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Why Fixed-Price Policy Prevails: The Effect of Trade Frictions and Competition

Cemil Selcuk

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Cardiff Business School
Cardiff University
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
business.cardiff.ac.uk

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Abstract: Fixed-price selling is common in today's markets. While previous research in marketing and economics literatures provide several intuitive reasons for the emergence of fixed-price selling (e.g. clarity and simplicity of managing the fixed-price process, reduced coordination and information costs) our study offers an entirely different rationale—based on market competition and trade frictions—that explains the prevalence of fixed-price selling. Using a market equilibrium approach, and employing a novel competitive search framework to account for a fully competitive and dynamic market, we offer a new and micro-founded account for the widespread use of fixed pricing policy. Considering three important market characteristics—customer risk aversion, the degree of trade frictions and the level of market competition—we explore the strategic choice between the fixed-price, best-offer, and over-the-sticker pricing policies. Unlike the standard models in the literature, which are based Hotelling, Cournot, Bertrand frameworks, the competitive search framework enables us to model competition with a large number of buyers and sellers, and to vary the degree of competition accordingly. We find that fixed pricing emerges as the unique or the de-facto selling rule in most parameter regions. Indeed, the only region where haggling matters is the case in which customers are risk neutral and trade frictions are significant and market competition is moderate.

Keywords: fixed-price selling, haggling, risk aversion, trade friction, competition

1 Introduction

Fixed-price selling appears to become the norm in many modern day markets (Phillips, 2012). Haggling, on the other hand, has traditionally been associated with markets involving large purchases including houses, cars, boats, high-end jewelry and so on. However, even in these haggling dominated markets, there is a notable trend for fixed-price selling. While companies like CarMax—a Fortune 500 used car superstore—have been offering no-haggle prices for some time, fixed-price selling has gained further momentum recently with Costco, AAA and Lexus announcing haggle-free selling programs (Halzack, 2015; Chappell, 2015) and with Sonic—the fourth-largest dealer group in the US with 105 stores—eliminating all haggling in its outlets with a decision to use a one-price no-haggle selling (Taylor III, 2014). In addition, in the housing market several realtors and home sellers report having moved to a no-haggle pricing approach which they suggest "seems to be working" (Ponce, 2015; Ludeman, 2008).

Does fixed price selling emerge just because it makes the buying process more convenient and hassle-free for the consumer, or could there be a deeper underlying mechanism explaining the adoption of fixed pricing? In order to better understand the drivers and prevalence of fixed-price policy, we specifically concentrate on large-purchase markets in which the dominant practice of haggling has been recently shifting towards a fixed-price policy. These markets are also notable as they often represent the most significant purchases in many consumers' lives. In addition, the search and matching nature of such markets—the fact that buyers and sellers need to spend considerable time and effort in order to buy or sell a limited inventory product—makes them a complex but highly important context to study pricing decisions.

Three factors appear to be critical in determining the emerging pricing policy in such markets. First, as these represent big purchases involving large sums, customers may naturally be risk averse. Second, trade frictions seem to be playing a significant role where shopping around requires considerable time, effort and other resources, and waiting is costly especially when a product (or a customer) available today may not be available tomorrow. Finally, market competition plays a key role in pricing decisions in that in highly competitive markets, sellers appear to give way for haggling policies, whereas in markets characterized by a lack of competition, customers may end up paying well above the posted price. These observations point to an important link between the equilibrium pricing policy, the degree of competition, as well as the degree of trade frictions in the market. In this paper, considering the aforementioned channels, we investigate the practice of three common pricing strategies—best-offer, fixed pricing and over-the-sticker pricing[1] —and explain why fixed pricing is the emergent and preferred strategy in most cases.

Previous research in marketing provides several reasons for the emergence of the fixed-price selling strategy. These include, among others, clarity and simplicity of managing the fixed-price process (or complexity of managing the negotiation process), reduced information costs (e.g., customers know how much they should pay in advance), reduced negotiation costs, economies of scale in pricing and reduced coordination costs (Riley and Zeckhauser, 1983; Phillips, 2012). Although these are all intuitive motives to employ fixed-price selling, our paper offers an entirely different rationale, based on competition and trade frictions, that explains the prevalence of fixed-price selling.

In studying our research questions, in order to account for fully competitive and dynamic markets which are characterized by trade frictions, we employ a unique directed search (or competitive search) setup. While previous work in marketing and pricing, notably Desai and Purohit (2004) and Kuo et al. (2011) , investigated fixed and flexible (i.e., haggling) pricing strategies, these studies were prone to several limitations. Firstly, despite the significant role of market competition in many real life buying-selling contexts, this line of work considered no-competition (i.e., a monopolist seller) or limited (e.g. duopolistic) competition as in Bertrand, Cournot, or Hotelling settings. Secondly, these work typically assume exogenous arrival to sellers, that is, customers arrive at the sellers at an exogenous rate and then make a purchasing decision. Our paper relaxes both of these commonly held assumptions by characterizing a fully competitive and dynamic market, and by endogenizing the

[1] In the automobile market, for instance, customers typically pay more than the sticker price if they are after a newly released model. Edmunds.com, a major online resource for automotive information, calls this practice "Over-the-Sticker Pricing". The same is true in the housing market as well. An article in Washington Times (May 1, 2013) discusses the surge in demand for residential real estate in DC area and how this affects the way properties are sold. It reports that many houses are sold the day they are listed, while some Capitol Hill homes fetched more than double the asking price.

expected demand depending on the list price and the pricing strategies posted by the sellers. As such, our study which introduces a competitive search framework and is based on a market equilibrium approach provides also an important methodological contribution to marketing literature.

The model presents a micro-founded account for the widespread use of fixed pricing policy and, along the way, provides several connected insights. A main conclusion is that haggling matters only if (i) customers are risk neutral, and (ii) trade frictions are significant, and (iii) the degree of market competition is moderate (not low, not high). In all other scenarios fixed price selling emerges either as the unique selling rule or as the de-facto selling rule. We reach this conclusion by investigating the model in various parameter regions (e.g., risk aversion/neutrality, high/low competition, high/low trade frictions).

If customers are risk averse, then we show that fixed-pricing emerges as the unique equilibrium pricing policy. The reason is this. With flexible rules (best-offer or over-the-sticker pricing) the sale price typically differs from the posted price because either the seller or the buyer ends up asking for a better deal. Customers, therefore, face some uncertainty in that they do not know in advance what the sale price is going to be—a notion we label as *price uncertainty.* Fixed pricing eliminates this uncertainty, which is why, all else equal, customers are more likely to shop at such stores. In a competitive market, this inclination gives an edge to fixed price sellers; hence in equilibrium all sellers end up being fully committed to charge what they post.

If customers are risk-neutral, then they do not mind the aforementioned price uncertainty, so there exists a continuum of equilibria where fixed and flexible rules are payoff equivalent and coexist in the same marketplace. Interestingly, the characteristic of each pricing rule permeates into the equilibrium price associated with it. Best-offer sellers, for instance, correctly anticipate the discounts to be conceded to the customers, so they inflate their list price in the first place. Similarly sellers trading via over-the-sticker pricing realize that they will end up charging more than the posted price, so they advertise lower prices up-front. Fixed-price sellers, on the other hand, charge what they post, so they advertise moderate prices.

Even though fixed and flexible rules may coexist, the difference between them is pronounced only if trade frictions are high. We prove that as trade frictions vanish, prices emerging from bargaining converge to the equilibrium fixed price. Indeed with no trade frictions, players face the same outlook ex-ante or ex-post (i.e. before a match or in a match); so, the ex-post bargained price approximates to the ex-ante fixed price. Since transactions are bound to be settled at the equilibrium fixed price anyway, the availability of haggling becomes immaterial, and therefore, fixed pricing emerges as the de-facto selling rule. Remarkably the convergence result is robust to the underlying bargaining protocol. Indeed, prices resulting both from Nash Bargaining as well as Strategic Bargaining—two most commonly used and distinct bargaining protocols—converge to the equilibrium fixed price.

The discussion so far reveals that the only area in the parameter space where fixed and flexible rules may coexist without being practically identical to each other is the region where buyers are risk averse and trade frictions are significant. But even in this region the difference between fixed and flexible rules, proxied by the degree of price dispersion,[2] is practically nonexistent if market

[2] The model implies equilibrium price dispersion both in *list prices* as well as in *sale prices*. The dispersion in list

competition is sufficiently high or sufficiently low; hence fixed pricing, again, emerges as the de-facto selling rule.

In summary, our investigation of the model in three independent dimensions (customer risk aversion, degree of competition, and degree of trade frictions) reveals that fixed pricing emerges as the unique or de-facto selling rule in most parameter regions. The only exception where haggling matters is the case in which customers are risk neutral, trade frictions are significant, and market competition is moderate.

2 Related Literature

Previous research in marketing have studied implications of haggling and bargaining (Pennington, 1968; Mathews et al., 1972; Tauber, 1972; Neslin and Greenhalgh, 1983; Dwyer et al., 1987). These include a large body of research in the context of channel relationships (Iyer and Villas-Boas, 2003; Dukes et al., 2006; Srivastava et al., 2000; Draganska et al., 2010; Guo and Iyer, 2013) as well as in consumer level transactions such as automobile purchases (Chen et al., 2008). Another stream of marketing research that investigates haggling is the practice of name-your-own-price retailers. Terwiesch et al. (2005) study online haggling in the context of a name-your-own-price retailer who waits for potential buyers to submit offers for a given product and then chooses to either accept or reject them. Consumers whose offers have been rejected may continue haggling with additional offers. In a similar domain, Hann and Terwiesch (2003) examine frictional costs of online transactions at a name-your-own-price retailer. Our model diverges from above by offering strategic insights to sellers' choice of pricing policies in fully competitive markets.

Pricing is an important tool for firms with which they can gain strategic advantage in the marketplace (see Özer and Phillips (2012) for a review) and to that end game-theoretic models of pricing have been widely employed in marketing literature to analyze these strategic decisions (Kopalle and Shumsky, 2012). Studying different selling and pricing policies, Wernerfelt (1994) provides a comparative analysis of three selling formats—price advertising, seller colocation, and bargaining—and evaluates their relative attractiveness. Among others, he finds that under high duopolistic competition, bargaining may be profit maximizing for the sellers as it helps them avoid the costly Bertrand competition. In other related works that investigate fixed-price and haggling policies, Riley and Zeckhauser (1983) examine a monopolist seller facing risk neutral customers and suggest that fixed pricing is optimal in comparison to negotiations. This is because while haggling may be advantageous in terms of price discrimination, the gains from haggling are more than offset when buyers refuse purchasing at higher prices. Wang (1995) creates a dynamic model and concludes that bargaining is preferable if it costs the same as fixed pricing or if the common costs are high enough. Bester (1993) on the other hand, focuses on the role of quality uncertainty in determining pricing policies.

prices is a corollary of the coexistence of multiple selling policies in the same marketplace, because sellers competing with different rules post different list prices. Sale prices, too, are dispersed. Under flexible rules, the sale price is transaction specific and depends on the local demand, which, along the equilibrium path, is stochastic; hence the dispersion in sale prices. We can analytically characterize the distribution of prices, and study how it is affected by different pricing rules and market features.

He proposes that fixed pricing is more competitive than negotiated pricing, however, it may lead to a deterioration in product quality. Also in a related work that examines take-it-or-leave-it and negotiation pricing, Kuo et al. (2011) model a dynamic pricing problem as a function of inventory and time (remaining selling season). They demonstrate that when the inventory level is high and/or the remaining selling season is short, negotiation could be an effective tool to achieve price discrimination. The focus in their work is operational (e.g., inventory) decisions and, unlike our setup, they consider a monopolist seller.

The closest work to ours is Desai and Purohit (2004) who similarly investigate fixed-price and best-offer strategies, and show that depending on the parameters, there may exist equilibria in which both firms choose fixed prices, both firms offer haggling, or where one firm offers haggling and the other charges fixed prices. An important finding of theirs is that the benefits of price discrimination in a monopoly setting do not necessarily transfer over to a competitive environment. Our model differs from Desai and Purohit (2004) in several aspects. First, while they consider a duopoly setting, we substantially extend their model by characterizing a fully competitive and dynamic market. Second, in our model customers arrive to stores endogenously and their arrival rates depend on the pricing policy chosen by a firm and its list price. Finally, in our model customers are homogeneous as our focus is on the role of degree of competition, customer risk aversion and search frictions. By examining the role of these three characteristics, our model sheds additional light on firms' choice of fixed and haggling price policies, and their strategic implications in competitive markets.

3 Model

3.1 Setup

Time is discrete and runs indefinitely. The market is populated by a continuum of buyers and sellers where each seller has one item that he is willing to sell above his reservation price, zero, and each buyer wants to purchase one item below his reservation price, one. The market is decentralized and operates via competitive search. At each period, sellers simultaneously and independently advertise a list price $r_{m,t} \in [0, 1]$ and a commitment declaration $m \in \mathbb{M} = \{b, f, o\}$ indicating whether and to what extent the list price is open to renegotiation. There are three commitment alternatives sellers can choose from:

- Fixed pricing (f). Sellers are fully committed to charge what they post; the transaction necessarily takes place at the list price.
- Best-offer pricing (b). Sellers commit not to request renegotiations but buyers have the option to do so. Specifically, if at the time of transaction a buyer foresees a lower price via bargaining, then he is free to make a counteroffer. Under this rule the sale price is less than or equal to the posted price.
- Over-the-sticker pricing (o). This is the opposite of best-offer pricing. Sellers reserve the right to renegotiate and ask for a higher price if, at the point of transaction, they believe such a price

is obtainable. With this scheme the sale price is greater than or equal to the list price.[3]

Buyers observe sellers' selections and independently choose to match with one seller. Since a seller might be visited by several buyers, we refer to $n = 0, 1, 2...$ as the realized demand. If $n \geq 2$, then each buyer has an equal chance $1/n$ of being served. In each match, a trade process takes place that might include renegotiation of the originally posted price (more details below). At the end of this process players realize their gains. Specifically, if transaction occurs at price p_t then the seller obtains payoff $\beta^{t-1}p_t$ and the buyer obtains $\beta^{t-1}v\,(1-p_t)$, where $\beta \in (0,1)$ is the common discount factor.

The market starts with a measure of s_1 sellers and b_1 buyers. At the end of each period players who have transacted exit the market while the remaining players replay the same game in the next period. At the beginning of each period $t = 2, 3...$ a new cohort of b_t^{new} buyers and s_t^{new} sellers enter the market joining the existing players. The buyer-seller ratio $\lambda_t \equiv b_t/s_t$, which is one of the key parameters of the model, proxies the degree of competition in the market: a low value of λ_t implies that the market is highly competitive whereas a high value of λ_t implies the opposite.

3.2 Discussion of the Model

Market Characteristics & the Competitive Search Model. Our modeling approach, based on a competitive search framework, provides unique advantages to study our research question. In examining the prevalence of fixed price policy, we concentrate on markets in which the common practice of haggling is being replaced by fixed-price selling (e.g., the housing market or the automobile market). Price competition is the primary driver of selecting pricing policies and associated prices in such markets[4] (Bitran and Caldentey, 2003). Despite the significance of market competition, previous papers in the pricing literature either consider a monopolist seller who receives customer exogenously, or they consider limited competition based on variations of Bertrand, Cournot or Hotelling models typically with only two firms; see, Kopalle and Shumsky (2012) for an overview of game theoretic pricing models. Unlike these approaches, our setup can encompass infinitely many buyers and sellers. In addition, as bargaining is a key selling mechanism that we investigate, our modelling approach allows us to examine both Nash Bargaining and Rubinstein Bargaining to establish robust results. To the best of our knowledge, our paper is the first in the literature that studies the selection of fixed and flexible pricing rules in a fully competitive and dynamic environment.[5]

[3] One can consider a fourth scenario where both the seller and the buyer may have the option to renegotiate, and therefore, the sale price may go above and below the list price. This case is a combination of the best-offer and over-the-sticker pricing rules and it can be analyzed similarly. However, the analysis does not produce significant new insights and therefore is omitted from our discussion.

[4] In a report analyzing the supply of new cars into the UK market, **?** highlights that competitors' price specifications is the most important factor in all major automakers' setting of their own list prices.

[5] The competitive search approach has its roots in search theory and, unlike the traditional random search, the demand at each store is endogenous and it strategically depends on the terms of trade each seller posts and how those terms compare with the rest of the market. Thanks to these features the competitive search paradigm has gained significant popularity within the search literature. Even though most of the studies using the competitive search approach have assumed that sellers compete via fixed pricing (Burdett et al., 2001; Shimer, 2005) there are several studies where sellers can compete with other pricing rules such as auctions or bargaining, e.g. (**??**Selcuk and Gokpinar,

Furthermore, in the aforementioned markets sellers typically have single inventories (e.g., an individual selling a house or a car), and buyers' visits are endogenous, depending on the posted price and the selling policy. Whether and to what extend the initial asking price goes up or down seems to depend on the level of competition. For example, in a highly "thick" market with many buyers and few sellers, buyers may well end up paying more than the asking price. Indeed, *The Independent* reports that in London's property market, which is notorious for its high demand and short supply, one in five buyers pay more than the asking price (Johnson, 2014). Some sellers demonstrate their intention of over-the-sticker pricing policy by adding words such as "from" or "offers exceeding" along with their list price; see for example, zoopla.co.uk. In contrast, in localities where there is relatively abundant supply and limited demand, the opposite happens, i.e., buyers manage to haggle down the list price. (These pricing practices are in line with our stylized definitions of over-the-sticker pricing and best-offer pricing.)

For making such large purchases, effectively, many buyers search and match with many competing sellers in a decentralized market with trade frictions. Our model incorporates all the aforementioned characteristics and allows us to examine implications of various pricing policies in an analytically tractable way.

Trade Frictions. The decentralized and search-and-matching nature of the directed search model coupled with sellers' limited inventories create trade frictions in that no one is guaranteed to trade immediately (Burdett et al., 2001). Multiple buyers may show up at the same seller, so, all but one walks out empty-handed, whereas another seller may well end up with no customer at all. If a player cannot trade today, then he needs to retry in the subsequent period and it may take several periods before one can actually buy or sell, but, of course, waiting is costly as future utilities are discounted. Frictions are exacerbated by the magnitude of the discount factor: the lower the value of β the higher the opportunity cost of not being able to trade, and therefore, the more pronounced is the impact of frictions on equilibrium objects. So, by varying β we are able to discern the relationship between the degree of trade frictions, the selling rule in place and the equilibrium prices.

Bargaining Costs. Before delving into the analysis, it is worth highlighting that throughout the paper we assume costless bargaining, and the reason is this. We demonstrate that fixed-price selling is the unique or de-facto selling rule in most parameter regions even in the absence of exogenous bargaining costs. So, if we included bargaining costs, then clearly this would favor the fixed-price policy, and therefore, it would confound our findings. In other words, our results are more conservative without bargaining costs. Furthermore, note that a common explanation offered for some firms' recent shift away from haggling and towards fixed-pricing is the notion that customers don't like haggling. Our results demonstrate that even in the absence of customers' dislike for bargaining, a competition-based mechanism can alone provide a rationale for fixed-price selling.

2015). Eeckhout and Kircher (2010) show that sellers can be indifferent to all "payoff-complete" pricing mechanisms, including fixed pricing and second price auctions. Our paper diverges from existing literature by considering negotiable posted prices, both up (i.e., over the sticker pricing) and down (i.e., best-offer pricing), in a dynamic set up. That is, while most existing studies have only considered one-shot setups, in our model, players who are unable to sell or buy in the first period try again in subsequent periods.

3.3 Bargaining, Commitment and Sale Price

We will move backwards to analyze the model. First we will study the determination of the bargained price in a match. Then we will examine the decision of buyers on where to shop. Finally, we will look at how sellers pick their prices and pricing policies.

Game theoretic analyses of bargaining adopt one of two approaches: the axiomatic Nash Bargaining and the strategic Rubinstein Bargaining. Below we study how the equilibrium bargaining price is pinned down under both methods.[6] One might wonder why we describe two separate methods in detail and not focus on, say, just Nash Bargaining. The reason is this. One of the key results in our paper is the fact that as trade frictions disappear, the equilibrium bargaining price converges to the equilibrium fixed price. The implication is that, since players would not agree on anything but the equilibrium fixed price anyway, the availability of bargaining becomes immaterial and fixed pricing emerges as the de-facto selling rule. The result is important, but one may wonder if it is specific to the underlying bargaining protocol. This is why we analyze the two most widely used protocols in the literature and show that the convergence result is indeed robust, i.e. it holds under both Nash Bargaining as well as Strategic Bargaining.

3.3.1 Nash Bargaining

Consider a seller with n customers and suppose that the sale price is determined via Nash Bargaining. Let $y_{n,t}$ denote the bargained price. In case of agreement the seller gets payoff $y_{n,t}$ and the buyer gets $v\left(1-y_{n,t}\right)$. The outside options are βu_{t+1} for buyers and $\beta \pi_{t+1}$ for sellers, which represent their values of search, i.e. the present value of being a buyer or seller in period $t+1$. We will pin down these expressions subsequently but for now we take them as given. The total payoff in a transaction cannot exceed the maximum possible surplus, one; thus we have

$$u_t + \pi_t \leq 1 \text{ for all } t.$$

Let $\theta_n \in (0,1)$ denote the buyer's bargaining power. The seller's bargaining power, therefore, is $1-\theta_n$. We assume that $1-\theta_n$ rises in n (or θ_n decreases in n), i.e. $1-\theta_{n+1} > 1-\theta_n$. In words, the larger the local demand n, the stronger the seller's bargaining power at the table.[7] The negotiated

[6] All players use the same bargaining method (Nash or Rubinstein) and this fact is common knowledge, i.e. there is no ambiguity regarding which method is going to be used in case someone wants to negotiate.

[7] The fact that θ_n falls in n is not an ad-hoc assertion. The assumption ensures that the bargained price rises in the local demand n, i.e. the more buyers demand the item, the stronger the seller's position at the table, and therefore, the higher the price. From a technical point of view, it is simpler to capture this property by assuming that negotiations takes place in one round but the seller's bargaining power rises with n, which is the approach we take in here. Alternatively, however, one can fix the bargaining power and assume that the seller negotiates with his customers sequentially over a number of sub-periods (bargaining rounds). Under this specification the local demand n filters into the bargained price not through the bargaining power but through the outside options: the higher the local demand n, the less likely is a buyer to be matched with the seller in the subsequent bargaining round, and therefore the smaller his outside option, and therefore the higher the price. In Appendix B we examine this alternative specification and pin down the equilibrium bargained price analytically. Remarkably the closed form solution of the equilibrium bargained price under both scenarios is the same, upto a relabelling of terms.

price can be found as the solution to the Nash product below:[8]

$$\max_{y_{n,t}} \ (v(1-y_{n,t})-\beta u_{t+1})^{\theta_n} \times (y_{n,t}-\beta\pi_{t+1})^{1-\theta_n} .$$

The FOC is given by

$$\frac{1-\theta_n}{\theta_n}=\frac{(y_{n,t}-\beta\pi_{t+1})\, v'\,(1-y_{n,t})}{v\,(1-y_{n,t})-\beta u_{t+1}}. \tag{1}$$

Let $y_{n,t}^{Nash}$ denote the unique value of $y_{n,t}$ solving (1). It is easy to show that $y_{n,t}^{Nash}$ rises in n, i.e. the higher the local demand, the higher the seller's share from the "pie". In addition, the degree of market competition affects bargained prices as well (through outside options u_{t+1} and π_{t+1}). Borrowing the expressions for π_{t+1} and u_{t+1}, which are given by (19) and (20), it is easy to verify that $dy_{n,t}^{Nash}/d\lambda_{t+1}>0$ i.e. the bargained price today rises if players expect a higher buyer-seller ratio tomorrow (see also Figure 2a and the subsequent discussion).

3.3.2 Strategic Bargaining

The alternative to the Nash Bargaining approach is the Strategic Bargaining or the Rubinstein Bargaining approach, where one specifies how players interact during the haggling process, and then derives the equilibrium of the game based on this specification. In what follows we consider a procedure where players alternate offers until an exogenous breakdown occurs, after which parties walk away with their outside options. (The exogenous-breakdown setup is standard in dynamic Strategic Bargaining models; see Muthoo (2000) for an extended discussion.)

WLOG, the process starts with the buyer making the initial offer $y_{n,t}^b$. If the offer is accepted then the seller gets $y_{n,t}^b$ and the buyer himself gets $v(1-y_{n,t}^b)$. If the seller rejects, then with probability $\delta\in(0,1)$ negotiations continue and now it is the seller's turn to make an offer. Following rejection, the initially selected buyer is no longer guaranteed to receive a counter offer from the seller; his chance to be selected is same as other buyers and equals to $1/n$. The process continues in this fashion until an agreement is reached, or until negotiations break down exogenously. It should be noted that bargaining rounds take place within the same search period i.e. they are sub-periods of a search period. At the end of the bargaining process players who have been unable to trade go back to the market to search again in the next period.

In Appendix B we analyze this game and show that it has a unique subgame perfect equilibrium where the initial offer is immediately accepted. The equilibrium offer $y_{n,t}^{Strategic}$ is the unique value of $y_{n,t}^b$ that solves

$$v\left(1-\tfrac{y_{n,t}^b}{\delta}+\tfrac{1-\delta}{\delta}\beta\pi_{t+1}\right)=\tfrac{\delta}{n}v\left(1-y_{n,t}^b\right)+\left(1-\tfrac{\delta}{n}\right)\beta u_{t+1}. \tag{2}$$

Similar to $y_{n,t}^{Nash}$, the bargained price $y_{n,t}^{Strategic}$ rises both in n as well as λ_{t+1}. In the rest of the paper we rarely need the closed form expressions for bargained prices; only in Section 4.2, where

[8] For detailed information on Nash Bargaining see Muthoo (2000) Chapter 2 or Osborne and Rubinstein (1990) Chapter 2.

we discuss risk neutral buyers, the closed form expressions will be needed, so for future reference substitute $v(x) = x$ (risk neutrality) into (1) and (2) to obtain:

$$y_{n,t}^{Nash} = 1 - \beta u_{t+1} - \theta_n \left(1 - \beta u_{t+1} - \beta \pi_{t+1}\right) \quad \text{and} \tag{3}$$

$$y_{n,t}^{Strategic} = \beta \pi_{t+1} + \left(1 - \beta u_{t+1} - \beta \pi_{t+1}\right) \left(\delta - \delta^2/n\right) / (1 - \delta^2/n). \tag{4}$$

We are going to need superscripts *Nash* or *Strategic* only for a few occasions; so they are dropped whenever understood. Also, for convenience we let $y_{0,t} \equiv 0$.

3.3.3 Sale Price

Given $y_{n,t}$ it is easy to pin down the subgame perfect sale price $p_{m,n,t}(r_{m,t})$. We have

$$\begin{array}{rl} \text{Fixed Pricing } (f): & p_{f,n,t}\left(r_{f,t}\right) = r_{f,t} \\ \text{Best-offer Pricing } (b): & p_{b,n,t}\left(r_{b,t}\right) = \min\{y_{n,t}, r_{b,t}\} \\ \text{Over-the-Sticker Pricing } (o): & p_{o,n,t}\left(r_{o,t}\right) = \max\{y_{n,t}, r_{o,t}\}. \end{array} \tag{5}$$

With fixed pricing no one negotiates; hence the sale price is equal to the list price $r_{f,t}$. With best-offer pricing all players expect $y_{n,t}$ to arise from the bargaining process, so at the point of transaction the buyer makes a counteroffer only if he expects to be able to negotiate a discount, i.e. if $y_{n,t}$ is less than $r_{b,t}$. Else he purchases at the list price. Specifically fix $r_{b,t} \in (y_{h,t}, y_{h+1,t}]$ for some unique $h = 0, 1, 2....$ It follows that

$$p_{b,n,t} = \begin{cases} y_{n,t} & \text{if } n \leq h \\ r_{b,t} & \text{if } n > h \end{cases}.$$

In words, the buyer manages to negotiate a discount if there are h or fewer customers present at the store; else he pays the list price. The cutoff h is endogenous and determines whether the transaction is settled at the posted price or through haggling. Since the bargained price $y_{n,t}$ increases in n, it is clear that the buyer haggles if n is small and purchases at the list price if n is large.

Over-the-sticker pricing is similar. Now the seller has the option to request bargaining; so, if n is small then he charges the list price, but if n is large then he asks for more. Technically if $r_{o,t} \in (y_{j,t}, y_{j+1,t}]$ for some unique $j = 0, 1, 2...$ then

$$p_{o,n,t} = \begin{cases} r_{o,t} & \text{if } n \leq j \\ y_{n,t} & \text{if } n > j \end{cases},$$

i.e. the seller charges the list price if he gets j or fewer customers and he ask for more otherwise. From a seller's point of view, choosing the list price is akin to selecting the probability of haggling. For instance, with over-the-sticker pricing a high $r_{o,t}$ is associated with a high threshold j, and therefore, a low probability of haggling. So, if $r_{o,t}$ is sufficiently large then the seller almost never renegotiates and over-the-sticker pricing becomes practically identical to fixed pricing. The mirror image of this argument applies for best-offer pricing (see the simulations in Section 5.1.1).

Finally note that, all else equal, over-the-sticker pricing delivers the highest expected surplus to sellers; so one might be tempted to think that sellers might collude on this pricing rule. However, the model is based on competitive search where customers' visits can be encouraged by switching to a more "buyer-friendly" pricing rule. For instance, an individual seller might corner the market and attract a disproportionate number of customers by switching to, say, best-offer pricing if all his competitors indeed use over-the-sticker pricing. So, it is not obvious at all which pricing rule sellers would choose or what price they would post.

3.4 Buyers

Following the competitive search literature we focus on visiting strategies that are anonymous and symmetric (Burdett et al., 2001; Shimer, 2005; Eeckhout and Kircher, 2010). Anonymity means that a buyer's visiting strategy ought to depend on what sellers post but not on sellers' identities i.e. if two sellers were to post the same list price $r_{m,t}$ and trade with the same rule m then buyers ought to visit them with identical probabilities. Symmetry, on the other hand, requires buyers to adopt the same (anonymous) visiting strategies.[9]

Given these assumptions, the demand at each store follows a Poisson distribution, though the arrival rates are endogenous. To understand why, first consider a finite setting with B buyers and S sellers, where the buyer seller ratio equals to $\lambda = B/S$. For now suppose that all sellers trade with fixed pricing and that they post the same list price, say, $r = 0.75$. Since all sellers post identical terms, symmetry and anonymity imply that each seller is visited with probability $1/S$ by any given buyer. Therefore, the probability that a particular seller is visited by n customers is equal to

$$\Pr[n] = \binom{B}{n} (1/S)^n (1 - 1/S)^{B-n},$$

i.e. his demand distribution is binomial with parameters B and $1/S$ and his expected demand equals to $B/S = \lambda$. Now fix λ and let the market size tend to infinity, i.e. fix $\lambda \in \mathbb{R}_+$ and let $B = \lambda S$ and $S \to \infty$ (recall that we have a continuum of buyers and sellers). As $S \to \infty$ the binomial distribution converges to (e.g. see **?**)

$$\Pr[n] = \frac{e^{-\lambda}\lambda^n}{n!}.$$

I.e. in a large market the distribution of demand of each seller can be approximated by a Poisson distribution with arrival rate λ. (We will use the terms "arrival rate" and "expected demand" interchangeably.) Now, if sellers were to adopt different pricing rules or post different prices, then, again because of symmetry and anonymity, the distribution of demand at each store would still be Poisson, but each distribution would have its own arrival rate that depends on what exactly the seller posts and how it compares with the rest of the market (Galenianos and Kircher, 2012). For example, in

[9] Imposing anonymity and symmetry on buyers' visiting strategies greatly facilitates the characterization of the equilibrium and leads to results that are analytically tractable, which explains why they are standard assumptions in the directed search literature. An exception can be found in Burdett et al. (2001) where they construct equilibria supported by asymmetric pure visiting strategies in a two-buyer-two-seller setting; however such equilibria require buyers to coordinate between themselves on who goes where. But even Burdett et al. (2001) admit that, though such coordination may be possible in a small market with few buyers and sellers, it is unlikely to be attainable in a large market with many buyers and sellers. Symmetric mixed strategies, on the other hand, require no such coordination.

the previous scenario if a seller were to post a lower price, say 0.5, then his his expected demand q would be higher than λ (more on this below).

In the full-fledged model, the expected demand q depends not only on the list price r, but also on the pricing rule m and the date t. Specifically, the probability that a seller with the terms $(r_{m,t}, m)$ meets $n = 0, 1, 2...$ customers is given by

$$\Pr[n] = \frac{e^{-q_{m,t}} q_{m,t}^n}{n!} \equiv z_n(q_{m,t}). \tag{6}$$

The parameter $q_{m,t}$ is the expected demand, which is endogenous and strategically depends on the list price $r_{m,t}$ as well as the selling rule m. In what follows, we discuss how $q_{m,t}$ is pinned down.

Expected Utility. A buyer's expected utility, conditional on being at a store displaying $(r_{m,t}, m)$, is given by

$$\begin{aligned} U_{m,t} &= \sum_{n=0}^{\infty} z_n(q_{m,t}) \frac{v\left(1 - p_{m,n+1,t}\left(r_{m,t}\right)\right)}{n+1} + \sum_{n=0}^{\infty} \frac{n}{n+1} z_n(q_{m,t}) \times \beta u_{t+1} \\ &= \frac{1}{q_{m,t}} \sum_{n=1}^{\infty} z_n(q_{m,t}) v\left(1 - p_{m,n,t}\left(r_{m,t}\right)\right) + \left[1 - \frac{1 - z_0\left(q_{m,t}\right)}{q_{m,t}}\right] \beta u_{t+1}. \end{aligned} \tag{7}$$

To understand the first line notice that with probability $z_n(q_{m,t})$ the buyer finds $n = 0, 1, \ldots$ other buyers at the same store. Since there are $n+1$ buyers in total, his chance of obtaining the item is $\frac{1}{n+1}$, the sale price is $p_{m,n+1,t}$, and therefore his expected payoff is $\frac{v(1-p_{m,n+1,t})}{n+1}$. With the complementary probability $\frac{n}{n+1}$ he is unable to purchase and walks away with his value of search βu_{t+1} (i.e. the present value of being a buyer in period $t+1$). The second line follows from the fact that $z_{n+1} = q_{m,t} \times z_n/(n+1)$.

It is straightforward to verify that $\partial U_{m,t}/\partial r_{m,t} < 0$ and $\partial U_{m,t}/\partial q_{m,t} < 0$ i.e. buyers dislike expensive and crowded stores. The sign of the first partial derivative is obvious; for the second, note that a larger $q_{m,t}$ shifts the probability mass from low to high demand realizations. Such a shift causes $U_{m,t}$ to decline because customers are less likely to be served if the realized demand is high.

Let $\overline{U}_t$ denote the maximum expected utility ("market utility") a customer can obtain in the entire market at time t. For now we treat $\overline{U}_t$ as given, subsequently it will be determined endogenously.[10] So, consider an individual seller who advertises the package $(r_{m,t}, m)$ and suppose that buyers respond to this with the expected demand $q_{m.t} \geq 0$. The expected demand satisfies

$$q_{m,t} > 0 \text{ if } U_{m,t}(r_{m,t}, q_{m,t}) = \overline{U}_t \text{ else } q_{m,t} = 0. \tag{8}$$

The indifference condition (8) says that if the price package generates an expected utility of $\overline{U}_t$ for customers, then they will visit this store with some positive probability, else they will stay

[10] The market utility approach is standard in the directed search literature (Burdett et al., 2001; Shimer, 2005; Menzio and Shi, 2010). Galenianos and Kircher (2012) provide game theoretic foundations for the use of the market utility paradigm in a variety of directed search setups.

away.[11] Furthermore, the indifference condition reveals a "law of demand": the expected demand $q_{m,t}$ decreases as the list price $r_{m,t}$ increases. To see why, apply the Implicit Function Theorem to the equality $U_{m,t}(r_{m,t}, q_{m,t}) = \overline{U}_t$ to obtain

$$\frac{dq_{m,t}}{dr_{m,t}} = -\frac{\partial U_{m,t}/\partial r_{m,t}}{\partial U_{m,t}/\partial q_{m,t}}.$$

The numerator and the denominator are both negative (see above); hence $dq_{m,t}/dr_{m,t}$ is also negative. This relationship implies that if the seller raises r then buyers respond by decreasing q. The seller ought to find a balance between these two opposing effects, which we study next.

3.5 Sellers

The expected profit of a seller, denoted by $\Pi_{m,t}(r_{m,t}, q_{m,t})$, depends on the pricing rule m, the list price $r_{m,t}$ and the expected demand $q_{m,t}$. We have

$$\Pi_{m,t} = \sum_{n=1}^{\infty} z_n(q_{m,t})\, p_{m,n,t}(r_{m,t}) + z_0\beta\pi_{t+1} \tag{9}$$

The expression is easy to interpret. With probability z_n the seller receives $n = 1, 2, \ldots$ customers, in which case he sells at price $p_{m,n,t}$; however, with probability z_0 he receives no customer at all and walks away with his value of search $\beta\pi_{t+1}$ (i.e. the present value of being a seller in period $t+1$). Before moving on, note that sellers are free to select any rule $m \in \mathbb{M}$ they wish to compete with and post any list price $r_{m,t} \in [0, 1]$ they wish to advertise, i.e. we do not impose symmetry on sellers' selections. As it turns out, sellers competing under the same rule will end up posting the same list price (e.g., all fixed price sellers will post the same price); however notice that this is a result, and not an assumption.

Fix some rule $m \in \mathbb{M} = \{b, f, o\}$. The seller's price posting problem is

$$\max_{r_{m,t}\in[0,1],\ q_{m,t}\in\mathbb{R}_+} \Pi_{m,t}(r_{m,t}, q_{m,t}) \text{ subject to } U_{m,t}(r_{m,t}, q_{m,t}) = \overline{U}_t. \tag{10}$$

The expected demand $q_{m,t}$ is pinned down by the indifference condition (8). Recall that $q_{m,t}$ falls in $r_{m,t}$, which means that when selecting the list price, the seller faces a trade-off between revenue (intensive margin) and expected demand (extensive margin): on the one hand there is the desire to sell at a high price, but on the other hand, there is the fear of not being able to trade today.

Let $\hat{r}_{m,t}$ denote the optimal list price that solves (10) and let $\widehat{\Pi}_{m,t}$ be the value of $\Pi_{m,t}$ evaluated at $\hat{r}_{m,t}$. The fraction of sellers adopting rule m, denoted by $\alpha_{m,t}$, satisfies

$$\alpha_{m,t} > 0 \text{ only if } \widehat{\Pi}_{m,t} = \max_{\tilde{m}\in\mathbb{M}} \widehat{\Pi}_{\tilde{m},t} \tag{11}$$

i.e. a rule is selected only if it is capable of delivering the highest expected profit. This condition

[11] The market utility, by construction, is either greater than or equal to the expected utility at each individual store; hence the case $U_{m,t} > \overline{U}_t$ is ruled out.

does not imply that a unique pricing rule will prevail. It is possible that, and indeed it is the case that, multiple rules coexist in equilibrium delivering equal profits, i.e. sellers adopt asymmetric yet payoff equivalent pricing rules.

Finally to close down the model, we need a consistency condition to ensure that the weighted sum of expected demands (per seller) equals to the market wide buyer-seller ratio λ_t:

$$\alpha_{f,t} q_{f,t} + \alpha_{b,t} q_{b,t} + \alpha_{o,t} q_{o,t} = \lambda_t. \tag{12}$$

Technically the expected demands in equation (12) need to be indexed not only by the pricing rule m but also by the list price $r_{m,t}$, because sellers are free to chose any price $r_{m,t} \in [0,1]$. However, below we show that sellers trading via the same rule will end up advertising the same list price, which is why the expected demands are not indexed by $r_{m,t}$. Now we can define the equilibrium.

Definition 1 *A competitive search equilibrium consists of prices $r^*_{f,t}$, $r^*_{b,t}$, $r^*_{o,t}$, queue lengths $q^*_{f,t}$, $q^*_{b,t}$, $q^*_{o,t}$ and fractions $\alpha^*_{f,t}$, $\alpha^*_{b,t}$, $\alpha^*_{o,t}$ satisfying the demand distribution (6), indifference (8), profit maximization (10), equal profits (11) and consistency (12).*

The evolution of the buyer seller ratio λ_t, also part of the equilibrium, is discussed in Section 5.

4 Analysis

In what follows, we investigate equilibrium outcomes by concentrating on three main dimensions that are likely to affect the pricing policy: (i) customers' risk preference (being risk averse or risk neutral), (ii) the degree of trade frictions, and (iii) the degree of competition. We conduct a step-by-step analysis of the parameter space starting with risk averse customers. Table 1 in the Conclusion lays out our roadmap of the analysis of the parameter space; we refer the reader to this table to visually inspect how we partition the parameter space and how we proceed in our analysis.

4.1 Risk Averse Buyers

Proposition 1 *If buyers are risk averse then fixed pricing emerges as the unique equilibrium pricing rule. All sellers advertise the same price $r^*_{f,t}$ that solves*

$$\frac{1 - z_0(\lambda_t) - z_1(\lambda_t)}{z_1(\lambda_t)} = \frac{v'(1 - r_{f,t})\left[r_{f,t} - \beta\pi_{t+1}\right]}{v(1 - r_{f,t}) - \beta u_{t+1}}.$$

The equilibrium expected demand of each seller is equal to λ_t.

The main message of the proposition is that when faced with risk averse customers sellers prefer to trade via fixed pricing. To understand why, note that with flexible rules (over-the-sticker pricing or best-offer pricing), the sale price differs from the posted price at least for some demand realizations. For instance, with over-the-sticker pricing the seller will ask for a higher price than the list price if demand turns out to be high.[12] The opposite is true with best-offer pricing. In either case, customers face an uncertainty in that they do not know in advance how much they will pay—a notion we label as *price uncertainty.* Fixed pricing eliminates this uncertainty; hence, all else equal, customers are more likely to shop at such stores. In a competitive setting this inclination gives an edge to fixed-price sellers, which is why in equilibrium fixed pricing emerges as the unique selling rule.

One might be tempted to think that buyers should not mind the price uncertainty if prices may only go down and never up, as it appears to be the case with best offer pricing. Notice, however, when deciding between fixed and flexible sellers, a buyer's reference point is the equilibrium fixed price i.e. one needs to compare the potential savings against what fixed price sellers charge and not against the already-inflated best offer price. Indeed in the next section we show that best offer sellers, anticipating potential discounts to customers, inflate their list prices upfront. So, even if it may seem that a customer haggles and purchases below the list price at a best offer seller, the savings may not be "real" in the sense that the final price may still exceed what fixed price sellers charge (the simulations in the next section show that obtaining a "real" discount is possible, but unlikely). Therefore, the notion of price uncertainty applies to best-offer stores as well.

The second part of the proposition characterizes the equilibrium fixed price. Putting some structure behind the utility function yields closed form expressions for prices and payoffs. Suppose, for instance, customers possess CRRA utility function $v(1-r) = (1-r)^{1-\varphi}/(1-\varphi)$ and that $\lambda_t = \lambda$ for all t i.e. the buyer-seller ratio remains constant. Since $\lambda_t = \lambda$, we have $u_t = u$ and $\pi_t = \pi$ for all t, where the expressions for u_t and π_t are given by (25) in the Appendix (we omit the subscript t when understood). Under this specification we have

$$r_f^* = \frac{1 - z_0(\lambda) - z_1(\lambda)[1 - \beta z_0(\lambda)]}{[1 - z_0(\lambda)][1 - \beta z_0(\lambda) - \varphi\beta z_0(\lambda)] - \varphi z_1(\lambda)(1-\beta)}.$$

It is easy to verify that that r_f^* rises both in λ and φ, where λ is the buyer-seller ratio and the parameter φ is a measure of risk aversion. In words, the price is higher if the market is not so competitive and/or if buyers are risk averse.

Notice that we highlight the notion of price uncertainty as a key mechanism that leads to the emergence of fixed pricing as the unique equilibrium policy when faced with risk-averse customers. While we do not explicitly introduce exogenous haggling costs into our model (please see our Discussion of the Model for details), one can consider price uncertainty as an integral component of haggling costs. To see why, note that a haggling process effectively involves two constituent dynamics and associated costs, one based on time and effort spent by the customer, and the other based on the uncertainty about the outcome of the negotiation process. By incorporating a concave utility function, our model with risk averse customers inherently takes into account the second, and arguably the more salient part of the haggling cost. Therefore one advantage of our model is that we provide insights into costs associated with the haggling process without resorting to introduce exogenous haggling costs.

[12] Note that the equilibrium expected demand of each seller is equal to λ_t; the realized demand n, however, is stochastic (it follows a Poisson distribution) and it differs from seller to seller.

4.2 Risk Neutral Buyers

We now turn to risk neutral customers, i.e. we let $v(x) = x$.[13] There are two main results in this section. First, unlike the previous section, customers do not mind the price uncertainty, so fixed and flexible rules may coexist in equilibrium; however a pricing rule emerges only if it is "unbiased", i.e. if it is capable of dividing the trade surplus between the seller and the buyer in a way that is commensurate with the degree of market competition. Second, if trade frictions disappear, then the equilibrium bargained price converges to the equilibrium fixed price; hence the availability of bargaining becomes immaterial and fixed pricing emerges as the de-facto selling rule.

To start, substitute $v(x) = x$ into (7) and combine it with (9) to get

$$\Pi_{m,t} = 1 - \beta u_{t+1} - z_0(q_{m,t}) \times (1 - \beta u_{t+1} - \beta \pi_{t+1}) - q_{m,t}(U_{m,t} - \beta u_{t+1}).$$

Substituting the constraint $U_{m,t}(r_{m,t}, q_{m,t}) = \overline{U}_t$ into the objective function, the seller's price selection problem in (10) becomes

$$\max_{q_{m,t} \in \mathbb{R}_+} 1 - \beta u_{t+1} - z_0(q_{m,t}) \times (1 - \beta u_{t+1} - \beta \pi_{t+1}) - q_{m,t}(\overline{U}_t - \beta u_{t+1}).$$

The first order condition is given by

$$z_0(q_{m,t}) = \frac{\overline{U}_t - \beta u_{t+1}}{1 - \beta u_{t+1} - \beta \pi_{t+1}} \equiv H_t. \tag{13}$$

It is easy to verify the second order condition; thus, the solution of FOC corresponds to the global maximum. Combine $U_{m,t} = \overline{U}_t$ with (7) and (13) to obtain

$$\underbrace{\sum_{n=1}^{\infty} z_n(q_{m,t}) \, p_{m,n,t}(r_{m,t})}_{R_{m,t}(q_{m,t}, r_{m,t})\text{: Expected Revenue}} = \mu(q_{m,t}), \tag{14}$$

where

$$\mu(q_{m,t}) \equiv [1 - z_0(q_{m,t})](1 - \beta u_{t+1}) - z_1(q_{m,t})(1 - \beta u_{t+1} - \beta \pi_{t+1}). \tag{15}$$

Inserting sale price functions, which are given in (5), into equation (14) and solving the equation for $r_{m,t}$ yields optimal list prices under all pricing rules.

Lemma 1 *Sellers trading with the same rule m must post the same list price $\widehat{r}_{m,t}$. Specifically, sellers using fixed pricing post*

$$\widehat{r}_{f,t} = 1 - \beta u_{t+1} - \frac{z_1(q_{f,t})}{1 - z_0(q_{f,t})}(1 - \beta u_{t+1} - \beta \pi_{t+1}) \equiv \varrho_f(q_{f,t}). \tag{16}$$

[13] For risk neutrality, $v(x)$ needs to be linear in x. Applying a monotonic transformation to a utility function representing a preference relation creates another utility function representing the same preference relation; so WLOG we use $v(x) = x$.

Sellers who compete via over-the-sticker pricing post

$$\widehat{r}_{o,t} = \begin{cases} 0 & \text{if } q_{o,t} \leq \overline{q}_t \\ \varrho_o\left(q_{o,t}\right) \in \left(y_j, y_{j+1}\right] \text{ for a unique } j = 1,2,3,... & \text{if } q_{o,t} > \overline{q}_t \end{cases},$$

whereas sellers competing via best-offer pricing post

$$\widehat{r}_{b,t} = \begin{cases} 1 & \text{if } q_{b,t} \geq \overline{q}_t \\ \varrho_b\left(q_{b,t}\right) \in \left(y_h, y_{h+1}\right] \text{ for a unique } h = 0,1,2,... & \text{if } q_{b,t} < \overline{q}_t \end{cases},$$

where $\overline{q}_t \in \mathbb{R}_{++}$ *is the unique value of* q_t *solving* $\sum_{n=1}^{\infty} z_n(q_t) y_{n,t} = \mu(q_t)$ *and*

$$\varrho_o\left(q_{o,t}\right) \equiv \frac{\mu\left(q_{o,t}\right) - \sum_{n=j+1}^{\infty} y_{n,t} z_n\left(q_{o,t}\right)}{\sum_{n=1}^{j} z_n\left(q_{o,t}\right)} \quad \text{and} \quad \varrho_b\left(q_{b,t}\right) \equiv \frac{\mu\left(q_{b,t}\right) - \sum_{n=1}^{h} y_{n,t} z_n\left(q_{b,t}\right)}{\sum_{n=h+1}^{\infty} z_n\left(q_{b,t}\right)}. \tag{17}$$

To understand the lemma, note that the equation (14) requires the expected revenue of a seller, $R_{m,t}$, to be equal to $\mu\left(q_{m,t}\right)$. With fixed pricing $R_{f,t}$ has a sufficiently small lower bound and a sufficiently large upper bound; hence for any given $q_{f,t}$ there exits an interior price ϱ_f satisfying $R_{f,t} = \mu\left(q_{f,t}\right)$[14]. With over-the-sticker pricing, $R_{o,t}$ has a non-zero lower bound, which may lead to a corner solution. Indeed, if the expected demand $q_{o,t}$ is small (i.e. less than threshold $\bar{q}_t$), then the seller earns more than $\mu\left(q_{o,t}\right)$ even if he posts the lowest possible list price $r_{o,t} = 0$, so we have a corner solution $\widehat{r}_{o,t} = 0$. An interior price ϱ_o obtains only if $q_{o,t}$ is large enough. The case with best-offer pricing is the opposite: the upper bound of the revenue can be low due to discounts conceded to haggling customers, which, in turn, may lead to a corner solution. Specifically if the expected demand $q_{b,t}$ is below threshold $\overline{q}_t$ then the profit maximizing price ϱ_b is interior; else we have a corner solution where the seller posts the highest possible list price, 1, but still earns less than $\mu\left(q_{b,t}\right)$.

Note that at this stage we do not know which pricing rule emerges in equilibrium; Lemma 1 only says that if a seller competes via rule m then he ought to post the price $\widehat{r}_{m,t}$. To study the selection of pricing rules we start by classifying them as "seller-biased", "buyer-biased" and "unbiased" depending on how they allocate the surplus between buyers and sellers. To do so, we borrow an upcoming result that if a rule m is active, then all sellers competing with this rule must have the same expected demand λ_t and their expected revenue must be equal to $\mu\left(\lambda_t\right)$. Fixing the expected demand at λ_t, the upper bound for a seller's expected revenue is $\overline{R}_{m,t}\left(\lambda_t, 1\right)$, which is obtained by substituting the highest list price $r_{m,t} = 1$ into $R_{m,t}$. Similarly the lower bound is equal to $\underline{R}_{m,t}\left(\lambda_t, 0\right)$.

We say that pricing rule m is *seller-biased* if $\underline{R}_{m,t} > \mu\left(\lambda_t\right)$ i.e. if the lower bound is too high (if it allocates too much surplus to the seller and too little surplus to the buyer). Similarly rule m is *buyer-biased* if $\overline{R}_{m,t} < \mu\left(\lambda_t\right)$, i.e. if the upper bound is too low. Finally the rule is *unbiased* if $\mu\left(\lambda_t\right)$ falls between the upper and lower bounds.

[14]The lower bound $\underline{R}_{f,t}(q_{f,t}, 0) = 0$ is obtained by substituting $r_{f,t} = 0$ into $R_{f,t}$. Similarly, the upper bound $\bar{R}_{f,t}(q_{f,t}, 1) = 1 - z_0(q_{f,t})$ is obtained by substituting $r_{f,t} = 1$. It is easy to show that $\underline{R}_{f,t} < \mu < \bar{R}_{f,t}$.

Lemma 2 *Fixed pricing is unbiased for any given λ_t. Best-offer pricing is unbiased if the market is sufficiently competitive i.e. if λ_t is small. Over-the-sticker pricing is unbiased if the market lacks competition i.e. if λ_t is large. Specifically:*

	Seller biased	*Unbiased*	*Buyer Biased*
Over-the-Sticker Pricing:	*if* $\lambda_t < \overline{\lambda}_t$	*if* $\lambda_t \geq \overline{\lambda}_t$	*never*
Best-offer Pricing:	*never*	*if* $\lambda_t \leq \overline{\lambda}$	*if* $\lambda_t > \overline{\lambda}_t$

where $\overline{\lambda}_t$ is the unique value of λ_t satisfying $\sum_{n=1}^{\infty} z_n(\lambda_t)\, y_{n,t} = \mu(\lambda_t)$.

To prove the Lemma one needs to apply the definition of unbiasedness to the scenarios studied in the proof of Lemma 1. The algebra is repetitive, so we skip the full-blown analysis. In words, a pricing rule is unbiased if it manages to divide the surplus between buyers and sellers in a way that is commensurate with the degree of market competition, proxied by λ_t. Consider, for instance, best-offer pricing, where buyers are able to get deductions off the list price. Such a rule is unbiased in a highly competitive thin market, where there are few buyers per seller ($\lambda_t \leq \overline{\lambda}_t$); however in a thick market flush with buyers $\left(\lambda_t > \overline{\lambda}_t\right)$ this selling practice over-rewards buyers and penalizes sellers, so it is buyer-biased. Mirror-image arguments apply to over-the-sticker pricing, where sellers are able to charge more than what they post. In markets that lack competition (large λ_t) this selling practice is unbiased because in such markets buyers' payoffs are small anyway. As the market becomes more competitive, though, the over-the-sticker rule starts to over-reward sellers; thus becoming seller-biased. Remarkably, fixed pricing is unbiased irrespective of the degree of market competition, i.e. it is always capable of dividing the surplus between buyers and sellers in a commensurate way.[15]

Proposition 2 *In equilibrium sellers compete via unbiased rules only, i.e. they never adopt a biased rule, be it seller-biased or buyer-biased. There exists a continuum of equilibria where unbiased rules coexist in the same market, specifically:*

- *If $\lambda_t > \overline{\lambda}_t$, then sellers are indifferent between fixed pricing and over-the-sticker pricing. No seller competes via best-offer pricing.*
- *If $\lambda_t < \overline{\lambda}_t$, then sellers are indifferent between fixed pricing and best-offer pricing. No seller picks over-the-sticker pricing.*
- *If $\lambda_t = \overline{\lambda}_t$, then sellers are indifferent to all three rules.*

Equilibrium list prices $r^*_{b,t} > r^*_{f,t} > r^*_{o,t}$ are given by

$$r^*_{f,t} = \varrho_f(\lambda_t), \quad r^*_{o,t} = \begin{cases} 0 & \text{if } \lambda_t = \overline{\lambda}_t \\ \varrho_o(\lambda_t) & \text{if } \lambda_t > \overline{\lambda}_t \end{cases} \quad \text{and} \quad r^*_{b,t} = \begin{cases} 1 & \text{if } \lambda_t = \overline{\lambda}_t \\ \varrho_b(\lambda_t) & \text{if } \lambda_t < \overline{\lambda}_t \end{cases}. \tag{18}$$

[15] The notion of unbiasedness in our paper is similar to the notion of a pricing rule being "payoff-complete" in Eeckhout and Kircher (2010), who, too, classify pricing rules based on how they divide the surplus.

Sellers receive the same expected demand λ_t and earn the same expected profit π_t no matter which rule they commit to, whereas buyers earn the same expected utility u_t no matter which seller's rule they join in, specifically:

$$\pi_t = 1 - \beta u_{t+1} - \left[z_0\left(\lambda_t\right) + z_1\left(\lambda_t\right)\right]\left(1 - \beta u_{t+1} - \beta \pi_{t+1}\right), \tag{19}$$

$$u_t = z_0\left(\lambda_t\right)\left[1 - \beta u_{t+1} - \beta \pi_{t+1}\right] + \beta u_{t+1}. \tag{20}$$

Risk neutral customers do not mind the aforementioned price uncertainty; as such fixed and flexible rules may coexist in the same market, i.e. sellers may adopt asymmetric yet payoff equivalent pricing rules. The emergence of a rule is endogenous and critically depends on how competitive the market is. Over-the-sticker pricing, for instance, can be adopted if the market is not quite competitive (many buyers, few sellers); so only in such markets may we observe sellers asking for more than the posted price. Best-offer pricing emerges only in highly competitive markets with few buyers and many sellers. Again, only in such markets may buyers refuse to pay the list price in an attempt to negotiate better deals. Fixed pricing, on the other hand, is always unbiased and emerges under any degree of competition.

The proposition further reveals that the equilibrium list price associated with best-offer pricing is the highest, the one associated with over-the-sticker pricing is the lowest and the one associated with fixed pricing lies in the middle. To understand why, note that sellers adopting best-offer pricing correctly anticipate the discounts they are bound to concede to the customers, so they offset the shortfall in revenue by raising the list price. Similarly sellers trading via over-the-sticker pricing realize that they will end up charging more than what they post, so they advertise lower prices up-front. Fixed-price sellers, on the other hand, charge what they post, so they advertise moderately. In expected terms, all sellers (and buyers, too) earn the same.

Furthermore, the model implies equilibrium price dispersion, both in list prices as well as in sale prices. Dispersion in list prices follows from coexistence of different pricing rules. For instance, when $\lambda_t > \overline{\lambda}_t$ the market exhibits two prices; a low price $r^*_{o,t}$ and a high price $r^*_{f,t}$. Similarly when $\lambda_t < \overline{\lambda}_t$ there are two distinct prices, a high $r^*_{b,t}$ and a low $r^*_{f,t}$. In the knife edge case $\lambda_t = \overline{\lambda}_t$ one observes three different prices, a low $r^*_{o,t}$, a moderate $r^*_{f,t}$ and a high $r^*_{b,t}$.

Sale prices, too, are dispersed. Under flexible rules, sale prices are transaction specific and depend on the local demand, which, along the equilibrium path, is stochastic and follows a Poisson distribution. Hence, sale prices, too, are stochastic and differ from seller to seller, even if those sellers trade via the same (flexible) rule and post the same list price. For instance, all sellers trading via over-the-sticker pricing post the same list price; but the ones who receive few customers charge what they post, whereas the ones who get many customers negotiate better prices; hence the dispersion.

In Section 5.1.1 we further discuss price dispersion via numerical simulations, however at this stage we need to answer a question triggered by price dispersion—that is, whether a customer should buy right away or whether, given the dispersion, he should walk away with the hope of obtaining a better deal in the future.

Remark 1 *Buyers and sellers are better off to transact immediately rather than waiting.*

The intuition is this. The market is characterized by trade frictions and no-one is guaranteed to find a suitable match in the next period: the seller may not get a customer at all, whereas the buyer may well end up in a crowded store and walk out empty handed as a result. Therefore, a sure transaction today, even if it involves paying more than the list price or conceding a discount to the customer, is still better than walking away and facing the prospect of not being able to buy or sell tomorrow. (The proof of the remark is in the Appendix.)

Trade frictions are exacerbated by the magnitude of the discount factor: the lower the value of β, the more costly it is to wait and search, and therefore, the more pronounced can be the difference between the ex-ante posted price and the ex-post transaction price. We will document this claim via numerical simulations in the next section, however at this stage we have a theoretical result that makes this point from the opposite angle. We show that as $\beta \to 1$, i.e. as trade frictions disappear, the difference between ex-ante posted prices and the ex-post bargained prices vanishes and fixed pricing emerges as the de-facto selling rule.

Proposition 3 *If $\beta \to 1$ then we have*

$$\lim_{\beta \to 1} y_{n,t}^{Nash} = \lim_{\beta \to 1} y_{n,t}^{Strategic} = \lim_{\beta \to 1} r_{f,t}^{*} \text{ for all } n, t$$

i.e. equilibrium bargained prices under Nash Bargaining or under Strategic Bargaining converge to the equilibrium fixed price. Consequently fixed pricing emerges as the de-facto selling rule because negotiations would not produce anything but the equilibrium fixed price anyway.

Due to trade frictions, players face different outlooks ex-ante and ex-post, i.e. before being matched with a counterparty and in a match. Players in a match realize that if they do not trade today, then they will have to search again, but of course, they are not guaranteed of a sure trade in the next period. This is why the player who holds the key for bargaining, i.e. who has the option to renegotiate, can take advantage of his position and obtain a better deal than what was posted earlier. The deal goes through because the equilibrium offer accounts for the "costs" associated with searching again (see the proof of Remark 1). However as $\beta \to 1$ players start to face the same outlook ex-ante or ex-post, and therefore, trade frictions start to lose their effect on the outcome of bargaining. Indeed if players can costlessly wait for the next period and search again without losing any surplus, then they would not agree on anything but what was initially posted, which is why the ex-post bargained price $y_{n,t}$ starts to converge to the ex-ante posted price $r_{f,t}^{*}$.

A corollary of the convergence result is that fixed pricing emerges as the de-facto selling rule, because even if players were to negotiate, the resulting price would be the equilibrium posted price anyway. What is remarkable, this outcome is robust to the underlying bargaining protocol: prices coming from Nash Bargaining as well as from Strategic Bargaining—two most commonly used and distinct protocols—both converge to the equilibrium fixed price.

5 Further Investigation via Numerical Simulations

The analysis so far has established that the only region in the parameter space in which flexible rules may coexist with fixed pricing, without being practically identical to it, is where buyers are risk neutral and trade frictions are pronounced. In what follows, via numerical simulations, we demonstrate that even in this region fixed pricing emerges as the de-facto selling rule, provided that the market is sufficiently thin (few buyers, many sellers) or sufficiently thick (many buyers, few sellers). In such markets, all sellers—irrespective of the rule they compete with—post almost identical list prices and players almost never attempt to renegotiate even if they have the option to so. As a result, transactions are almost always settled at the posted price and, again, fixed pricing emerges as the de-facto selling rule.

The key parameter in the simulations below is the buyer seller ratio λ_t, which inversely proxies the degree of competition in the market. To verify the robustness of our insights and conclusions, the simulations are run under both constant and fluctuating trajectories of λ_t. Specifically, in Case 1 we let $\lambda_t = \lambda$ for all t, which can be thought of as a steady and mature market in which the buyer-to-seller ratio tends to remain constant. In Case 2 we consider a fluctuating λ_t over time, which is reminiscent of markets with seasonal trends.

Recall that the market begins with a measure of s_1 sellers and b_1 buyers. At the end of each period, trading players leave the market whereas the ones who could not trade move to the next period to replay the same game. In addition, at the beginning of each period $t = 2, 3...$ a new cohort of s_t^{new} sellers and b_t^{new} buyers enter the market joining the existing players. Now consider the first period. The expected demand along the equilibrium path is equal to λ_1, so each seller trades with probability $1 - z_0(\lambda_1)$. The law of large numbers implies that $s_1(1 - z_0(\lambda_1))$ sellers trade and exit the market. Each transaction involves one seller and one buyer, so the total number of buyers who trade and exit is also $s_1(1 - z_0(\lambda_1))$. The number of sellers present in the second period, then, is equal to

$$s_2 = s_2^{new} + s_1 z_0(\lambda_1),$$

whereas the number of buyers is equal to

$$b_2 = b_2^{new} + b_1 - s_1(1 - z_0(\lambda_1)).$$

Iterating on this procedure, we have

$$s_t = s_t^{new} + s_{t-1} z_0(\lambda_{t-1}) \quad \text{and} \quad b_t = b_t^{new} + b_{t-1} - s_{t-1}(1 - z_0(\lambda_{t-1})) \text{ for } t \geq 2.$$

The buyer seller ratio λ_t, therefore, equals to

$$\lambda_t = \frac{b_t^{new} + b_{t-1} - s_{t-1}(1 - z_0(\lambda_{t-1}))}{s_t^{new} + s_{t-1} z_0(\lambda_{t-1})} \text{ for } t \geq 2. \tag{21}$$

5.1 Case 1: Constant λ_t

Suppose $\lambda_t = \lambda$ for all t. This case corresponds to a perfect replacement scenario where each outgoing player is replaced by a clone.[16] Dashed lines in Fig 1a and 1b depict bargained prices: y_1 obtains if a single customer is present at the store, y_2 obtains if there are two customers, and so on. Solid lines, on the other hand, depict list prices. The horizontal axis traces the buyer seller ratio λ, which inversely proxies the degree of competition in the market: a low value of λ corresponds to a high degree of competition and vice versa. Cutoffs λ_1^-, λ_2^+, etc., which are defined subsequently, help us determine the likelihood of transactions being settled via bargaining. In panel 1a the opportunity cost of search is high ($\beta = 0.2$) whereas in panel 1b it is less so ($\beta = 0.95$).

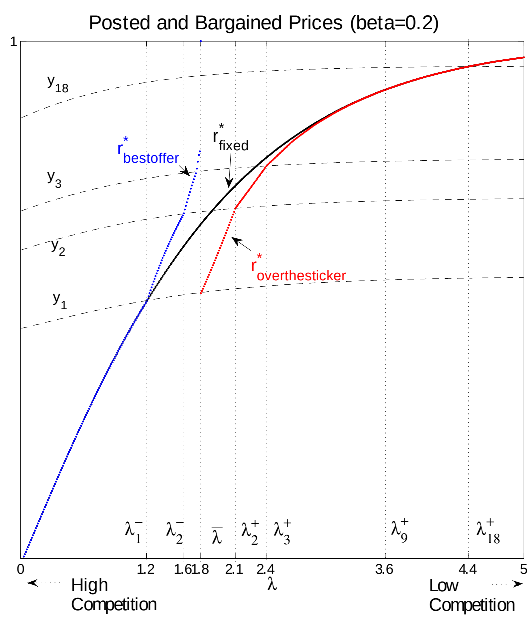

Fig 1a

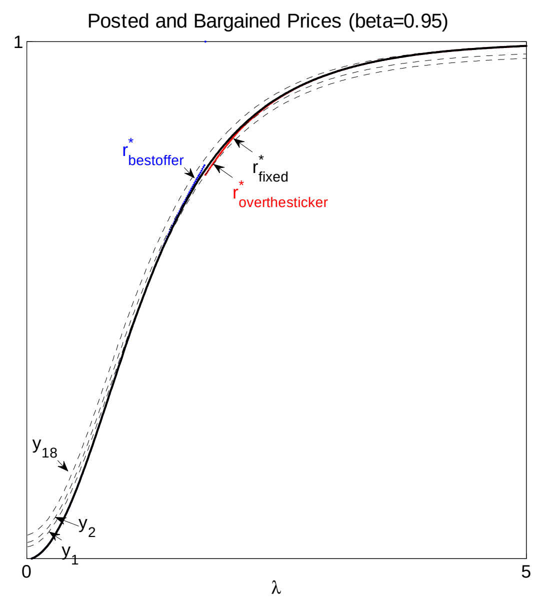

Fig 1b

Convergence of Prices. In 1a prices are somewhat dispersed but in 1b they are considerably squeezed around r^*_{fixed}. The reason is that as one moves from 1a to 1b trade frictions start to fade away (β rises from 0.2 to 0.95) and, as predicted by Proposition 3, prices start to converge to the equilibrium fixed price.

Coexistence of Multiple Selling Rules. Focus on panel 1a, where the critical threshold for unbiasedness is equal to $\overline{\lambda} = 1.8$.[17] Fixed pricing is always unbiased, hence r^*_{fixed} exists for any λ. If $\lambda \leq \overline{\lambda}$ then best-offer pricing is also unbiased, so $r^*_{\text{best-offer}}$ coexists with r^*_{fixed}. Similarly if $\lambda \geq \overline{\lambda}$

[16] Technically, substituting $s_t^{new} = b_t^{new} = s_{t-1}(1 - z_0(\lambda_{t-1}))$ into the law of motion (21) yields $\lambda_t = \lambda$ for all t.

[17] The threshold $\bar{\lambda}$ solves $\sum_{n=1}^{\infty} z_n(\lambda)\, y_n = \mu(\lambda)$. In the simulations the bargained prices are assumed to be determined via Nash bargaining with $\theta_n = 1/n$, so the closed form of y_n can be obtained from (3). Substituting for y_n and β and solving the equation above for λ, one can obtain the threshold $\bar{\lambda}$.

then over-the-sticker pricing is unbiased and therefore $r^*_{\text{overthesticker}}$ coexists with r^*_{fixed}. Observe that if λ is small enough (i.e. if the market is sufficiently competitive) then sellers post almost identical (and low) prices no matter which rule they compete with. For instance $r^*_{\text{best-offer}} \approx r^*_{\text{fixed}}$ when $\lambda < 1.2$. In the opposite extreme where λ is large, again, sellers post almost identical prices. Indeed $r^*_{\text{overthesticker}} \approx r^*_{\text{fixed}}$ when $\lambda > 3.6$. It follows that the dispersion in posted prices is pronounced only for moderate values of λ and it fades away otherwise (we will come back to this point later). Incidentally observe that $r^*_{\text{best-offer}} > r^*_{\text{fixed}} > r^*_{\text{overthesticker}}$ whenever they coexist, which confirms our previous finding that sellers competing with best-offer pricing advertise high, the ones competing with over-the-sticker pricing advertise low, and the ones competing with fixed pricing advertise moderate list prices.

Discounts. One might be tempted to think that if a customer shops at a best offer seller and if he manages to obtain a discount then he will be better off than someone who shops, say, at a fixed price seller. A quick investigation of panel $1a$ reveals that this is not necessarily the case. E.g. fix $\lambda = 1.7$ and observe that best-offer sellers advertise $r^*_b = 0.73$ whereas fixed price sellers advertise $r^*_f = 0.63$. A buyer visiting a best-offer seller faces the following prospect: he can negotiate a substantial discount and pay only $y_1 = 0.52$ if he is the sole customer there (the probability of this outcome is equal to $e^{-\lambda} = 18\%$). If, on the other hand, there is another buyer at the same seller, then he can still negotiate $y_2 = 0.67$; however, this amount is more than what fixed price sellers charge. Finally, if two or more other customers are present, then he cannot even get a discount because y_3 exceeds the list price 0.73, so he will pay the full price $r^*_b = 0.73$. It is clear that obtaining a "real" discount is possible, but highly unlikely.[18]

Incidentally this observation sheds further light on why risk averse customers prefer fixed pricing instead of flexible pricing, even if the flexibility may involve a potential discount. As discussed earlier, one may think that risk averse buyers ought not to despise price fluctuations if the direction of the fluctuation is down and not up, as it seems to be the case with best offer pricing. Notice, however, when deciding between fixed and best-offer sellers, a buyer compares his expected utility at both stores, so the potential savings are judged against the equilibrium fixed price and not against the already-inflated best offer price. Given the reference point, it is clear that the direction of the fluctuation is rarely down, which explains why risk averse customers stay away from flexible stores even if they may be promising potential savings.

5.1.1 Price Dispersion

As noted earlier the model implies equilibrium price dispersion, both in list prices as well as in sale prices. Below we discuss both types of price dispersion and conclude that they are pronounced only when λ is moderate. If λ is small enough or large enough then price dispersion fades away and fixed pricing emerges as the de-facto selling rule.

[18] Despite the differences in sale prices, players still prefer to transact immediately rather than waiting. Please see Remark 1 and the subsequent discussion.

Dispersion in Posted Prices. With risk neutral buyers the equilibrium choice of pricing rules is indeterminate; hence the distribution of the list prices, too, is indeterminate. To get around this issue we take the following approach. In the region $\lambda < \overline{\lambda}$, where best-offer pricing and fixed pricing coexist, we fix $\alpha_f^* = \alpha_b^* = 1/2$, i.e. we assume that half of the sellers adopt fixed pricing and the other half adopt best offer pricing. Similarly, in the region $\lambda > \overline{\lambda}$ we set $\alpha_f^* = \alpha_o^* = 1/2$. Finally in the knife edge case $\lambda = \overline{\lambda}$, where all three rules may coexist, we fix $\alpha_f^* = \alpha_b^* = \alpha_o^* = 1/3$. Based on these parameters we plot the average list price as well as the standard deviation. (Putting equal weights behind each active rule maximizes the standard deviation and amplifies the dispersion.)

The average list price in panel 2a generally rises in λ, since each individual list price rises in λ. The discontinuity at $\overline{\lambda}$ is due to the switch in the pricing regime, where best-offer pricing is replaced with over-the-sticker pricing. More importantly, the standard deviation in 2b is hump-shaped implying that price dispersion is pronounced only for intermediate values of λ and it vanishes elsewhere. Said differently, if there are too few or too many buyers in the market then sellers post almost identical prices irrespective of the pricing rule they compete with.

The fact that sellers post almost identical prices does not necessarily imply that transactions are settled at similar prices because some transactions may involve bargaining. In what follows we investigate the dispersion in *sale prices* and conclude that if λ is sufficiently small or sufficiently large, then sale prices, too, are almost identical.

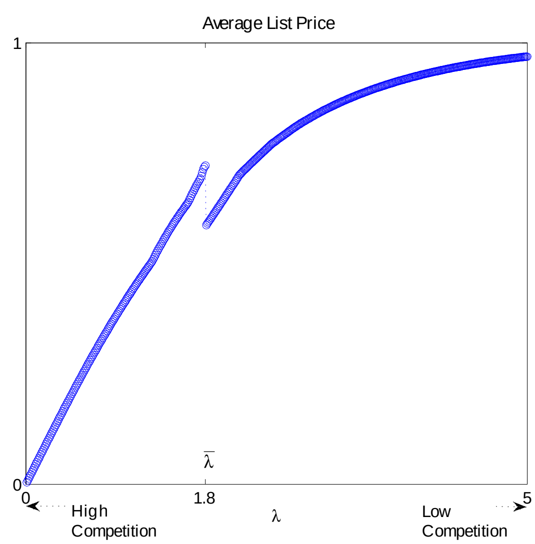

Fig 2a

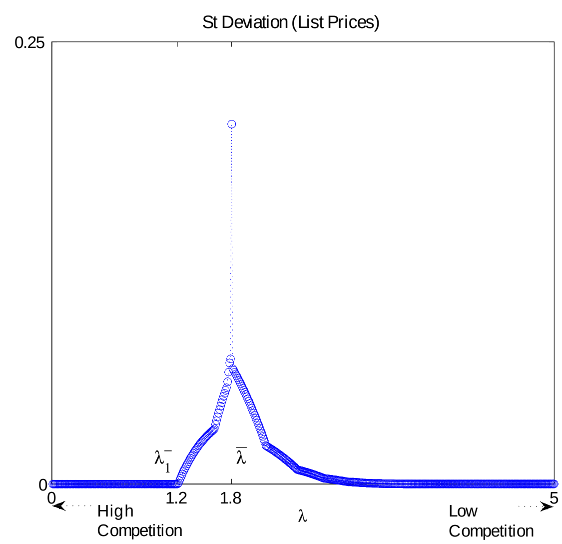

Fig 2b

Dispersion in Sale Prices. In equilibrium sellers competing via the same flexible rule post the same list price, however the equilibrium demand is stochastic, so the ones who receive few customers charge less and the ones who receive many customers charge more; hence the dispersion in sale prices. (Sale price dispersion applies only to flexible rules.) To quantify this dispersion we will pin down the probability of a transaction being settled via bargaining. The higher this probability, the more dispersed are the sale prices. We will show that this probability is hump-shaped in λ, implying that the dispersion fades away and fixed pricing emerges as the de-facto selling rule if λ is sufficiently large or sufficiently small.

To start, note that the equilibrium probability of renegotiation hinges on the pricing rule, the list price, as well as the local demand n. Start with over-the-sticker pricing where the sale price is equal to $\max\{r_o^*, y_n\}$. Assuming that $r_o^* \in (y_j, y_{j+1}]$ for some $j = 1, 2, ..$ the transaction takes place at the posted price r_o^* if there are j or fewer customers present; else, the seller negotiates a better price. Let λ_j^+ denote the unique positive value of λ that satisfies $\varrho_o(\lambda_j^+) = y_j$ for $j = 1, 2, 3, ...$ and note that $\lambda_1^+ = \overline{\lambda}$. (The subsequent discussion is best understood with the aid Figure 1a.) We have

$$\Pr[\text{bargaining} \mid \text{over-the-sticker pricing}] = \begin{cases} 1 & \text{if} \quad \lambda = \overline{\lambda} \\ \sum_{n=j+1}^{\infty} \frac{z_n(\lambda)}{1-z_0(\lambda)} \in (0,1) & \text{if} \quad \lambda \in (\lambda_j^+, \lambda_{j+1}^+] \\ 0 & \text{if} \quad \lambda = \infty \end{cases} \tag{22}$$

The probability that the transaction is settled via bargaining equals to the probability that the seller gets more than j customers (conditional, of course, on making a sale), which is given by $\sum_{n=j+1}^{\infty} z_n(\lambda) / [1 - z_0(\lambda)]$. This expression falls as λ grows. Indeed the growing λ raises the list price r_o^*, which in turn means that the seller is not likely to negotiate a better deal than the already high list price r_o^*. In the extreme scenario where $\lambda \to \infty$ (practically $\lambda > 3.6$ in Fig 1a) this probability converges to zero; hence over-the-sticker pricing virtually turns into fixed pricing.

The case with best-offer pricing is similar. Let λ_h^- be the value of λ satisfying $\varrho_b(\lambda_h^-) = y_h$ for $h = 1, 2, 3, ...$ and observe that $\lambda_\infty^- = \overline{\lambda}$. It follows that

$$\Pr[\text{bargaining} \mid \text{best-offer pricing}] = \begin{cases} 0 & \text{if} \quad \lambda \leq \lambda_1^- \\ \sum_{n=1}^{h} \frac{z_n(\lambda)}{1-z_0(\lambda)} \in (0,1) & \text{if} \quad \lambda \in (\lambda_h^-, \lambda_{h+1}^-] \\ 1 & \text{if} \quad \lambda = \overline{\lambda} \end{cases} \tag{23}$$

Here the probability that the transaction is settled via bargaining equals to the probability of getting h or fewer customers, which is given by the expression $\sum_{n=1}^{h} z_n(\lambda) / [1 - z_0(\lambda)]$. The expression rises in λ. Specifically, if λ is sufficiently small (i.e. $\lambda \leq \lambda_1^- = 1.2$ in figure 1a) then the list price r_b^* falls below y_1 and no buyer manages to negotiate a better deal than the already low list price. Thus, the probability of haggling falls down to zero and best-offer pricing becomes practically identical to fixed pricing.

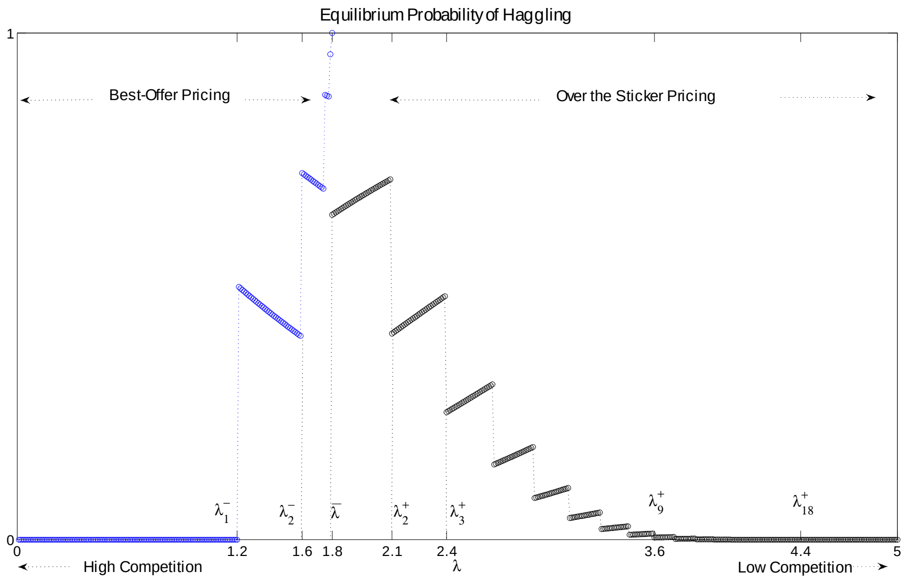

Fig 3

A glance at Figure 3, which plots probability functions (22) and (23) against λ, reveals that, overall, the equilibrium probability of haggling is hump-shaped, which implies that the dispersion in sale prices, like the dispersion in list prices, is pronounced only for moderate values of λ and it fades away as λ grows or shrinks.[19] The following Remark summarizes the results so far.

Remark 2 *If λ is sufficiently small or sufficiently large, then all sellers, irrespective of the pricing rule they compete with, post almost identical list prices. In addition, players almost never attempt to renegotiate even if they have the option to so. Therefore, transactions are almost always settled at the posted price and fixed pricing emerges as the de-facto selling rule.*

Before we move on to discuss Case 2, note that in our model ex-ante heterogeneity, informational asymmetries or exogenous matching frictions—prominent assumptions in earlier investigations—do not play a role in generating price dispersion.[20] Equilibrium dispersion arises simply because different sellers may have different types of commitment to what they post and therefore transactions may be

[19] The function is non-monotonic, because both the list price and the expected demand grow with λ, which creates opposing effects on the probability of bargaining. Consider, for instance, over-the-sticker pricing where $p_{o,n} = \max\{r_o^*, y_n\}$ and suppose for a moment that $\lambda_2^+ < \lambda < \lambda_3^+ \Leftrightarrow y_2 < r_o^* < y_3$ i.e. the seller negotiates if he gets more than two customers. A rise in λ causes r_o^* to grow, but as long as r_o^* remains between y_2 and y_3 the rise in λ increases only the probability of renegotiation, which explains why in Fig 3 the probability keeps going up as λ moves from λ_2^+ to λ_3^+. But once λ exceeds λ_3^+, the list price r_o^* grows beyond y_3 and the probability jumps down because now the seller negotiates if he gets more than three customers (not two).

[20] The existing literature cites various sources of price dispersion, namely information heterogeneity as in Baye et al. (1992), Varian (1980); costly search as in Carlson and McAfee (1983); firm heterogeneity as in Reinganum (1979); random search that limits price information as in Burdett and Judd (1983). See also Camera and Selcuk (2009).

settled via bargaining. We show that even in the absence of all the previously mentioned sources, this is enough to generate price dispersion, both in list prices as well as in sale prices.

5.2 Case 2: Fluctuating λ_t

To verify the robustness of our findings in the previous case, here we consider a fluctuating trajectory for λ_t. Specifically we consider periodic cycles where each cycle lasts τ periods, that is $\lambda_t = \lambda_{t+\tau}$ for some integer τ. The trajectory of λ_t is endogenous but given the law of motion (21) one can reverse-engineer and pick the starting cohort $\{b_1, s_1\}$ as well as the incoming cohorts $\{b_t^{new}, s_t^{new}\}_{t=2}^{\tau+1}$ in such a way that the resulting λ_t exhibits the trajectory one has in mind. For instance let $\tau = 2$ and suppose that the market alternates between episodes of high demand and low demand, where in odd periods $\lambda_{odd} = 0.5$ and in even periods $\lambda_{even} = 1$. One can generate such cycles by picking $\{b_1, s_1\} = \{2, 4\}$, $\{b_{t=even}^{new}, s_{t=even}^{new}\} = \{2.5, 0.5\}$ and $\{b_{t=odd}^{new}, s_{t=odd}^{new}\} = \{0.92, 2.92\}$.[21]

In the simulations below we set $\tau = 12$ and pick the starting and entering cohorts so that the expected demand λ_t starts at its lowest value 0.5, then it reaches its maximum at 3, then it declines back to 0.5 to start again (see 4a). Solid lines in 4b, 4c, and 4d depict equilibrium list prices whereas dashed lines depict bargained prices. The lessons in previous simulations remain valid here as well (confirming their robustness), so our discussion will be somewhat brief.

First, recall that all prices, listed and bargained, rise as λ_t rises and they fall as λ_t falls. This is why price trajectories resemble the trajectory of λ_t, i.e. they start low at the beginning of the cycle, peak in the middle of the cycle and subside towards the end of the cycle.

Second, the simulations confirm the convergence result reported in Proposition 3. Prices are most dispersed in 4b but they gradually get squeezed and converge to the equilibrium fixed price as one moves to 4c and 4d; indeed, in 4d all prices are almost equivalent to what fixed price sellers charge. The reason is that as we move towards 4d, trade frictions start to fade away as β rises from 0.5 to 0.95.

[21] The buyer seller ratio in the first period equals to $\lambda_1 = b_1/s_1 = 0.5$. At the end of the period $s_1\left(1 - e^{-\lambda_1}\right) = 1.57$ buyers and sellers trade and exit, which means that a measure of 2.43 sellers and 0.43 buyers are unable to trade so they move to the next period. At the beginning of period 2 we have $b_{t=even}^{new} = 2.5, s_{t=even}^{new} = 0.5$; thus $b_2 = s_2 = 2.93$ and therefore $\lambda_2 = 1$. At the end of period 2 $s_2\left(1 - e^{-\lambda_2}\right) = 1.85$ buyers and sellers trade and exit; hence a measure of 1.08 sellers and 1.08 buyers move to period 3. At the beginning of period 3 we have $b_{t=odd}^{new} = 0.92$ and $s_{t=odd}^{new} = 2.92$; thus $b_3 = 2$ and $s_3 = 4$ and therefore $\lambda_3 = 1$. And so on. Note that this solution is not unique as there is a continuum of other pairs of $\{b_1, s_1\}$ and $\{b_t^{new}, s_t^{new}\}_{t=2}^{\tau+1}$ generating the same cycle.

Fig 4a

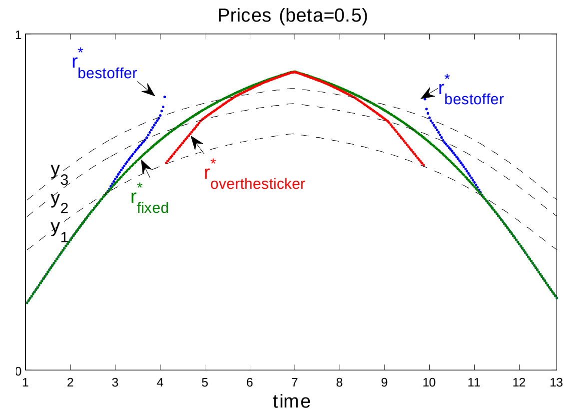

Fig 4b

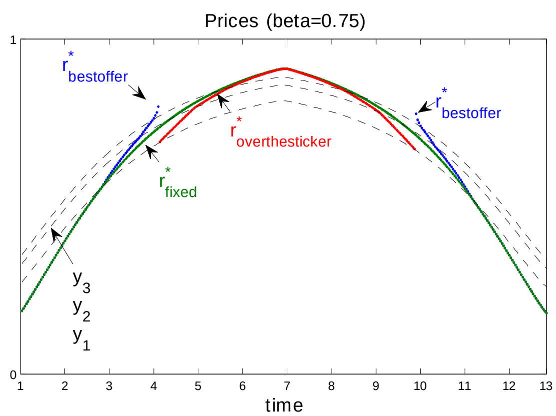

Fig 4c

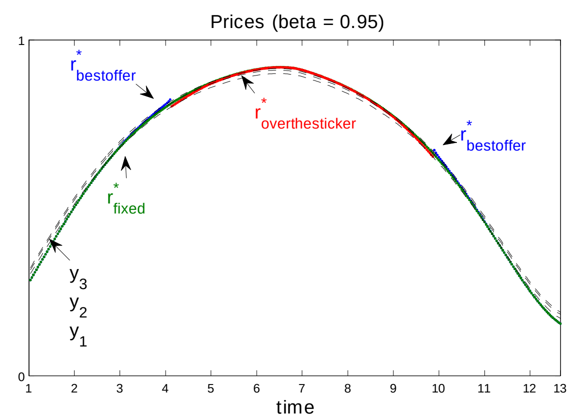

Fig 4d

Third, the emergence of a pricing rule is time-dependent. Fixed pricing is always unbiased, hence r^*_{fixed} exists throughout. Best-offer pricing is unbiased if λ_t is small; so it emerges at the beginning and at the end of the cycle where demand is scarce (the relevant regions in panel 4b are $t < 4$ and $t > 10$). Over-the-sticker pricing, on the other hand, is unbiased if λ_t is large, so it emerges in the middle of the cycle where the demand peaks (e.g. $4 < t < 10$ in panel 4b).

Finally, it is easy to verify that price dispersion is pronounced if λ_t is moderate and diminishes as λ_t grows or shrinks, confirming the findings of the previous section. Consider, for instance, the dispersion in posted prices and focus on panel 4b where prices are most dispersed. In regions $t < 3$ or $t > 11$, where λ_t is small, posted prices are almost identical: observe that $r_{\text{fixed}} \approx r_{\text{best-offer}}$. Similarly in the region $6 < t < 8$, where λ_t is large, we have $r_{\text{fixed}} \approx r_{\text{over-the-sticker}}$. Dispersion is pronounced only if λ_t is moderate, i.e. when $3 < t < 6$ or $8 < t < 11$. The dispersion in sale prices, too, displays the same pattern, i.e. it is pronounced only for moderate values of λ_t and vanishes otherwise. The implication is that the availability of haggling matters only towards the beginning and towards the end of the mid-season where the demand is moderate. In other times fixed pricing emerges either as the unique or the de-facto selling rule.

6 Conclusion

Pricing policies (e.g. fixed pricing, haggling, etc.) that we normally take for granted in many markets are not immutable. The majority of American retailers, for instance, moved from bargaining to fixed pricing within two decades towards the end of the nineteenth century (Phillips, 2012). Recently, we observe a further surge in the practice of fixed-price selling even in traditional haggling markets such as the automobile market or the housing market in some localities. What are the drivers of this phenomenon? Why does fixed price selling emerge even in these markets? We set to explore these questions with a rigorous analytical model based on competitive search which allows us to study the selection of pricing policies in a fully competitive environment. In doing so, we examine three different pricing policies—fixed pricing, best offer pricing and over the sticker pricing—which are, arguably, the most commonly used pricing policies in markets that we concentrate on.

While fixed pricing offers some intuitive benefits such as the sense of fairness, money-back guarantee options, centralization and economies of scale in pricing, and increased efficiency (Phillips, 2012), our model, using a market equilibrium approach, offers a new micro-founded account for the widespread use of the fixed price policy. Investigating our setup in three main dimensions including customer risk aversion, the degree of competition, and the degree of trade frictions; we address questions such as when and how the sellers gain a strategic advantage by posting fixed prices, and we find that fixed pricing prevails in most cases. These include when customers are risk averse, or when market competition is very high or very low, or when trade frictions are low (see Table 1 for an extended overview of our results).

In addition, we investigate the evolution of list and sales prices. An interesting finding is that in markets where fixed and flexible policies coexist, best offer sellers advertise high list prices, over-the-sticker sellers advertise low list prices, and fixed price sellers advertise moderate list prices. This is because sellers using over-the-sticker pricing realize that they will end up charging more than the list price so they post low, whereas best offer sellers anticipate subsequent price reductions during negotiations so they inflate the list price in the first place. Fixed price sellers, on the other hand, charge what they post, so they advertise moderately. An implication is that, discounts obtained at best offer sellers may not be "real" i.e., even if a customer manages to obtain a discount at a best offer seller, the sale price may still exceed what fixed price sellers charge.

Market Characteristics	**Findings**
A. Risk Averse Customers	Risk averse customers dislike “price uncertainty”, the uncertainty of not knowing how much to pay in advance, which is why fixed pricing emerges as the unique selling rule.
B. Risk Neutral Customers	Risk neutral customers do not mind the price uncertainty, so fixed and flexible rules may coexist provided that they are “unbiased”, i.e. they are capable of dividing the surplus commensurate with the degree of market competition. These rules, however, may be practically equivalent; see below.
B1. Low Trade Frictions	The outcome of bargaining converges to the equilibrium fixed price; hence fixed pricing becomes the de-facto selling rule as players cannot negotiate a different price anyway.
B2. High Trade Frictions	Whether buyers or sellers end up exercising their option to renegotiate depends on how competitive the market is; see below
B2(i). High Competition	The list price associated with best-offer pricing is quite low, so buyers are highly unlikely to ask for less. Fixed pricing is the de-facto selling rule.
B2(ii). Low Competition	The list price associated with over–the-sticker pricing is quite high, so sellers are highly unlikely to ask for more. Fixed pricing is the de-facto rule.
B2(iii). Moderate Competition	This is the only region where haggling matters as transactions are likely to be settled via bargaining, the outcome of which may be different than what fixed price sellers charge.

Table 1

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Appendix

Proof of Proposition 1. The proof consists of two steps.
Step 1. We will show that in equilibrium all sellers adopt fixed pricing. The proof is by contradiction; so, suppose that there exists an equilibrium where a seller adopts a flexible rule (over the sticker pricing (o) or best-offer pricing (b)). Let $m = \{o, b\}$ denote the rule the seller competes with, $r_{m,t}$ the list price he posts, $p_{m,n,t}(r_{m,t})$ the resulting sale price and $q_{m,t}$ the expected demand. We have

$$0 < \sum_{n=1}^{\infty} \frac{z_n(q_{m,t})}{q_{m,t}} v(1 - p_{m,n,t}(r_{m,t})) < \frac{1 - z_0(q_{m,t})}{q_{m,t}} v(1). \tag{24}$$

First note that since customers are risk averse the utility function v satisfies $v(0) = 0$, $v' > 0$, $v'' < 0$. The term on the far right in (24) is obtained by substituting $p_{m,n,t} = 0$ for all n into the term in the middle. The term on the far left (zero) is obtained by substituting $p_{m,n,t} = 1$ into the term in the middle and noting that $v(0) = 0$. Both inequalities are strict because $p_{m,n,t} \neq p_{m,\tilde{n},t}$ at least for some n and $\tilde{n}$ as the rule m is flexible.

The seller must provide buyers with market utility $\overline{U}_t$, so $q_{m,t}$ satisfies

$$U_{m,t}(r_{m,t}, q_{m,t}) = \overline{U}_t.$$

We argue that if the seller switches to fixed pricing then he can earn more, thus the conjectured outcome cannot be an equilibrium. The main obstacle in reaching this conclusion is that, if the seller changes the pricing rule, then his expected demand changes as well, which, of course, renders the comparison of expected profits non-trivial. To get around this issue we show that there exists a unique fixed price $\widehat{r} \in (0, 1)$ that satisfies $U_{f,t}(\widehat{r}, q_{m,t}) = \overline{U}_t$. I.e. if the seller switches to fixed pricing and posts this specific $\widehat{r}$, then he can provide buyers with the same market utility $\overline{U}_t$ *and* he would still keep his previous expected demand $q_{m,t}$. Once $q_{m,t}$ is controlled for, we can compare profits and show that the seller indeed would earn more if he were to switch to fixed pricing.

To start, fix some generic price $r_{f,t}$ and observe that

$$U_{f,t}(r_{f,t}, q_{m,t}) = \frac{1 - z_0(q_{m,t})}{q_{m,t}} v(1 - r_{f,t}) + \left[1 - \frac{1 - z_0(q_{m,t})}{q_{m,t}}\right] \beta u_{t+1}.$$

Let $\Delta(r_{f,t}) \equiv U_{f,t}(r_{f,t}, q_m) - \overline{U}_t$ and recall that $U_{m,t}(r_{m,t}, q_{m,t}) = \overline{U}_t$ where $U_{m,t}$ is given by (7). Substituting $U_{m,t}$ for $\overline{U}_t$ yields

$$\Delta(r_{f,t}) = \frac{1 - z_0(q_{m,t})}{q_{m,t}} v(1 - r_{f,t}) - \sum_{n=1}^{\infty} \frac{z_n(q_{m,t})}{q_{m,t}} v(1 - p_{m,n,t}(r_{m,t})).$$

Note that Δ decreases in $r_{f,t}$. Furthermore, the inequalities in (24) imply that $\Delta(0) < 0$ and $\Delta(1) > 0$. The Intermediate Value Theorem, therefore, implies that there exists some $\widehat{r} \in (0, 1)$ satisfying $U_{f,t}(\widehat{r}, q_{m,t}) = \overline{U}_t$.

Given that $U_{m,t}(r_{m,t}, q_{m,t}) = U_{f,t}(\widehat{r}, q_{m,t})$, we now show that $\Pi_{f,t}(\widehat{r}, q_{m,t}) > \Pi_{m,t}(r_{m,t}, q_{m,t})$, i.e.

the seller is better off by switching. To start, observe that

$$U_{f,t}\left(\widehat{r},q_{m,t}\right)=U_{m,t}\left(r_{m,t},q_{m,t}\right)\Rightarrow v\left(1-\widehat{r}\right)=\sum_{n=1}^{\infty}\zeta_{n}v\left(1-p_{m,n,t}\left(r_{m,t}\right)\right),$$

where

$$\zeta_{n}\equiv\frac{z_{n}\left(q_{m,t}\right)}{1-z_{0}\left(q_{m,t}\right)}\in\left(0,1\right).$$

Since (i) $\zeta_n\in(0,1)$ for all n, (ii) $\sum_{n=1}^{\infty}\zeta_n=1$ and (iii) $v(\cdot)$ is concave we have (Jensen's Inequality)

$$\sum_{n=1}^{\infty}\zeta_{n}v\left(1-p_{m,n,t}\right)<v\left(\sum_{n=1}^{\infty}\zeta_{n}\left(1-p_{m,n,t}\right)\right).$$

The inequality is strict because $p_{m,n,t}\neq p_{m,\widetilde{n},t}$ at least for some n and $\widetilde{n}$, since rule m is flexible. It follows that

$$v\left(\sum_{n=1}^{\infty}\zeta_{n}\left(1-p_{m,n,t}\right)\right)>v\left(1-\widehat{r}\right)\Rightarrow\sum_{n=1}^{\infty}\zeta_{n}\left(1-p_{m,n,t}\right)>1-\widehat{r}.$$

Substituting for ζ_n yields

$$\sum_{n=1}^{\infty}z_{n}\left(q_{m,t}\right)\left(1-p_{m,n,t}\right)>\left[1-z_{0}\left(q_{m,t}\right)\right]\left(1-\widehat{r}\right),$$

which in turn implies that

$$\Pi_{f,t}\left(\widehat{r},q_{m,t}\right)>\Pi_{m,t}\left(r_{m,t},q_{m,t}\right).$$

I.e. there is a profitable deviation; hence there cannot be an equilibrium where flexible rules are adopted. In other words, a market with risk averse buyers can exhibit only a fixed price equilibrium, which we discuss next.

Step 2. We now characterize the fixed price equilibrium. Consider a seller with price $r_{f,t}$ and expected demand $q_{f,t}$. We have

$$\begin{aligned}\Pi_{f,t}&=\left(1-z_{0}\left(q_{f,t}\right)\right)r_{f,t}+z_{0}\left(q_{f,t}\right)\beta\pi_{t+1},\quad\text{and}\\U_{f,t}&=\frac{1-z_{0}\left(q_{f,t}\right)}{q_{f,t}}v\left(1-r_{f,t}\right)+\left[1-\frac{1-z_{0}\left(q_{f,t}\right)}{q_{f,t}}\right]\beta u_{t+1}.\end{aligned}$$

The seller's problem is

$$\max_{r_{f,t}}\Pi_{f,t}\ \text{ subject to }\ U_{f,t}=\overline{U}_{t}.$$

The FOC is given by

$$1-z_{0}\left(q_{f,t}\right)+r_{f,t}z_{0}\left(q_{f,t}\right)\frac{dq_{f,t}}{dr_{f,t}}-\beta z_{0}\left(q_{f,t}\right)\pi_{t+1}\frac{dq_{f,t}}{dr_{f,t}}=0.$$

The indifference condition $U_{f,t} = \overline{U}_t$ implies that (Implicit Function Theorem)

$$\frac{dq_{f,t}}{dr_{f,t}} = -\frac{\partial U_{f,t}/\partial r_{f,t}}{\partial U_{f,t}/\partial q_{f,t}} = -\frac{q_{f,t}\left(1 - z_0\left(q_{f,t}\right)\right) v'\left(1 - r_{f,t}\right)}{\left(1 - z_0\left(q_{f,t}\right) - z_1\left(q_{f,t}\right)\right)\left(v\left(1 - r_{f,t}\right) - \beta u_{t+1}\right)};$$

hence the FOC is equivalent to $\Gamma\left(r_{f,t}, q_{f,t}\right) = 0$, where

$$\Gamma\left(r_{f,t}, q_{f,t}\right) \equiv \frac{\left[r_{f,t} - \beta \pi_{t+1}\right] v'\left(1 - r_{f,t}\right)}{v\left(1 - r_{f,t}\right) - \beta u_{t+1}} - \frac{1 - z_0\left(q_{f,t}\right) - z_1\left(q_{f,t}\right)}{z_1\left(q_{f,t}\right)}.$$

Notice that $r_{f,t} \geq \beta \pi_{t+1}$ and $v\left(1 - r_{f,t}\right) \geq \beta u_{t+1}$. These inequalities imply an upper bound $\overline{r}_{f,t}$ and a lower bound $\underline{r}_{f,t}$ for the list price $r_{f,t}$ where (i) $v\left(1 - \overline{r}_{f,t}\right) = \beta u_{t+1}$ and (ii) $\underline{r}_{f,t} = \beta \pi_{t+1}$. Recall that $v' > 0$, $v'' < 0$. Using these inequalities, along with (i) and (ii), one can show that Γ increases in $r_{f,t}$, and that $\Gamma(\underline{r}_{f,t}, q_{f,t}) < 0$ and $\Gamma(\overline{r}_{f,t}, q_{f,t}) > 0$. Thus, there exists a unique price $r_{f,t} = \rho\left(q_{f,t}\right) \in (\underline{r}_{f,t}, \overline{r}_{f,t})$ satisfying the FOC, i.e. satisfying $\Gamma\left(\rho\left(q_{f,t}\right), q_{f,t}\right) = 0$, where $\rho\left(\cdot\right)$ is an increasing function of $q_{f,t}$.

To establish that all sellers post the same list price, consider another seller with price $r'_{f,t}$ and expected demand $q'_{f,t}$. His problem is similar, so he optimally posts $\rho(q'_{f,t})$. Since in equilibrium all sellers must earn equal profits, we need $\Pi_{f,t}(\rho(q'_{f,t}), q'_{f,t}) = \Pi_{f,t}(\rho\left(q_{f,t}\right), q_{f,t})$ hence $q_{f,t} = q'_{f,t}$ and therefore $r'_{f,t} = r_{f,t}$ because ρ is one-to-one.

Since all sellers post the same list price, symmetry in buyers visiting strategies implies that the expected demand of each seller must be equal to λ_t; thus the equilibrium price $r^*_{f,t}$ uniquely solves $\Gamma(r^*_{f,t}, \lambda_t) = 0$. Substituting $r_{f,t} = r^*_{f,t}$ and $q_{f,t} = \lambda_t$ into $U_{f,t}$ and $\Pi_{f,t}$ yields the equilibrium payoffs

$$u_t = \frac{1 - z_0\left(\lambda_t\right)}{\lambda_t} v(1 - r^*_{f,t}) + \beta\left[1 - \frac{1 - z_0\left(\lambda_t\right)}{\lambda_t}\right] u_{t+1} \quad \text{and} \quad \pi_t = \left(1 - z_0\left(\lambda_t\right)\right) r^*_{f,t} + \beta z_0\left(\lambda_t\right) \pi_{t+1}. \quad (25)$$

This completes the proof. ■

Proof of Lemma 1. The proof consists of three steps.

Step 1. This step deals with obtaining the profit maximizing price $\hat{r}_{m,t}$. Start with fixed pricing (f): Substituting $p_{f,n,t} = r_{f,t}$ into the FOC (14) yields

$$\Delta_f\left(r_{f,t}\right) \equiv r_{f,t}\left[1 - z_0\left(q_{f,t}\right)\right] - \mu\left(q_{f,t}\right) = 0.$$

Observe that $\Delta_f\left(0\right) < 0$ and $\Delta_f\left(1\right) > 0$. Indeed $\Delta_f\left(0\right) < 0$ if $\mu\left(q_{f,t}\right) > 0$, which is true if

$$\left(1 - \beta u_{t+1}\right)\left\{1 - z_0\left(q_{m,t}\right) - z_1\left(q_{m,t}\right)\right\} + z_1 \beta \pi_{t+1} > 0.$$

The term $1 - \beta u_{t+1}$ is positive because u_{t+1} cannot exceed the maximum possible surplus, 1. The expression inside the curly brackets is positive since $\sum_{n=0}^{\infty} z_n = 1$; hence $\Delta_f\left(0\right) < 0$. In addition $\Delta_f\left(1\right) > 0$ if

$$\beta u_{t+1}\left[1 - z_0\left(q_{f,t}\right)\right] + z_1\left(q_{f,t}\right)\left(1 - \beta u_{t+1} - \beta \pi_{t+1}\right) > 0,$$

which is true because of the same reasons above. Since Δ_f rises in $r_{f,t}$, the Intermediate Value Theorem implies that there exists a unique price $\hat{r}_{f,t}$ in the unit interval satisfying $\Delta_f(\hat{r}_{f,t}) = 0$. Basic algebra yields $\hat{r}_{f,t} = \varrho_f$, where ϱ_f is given by (16).

With over-the-sticker pricing (o) we have $p_{o,n,t} = \max\{r_{o,t}, y_{n,t}\}$; thus the FOC (14) becomes

$$\Delta_o(r_{o,t}) \equiv \sum_{n=1}^{\infty} z_n(q_{o,t}) \max\{r_{o,t}, y_{n,t}\} - \mu(q_{o,t}) = 0.$$

Using the steps above it is easy to show that $\Delta_o(1) > 0$. However $\Delta_o(0)$ is bounded below by $\sum_{n=1}^{\infty} z_n(q_{o,t}) y_{n,t} - \mu(q_{o,t})$, hence there are two cases:

- If $\sum_{n=1}^{\infty} z_n(q_{o,t}) y_{n,t} - \mu(q_{o,t}) < 0$ i.e. if $q_{o,t} > \overline{q}_t$, where $\overline{q}_t \in \mathbb{R}_{++}$ is the unique value of q that solves $\sum_{n=1}^{\infty} z_n(q) y_{n,t} = \mu(q)$, then there exists an interior price $r_{o,t} \in (y_j, y_{j+1}]$ for a unique $j(t) = 1, 2, 3, ...$ satisfying the FOC. Solving for $r_{o,t}$ yields $\widehat{r}_{o,t} = \varrho_o(q_{o,t})$, where ϱ_o is given by (17).

- If $\sum_{n=1}^{\infty} z_n(q_{o,t}) y_{n,t} - \mu(q_{o,t}) \geq 0$ i.e. if $q_{o,t} \leq \overline{q}_t$ then no interior price satisfies the FOC, hence we have a corner solution where $\widehat{r}_{o,t} = 0$.

The case with best-offer pricing (b) is similar. Substituting $p_{b,n,t} = \min\{r_{b,t}, y_{n,t}\}$ into (14) yields

$$\Delta_b(r_{b,t}) \equiv \sum_{n=1}^{\infty} z_n(q_{b,t}) \min\{r_{b,t}, y_{n,t}\} - \mu(q_{b,t}) = 0.$$

Using the steps above it is easy to show that $\Delta_b(0) < 0$ however $\Delta_b(1)$ is bounded above by $\sum_{n=1}^{\infty} z_n(q_{b,t}) y_{n,t} - \mu(q_{b,t})$, so, again, there are two cases:

- If $\sum_{n=1}^{\infty} z_n(q_{b,t}) y_{n,t} - \mu(q_{b,t}) > 0$ i.e. if $q_{b,t} < \overline{q}_t$ then there exists a unique interior price $r_{b,t} \in (y_{h,t}, y_{h+1,t}]$ for a unique $h(t) = 0, 1, 2, ...$ satisfying the FOC (recall that $y_{0,t} = 0$). Solving for $r_{b,t}$ yields $\widehat{r}_{b,t} = \varrho_b(q_{b,t})$, where ϱ_b is given by (17).

- If $\sum_{n=1}^{\infty} z_n(q_{b,t}) y_{n,t} - \mu(q_{b,t}) \leq 0$ i.e. if $q_{b,t} \geq \overline{q}_t$ then the FOC cannot hold with equality, so we have a corner solution where $\widehat{r}_{b,t} = 1$.

Step 2. This step involves proving the following Lemma.

Lemma 3 *Suppose that the FOC (13) associated with the price posting problem binds for one seller (so he posts an interior price) and does not bind for another seller (so he posts a corner price). The former seller's expected profit exceeds the latter's.*

Proof. WLOG, consider two sellers competing via different rules; one with rule m and the other with $\tilde{m}$.[22] Suppose the FOC (13) associated with $\tilde{m}$ does not bind while the one associated with m

[22] The case where both sellers compete with the same rule is identical except for a slight modification in notation.

binds, i.e. suppose $z_0(q_{\tilde{m},t}) \neq H_t$ and $z_0(q_{m,t}) = H_t$, where H_t is given by (13). This means that $q_{m,t} \neq q_{\tilde{m},t}$. Expression (9) implies that

$$\Pi_{m,t} = 1 - \beta u_{t+1} - z_0(q_{m,t})(1 - \beta u_{t+1} - \beta\pi_{t+1}) - q_{m,t}(U_{m,t} - \beta u_{t+1}),$$
$$\Pi_{\tilde{m},t} = 1 - \beta u_{t+1} - z_0(q_{\tilde{m},t})(1 - \beta u_{t+1} - \beta\pi_{t+1}) - q_{\tilde{m},t}(U_{\tilde{m},t} - \beta u_{t+1}),$$

where $\Pi_{m,t}$ and $\Pi_{\tilde{m},t}$ are sellers' expected profits and $U_{m,t}$ and $U_{\tilde{m},t}$ are buyers' expected utilities. Both sellers must be providing their customers with the same market utility $\overline{U}_t$, that is $U_{m,t} = U_{\tilde{m},t} = \overline{U}_t$. It follows that

$$\begin{aligned}
\Pi_{m,t} - \Pi_{\tilde{m},t} &= [z_0(q_{\tilde{m},t}) - z_0(q_{m,t})](1 - \beta u_{t+1} - \beta\pi_{t+1}) + (q_{\tilde{m},t} - q_{m,t})(\overline{U}_t - \beta u_{t+1}) \\
&= (1 - \beta u_{t+1} - \beta\pi_{t+1}) \times \{z_0(q_{\tilde{m},t}) - z_0(q_{m,t}) + (q_{\tilde{m},t} - q_{m,t}) z_0(q_{m,t})\} \\
&= (1 - \beta u_{t+1} - \beta\pi_{t+1}) \times z_0(q_{m,t}) \times \left\{e^{-\chi} - 1 + \chi\right\},
\end{aligned}$$

where $\chi \equiv q_{\tilde{m},t} - q_{m,t}$ and note that $\chi \neq 0$ since $q_{m,t} \neq q_{\tilde{m},t}$. In the second step we used the relationship $\overline{U}_t - \beta u_{t+1} = z_0(q_{m,t})(1 - \beta u_{t+1} - \pi_{t+1})$ which follows from (13). In the third step, note that the expression in the curly brackets is positive for all $\chi \neq 0$. It follows that $\Pi_{m,t} > \Pi_{\tilde{m},t}$ i.e. the seller whose FOC holds with equality earns more than the other seller. *Q.E.D.*

Step 3. In this step we argue that any two sellers competing with the same rule m ought to post the same list price. Again, there are three cases:

- Fixed pricing. Suppose that another seller, still competing with fixed pricing, posts a different price $r'_{f,t}$ and has a queue length $q'_{f,t}$. The FOC of his price posting problem is given by $z_0(q'_{f,t}) = H_t$, which in turn means that he ought to post $r'_{f,t} = \varrho_f(q'_{f,t})$ (from Step 1 above). Combining the FOCs we have $z_0(q'_{f,t}) = z_0(q_{f,t}) = H_t$ which implies that $q_{f,t} = q'_{f,t}$, since the expression H_t is independent of $q_{f,t}$ and $q'_{f,t}$. It follows that $r_{f,t} = r'_{f,t}$ since ϱ_f is one-to-one.
- Best-offer pricing. Again suppose that another seller competing with the same rule posts $r'_{b,t}$ and has queue length $q'_{b,t}$. There are three possibilities:
    1. Both $r'_{b,t}$ and $r_{b,t}$ are interior. This case is similar to the case of fixed pricing. The FOCs together imply that $z_0(q'_{b,t}) = z_0(q_{b,t}) = H_t$; hence $q_{b,t} = q'_{b,t}$, and therefore $r_{b,t} = r'_{b,t}$ since ϱ_b is one-to-one.
    2. Both $r'_{b,t}$ and $r_{b,t}$ are corner. Then $r_{b,t} = r'_{b,t} = 1$, so both prices are identical.
    3. One price is corner and the other is interior. This case is impossible as it violates the equal profit condition (Lemma 3).
- Over-the-sticker pricing. This case is similar to best-offer pricing; hence it is skipped. ■

Proof of Proposition 2. The proof consists of three steps.

Step 1. We prove that sellers compete via unbiased rules only, i.e. they never commit to a biased rule, be it seller biased or buyer biased. Let $M_t^* \subseteq \mathbb{M}$ be the set of active pricing rules in equilibrium at time t, i.e. let $M_t^* = \left\{ m \in \mathbb{M} | \, \alpha_{m,t}^* > 0 \right\}$. By way of contradiction, suppose that M_t^* includes a biased rule. Then the FOC, given by (13), or alternatively by (14), cannot hold with equality for all rules in M_t^*. To see why, suppose it does. Then we would have $z_0 \left(q_{m,t} \right) = z_0 \left(q_{m',t} \right) = H_t$ for all $m, m' \in M_t^*$, and therefore $q_{m,t} = q_{m',t}$. Inserting this equality into (12) yields $q_{m,t} = \lambda_t$ for all $m \in M_t^*$. Substituting $q_{m,t} = \lambda_t$ into (14) yields $\sum_{n=1}^{\infty} z_n \left(\lambda_t \right) p_{m,n,t}(r_{m,t}) = \mu \left(\lambda_t \right)$, for all $m \in M_t^*$, which implies that all rules in M_t^* are unbiased; a contradiction.

So, let $\tilde{m}$ denote the pricing rule for which the FOC does not hold, that is $z_0 \left(q_{\tilde{m},t} \right) \neq H_t$. Lemma 3 above implies that a seller who adopts rule m can earn more if he were to switch to a rule for which the FOC holds with equality. Fixed pricing is a candidate for such a switch. Indeed, with fixed pricing we have $z_0 \left(q_{f,t} \right) = H_t$ for any given $q_{f,t}$ (Lemma 1). It follows that the seller would earn more if he were to switch to fixed pricing instead of sticking with rule $\tilde{m}$. Hence the outcome where M_t^* includes a biased rule cannot correspond an equilibrium.

Step 2. We show that unbiased pricing rules are payoff equivalent in equilibrium and there exits a continuum of equilibria where such rules may coexist. To start suppose that $\lambda_t > \overline{\lambda}_t$. In this parameter region best-offer pricing is buyer-biased (Lemma 2), so it will not be offered in equilibrium i.e. $\alpha_{b,t}^* = 0$ (Step 1). Now consider an outcome where some arbitrary fraction of sellers compete via fixed pricing while the rest compete via over-the-sticker pricing, i.e. fix some arbitrary $\alpha_{f,t}^* \geq 0$ and $\alpha_{o,t}^* \geq 0$ such that $\alpha_{f,t}^* + \alpha_{o,t}^* = 1$. We will show that over-the-sticker pricing and fixed pricing are both payoff equivalent; so, no seller has a profitable deviation from the pricing rule he is randomly assigned to.

To start, conjecture that (to be verified) the FOC (13) holds with equality under both rules, i.e. conjecture that $z_0 \left(q_{f,t} \right) = z_0 \left(q_{o,t} \right) = H_t$, which in turn means that $q_{f,t} = q_{o,t}$. Substituting this equality, along with the fact that $\alpha_{b,t}^* = 0$, into (12) yields $q_{f,t} = q_{o,t} = \lambda_t$. Therefore equation (14) becomes

$$\sum_{n=1}^{\infty} z_n \left(\lambda_t \right) p_{m,n,t}(r_{m,t}) = \mu \left(\lambda_t \right) \text{ for } m = f, o. \tag{26}$$

Since $\lambda_t > \overline{\lambda}_t$, both rules, f and o, are unbiased; so, there exists unique interior list prices $r_{f,t} = \varrho_f \left(\lambda_t \right)$ and $r_{o,t} = \varrho_o \left(\lambda_t \right)$ (characterized earlier in Lemma 1) satisfying the above equality for both pricing rules and verifying the conjecture above. Combining (26) with (9) reveals that along this outcome sellers earn the same expected profit

$$\Pi_{m,t} = 1 - \beta u_{t+1} - \left[z_0 \left(\lambda_t \right) + z_1 \left(\lambda_t \right) \right] \left(1 - \beta u_{t+1} - \beta \pi_{t+1} \right) \equiv \pi_t,$$

whether they compete with fixed pricing or over-the-sticker pricing. Similarly combining (26) with (7) shows that buyers earn the same expected utility

$$U_{m,t} = z_0 \left(\lambda_t \right) \left[1 - \beta u_{t+1} - \beta \pi_{t+1} \right] + \beta u_{t+1} \equiv u_t$$

whether they shop at fixed price sellers or over-the-sticker sellers. The arbitrary selection of $\alpha^*_{f,t} \geq 0$ and $\alpha^*_{o,t} \geq 0$ corresponds to payoff equivalence, which means that there exists a continuum of equilibria where either rule may be offered by any fraction of sellers.

Now consider the case $\lambda_t < \overline{\lambda}_t$. In this region, over-the-sticker pricing is seller-biased (Lemma 2) and cannot be offered (Step 1). Best-offer pricing and fixed pricing, on the other hand, are unbiased. Going through similar steps, one can show that both of these rules are payoff equivalent and, therefore, there exits a continuum of equilibria where either rule may be offered by any fraction of sellers. The analysis is same as above; hence it is skipped. Finally, in the knife edge case $\lambda_t = \overline{\lambda}_t$, where all rules are unbiased, one can similarly show that there exits a continuum of equilibria where any one of the three rules may be offered by any fraction of sellers.

Step 3. To obtain the relationship $r^*_{o,t} < r^*_{f,t}$ fix some $\lambda_t \geq \overline{\lambda}_t$ and some generic list price $r \geq y_{1,t}$. For the given list price r, the sale price with over-the-sticker pricing exceeds the sale price with fixed pricing i.e. $r \leq \max(r, y_{n,t})$ where the inequality is strict at least for some n. Recall that in equilibrium both rules must attract the same demand λ_t and must be payoff equivalent i.e. $\Pi_{o,t}(r, \lambda_t) = \Pi_{f,t}(r, \lambda_t)$. Both profit functions increase in r. Since λ_t is already controlled for, payoff equivalence is possible only if $r^*_{o,t} < r^*_{f,t}$. The fact that $r^*_{b,t} > r^*_{f,t}$ is proved using mirror image arguments.■

Proof of Remark 1. We establish that players prefer to trade immediately rather than waiting.
1. Fixed Pricing. A seller prefers to sell if

$$r^*_{f,t} \geq \beta\pi_{t+1},$$

i.e. if the equilibrium fixed price is greater than or equal to the discounted value of continuing to search as a seller. Recall that $r^*_{f,t} = \varrho_f(\lambda_t)$, where ϱ_f is given by (16). One can verify that

$$\varrho_f(\lambda_t) > \beta\pi_{t+1} \text{ if } 1 - z_0(\lambda_t) - z_1(\lambda_t) > 0.$$

The inequality is true since $\sum_{n=0}^{\infty} z_n(\lambda_t) = 1$; hence the seller is better off trading right away. Now consider a buyer. If he transacts immediately then he gets $1 - \varrho_f(\lambda_t)$, but if he waits then he obtains βu_{t+1}. Note that

$$1 - \varrho_f(\lambda_t) > \beta u_{t+1} \text{ if } 1 - \beta u_{t+1} - \beta\pi_{t+1} > 0.$$

Recall that $u_t + \pi_t \leq 1$. Hence the above inequality is true and the buyer, too, is better off purchasing immediately.

2. Over the Sticker Pricing. Recall that $y_{n,t}$ rises in n. Since the sale price is equal to $\max\{r^*_{o,t}, y_{n,t}\}$ the lowest possible sale price is $y_{1,t}$. If the seller trades at this price then he will certainly trade at other (higher) prices. Note that $y_{1,t}$ is equal to

$$\begin{aligned} y_{1,t}^{Nash} &= 1-\beta u_{t+1}-\theta_1\left(1-\beta u_{t+1}-\beta \pi_{t+1}\right) \\ y_{1,t}^{Strategic} &= \frac{\delta\left(1-\beta u_{t+1}\right)+\beta \pi_{t+1}}{1+\delta}, \end{aligned}$$

depending on whether players use Nash Bargaining or Strategic Bargaining. These expressions are obtained by substituting $n=1$ into (3) and (4). The seller will trade if $y_{1,t} \geq \beta \pi_{t+1}$. It is straightforward to show that

$$y_{1,t}^{Nash} \geq \beta \pi_{t+1} \text{ if } \theta_1 \leq 1 \text{ and}$$

$$y_{1,t}^{Strategic} \geq \beta \pi_{t+1} \text{ if } \delta(1-\beta u_{t+1}-\beta \pi_{t+1}) \geq 0.$$

The first inequality is satisfied because $\theta_n < 1$ for all n. The second inequality is satisfied because $u_{t+1}+\pi_{t+1} \leq 1$ and $\delta \in (0,1)$. Hence the seller is better off transacting immediately.

Now consider the buyer. With over-the-sticker pricing the buyer will end up paying more than the list price if there are a number of other buyers at the same store. The worst case scenario is where $n \rightarrow \infty$, and therefore having to pay $\lim_{n \rightarrow \infty} y_{n,t} \equiv y_{\infty,t}$. If the buyer purchases at this extreme price then he will certainly purchase at lower prices. Note that $y_{\infty,t}$ is equal to

$$y_{\infty,t}^{Nash} = 1-\beta u_{t+1}-\theta_\infty\left(1-\beta u_{t+1}-\beta \pi_{t+1}\right) \quad \text{and}$$

$$y_{\infty,t}^{Strategic} = \delta\left(1-\beta u_{t+1}\right)+(1-\delta)\, \beta \pi_{t+1}.$$

The buyer purchases if $1-y_{\infty,t} \geq \beta u_{t+1}$. It is straightforward to show that

$$1-y_{\infty,t}^{Nash} \geq \beta u_{t+1} \text{ if } \theta_\infty \geq 0 \text{ and}$$

$$1-y_{\infty,t}^{Strategic} \geq \beta u_{t+1} \text{ if } \delta \leq 1.$$

The first inequality is satisfied because $\theta_n \geq 0$ for all n. The second inequality is satisfied because $\delta \in (0,1)$. Therefore the buyer, too, is better off transacting immediately.

3. Best-offer Pricing. This case is similar to over-the-sticker pricing; hence it is skipped. ■

Proof of Proposition 3. Start by rearranging expressions (19) and (20) to obtain

$$\pi_t = 1-\beta u_{t+1}-\left[z_0\left(\lambda_t\right)+z_1\left(\lambda_t\right)\right] \Delta_{t+1} \text{ and } u_t = \beta u_{t+1}+z_0\left(\lambda_t\right) \Delta_{t+1}$$

where

$$\Delta_{t+1} \equiv 1-\beta u_{t+1}-\beta \pi_{t+1}.$$

Given Δ_{t+1} the expressions for $y_{n,t}^{Nash}$, $y_{n,t}^{Strategic}$ and $r_{f,t}^*$, which are given by (3), (4) and (18), can be re-written as follows.

$$y_{n,t}^{Nash} = 1-\beta u_{t+1}-\theta_n\Delta_{t+1},$$
$$y_{n,t}^{Strategic} = \beta\pi_{t+1}+\frac{\delta-\delta^2/n}{1-\delta^2/n}\Delta_{t+1},$$
$$r_{f,t}^{*} = 1-\beta u_{t+1}-\frac{z_1(\lambda_t)}{1-z_0(\lambda_t)}\Delta_{t+1}.$$

Now, pick an arbitrary integer $s\in\mathbb{N}_+$. Iteration on t yields

$$\Delta_{t+1}=(1-\beta)\times\left[1+\sum_{i=1}^{s-1}\beta^i\prod_{j=1}^{i}z_1(\lambda_{t+j})\right]+\underbrace{\beta^s\prod_{j=1}^{s}z_1(\lambda_{t+j})\times\Delta_{t+1+s}}_{O(s)}. \tag{27}$$

The terms $z_1(\lambda_{t+j})$ are all strictly less than 1. Since time runs indefinitely (i.e. the terminal period $T=\infty$), one can pick s large enough to ensure that $O(s)\approx 0$; hence

$$\Delta_{t+1}\approx(1-\beta)\times\left[1+\sum_{i=1}^{s-1}\beta^i\prod_{j=1}^{i}z_1(\lambda_{t+j})\right]$$

Consequently we have $\lim_{\beta\to1}\Delta_{t+1}=0$; and therefore $\lim_{\beta\to1}\pi_t=1-u_{t+1}$ and $\lim_{\beta\to1}u_t=u_{t+1}$. It follows that, as β tends to 1, we have $u_t=\bar{u}$, $\pi_t=1-\bar{u}$ for all t, and therefore

$$\lim_{\beta\to1}y_{n,t}^{Nash}=\lim_{\beta\to1}y_{n,t}^{Strategic}=\lim_{\beta\to1}r_{f,t}^{*}=1-\bar{u}.$$

This completes the proof. ■

7 Appendix B: Bargaining (Not Intended for Publication)

7.1 Nash Bargaining

In the main body of the text we assume that the bargaining power of a buyer θ_n drops in n and based on this assumption we derive the equilibrium bargained price. Here we consider an alternative scenario where the bargaining power θ is constant but negotiations take place over a number of bargaining rounds, where the seller sequentially negotiates with buyers. Specifically consider a seller with n customers and, WLOG, suppose that negotiations take place over two rounds.[23] In round one, the seller is randomly matched with a buyer and they negotiate via Nash Bargaining. In case of disagreement negotiations continue to the second round where the seller, again, is matched randomly with a buyer. The initially selected buyer has the same chance as the other buyers to be reselected

[23] It is straightforward to increase the number of rounds, however this requires computing intermediate prices at each round, which makes the analysis burdensome and yet does not add any additional insight when compared to the case with two rounds.

$(1/n)$. If negotiations fail yet again then all players go back to the market empty-handed to search again in the next period. Bargaining rounds are sub-periods within the same search period, and in order to keep things simple we assume that there is no discounting between the bargaining rounds.

Start with round 2. Let y_2 denote the bargaining price that would emerge in this round and note that at this stage the outside options are βu_{t+1} for the buyer and $\beta \pi_{t+1}$ for the seller. Thus the Nash product is given by

$$\max_{y_2} \; \left(v\left(1-y_2\right)-\beta u_{t+1}\right)^{\theta}\left(y_2-\beta \pi_{t+1}\right)^{1-\theta}$$

Note that the bargaining power θ is fixed and it is no longer indexed by n. The bargained price y_2 solves the FOC, which is given by

$$\frac{\left(y_2-\beta \pi_{t+1}\right) v^{\prime}\left(1-y_2\right)}{v\left(1-y_2\right)-\beta u_{t+1}}=\frac{1-\theta}{\theta}.$$

Note that y_2 must satisfy $v\left(1-y_2\right) \geq \beta u_{t+1}$ and $y_2 \geq \beta \pi_{t+1}$, i.e. each player ought to get a payoff that is greater than or equal to his outside option. Now consider round 1. The outside option of the buyer who negotiates with the seller is equal to

$$\frac{1}{n} v\left(1-y_2\right)+\frac{n-1}{n} \beta u_{t+1}=\beta u_{t+1}+\frac{v\left(1-y_2\right)-\beta u_{t+1}}{n}.$$

To see why, note that if bargaining results in disagreement in the first round, then in the second round the buyer will be reselected to deal with the seller with probability $\frac{1}{n}$, in which case he would obtain payoff $v\left(1-y_2\right)$. With the complementary probability $\frac{n-1}{n}$, however, he will not be reselected and he will walk away with his value of search, βu_{t+1}. Observe that the buyer's outside option falls as n increases because the larger n, the less likely is the buyer to be re-selected in the next round. As for the seller, his outside option in this round is simply y_2, which is the price he would obtain in the next round. Unlike the buyer, the seller does not worry about not being reselected as he is always a counterparty in bargaining. (He is the central player whereas buyers are peripheral players.) The Nash product in round 1 is given by

$$\max_{y_1} \; \left(v\left(1-y_1\right)-\beta u_{t+1}-\frac{v\left(1-y_2\right)-\beta u_{t+1}}{n}\right)^{\theta}\left(y_1-y_2\right)^{1-\theta}$$

The bargained price y_1 solves the FOC

$$\frac{\left(y_1-y_2\right) v^{\prime}\left(1-y_1\right)}{v\left(1-y_1\right)-\beta u_{t+1}-\frac{v\left(1-y_2\right)-\beta u_{t+1}}{n}}=\frac{1-\theta}{\theta}.$$

Call the expression on the left hand side $M\left(y_1, n\right)$ and note that $\frac{1-\theta}{\theta}$ is a constant. Applying the Implicit Function Theorem to the equality above we have

$$\frac{d y_1}{d n}=-\frac{M_n}{M_{y_1}},$$

where M_n and M_{y_1} are partial derivatives. Note that M_n is negative because y_2 is independent of n whereas the term $\frac{v(1-y_2)-\beta u_{t+1}}{n}$ is positive and falls in n. Furthermore M_{y_1} is positive because v is weakly concave (as the individual is either risk neutral or risk averse). It follows that $\frac{dy_1}{dn}$ is positive i.e. the larger the local demand n the higher the equilibrium bargained price y_1. This completes the first part of the claim—that is, even though the bargaining power may be fixed, one can obtain a positive relationship between the bargained price and the local demand n by considering sequential rounds of negotiations

Now we show that the closed form solution of the equilibrium bargained price in this scenario is indeed identical to its counterpart in the main body of the text, given by equation (3), up to a relabelling of the bargaining power. Recall that expression in (3) is derived when buyers were risk neutral; so, in order to compare the closed form solutions we will consider risk neutral buyers here as well. Substituting $v(x) = x$ into the first order conditions above, we have

$$y_2 = 1 - \beta u_{t+1} - \theta (1 - \beta u_{t+1} - \beta \pi_{t+1})$$

and therefore

$$y_1 = 1 - \beta u_{t+1} - \underbrace{\frac{\theta^2 (n-1) + \theta}{n}}_{\theta_n} (1 - \beta u_{t+1} - \beta \pi_{t+1}).$$

Note that $\frac{\theta^2(n-1)+\theta}{n}$ falls in n. If we label this term as θ_n and interpret it as the buyer's "composite bargaining power" (which not only depends on the constant factor θ but also on the realized demand n) then indeed we have $\theta_n > \theta_{n+1}$ justifying the assumption in the main body—that the buyer's bargaining power falls (and the seller's bargaining power rises) in n . What is remarkable, after this relabelling, the closed form solution of the equilibrium bargained price y_1 is the same as its counterpart in the main text, given by expression (3).[24] ■

7.2 Strategic Bargaining

Consider the strategic bargaining setup described in Section 3.3. In what follows we focus on a subgame perfect equilibrium (SPE) where offers are stationary and are accepted without delay. So, let $y^s_{n,t}$ denote the seller's offer to a buyer, and let $y^b_{n,t}$ denote a buyer's offer to the seller. (Note that both $y^s_{n,t}$ and $y^b_{n,t}$ are prices.) The indifference conditions are given by

$$y^b_{n,t} = \delta y^s_{n,t} + (1-\delta) \beta \pi_{t+1} \quad \text{and} \quad v(1 - y^s_{n,t}) = \frac{\delta}{n} v(1 - y^b_{n,t}) + (1 - \frac{\delta}{n}) \beta u_{t+1}.$$

Substituting for $y^s_{n,t}$ yields

$$\Theta(y^b_{n,t}) \equiv v(1 - y^b_{n,t}/\delta + (1-\delta) \beta \pi_{t+1}/\delta) - \delta v(1 - y^b_{n,t})/n - (1 - \delta/n) \beta u_{t+1} = 0.$$

[24] Along the equilibrium path negotiations result in agreement in round 1, so the relevant price is y_1, and not y_2.

Below we show that there exists a unique offer $\widehat{y}^b_{n,t}$ satisfying $\Theta(\widehat{y}^b_{n,t}) = 0$. Recall that π_{t+1} and u_{t+1} are sellers' and buyers' expected payoffs from the next period. It follows that $y^b_{n,t} \in [\underline{y}^b_{n,t}, \overline{y}^b_{n,t}]$, where

$$\underline{y}^b_{n,t} = \beta\pi_{t+1} \quad \text{and} \quad v(1 - \overline{y}^b_{n,t}) = \beta u_{t+1}.$$

Using these relationships it is easy to verify that

$$\Theta(\underline{y}^b_{n,t}) = [v(1 - \underline{y}^b_{n,t}) - \beta u_{t+1}]\,(1 - \delta/n) > 0 \text{ and}$$
$$\Theta(\overline{y}^b_{n,t}) = v(1 - \overline{y}^b_{n,t}/\delta + (1-\delta)\,\beta\pi_{t+1}/\delta) - v(1 - \overline{y}^b_{n,t}) < 0.$$

In addition, notice that $\partial\Theta/\partial y^b_{n,t} < 0$, where

$$\partial\Theta/\partial y^b_{n,t} = -\frac{v'(1 - y^b_{n,t}/\delta + (1-\delta)\,\beta\pi_{t+1}/\delta)}{\delta} + \frac{\delta v'(1 - y^b_{n,t})}{n}.$$

The expression is negative since v' is non-increasing and

$$1 - \frac{y^b_{n,t}}{\delta} + \frac{(1-\delta)\,\beta\pi_{t+1}}{\delta} < 1 - y^b_{n,t}.$$

Given that Θ is decreasing and that $\Theta(\underline{y}^b_{n,t}) > 0 > \Theta(\overline{y}^b_{n,t})$, the Intermediate Value Theorem implies that there exits a unique $\widehat{y}^b_{n,t} \in [\underline{y}^b_{n,t}, \overline{y}^b_{n,t}]$ satisfying $\Theta(\widehat{y}^b_{n,t}) = 0$. Once $\widehat{y}^b_{n,t}$ is obtained, the seller's optimal offer $\hat{y}^s_{n,t}$ can be recovered from the indifference conditions above.

Existence of the unique pair $\widehat{y}^b_{n,t}$ and $\widehat{y}^s_{n,t}$ implies that there is a unique SPE satisfying stationarity and no-delay. It is easy to check that the following strategies are indeed subgame perfect. (i) Each buyer offers to buy the good at price $\widehat{y}^b_{n,t}$ and accepts any offer $y^s_{n,t}$ from the seller as long as $y^s_{n,t} \leq \widehat{y}^s_{n,t}$. (ii) The seller offers to sell the good at price $\widehat{y}^s_{n,t}$ and accepts any offer $y^b_{n,t}$ from the buyers as long as $y^b_{n,t} \geq \widehat{y}^b_{n,t}$.

Finally, we verify that $\widehat{y}^b_{n,t}$ rises in n. Notice that

$$\partial\Theta/\partial n = \delta[v(1 - \hat{y}^b_{n,t}) - \beta u_{t+1}]/n^2 > 0.$$

The Implicit Function Theorem suggests that

$$d\widehat{y}^b_{n,t}/dn = -\frac{\partial\Theta/\partial n}{\partial\Theta/\partial\hat{y}^b_{n,t}} > 0.$$

The numerator is positive and the denominator is negative (see above); hence $d\widehat{y}^b_{n,t}/dn$ is positive. ■