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On the determination of the real exchange rate in free markets: do consumer risk-pooling and uncovered interest parity differ and fit?

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Abstract

We revisit the 'puzzle' in open economy studies that evidence of international risk-sharing is hardly seen despite the completeness of the financial market. We reassess both risk-pooling via state-contingent bonds, and uncovered interest parity – both were believed to be different, and spuriously rejected, in previous work – in the context of a full DSGE model. We prove that the two models are identical, both analytically and numerically. When tested as part of the full DSGE model by indirect inference which circumvents the bias of single-equation tests, we find strong and wide evidence of international risk-sharing.

Keywords: consumer risk-pooling; UIP; two-country DSGE model; indirect inference test

JEL Classification: C12, E12, F41

1 Introduction

Given the depth and wide activity in international asset markets, it has seemed paradoxical that consumer risk-pooling via these markets or even a weaker version of it in the form of uncovered interest parity (UIP) has been difficult to find empirically – some recent examples include Hess and Shin (2000), Delcoure et al. (2003), Isard (2006) and Burnside (2019). The empirical testing in this work has been via predictive tests on the exchange rate based on single-equation regressions, where among others one of the main difficulties in assessing this evidence has been that all the variables in these regressions are endogenous. A notable recent example is Burnside (2019) who rejects UIP for a dozen pairs of industrialised economies on single-equation tests. The paper joins an 'empirical consensus' – now barely questioned – that UIP fails to fit, which is a 'puzzle' many including Burnside attempt to solve with a variety of model features.

However, the difficulties with the single-equation tests used by Burnside are circumvented by Minford, Ou and Zhu (2021) (MOZ hereafter) who embed the risk-pooling hypothesis and its UIP variant in a full model and test the model as a whole. The model takes the familiar IS curve, Phillips curve and Taylor rule New Keynesian set-up of Clarida, Gali and Gertler (1999) extended to embrace the US, Europe and the rest of the world, essentially a two-country model for the US and EU. They use the method of Indirect Inference to estimate and test the two model versions for the US and the EU pair of economies. What they find is

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that neither hypothesis is rejected on the test, with risk-pooling being more probable. They account for the discrepancy between these findings and the rejection of both hypotheses in conventional single-equation tests by showing, in a Monte Carlo experiment on that two-country model, when either hypothesis was true, that certain widely-used single-equation tests would be heavily biased towards the hypothesis' rejection.

In this paper, we revisit the risk-pooling hypothesis within a full DSGE model where the consumption Euler equations are explicitly included, instead of being substituted out for the forward-looking IS curves. It turns out that using IS curves as in MOZ creates a distinction between risk-pooling through state-contingent assets, and UIP, where in fact none exists; UIP, which relies on arbitrage between non-contingent bonds, provides the same scope for consumers to smooth their consumption over time and across borders, in a way we will demonstrate below. The MOZ finding that the two hypotheses differ somewhat in their test performance comes about because the errors used in each hypothesis differ in their detailed application using the IS curves; when the model version including the Euler equations is used, such a difference in errors disappears.

Our two-fold contribution in this paper, which resolves a long-standing 'puzzle' in open economy research on international risk-sharing and exchange rate behaviour, is therefore: first, we prove that full risk-pooling is always obtainable in free markets, whether state-contingent bonds are available or not; a standard DSGE model with 'complete' structure will behave in the same manner whether the exchange rate is assumed to reflect the consumption gap between two economies as a result of international risk-sharing via state-contingent bonds, or to adjust to clear the interest rate gap between them as a result of international arbitrage. Second, we correct the spurious 'empirical consensus' that international risk-sharing is not supported by the data; we show that, once the correct assessment method is used on a full model, the data suggests that it exists universally.

The rest of this paper is organised as follows: in Section 2 we demonstrate the formal equivalence of risk-pooling and UIP when the Euler equation is present; in Section 3 we set out our full DSGE model in its two versions, of risk-pooling and UIP, and side by side with it we verify their numerical equivalence; in Section 4 we explain our indirect inference empirical test procedure for carrying out our empirical tests; in Section 5 we report our test result for the 10 pairs of economies assessed by Burnside (2019); Section 6 concludes.

2 The equivalence of risk-pooling and UIP

These two models of consumer behaviour in the open economy, risk-pooling via state-contingent assets and plain UIP, can be derived following Chari, Kehoe and McGrattan (2002), as follows:

Case A: full risk-pooling via state-contingent nominal bonds

Let the price at time $t = 0$ (when the state was s_0) of a home nominal state-contingent bond paying 1 unit of home currency in state s_t be:

$$n(s_t|s_0) = \beta f(s_t|s_0) \frac{U_c(s_t|s_0)}{P(s_t|s_0)} / \frac{U_c(s_0)}{P(s_0)} \tag{1}$$

where β is time discount factor, $f(s_t|s_0)$ is the probability of s_t occurring given s_0 has occurred, U_c is the marginal utility of consumption, P is the general price level. Now note that foreign consumers can also buy this bond freely via the foreign exchange market and its value as set by them will be:

$$n(s_t|s_0) = \beta^* f(s_t|s_0) \frac{U_c^*(s_t|s_0)}{P^*(s_t|s_0)Q(s_t|s_0)} / \frac{U_c^*(s_0)}{P^*(s_0)Q(s_0)} \tag{2}$$

where '$*$' denotes foreign variables, Q is the nominal exchange rate. Here they are equating the expected marginal utility of acquiring this bond with foreign currency, with the marginal utility of a unit of foreign currency at time 0. Plainly the price paid by foreign consumers must be equal by arbitrage to the price paid by home consumers. Equating these two equations yields:

$$\beta \frac{U_c(s_t|s_0)}{P(s_t|s_0)} / \frac{U_c(s_0)}{P(s_0)} = \beta^* \frac{U_c^*(s_t|s_0)}{P^*(s_t|s_0)Q(s_t|s_0)} / \frac{U_c^*(s_0)}{P^*(s_0)Q(s_0)} \tag{3}$$

Now we note that the terms for the period $t = 0$ are the same for all s_t so that for all t from $t = 0$ onwards:

$$\frac{U_c(s_t|s_0)}{U_c^*(s_t|s_0)} = \Theta \frac{P(s_t|s_0)}{P^*(s_t|s_0)Q(s_t|s_0)} \tag{4}$$

where $\Theta = \frac{U_c(s_0)}{P(s_0)} / \frac{U_c^*(s_0)\beta}{P^*(s_0)Q(s_0)\beta^*}$ is a constant.

Let the utility function be $U = C_t^{(1-\sigma)}\varepsilon_{j,t}/(1-\sigma)$ where σ is the inverse of the consumption elasticity and $\varepsilon_{j,t}$ is the time-preference shock, and $\hat{q}_t = -\hat{P}_t + \hat{P}_t^* + \hat{Q}_t$ is the real exchange rate (with '$\widehat{x_t}$' denoting a variable x_t in percentage deviation from its steady-state value). Equation (4) implies:

$$\sigma(\hat{c}_t - \hat{c}_t^*) = \hat{q}_t - v_t \tag{5}$$

ignoring the constant, which is the risk-pooling condition; v_t is the difference between the logs of the two countries' time-preference shocks[1].

To see that this implies the UIP relationship, use the Euler equations for consumption (e.g. for home consumers $\hat{c}_t = -\frac{1}{\sigma}\left(\frac{r_t}{1-B^{-1}} - \hat{\varepsilon}_{j,t}\right)$ where B^{-1} is the forward operator keeping the date of expectations constant). Substituting for consumption into (5) gives us UIP:

$$E_t\hat{q}_{t+1} - \hat{q}_t = r_t - r_t^* \tag{6}$$

Case B: when there are only non-contingent bonds

In this case arbitrage forces the UIP equation. When (6) is substituted back into the Euler equations, it yields:

$$(1 - B^{-1})\sigma(\hat{c}_t - \hat{c}_t^*) = (1 - B^{-1})(\hat{q}_t - v_t) \tag{7}$$

Thus the risk-pooling condition occurs in expected form from where it currently is. But it can be shown to yield the same risk-pooling outcome period by period – exactly as (5) – most easily by dividing through the equation by $(1 - B^{-1})$[2].

What these two cases have illustrated is that, whether there are state-contingent bonds or simple borrowing with non-contingent bonds, relative consumption is exactly correlated with the real exchange rate and time-preference shocks. Thus, *even without explicit insurance contracts, consumers can insure themselves by borrowing from foreigners, smoothing out consumption across good and bad times; we do not need explicit future contingent contracts to supplement the workings of free markets.* Indeed, these futures can be thought of as copying the market solution in advance, much like Arrow-Debreu contracts map out the future

[1] An implicit simplifying assumption here is that home and foreign consumers share the same consumption elasticity, such that σ is the same for both economies. Allowing σ to take different values for the two economies does not change the implication.

[2] An alternative way to show this equivalence is to first write UIP as $\hat{q}_t = -(\hat{r}_t - \hat{r}_t^*)/(1 - B^{-1})$, where in effect the real exchange rate mirrors the whole expected future path of the real interest rate; then using directly the Euler equations, in which also current consumption reflects the same whole expected path of real rates, which yields: $\hat{q}_t = \sigma(\hat{c}_t - \hat{c}_t^*) + v_t$, so $\sigma(\hat{c}_t - \hat{c}_t^*) = \hat{q}_t - v_t$.

of the economy as it will respond freely to shocks. It follows that *an open economy model with optimising consumers will behave the same under risk-pooling via state-contingent contracts as under UIP; so the two models are identical*.

Note however that if there is no explicit Euler equation in the model and instead there is a forward-looking IS curve reflecting a variety of demand shocks (as in MOZ), the IS curve implies: $\hat{y}_t - \hat{y}_t^* = -\frac{1}{\sigma}\frac{c}{y}\frac{r_t - r_t^*}{1-B^{-1}} + err_t$ (where $\frac{c}{y}$ is the steady-state consumption-to-output ratio and err_t includes the effect of v_t). If we impose UIP now we will get a relationship between relative outputs and the real exchange rate under UIP as: $\hat{y}_t - \hat{y}_t^* = \frac{1}{\sigma}\frac{c}{y}\hat{q}_t + err_t$.

If instead of imposing UIP we impose risk-pooling, then $\hat{q}$ will be solved from the risk-pooling equation (5) conditional on output and market-clearing consumption. This generally will not deliver the same real exchange rate as under UIP, because the consumption derived from the market-clearing condition will not generally be the one implied by the IS curve used[3]. This explains why Minford, Ou and Zhu (2022) – who compared risk-pooling and UIP using an IS curve to abstract the demand side – found a differential ability of the two models to fit the facts of different country pairs' behaviour. In effect their model with UIP only will not produce risk-pooling for consumers, because there is not a tight relationship between relative output minus the shock vector err, and relative consumption. Yet, empirically the results for full risk-pooling and UIP were fairly close, as one would expect.

However, if we set out a full DSGE model with an explicit Euler equation, then whether we include fully state-contingent bonds or UIP will give us full risk-pooling. We can test such a model by indirect inference and this model will test in a tight way for risk-pooling. We carry out this 'tight test' in this note as a supplement to MOZ. Since risk-pooling and UIP imply each other, it is also a tight test for UIP in contrast to Burnside (2019).

3 A full, two-country DSGE model

We start by constructing a two-country model without state-contingent bonds which is 'standard' in the open economy set-up. UIP in this benchmark setting is enforced by international arbitrage as households in both home and foreign economies are allowed to buy bonds issued by any country. We then construct the risk-pooling (RP) model equivalent where, as illustrated above, the allowance for state-contingent bonds implies an RP equation for real exchange rate determination, which replaces the UIP equation. To save space we only present the home economy equations; the foreign economy is symmetrical, and connected with the home economy via international capital movements and trades. Variables/parameters of the home economy are unmarked; those of the foreign economy are asterisked. Variables without a time subscript denote the steady-state value of them. '$\widehat{x_t}$' continues to denote the percentage deviation of a variable x_t from its steady-state value. We outline the model structure in the main text. The linearised model equations are listed in the appendix.

[3] Under risk-pooling, $\hat{q}_t = \sigma(\hat{c}_t - \hat{c}_t^*) + v_t$, where the consumption is calculated from market-clearing output. Since the Euler equation was only loosely imposed in the model in deriving the IS curve, the consumption derived from the market-clearing condition will not be strictly the same as that implied by the IS curve. Thus we will have $\hat{q}_t = \sigma(\hat{c}_t - \hat{c}_t^*) + v_t = \sigma\frac{y}{c}\left[(\hat{y}_t - \hat{y}_t^*) - \hat{x}_t\right] + v_t$ where $\hat{x}_t$ is given by the difference between outputs and consumptions. This solution will only be the same as UIP if $\sigma\frac{y}{c}(-\hat{x}_t) + v_t = -\left(\frac{1}{\sigma}\frac{c}{y}\right)^{-1} err_t$, i.e., when $err_t = \hat{x}_t - \sigma\frac{y}{c}v_t$ holds, which it will not do in general since err_t includes elements in demand that are not explicitly in the model – such as investment and government spending on this ocassion.

3.1 The standard UIP version

3.1.1 Households

There is a continuum of measure one of representative households who work, consume and save; and have life-time utility:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t \varepsilon_{j,t} \left(\ln c_t - \psi \frac{n_t^{1+\eta}}{1+\eta} \right) \tag{8}$$

where c_t is consumption, n_t is labour hour, ψ is the preference of leisure, η is the inverse of wage elasticity, β is the time discount factor, and $\varepsilon_{j,t}$ is the time preference shock. The composite consumption index is defined by:

$$c_t \equiv [(1-\alpha)^{\frac{1}{\nu}} c_{h,t}^{\frac{v-1}{v}} + \alpha^{\frac{1}{\nu}} im_t^{\frac{v-1}{v}}]^{\frac{v}{v-1}} \tag{9}$$

where $c_{h.t}$ is the consumption on domestic goods, im_t is imports, v (> 0) is the substitutability between $c_{h.t}$ and im_t, α is the degree of openness.

The household budget constraint is:

$$c_{h,t} + q_t im_t + b_t + q_t bf_t + t_t = w_t n_t + (1 + r_{t-1})b_{t-1} + (1 + r^*_{t-1})q_t bf_{t-1} + \Pi_t \tag{10}$$

where q_t is the real exchange rate (defined as the units of domestic goods per unit of foreign goods), b_t and bf_t are holdings of home and foreign bonds, respectively, r_{t-1} and r^*_{t-1} are the home and foreign real interest rates, w_t is the real wage rate, t_t and Π_t are lump-sum tax payment and profit received, respectively.

The household problem is to maximise (8) by choosing $c_{h,t}$, im_t, n_t, b_t and bf_t, subject to (10). The first order conditions imply the demand for domestic and foreign goods, the labour supply, and the UIP condition:

$$E_t \hat{q}_{t+1} - \hat{q}_t = r_t - r^*_t \tag{11}$$

which suggests that home currency must depreciate/appreciate to eliminate any arbitrage opportunity should there be a positive/negative margin between the home and foreign interest rates.

3.1.2 Firms

There is a continuum of measure one of representative firms which produce differentiated goods using the same technology; for simplicity we assume a labour-only production function:

$$y_t = \varepsilon_{z,t} n_t \tag{12}$$

where y_t is the aggregate output, $\varepsilon_{z,t}$ is productivity.

Under Calvo (1983) pricing, which assumes that in each period only a fraction $(1 - \omega)$ of the firms are able to reset prices, the standard profit maximisation problem under the assumptions of a zero-inflation steady state and no past-inflation indexation implies the Phillips curve for domestic price inflation:

$$\pi_{h,t} = \beta E_t \pi_{h,t+1} + \kappa \widehat{mc}_t + \hat{\varepsilon}_{\pi,t} \tag{13}$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$, $mc_t (= w_t/\varepsilon_{z,t})$ is the real marginal cost of production, $\hat{\varepsilon}_{\pi,t}$ is price mark-up shock. Given the definition of CPI:

$$P_t = [(1-\alpha) P_{h,t}^{1-v} + \alpha (Q_t P^*_{h,t})^{1-v}]^{\frac{1}{1-v}} \tag{14}$$

where $P_{h,t}$ and $P^*_{h,t}$ are the price levels of domestic and imported goods, respectively, and Q_t is the nominal exchange rate (defined as the units of home currency per unit of foreign currency), CPI inflation may be shown as:

$$\pi_t = (1-\alpha)\pi_{h,t} + \alpha\pi^*_{h,t} + \alpha\Delta\hat{Q}_t \tag{15}$$

which is the weighted average of the domestic and foreign inflation, adjusted by the nominal exchange rate movement[4].

The firm profit in each period ($\Pi_t = y_t - w_t n_t$) is transferred to households, who are assumed to own these firms, as a lump-sum.

3.1.3 Monetary and fiscal policies

The central bank adjusts the nominal interest rate following a Taylor rule:

$$1 + R_t = (1 + R_{t-1})^{\rho_R}(1+\pi_t)^{(1-\rho_R)\phi_\pi}\left(\frac{y_t}{y_{t-1}}\right)^{(1-\rho_R)\phi_y}(1+r)^{(1-\rho_R)}\varepsilon_{R,t} \tag{16}$$

where the rate responds to both inflation (ϕ_π) and growth (ϕ_y), subject to inertia (ρ_R) and a monetary policy shock ($\varepsilon_{R,t}$).

The fiscal authority adjusts government spending, which is assumed to be a stationary exogenous process around its steady-state level:

$$g_t = \varepsilon_{g,t} g \tag{17}$$

where $\varepsilon_{g,t}$ is the shock to the spending.

3.1.4 Identities and shock processes

The goods market clearing requires:

$$y_t = c_{h,t} + g_t + im^*_t \tag{18}$$

where im^*_t is imports by the foreign economy, hence exports of the home economy.

The balance of international payments requires:

$$q_t\left[bf_t + im_t - (1 + r^*_{t-1})bf_{t-1}\right] = im^*_t \tag{19}$$

where in solving the model we impose the terminal condition that $\Delta bf = 0$ to find the equilibrium real exchange rate.

The real exchange rate is defined as:

$$q_t = \frac{Q_t P^*_{h,t}}{P_{h,t}} \tag{20}$$

The real interest rate is calculated by the Fisher equation:

$$r_t = R_t - E_t\pi_{t+1} \tag{21}$$

All shocks of the model, except for the productivity shock, are assumed to follow an AR(1) process in natural logarithm:

$$\hat{\varepsilon}_{i,t} = \rho_i\hat{\varepsilon}_{i,t-1} + u_{i,t} \tag{22}$$

[4] In deriving this, it is assumed that full PPP holds in the long run, such that $\frac{P_h}{P} = \frac{QP^*_h}{P} = 1$.

where $i = j, \pi, R, g$. The productivity shock, whose impact is assumed to be permanent, is let follow a simple ARIMA (1,1,0) process:

$$\hat{\varepsilon}_{z,t} - \hat{\varepsilon}_{z,t-1} = \Gamma - \delta(\hat{\varepsilon}_{z,t-1} - \hat{\varepsilon}_{z,t-2}) + u_{z,t} \tag{23}$$

where Γ is a constant, δ is the mean-reversing parameter. All $u's$ in the shock processes are *iid*.

3.2 The RP model equivalent

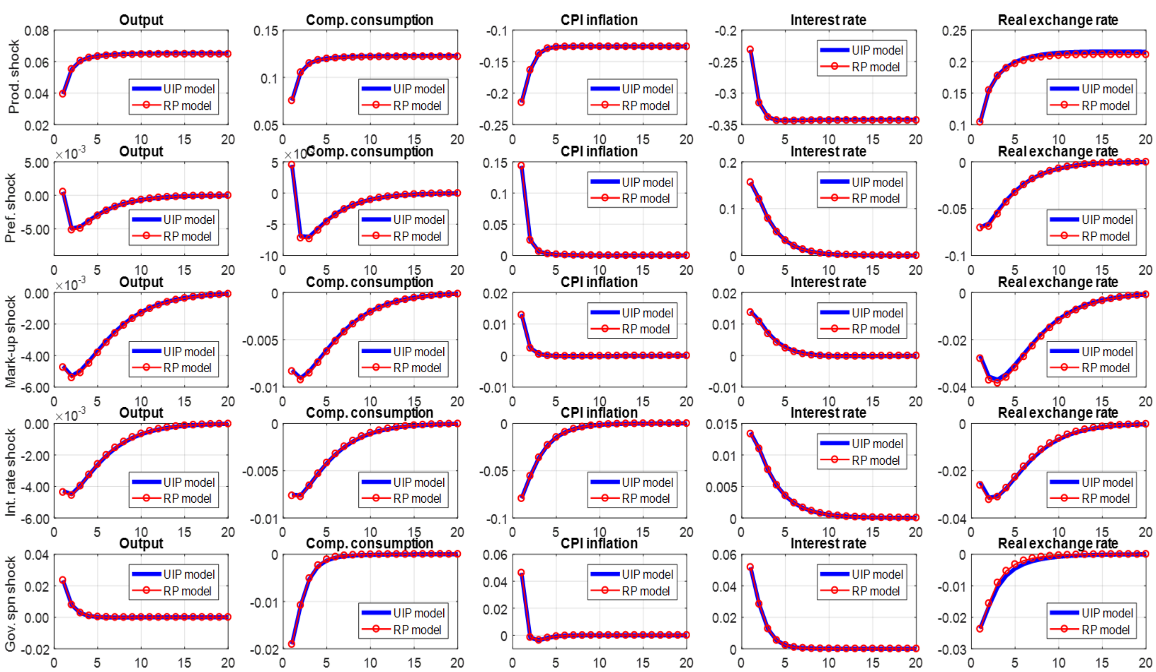
To construct the RP model equivalent, recall as reviewed earlier that arbitrage and the law of one price in a world with state-contingent nominal bonds implies $\frac{U_c(s_t,s_0)}{U_c^*(s_t,s_0)} = \Theta \frac{P(s_t,s_0)}{P^*(s_t,s_0)Q(s_t,s_0)}$ (Equ. (4) in the last section). Given that $U_t = \varepsilon_{j,t}\left(\ln c_t - \psi \frac{n_t^{1+\eta}}{1+\eta}\right)$ and hence $U_c = \varepsilon_{j,t} c_t^{-1}$ with our model, international risk sharing implies the RP condition:

$$\hat{q}_t = (\hat{c}_t - \hat{c}_t^*) - \left(\hat{\varepsilon}_{j,t} - \hat{\varepsilon}_{j,t}^*\right) \tag{24}$$

which ties the real exchange rate to the relative consumption of the two economies, subject to the difference in the two economies' time preference shocks. Hence, the RP equivalent of the standard UIP model simply replaces the UIP equation (11) with (24), *ceteris paribus*.

Figure 1 compares the key impulse response functions of the two models assigned the same parameter values[5]. Plainly the two models solve identically, as predicted by the theory; so RP (red, marked) and UIP (blue, unmarked) work in exactly the same manner. This verification is momentous, as it has been widely believed in previous work (e.g., Chari, Kehoe and McGrattan, 2002) that the two models are different. As noted earlier, in incomplete models where some parts of the structure are omitted, they indeed are, as found in MOZ. However with a full structural model as here, they are identical.

Figure 1: Key IRFs of the UIP and RP models



[5] For the purpose of illustration we only show the IRFs of a selected set of home variables in response to the home shocks. The other IRFs are available on request.

4 Testing the model – the method of Indirect Inference

We now go on to explain our empirical test procedure for evaluating the model's fit, the method of Indirect Inference, which has been widely used in applied macroeconomics.

Early applications of the method can be dated back to Smith (1993), Gregory and Smith (1991, 1993), Gourieroux, Monfort and Renault (1993), Gourieroux and Monfort (1996) and Canova (2005). The method was originally designed for estimating a structural model when the model's likelihood function (based on which 'direct' inferences could be implied) is too complex for regular algorithms to find the optimal parameter values. The basic idea is to first use an auxiliary model whose likelihood function is relatively simple for referential, indirect inferences to be found; the algorithm then searches for the parameter values of the structural model that enable the structural model to best replicate the inferences implied by the auxiliary model.

The method has been substantially developed by Minford, Theodoridis and Meenagh (2008), Meenagh, Minford and Wickens (2009), Le et al. (2011, 2016) and Minford, Wickens and Xu (2019) in recent years for it to be used as a formal statistical test on an already estimated or calibrated model. The idea in this task of testing is to first describe the data's behaviour in the sample by the auxiliary model (for which we use a VARX below). It then simulates the structural model by bootstrapping its innovations to create parallel simulations from each of which the counterpart auxiliary model estimates are found, generating a distribution of them under the null hypothesis that 'the structural model is the true model'; the innovations are calculated using the model equations and the data, given any potential set of model parameter values. It then asks whether the auxiliary model estimates found with the sample data came from this distribution with a high-enough probability according to the Wald test, where the structural model is considered to be rejected/not rejected by the data should the p-value of the test be below/above the usual 1%, 5% or 10% threshold[6].

In our practice of testing risk-pooling here we are interested in the models' ability to explain the international business cycle and real exchange rate dynamics; so we use a VARX(1) of both the home and foreign outputs, and the real exchange rate, as the auxiliary model:

$$Y_t = AY_{t-1} + BX_{t-1} + e_t \tag{25}$$

where $Y_t \equiv (\hat{y}_t, \hat{y}_t^*, \hat{q}_t)'$; the vector of exogenous variables is set to be $X_t \equiv (\hat{\varepsilon}_{z,t}, \hat{\varepsilon}_{z,t}^*, t)'$ under the assumption that both these outputs are cointegrated with productivity and a deterministic time factor; e_t is a vector of errors; A and B are matrices of coefficients.

The Wald test statistic is calculated by:

$$Wald = (\Phi^{Act} - \overline{\Phi})' \sum\nolimits_{(\Phi\Phi)}^{-1} (\Phi^{Act} - \overline{\Phi}) \tag{26}$$

where Φ^{Act} is a vector of the VARX estimates found with the actual data, and $\overline{\Phi}$ and $\sum_{(\Phi\Phi)}$ are, respectively, a vector of the mean values, and the variance-covariance matrix, of the same set of VARX estimates found

[6] It is worth pointing that the widely used Bayesian method, which is a method of estimating but not testing models, does not suit our model evaluation task. The Bayesian method does not test whether a model fits the data; it only tries to find parameter values (with set priors of them) that maximise the model's likelihood as much as possible, which may still be rejected by the data. The DSGE-VAR method due to Del Negro and Schorfheide (2006) does evaluate a model's absolute fit; however it is not a formal statistical test and therefore, provides no indication as to when a model should/should not be rejected. The Maximum Likelihood method does provide a likelihood test of data fit, which could be used for our task. But as Le et al. (2016) have shown, by Monte Carlo experiment on macro models, likelihood tests generally suffer from low power compared with indirect inference tests, and ML estimates are highly biased in small samples, as is well-known.

with the parallel simulations. In particular, we let these VARX estimates be the autoregressive coefficients and the variances of the equation residuals of (25), such that in effect the test evaluates the DSGE model's capacity in fitting the data's dynamic behaviour and volatility.

The null hypothesis of the test, H_0, is that 'the DSGE model is true'. The p-value of it is calculated by:

$$p = (100 - WP)/100 \tag{27}$$

where WP is the percentile of the Wald statistic found with the actual data under the distribution of it generated by the parallel simulations.

5 Is there consumer risk-pooling in the data?

As reviewed earlier, by single-equation tests Burnside (2019) rejected UIP – proven here as identical to having full risk-pooling – for a dozen pairs of industrialised economies. In this section we report our indirect inference, full-model test results for the same currency pairs against the US dollar based on pretty much the same sample period. The data, observed between 1971Q1 and 2018Q4, are collected from Euro-area-statistics, FRED, the IMF and the OECD; and are processed in the standard manner for them to be used by DSGE models. Values of the structural parameters are selected by a grid search over the permissible parameter space for them to deliver the minimum Wald statistic, (26), such that they fit the model to the data as much as possible[7]. The p-values of the tests are reported in comparison with Burnside's in Table 1.

Table 1: Model p-value by currency pair against USD

Currency	Country	Indirect inference full-model test of risk-pooling/UIP	Burnside (2019) single-equation test of UIP
AUD	Australia	0.150	0.006
CAD	Canada	0.099	0.009
DKK	Denmark	0.090	0.001
EUR	Euro Area	0.117	0.093
JPY	Japan	0.079	0.003
NOK	Norway	0.063	0.047
NZD	New Zealand	0.102	0.000
SEK	Sweden	0.079	0.904
CHF	Switzerland	0.098	0.014
GBP	UK	0.133	0.002

While only two out of the 10 currency pairs (i.e., EUR and SEK) in the Burnside test were found to comply with UIP at the 5% level, we find this to be the victim of bias to over-rejection by the single-equation method. When this bias is circumvented by our indirect inference full-model test, as our new evidence here shows, the hypothesisof UIP/risk-pooling is upheld – for all the currency pairs – generally with a high p-value showing a good fit. Hence, there is strong evidence of the wide existence of international risk-sharing

[7] This is in essence the indirect inference method for estimation. Due to the large number of economy pairs we estimate, we omit the sets of the estimated parameter values, which are available on request, for conciseness.

for consumption smoothing. The earlier 'empirical consensus' that UIP fails to fit, or the 'puzzle' that consumer risk-pooling is hard to find in the data despite the completeness of the international financial market, appears to be a statistical artefact that came from the misuse of single-regression tests on this issue. Open economy macro models in their 'standard' form, as assessed here, suffice to explain the key data features including those of the real exchange rate in a formal probability test; the 'more advanced' models that attempt to resolve the 'puzzle' by complicating the model structure in various ways (such as Chari, Kehoe and McGrattan, 2002) only complicate it unnecessarily and may damage the fit to the data in the process.

6 Conclusion

In this paper we have set out a full DSGE model of an open economy, paired with a model of another economy with which it trades and shares capital transactions – this economy is in all cases the US. We have shown that the model behaves identically under risk-pooling with contingent assets and uncovered interest parity, UIP, with non-contingent bonds. We have tested the risk-pooling/UIP hypothesis in 10 developed country pairs under the powerful indirect inference testing procedure and in all cases we do not reject the hypothesis. While it has been usual to consider the hypothesis as not holding up empirically, this view has emerged from tests on single-equation studies which do not impose the full set of restrictions that bind in a full structural DSGE model. Thus when these are imposed, as is appropriate, the hypothesis is upheld by the data universally.

References

[1] Burnside, C. (2019), 'Exchange Rates, Interest Parity, and the Carry Trade', *Oxford Research Encyclopedia of Finance and Economics*, DOI: 10.1093/acrefore/9780190625979.013.315.

[2] Calvo, G.A. (1983), 'Staggered Prices in a Utility-Maximizing Framework', *Journal of Monetary Economics,* Vol. 12(3), pp.383–398.

[3] Canova, F. (2005), *Methods for Applied Macroeconomic Research*, Princeton University Press, Princeton.

[4] Chari, V., Kehoe, P.J. & McGrattan, E. (2002), 'Can sticky price models generate volatile and persistent real exchange rates?', *Review of Economic Studies*, Vol. 69(3), pp. 533–563.

[5] Clarida, R., Gali, J. & Gertler, M. L. (1999), 'The Science of Monetary Policy: a New Keynesian Perspective', *Journal of Economic Literature*, Vol. 37(4), pp.1661-1707.

[6] Delcoure, N., Barkoulas, J., Baum, C.F. & Chakraborty, A. (2003), 'The forward rate unbiasedness hypothesis reexamined: evidence from a new test', *Global Finance Journal*, Vol.14, pp.83– 93.

[7] Del Negro, M. & Schorfheide, F. (2006), 'How Good Is What You've Got? DSGE-VAR as a Toolkit for Evaluating DSGE Models', *Economic Review*, Vol. 91(2), pp.21-37, Federal Reserve Bank of Atlanta.

[8] Gourieroux, C. & Monfort, A. (1996), *Simulation Based Econometric Methods*, CORE Lectures Series, Oxford University Press, Oxford.

[9] Gourieroux, C., Monfort, A. & Renault, E. (1993), 'Indirect inference', *Journal of Applied Econometrics*, Vol. 8(S), pp.85-118.

[10] Gregory, A.W. & Smith, G.W. (1991), 'Calibration as Testing: Inference in Simulated Macroeconomic Models', *Journal of Business and Economic Statistics*, Vol. 9(3), pp.297-303.

[11] Gregory, A.W. & Smith, G.W. (1993), 'Calibration in macroeconomics', *Handbook of Statistics*, G. S. Maddala, ed., Chapter 11, pp.703-719, St. Louis, MO.: Elsevier.

[12] Hess, G. & K. Shin, (2000), 'Risk Sharing within and Across Regions and Industries,' *Journal of Monetary Economics*, Vol.45, pp.533-560.

[13] Isard, P. (2006), 'Uncovered Interest Parity', IMF working paper, 06/96, April.

[14] Le, M., Meenagh, D., Minford, P. & Wickens, M. (2011), 'How much nominal rigidity is there in the US economy? Testing a new Keynesian DSGE model using indirect inference', *Journal of Economic Dynamics and Control*, Vol. 35(12), pp.2078-2104.

[15] Le, M., Meenagh, D., Minford. P., Wickens, M. & Xu, Y. (2016), 'Testing Macro Models by Indirect Inference: A Survey for Users', *Open Economies Review*, Vol. 27(1), pp.1–38.

[16] Meenagh, D., Minford, P. & Wickens, M. (2009), 'Testing a DSGE Model of the EU Using Indirect Inference', *Open Economies Review*, Vol. 20(4), pp.435-471.

[17] Minford, P., Ou, Z. & Zhu, Z. (2021), 'Can a small New Keynesian model of the world economy with risk-pooling match the facts?', *International Journal of Finance and Economics*, Vol. 26(2), pp.1993-2021.

[18] Minford, P., Ou, Z. & Zhu, Z. (2022), 'Is there consumer risk-pooling in the open economy? The evidence reconsidered', *Open Economies Review*, Vol. 33, pp.109-120.

[19] Minford, P., Theodoridis, K. & Meenagh, D. (2008), 'Testing a Model of the UK by the Method of Indirect Inference', *Open Economies Review*, Vol. 20(2), pp.265–291.

[20] Minford, P., Wickens, M. & Xu, Y. (2019), 'Testing Part of a DSGE Model by Indirect Inference', *Oxford Bulletin of Economics and Statistics*, Vol. 81(1), pp.178-194.

[21] Smith, A. (1993), 'Estimating nonlinear time-series models using simulated vector autoregressions', *Journal of Applied Econometrics*, Vol. 8(S), pp.63-84.

Appendix

Listing of the linearised model

Home economy

- Demand for home goods:

$$\hat{c}_{h,t} = E_t\hat{c}_{h,t+1} + \Upsilon_1(E_t\widehat{im}_{t+1} - \widehat{im}_t) - \Upsilon_2\frac{1}{1+r}(r_t - r) - \Upsilon_2\left(E_t\hat{\varepsilon}_{j,t+1} - \hat{\varepsilon}_{j,t}\right) \tag{A.1}$$

where $\Upsilon_1 = \left[1 + \frac{1+\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\nu}}\left(\frac{im}{y}\right)^{\frac{v-1}{v}}\left(\frac{c_h}{y}\right)^{\frac{1-v}{v}}}{\left(\frac{v-1}{v}\right)\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\nu}}\left(\frac{im}{y}\right)^{\frac{v-1}{v}}\left(\frac{c_h}{y}\right)^{\frac{1-v}{v}}}\right]^{-1}$, $\Upsilon_2 = \left[1 + \frac{\left(\frac{v-1}{v}\right)\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\nu}}\left(\frac{im}{y}\right)^{\frac{v-1}{v}}\left(\frac{c_h}{y}\right)^{\frac{1-v}{v}}}{1+\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\nu}}\left(\frac{im}{y}\right)^{\frac{v-1}{v}}\left(\frac{c_h}{y}\right)^{\frac{1-v}{v}}}\right]^{-1}$.

- Demand for imports:

$$\widehat{im}_t = \Psi_1\hat{c}_{h,t} - \Psi_2\hat{q}_t \tag{A.2}$$

where $\Psi_1 = \left[\frac{\left(\frac{1}{q}\right)\left(\frac{c_h}{y}\right)\left(\frac{im}{y}\right)^{-1}+\left(\frac{1}{q}\right)\left(\frac{1}{v}\right)\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\nu}}\left(\frac{c_h}{y}\right)^{\frac{1}{v}}\left(\frac{im}{y}\right)^{-\frac{1}{v}}-\left(\frac{v-1}{v}\right)\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\nu}}\left(\frac{c_h}{y}\right)^{\frac{v-1}{v}}\left(\frac{im}{y}\right)^{\frac{1-v}{v}}}{1+\frac{1}{v}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\nu}}\left(\frac{c_h}{y}\right)^{\frac{v-1}{v}}\left(\frac{im}{y}\right)^{\frac{1-v}{v}}-\frac{1}{q}\left(\frac{v-1}{v}\right)\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\nu}}\left(\frac{c_h}{y}\right)^{\frac{1}{v}}\left(\frac{im}{y}\right)^{-\frac{1}{v}}}\right]$,

$\Psi_2 = \left[\frac{1+\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\nu}}\left(\frac{c_h}{y}\right)^{\frac{v-1}{v}}\left(\frac{im}{y}\right)^{\frac{1-v}{v}}}{1+\frac{1}{v}\left(\frac{1-\alpha}{\alpha}\right)^{\frac{1}{\nu}}\left(\frac{c_h}{y}\right)^{\frac{v-1}{v}}\left(\frac{im}{y}\right)^{\frac{1-v}{v}}-\frac{1}{q}\left(\frac{v-1}{v}\right)\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\nu}}\left(\frac{c_h}{y}\right)^{\frac{1}{v}}\left(\frac{im}{y}\right)^{-\frac{1}{v}}}\right]$.

- Supply of labour:

$$\hat{n}_t = \frac{1}{\eta}\hat{w}_t - \frac{1}{\eta}\Xi\hat{c}_{h,t} - \frac{1}{\eta}\left(\frac{1}{\Upsilon_2} - 1\right)\widehat{im}_t \tag{A.3}$$

where $\Xi = \left[\frac{1+\frac{1}{v}\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\nu}}\left(\frac{im}{y}\right)^{\frac{v-1}{v}}\left(\frac{c_h}{y}\right)^{\frac{1-v}{v}}}{1+\left(\frac{\alpha}{1-\alpha}\right)^{\frac{1}{\nu}}\left(\frac{im}{y}\right)^{\frac{v-1}{v}}\left(\frac{c_h}{y}\right)^{\frac{1-v}{v}}}\right]$.

- Production function:

$$\hat{y}_t = \hat{\varepsilon}_{z,t} + \hat{n}_t \tag{A.4}$$

- Real marginal cost:

$$\widehat{mc}_t = \hat{w}_t - \hat{\varepsilon}_{z,t} \tag{A.5}$$

- Domestic price inflation:

$$\pi_{h,t} = \beta E_t\pi_{h,t+1} + \kappa\widehat{mc}_t + \hat{\varepsilon}_{\pi,t} \tag{A.6}$$

where $\kappa = \frac{(1-\omega)(1-\beta\omega)}{\omega}$.

- CPI inflation:

$$\pi_t = (1-\alpha)\pi_{h,t} + \alpha\pi^*_{h,t} + \alpha\Delta\hat{Q}_t \tag{A.7}$$

where α is openness.

- Taylor rule:

$$R_t = \rho_R R_{t-1} + (1-\rho_R)[r + \phi_\pi\pi_t + \phi_y(\hat{y}_t - \hat{y}_{t-1})] + \hat{\varepsilon}_{R,t} \tag{A.8}$$

- Government spending:

$$\hat{g}_t = \hat{\varepsilon}_{g,t} \tag{A.9}$$

- Market clearing:

$$\hat{y}_t = \frac{c}{y}\hat{c}_{h,t} + \frac{g}{y}\widehat{g}_t + \frac{im^*}{y}\widehat{im_t^*} \tag{A.10}$$

- Fisher equation:

$$r_t = R_t - E_t\pi_{t+1} \tag{A.11}$$

- Balance of payment equation:

$$\frac{bf}{y}\widehat{bf}_t = \frac{bf}{y}(r_{t-1}^* - r^*) + (1+r^*)\frac{bf}{y}\widehat{bf}_{t-1} + \frac{1}{q}\frac{im^*}{y}\left(\widehat{im_t^*} - \hat{q}_t\right) - \frac{im}{y}\widehat{im}_t \tag{A.12}$$

- Uncovered interest parity:

$$\hat{q}_t - E_t\hat{q}_{t+1} = R_t^* - R_t - (E_t\pi_{t+1}^* - E_t\pi_{t+1}) \tag{A.13}$$

- Nominal exchange rate:

$$\Delta\hat{Q}_t = \Delta\hat{q}_t - \pi_{h,t}^* + \pi_{h,t} \tag{A.14}$$

Foreign economy

- The foreign economy equations are in-form the same as the home economy's, except that where the exchange rate terms are involved, the terms take an opposite sign. Similarly, all the steady-state exchange rate terms are inverted.