 industrialised economy pairs

As reviewed earlier, by single-equation tests Burnside (2019) rejected UIP – proved above to be identical to full risk-pooling – for a dozen pairs of industrialised economies. Here we report our indirect inference, full-model test results for the same currency pairs against the US dollar based on pretty much the same sample period (1971Q1-2018Q4)[6]. Values of the structural parameters are selected by a grid search over the permissible parameter space for them to deliver the minimum Wald statistic, (9), such that they fit the model to the data as much as possible[7]. The p-values of the tests are reported in comparison with Burnside's in Table 1.

[6] The data are collected from Euro-area-statistics, FRED, the IMF and the OECD; and are processed in the standard manner for them to be used by DSGE models.

[7] This is in essence the indirect inference method for estimation. Due to the large number of economy pairs we estimate, we omit the sets of the estimated parameter values, which are available on request, for conciseness.

Table 1: Model p-value by currency pair against USD

Currency	Country	Indirect inference full-model test of risk-pooling/UIP	Burnside (2019) single-equation test of UIP
AUD	Australia	0.150	0.006
CAD	Canada	0.099	0.009
DKK	Denmark	0.090	0.001
EUR	Euro Area	0.117	0.093
JPY	Japan	0.079	0.003
NOK	Norway	0.063	0.047
NZD	New Zealand	0.102	0.000
SEK	Sweden	0.079	0.904
CHF	Switzerland	0.098	0.014
GBP	UK	0.133	0.002

While only two out of the 10 currency pairs (i.e., EUR and SEK) in the Burnside test were found to comply with UIP at the 5% level, we find this evidence to be a victim of over-rejection bias in the single-equation method. When this bias is circumvented by our indirect inference full-model test, as our new evidence here shows, the hypothesis is upheld – for all the currency pairs – generally with a high p-value showing its good fit to the data. Hence there is strong and wide evidence of international risk-sharing in the data; the earlier 'empirical consensus' that the hypothesis fails to fit appears to be a statistical artefact brought about by the misuse of single-regression tests on this issue. It follows that open economy macro models in their 'standard' form can explain the key data features including those of the real exchange rate; we do not need a 'more advanced' model, which would only weigh on the model's fit unnecessarily by complicating its structure, in order to resolve the so-called 'exchange rate puzzle'.

5 Conclusion

In this paper we have constructed a full, two-country DSGE model in its two versions, one allowing for risk-pooling via state-contingent assets, the other with non-contingent assets enforcing UIP. We proved that these versions were equivalent in theory, and verified that they behaved identically numerically. We also tested the model as a whole using the method of indirect inference, and found that risk-pooling/UIP in fact existed universally despite being rejected spuriously by single-equation tests. While it has been usual to consider the hypothesis as not holding up empirically, this spurious consensus seems to have emerged from the fact that these single-equation tests failed to impose the full set of restrictions that bind in a full structural DSGE model.

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Appendix

The full DSGE model

We only present the home economy equations for conciseness; the foreign economy is symmetrical, and connected with the home economy via international capital movements and trades. Variables/parameters of the home economy are unmarked; those of the foreign economy are asterisked. Variables without a time subscript denote the steady-state value of them. '$\widehat{x_t}$' continues to denote the percentage deviation of a variable x_t from its steady-state value.

The standard UIP version

Households

Households work, consume and save; and have life-time utility:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_{j,t} \left(\ln c_t - \psi \frac{n_t^{1+\eta}}{1+\eta} \right) \tag{A.1}$$

where c_t is consumption, n_t is labour hour, ψ is the preference of leisure, η is the inverse of wage elasticity, β is the time discount factor, and $\varepsilon_{j,t}$ is the time preference shock. The composite consumption index is defined by:

$$c_t \equiv [(1-\alpha)^{\frac{1}{\nu}} c_{h,t}^{\frac{v-1}{v}} + \alpha^{\frac{1}{\nu}} im_t^{\frac{v-1}{v}}]^{\frac{v}{v-1}} \tag{A.2}$$

where $c_{h.t}$ is the consumption on domestic goods, im_t is imports, v (> 0) is the substitutability between $c_{h.t}$ and im_t, α is the degree of openness.

The household budget constraint is:

$$c_{h,t} + q_t im_t + b_t + q_t bf_t + t_t = w_t n_t + (1 + r_{t-1}) b_{t-1} + (1 + r^*_{t-1}) q_t bf_{t-1} + \Pi_t \tag{A.3}$$

where q_t is the real exchange rate, b_t and bf_t are holdings of home and foreign bonds, r_{t-1} and r^*_{t-1} are the home and foreign real interest rates, w_t is the real wage rate, t_t and Π_t are lump-sum tax payment and profit received, respectively.

The household problem is to maximise (A.1) by choosing $c_{h,t}$, im_t, n_t, b_t and bf_t, subject to (A.3). The first order conditions imply the demand for domestic and foreign goods, the labour supply, and the UIP condition:

$$E_t \hat{q}_{t+1} - \hat{q}_t = r_t - r^*_t \tag{A.4}$$

Firms

Firms produce using the same technology; for simplicity we assume a labour-only production function:

$$y_t = \varepsilon_{z,t} n_t \tag{A.5}$$

where y_t is the aggregate output, $\varepsilon_{z,t}$ is productivity.

Under Calvo pricing (where a fraction, $1-\omega$, of the firms are assumed to be able to reset prices) the standard profit maximisation problem under the assumptions of a zero-inflation steady state and no past-inflation indexation implies the Phillips curve for domestic price inflation:

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \kappa \widehat{mc}_t + \hat{\varepsilon}_{\pi,t} \tag{A.6}$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$, $mc_t(= w_t/\varepsilon_{z,t})$ is the real marginal cost of production, $\hat{\varepsilon}_{\pi,t}$ is price mark-up shock.

Let CPI be defined as:

$$P_t = [(1-\alpha)P_{h,t}^{1-v} + \alpha(Q_t P_{h,t}^*)^{1-v}]^{\frac{1}{1-v}} \tag{A.7}$$

where $P_{h,t}$ and $P_{h,t}^*$ are prices of domestic and imported goods, respectively, and Q_t is the nominal exchange rate, CPI inflation may be shown as[8]:

$$\pi_t = (1-\alpha)\pi_{h,t} + \alpha\pi_{h,t}^* + \alpha\Delta\hat{Q}_t \tag{A.8}$$

The firm profit in each period ($\Pi_t = y_t - w_t n_t$) is transferred to households, who are assumed to own these firms, as a lump-sum.

Monetary and fiscal policies

The central bank adjusts the nominal interest rate following a Taylor rule:

$$1 + R_t = (1 + R_{t-1})^{\rho_R}(1+\pi_t)^{(1-\rho_R)\phi_\pi}\left(\frac{y_t}{y_{t-1}}\right)^{(1-\rho_R)\phi_y}(1+r)^{(1-\rho_R)}\varepsilon_{R,t} \tag{A.9}$$

where the rate responds to both inflation (ϕ_π) and growth (ϕ_y), subject to inertia (ρ_R) and a monetary policy shock ($\varepsilon_{R,t}$).

The fiscal authority adjusts government spending, which is assumed to be a stationary exogenous process around its steady-state level:

$$g_t = \varepsilon_{g,t} g \tag{A.10}$$

where $\varepsilon_{g,t}$ is the shock to the spending.

Identities and shock processes

The goods market clearing requires:

$$y_t = c_{h,t} + g_t + im_t^* \tag{A.11}$$

where im_t^* is imports by the foreign economy, hence exports of the home economy.

The balance of international payments requires:

$$q_t\left[bf_t + im_t - (1 + r_{t-1}^*)bf_{t-1}\right] = im_t^* \tag{A.12}$$

where in solving the model we impose the terminal condition that $\Delta bf = 0$ to find the equilibrium real exchange rate.

The real exchange rate is defined as:

$$q_t = \frac{Q_t P_{h,t}^*}{P_{h,t}} \tag{A.13}$$

The real interest rate is calculated by the Fisher equation:

$$r_t = R_t - E_t\pi_{t+1} \tag{A.14}$$

All shocks of the model, except for the productivity shock, are assumed to follow an AR(1) process in natural logarithm:

$$\hat{\varepsilon}_{i,t} = \rho_i \hat{\varepsilon}_{i,t-1} + u_{i,t} \tag{A.15}$$

[8] In deriving this, it is assumed that full PPP holds in the long run, such that $\frac{P_h}{P} = \frac{QP_h^*}{P} = 1$.

where $i = j, \pi, R, g$. The productivity shock, whose impact is assumed to be permanent, is let follow a simple ARIMA (1,1,0) process:

$$\hat{\varepsilon}_{z,t} - \hat{\varepsilon}_{z,t-1} = \Gamma - \delta(\hat{\varepsilon}_{z,t-1} - \hat{\varepsilon}_{z,t-2}) + u_{z,t} \tag{A.16}$$

where Γ is a constant, δ is the mean-reversing parameter. All $u's$ in the shock processes are *iid*.

The RP model equivalent

To construct the RP model equivalent, note that arbitrage and the law of one price in a world with state-contingent nominal bonds implies $\frac{U_c(s_t,s_0)}{U_c^*(s_t,s_0)} = \Theta\frac{P(s_t,s_0)}{P^*(s_t,s_0)Q(s_t,s_0)}$ (Equ. (4) in the main text). Given that $U_t = \varepsilon_{j,t}\left(\ln c_t - \psi\frac{n_t^{1+\eta}}{1+\eta}\right)$ and hence $U_c = \varepsilon_{j,t}c_t^{-1}$ with our model, international risk sharing implies the RP condition:

$$\hat{q}_t = (\hat{c}_t - \hat{c}_t^*) - \left(\hat{\varepsilon}_{j,t} - \hat{\varepsilon}_{j,t}^*\right) \tag{A.17}$$

which ties the real exchange rate to the relative consumption of the two economies, subject to the difference in the two economies' time preference shocks. The RP equivalent of the standard UIP model simply replaces the UIP equation (A.4) with (A.17), *ceteris paribus.*