Cardiff Economics Working Papers

Working Paper No. E2022/7

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March 2022

ISSN 1749-6010

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February 2022

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Abstract

The purpose of this paper is to investigate the nature of professionals' inflation forecasts inattentiveness. We introduce and empirically investigate a new generalized model of inattentiveness due to informational rigidity. In doing so, we outline a novel model that considers the non-linear relationship between inattentiveness and aggregate uncertainty, which crucially distinguishes between macro-economic and data (measurement error) uncertainty. The empirical analysis uses the Survey of Professional Forecasters data and indicates that inattentiveness due to imperfect information explains professional forecasts' dynamics.

Keywords: Inflation Forecasts, Information Rigidities, Inattentiveness, Uncertainty, Survey Forecasts

JEL classification: E3, E4, E5.

I: Introduction:

Inflation forecasts and, indeed, macroeconomic forecasts are permeated by inattentiveness. Professional forecasters, or experts, are no less susceptible to this issue, as established clearly in recent investigations (see Andrade and Le Bihan (2013), Coibion and Gorodnichenko (2015), Dovern et al (2015), Jain (2019), and Easaw and Golinelli (2021)). The purpose of the current paper is to investigate how aggregate uncertainty affects the inattentiveness of professionals' inflation forecasts.

A number of models have focused on deviations from full-information rational expectations due to informational rigidities (see, for example, Mankiw and Reis (2002), Woodford (2003) and Sims (2003)). The different forms of information rigidities, or agent's inattentiveness, form the basis of the competing rational expectations models with alternative types of informational frictions[1]. Coibion and Gorodnichenko (2015) contend that both types of models predict quantitatively similar forecast errors.

Using their simple framework where forecast errors are investigated empirically as deviations from the full-information rational expectations (FIRE), we consider a generalized framework that encompasses both the noisy and sticky information models. The paper extends the existing literature in three ways. Firstly, we introduce a generalized model of inflation forecast which encompasses the two main forms of inattentiveness or information rigidities, notably due to sticky and imperfect (noisy) information. In the generalized model, information rigidities due the sticky information model is nested within the imperfect information model and, hence, the features of both can be verified by testing the relevant parameters. The empirical analysis finds the significance of the additional explanatory variable of our model, which measures the noise capturing measurement errors due to data revisions. Secondly, we introduce a novel model that explicitly derives the non-linear relationship between inattentiveness and uncertainty. This non-linear relationship is modelled in two ways: as a multiplicative-interactions model and as a state-dependent model, which does not need to specify a functional form. Thirdly, we specifically introduce the concept of measurement error (e.g., data revisions) whose variance (i.e. data uncertainty) is distinct from macroeconomic uncertainty. In an

[1] The first type of informational friction models is the sticky-information model of Mankiw and Reis (2002) where agents update their information set sporadically. Such sticky information expectations have been used to explain not only inflation dynamics (Mankiw and Reis (2002)) but also aggregate outcomes in general (Mankiw and Reis (2006)) and the implications for monetary policy (Ball et al., 2005). The second type of informational friction models is the noisy information model of Woodford (2003) and Sims (2003) where the agents continuously update their information set, but can never fully observe the true state because of signal extraction problems.

innovative approach, we distinguish the effect of macroeconomic and data uncertainty on the inattentiveness of professional forecasters when forecasting inflation.

Recently, too, there has been a heighten interest in understanding and explaining the role macroeconomic uncertainty when forming subjective macroeconomic expectations and forecasts (see, for instances, Dick et al (2013) and Clements (2021)). Indeed, as Lahiri and Sheng (2010) and Dovern (2015) show, even disagreements amongst professional forecasters can be related to macroeconomic uncertainty. The issue of uncertainty is a pertinent one for both forms of inattentiveness. Regardless, the distinct micro-foundations of the two models suggests that the source and nature of the uncertainty are crucial. Hence, an important contribution of this paper is to distinguish between uncertainty resulting from macroeconomic shocks and noisy data, and resultant measurement error. We are, thereby, able to assess their different impact on inattentiveness.

Our empirical results indicate that the professionals' forecast errors display micro-foundations and dynamics that are consistent with imperfect, or noisy, information. This result is robust to different definitions of measurement error. When assessing the impact of uncertainty on professionals' inattentiveness, the distinction between macro and data uncertainty is both revealing and pertinent. We find that during periods of high macro uncertainty inattentiveness reduces, and that increasing data uncertainty leads to greater inattentiveness, as predicted by our theoretical model.

The structure of the paper is as follows. The next section outlines a simple generalized model encompassing both imperfect and sticky information models. The ensuing empirical investigations are outlined and discussed. Section III then introduces a new model that explicitly sets the relationship between inattentiveness due to information rigidities and uncertainty. Section IV estimates the relationship by using both multiplicative-interactions and state-dependent models. Finally, Section V outlines the summary of the key results and draws the concluding remarks.

II: Imperfect Information and Sticky Information Model: A Generalized Aggregation of Individual Forecasters:

In this section, we introduce a generalized model of professionals' inflation forecasts which nests both the imperfect, or noisy, information (NI) and sticky information (SI) models. In an influential paper, Nordhaus (1987) introduced the concept of 'weak efficiency' forecast. Following Coibion and Gorodnichenko (2015) extension and adapting this concept into the recent inattentiveness literature, we introduce a generalized version here. Specifically, our model outlines an aggregated version of the individual forecaster behaviours. The model considers the distinct microfoundations of the NI and SI models and, consequently, the dynamics of the ensuing forecast error. This is the basis for empirically investigating the main features of both the NI and SI models.

In the case of the NI model, agents know the structure of the model and its parameters and keep updated information sets, but never observe the actual state of inflation x_t (they only receive a noisy signal of it). Regarding the model, Coibion and Gorodnichenko (2015) assume that inflation evolves as a stationary AR model, and more recent models have assumed different versions of this. For instance, Jain (2019) assumes a stationary version of the UC model. Ryngaert (2017) also assumes that inflation evolves as a stationary AR(1) model with a constant. Of course, the stationarity assumption is better suited to representing the inflation dynamics in short samples, where inflation is deemed to revert to a constant long-run (core) inflation with no regime breaks or changes to core inflation.

Following Easaw and Golinelli (2021), we start from a general specification – more appropriate for longer sample periods – that crucially allows for breaks to the core inflation level.[2] Since the seminal Perron (1989), if a limited number of m fundamental shocks occur in T_j^B ($j = 1, 2, \ldots, m$), the inflation dynamics will be affected by m deterministic shifts (producing m+1 regimes) which are interpretable as fluctuations of the core inflation τ_{m+1}, plus a sequence of transitory shocks to the stationary inflation gap ξ_t. In this context, we assume that the forecaster estimates (either formally or qualitatively) the core inflation as τ_{m+1}, and that uses the most recent regime estimate to forecast inflation over long horizons.

[2] Nevertheless, for shorter sample periods and/or without significant breaks, this model collapses to the Ryngaert (2017) specification.

Formally, the unobservable inflation state x_t is

$$x_t = \tau_{m+1} + \xi_t \qquad (1)$$

where τ_{m+1} is a series of *m*+1 constants along *m*+1 inflation regimes, and the inflation gap ξ_t is assumed to evolve according to the stationary AR(1) process $\xi_t = \rho\xi_{t-1} + \upsilon_t$ (the shock υ_t is a zero-mean martingale difference with $Var(\upsilon_t) = \sigma_\upsilon^2$ and can be heteroskedastic, as σ_υ^2 may vary over time).[3]

The specification assumes that the i^{th} professional forecaster knows the structure of model (1) and its underlying parameters but does not directly observe the actual state of inflation at *t*, while individual agents only receive the signals y_{it} pertaining to x_t. The measurement equation (2) defines that the individual forecaster *i* can only observe the sum of the state x_t plus the *iid* zero-mean individual measurement noise ω_{it}:

$$y_{it} = \tau_{m+1} + \xi_t + \omega_{it} \qquad (2)$$

where the measurement noise is such that $Cov(\omega_{it}\omega_{jt-k})$ is equal to zero when *i*=*j* and *k*>0 (i.e. individual noises are not serially correlated) while, when *i*=*j* and *k*=0, the variance of noises is equal to σ_ω^2, which may also be generalized, i.e. varying by individual *i* and period *t*.

Measurement noise may be correlated simultaneously across agents in period *t*. Hence, when *i*≠*j*, $Cov(\omega_{it}\omega_{jt-k}) = \sigma_{ij}^2$ if $k = 0$, and $Cov(\omega_{it}\omega_{jt-k}) = 0$ if $k > 0$. The existence of simultaneous covariances across agents suggests that the individual perceptions of the state variable *x* can be contemporaneously related across agents (for instance, individuals' perception of *x* can be influenced by the same provisional data releases). In the absence of noise, that is when σ_ω^2 tends to zero, agents' measurements are closer to each other, and they are perfectly informed about the state *x*. Finally, individual measurement errors ω_{it} are assumed unrelated with the macroeconomic shocks υ_t (for example, the measurement errors due to preliminary data releases are not related with the macroeconomic shocks).

Given that the state x_t $F_{it}x_t$

[3] This point will be explored further in Section III below. When forecasting inflation, Stock and Watson (2007) explicitly model the variability of permanent and transitory disturbances to their UC model with stochastic volatility processes.

$F_{it-1}x_t$, only depends on the knowledge of the process prior to *t*. The *a posteriori* state estimate in *t*, $F_{it}x_t$, additionally requires the knowledge of the measurement y_{it}.

In the Kalman filter framework, the objective is to estimate the optimal *a posteriori* state $F_{it}x_t$ as a linear combination of the *a priori* estimate $F_{it-1}x_t$ and the weighted difference between the actual measurement y_{it} available in *t* and its prediction made in *t-1*:

$$F_{it}x_t = F_{it-1}x_t + G(y_{it} - F_{it-1}x_t) \tag{3}$$

where the Kalman gain *G* is the optimal weight, assumed to be the same across *i* and over time, with $0 \leq G \leq 1$.[4] Equation (3) can also be rearranged as the weighted sum of the new information (2) and the past forecast:

$$F_{it}x_t = G\left(\tau_{m+1} + \xi_t + \omega_{it}\right) + (1-G)F_{it-1}x_t \tag{4}$$

Using the *a posteriori* state estimate $F_{it}x_t$ depicted in equation (4), the agent may iterate it forward *h* steps ahead using the information about the data generation process of *x* given in equation (1):

$$F_{it}x_{t+h} = \tau_{m+1} + G\ (\rho^h \xi_t + \vartheta\omega_{it}) + (1-G)\rho^h\left(F_{it-1}x_t - \tau_{m+1}\right) \tag{5}$$

where ϑ parameter measures the rate at which the measurement error in *t* is forecast forward.[5] Since, $\rho^h\left(F_{it-1}x_t - \tau_{m+1}\right) = F_{it-1}x_{t+h} - \tau_{m+1}$, equation (5) now becomes:

$$F_{it}x_{t+h} = G\ (\tau_{m+1} + \rho^h \xi_t + \vartheta\omega_{it}) + (1-G)F_{it-1}x_{t+h} \tag{5'}$$

Following the definition of the FIRE forecast error *h*-steps ahead, $e^{FIRE}_{t+h|t}$,[6] we can substitute in equation (5') the FIRE forecast *h*-steps ahead, $\tau_{m+1} + \rho^h \xi_t$, with the difference between the actual values of *x* in *t+h* and the FIRE error, $x_{t+h} - e^{FIRE}_{t+h|t}$, to obtain:

$$F_{it}x_{t+h} = G\left[(x_{t+h} - e^{FIRE}_{t+h|t}) + \vartheta\omega_{it}\right] + (1-G)F_{it-1}x_{t+h} \tag{6}$$

Model (6) can be rearranged to define the i^{th} agent's forecast error as follows:

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[4] The assumption of constant *G* is dropped in Section III.

[5] If we assume the NI agents iterate forward their forecasts by treating the measurement error as a part of the inflation gap, we have $\vartheta = \rho^h$.

[6] See equations (A1.3) and (A1.4) in Appendix A1.

$$x_{t+h} - F_{it}x_{t+h} = \frac{1-G}{G}(F_{it}x_{t+h} - F_{it-1}x_{t+h}) + \varsigma_{it+h|t} \quad (7)$$

where the individual error $\varsigma_{it+h|t} = e^{FIRE}_{t+h|t} - \vartheta\omega_{it}$ consists of two orthogonal components: the FIRE forecast error (the same across agents) and the individual noise ω_{it}.

We derive the aggregate specification of this model by averaging individual forecasts across agents, with t = 1, 2, ..., T denoting the time when the forecast is made:

$$x_{t+h} - \sum_{i=1}^{N}\frac{F_{it}x_{t+h}}{N} = \frac{1-G}{G}\left[\sum_{i=1}^{N}\frac{F_{it}x_{t+h}}{N} - \sum_{i=1}^{N}\frac{F_{it-1}x_{t+h}}{N}\right] + \sum_{i=1}^{N}\frac{\varsigma_{it+h|t}}{N} \quad (8)$$

As stressed above, the Kalman gain G is assumed to be the same across individuals and over time.[7] Equation (8) represents the aggregate linear model, whose average error term depends on both the FIRE error over the forecast horizon and the average across all forecasters of the individual noise ω_{it} at the time of the survey:

$$\sum_{i=1}^{N}\frac{\varsigma_{it+h|t}}{N} = \sum_{i=1}^{N}\frac{e^{FIRE}_{t+h|t} - \vartheta\omega_{it}}{N} = e^{FIRE}_{t+h|t} - \vartheta\sum_{i=1}^{N}\frac{\omega_{it}}{N} \quad (8')$$

Therefore, the aggregate NI linear model can be written more explicitly as:

$$x_{t+h} - \sum_{i=1}^{N}\frac{F_{it}x_{t+h}}{N} = \frac{1-G}{G}\left[\sum_{i=1}^{N}\frac{F_{it}x_{t+h}}{N} - \sum_{i=1}^{N}\frac{F_{it-1}x_{t+h}}{N}\right] - \vartheta\sum_{i=1}^{N}\frac{\omega_{it}}{N} + e^{FIRE}_{t+h|t} \quad (9)$$

The individual noise ω_{it} can be further decomposed into two parts:

$$\omega_{it} = \mu_{it} - c_t \quad (10)$$

where c_t is the noise component which varies only over time as the result of measurement error, and that can be defined as: $c_t = x_t - y_t^1$, where x_t is the "final" estimate of inflation in t, and y_t^1 its first release, known by the forecaster at the time of the forecast. In the context of the decomposition (10), the inflation signal can be re-defined as: $y_{it} = x_t + \mu_{it} - (x_t - y_t^1)$ $= y_t^1 + \mu_{it}$. In short, the individual information about the variable to be forecasted is given by the sum of the first release y_t^1 and the idiosyncratic noise μ_{it} representing subjective interpretation errors. Taking the average of equation (10), we can express the aggregate noise measure $\overline{\omega}_t$ as the sum of two parts:

[7] We relax the assumption of time invariance in Section III by allowing the Kalman gain to be related to the time-varying uncertainty.

$$\overline{\varpi}_t = \sum_{i=1}^{N} \frac{\varpi_{it}}{N} = \sum_{i=1}^{N} \frac{\mu_{it} - c_t}{N} = \sum_{i=1}^{N} \frac{\mu_{it}}{N} - c_t = \overline{\mu}_t - c_t \qquad (10')$$

where $\overline{\mu}_t = \sum_{i=1}^{N} \frac{\mu_{it}}{N}$ is the average of the individual idiosyncratic noise. Therefore, if we focus on the aggregate individual noise $\overline{\varpi}_t$, the linear NI model (9) becomes:

$$x_{t+h} - F_t x_{t+h} = \frac{1-G}{G}\left[F_t x_{t+h} - F_{t-1} x_{t+h}\right] - \vartheta \overline{\varpi}_t + e_{t+h|t}^{FIRE} \qquad (11)$$

where $F_t x_{t+h} = \sum_{i=1}^{N} \frac{F_{it} x_{t+h}}{N}$ and $F_{t-1} x_{t+h} = \sum_{i=1}^{N} \frac{F_{it-1} x_{t+h}}{N}$ respectively.

While, if we explicitly account for both the parts $\overline{\mu}_t$ and c_t of the average noise, the linear NI model (9) becomes:

$$x_{t+h} - F_t x_{t+h} = \frac{1-G}{G}\left[F_t x_{t+h} - F_{t-1} x_{t+h}\right] - \vartheta \overline{\mu}_t + \vartheta c_t + e_{t+h|t}^{FIRE} \qquad (12)$$

Further, if we assume that: $\overline{\mu}_t = \sum_{i=1}^{N} \frac{\mu_{it}}{N} = 0$ (i.e. that the statistical average removes the idiosyncratic noises) the aggregate specification of the linear NI model (9) becomes:

$$x_{t+h} - F_t x_{t+h} = \frac{1-G}{G}\left[F_t x_{t+h} - F_{t-1} x_{t+h}\right] + \vartheta c_t + e_{t+h|t}^{FIRE} \qquad (13)$$

The SI model nests in the aggregate NI model (13). Following Coibion and Gorodnichenko (2015) and Carrol (2003, 2006), the sticky information (SI) model inflation forecast – where the measurement noise is unaccounted for – is defined:

$$F_t\, x_{t+h} = (1-\lambda) F_t^{FIRE} x_{t+h} + \lambda\{(1-\lambda) F_{t-1}^{FIRE} x_{t+h} + \lambda[(1-\lambda) F_{t-2}^{FIRE} x_{t+h} + ...]\} \qquad (14)$$

where $F_t^{FIRE} x_{t+h}$ is the FIRE *h*-steps ahead forecast dated at *t*, and λ is a parameter measuring the probability of agents of acquiring no new information (that is, the degree of information rigidity). By substituting the definition of the FIRE forecast into equation (14) and rearranging, we derive the following forecasts error structure for the SI model:

$$x_{t+h} - F_t\, x_{t+h} = \frac{\lambda}{1-\lambda}\left[F_t x_{t+h} - F_{t-1} x_{t+h}\right] + e_{t+h|t}^{FIRE} \qquad (14')$$

A crucial distinction between the linear NI and the SI models, respectively in (13) and (14'), is the inclusion of the time-varying common noise component c_t in the NI model[8]. Therefore, the SI model is a valid representation of the forecast errors and parsimoniously encompasses NI only if $\vartheta = 0$. Conversely, if $\vartheta \neq 0$, the SI model omits the common noise c_t.

The empirical investigation and comparison of the linear NI and SI models is based on the Survey of Professional Forecasters (SPF) data[9]. We use surveyed professionals' inflation forecast pertaining to GDP deflator inflation, *x*. The forecast horizon, *h*, is set to 3, that is, one-year ahead prediction from *t-1* (i.e., starting from the most recent information available at the survey date *t*). The one-year ahead forecast error $x_{t+h} - F_t x_{t+h}$, the dependent variable of models (11) to (13), is regressed against two explanatory variables. The first, common to all models, is the forecast revision from quarter *t-1* to *t*, $F_t x_{t+h} - F_{t-1} x_{t+h}$.[10] The second explanatory variable is a measure of the aggregate individual noise related to the process of data revisions. The presence of the noise as a regressor in models (11) to (13) supports the validity of the NI with a noise component which we measure in three different ways.[11]

Firstly, we estimate $\overline{\omega}_t$ in model (11) with the average of the individual measures of the forward noise which are defined as the revisions of the individually perceived GDP deflator levels for *t-1* from SPF vintage *t* to *t+4*. This measure has the advantage of embodying the idiosyncratic noise component μ_{it} too. In addition, we provide two aggregate measures of data revisions (one backward-looking, c_t^B, and the other forward-looking, c_t^F) that can be used in model (13) under the assumption that $\overline{\mu}_t = 0$ (i.e. that the idiosyncratic noise vanishes when averaging).

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[8] Indeed, in a recent paper Croushore (2020) concluded that bias found in survey-based forecasts, due to their choice of actuals, tend to depend considerably on data that are subject to revisions.

[9] The SPF survey is currently conducted by the Federal Reserve Bank of Philadelphia; individual data about inflation forecasts are available from the survey of 1968q4. For further information and data downloads, see the website: **http://www.phil.frb.org/research-and-data/real-time-center/survey-of-professional-forecasters/**

[10] Details about data sources and definitions of the forecast error and of the forecast revision are in Appendix A2.

[11] Note that the relevant concept of noise here pertains to the quality of the forecasters' information set and not the concept of noise (and news) to model the data revisions of the statistical agencies (see e.g. see Jacobs and van Norden (2011)). In other terms, the point of interest here is to quantify the noise affecting the forecasters' knowledge of the state (and so, their predictive ability as shown in Finzen and Stekler (1999)).

The fluctuations of the aggregate forward-looking measure of noise c_t^F are similar to those of $\overline{\omega}_t$.[12] The only difference is that c_t^F measures the first data release of the GDP deflator in *t* for *t-1* using the official NIPA data, while $\overline{\omega}_t$ measures it with the individually perceived data as they are reported by the forecaster in the survey *t*. The forward-looking measures of noise component c_t^F and $\overline{\omega}_t$, by definition, use information that is available only after one year. Instead, the aggregate backward-looking c_t^B is the only measure of noise that is known when the forecasters are surveyed. It averages the four most recent inflation revisions at the time of survey *t*, specifically during the past four quarters.

Both the forward- and backward-looking definitions of noise are of interest for the present paper. On the one hand, the two forward-looking definitions of noise, c_t^F and $\overline{\omega}_t$, have the advantage of quantifying the contribution of the actual measurement error faced by the forecaster at the time when the forecast is released. This unavoidably becomes part of the ex post forecast error, if the NI model is an accurate representation of the forecaster behaviour. On the other hand, the backward-looking definition of noise c_t^B is more "realistic" as it represents the forecasters' knowledge of the most recent data revisions when the forecast is released. Consequently, the knowledge of c_t^B can lead to better forecasts of the target.[13]

The availability of these three alternative noise measures allows us to assess the robustness of the empirical findings. The preliminary assessment of the univariate properties of the forecast error, forecast revision, and three measures of aggregate noise are undertaken using both visual inspection of the time patterns and the Elliott et al (1996) unit root test.[14] Evidence suggests that all the series are stationary, and that the OLS estimator (with Heteroskedasticity and Autocorrelation Consistent, HAC, standard errors of Newey and West, 1987) can be used to estimate the parameters of models (11) and (13). In fact, the residuals, which are orthogonal to both forecast revisions and noise, are consistent estimates of the FIRE errors. Thereby, assumed to be unrelated to the whole information set dated *t* or earlier. More explicitly, the specification of models (11) and (13) circumvents the potential pitfall of

[12] As clearly shown in Figure A4.2 of Appendix A4.

[13] The detailed description of our three measures of noise is in Appendix A3.

[14] Detailed results are in Appendix A4.

endogenous explanatory forecast revisions due to the omission of the explanatory noise (see Coibion and Gorodnichenko (1995), and Easaw and Golinelli (2021)).

The three columns in Table 1 report different OLS estimates corresponding to the alternative noise measures: the aggregate backward noise $\hat{c}_t^B$, the aggregate forward noise $\hat{c}_t^F$, and the average of forward individual noise $\hat{\bar{\omega}}_t$.

Table 1 here

Results, reporting significant "forecast revisions" parameter estimates, clearly reject the FIRE expectations, but are not sufficient to support any alternative specific model. The implicit estimates of $\hat{G}$ (the constant Kalman gain parameter) are remarkably similar (in the 0.47-0.49 range).

The "noise measures" parameter estimates ($\hat{\vartheta}$) are at least 10% significant. Additionally, the point estimates are also remarkably close to each other and to one. The latter finding supports a representation of actual inflation where the core dynamics is highly persistent.

III: Inflation Expectations, Imperfect Information and Uncertainty: Macroeconomic Uncertainty versus Data Uncertainty:

Having established the importance of measurement errors due to data revisions, in the remainder of the paper we outline a theoretical model that considers the impact of uncertainty on inattentiveness in the context of the NI model.

We distinguish between two sources of uncertainty faced by a professional forecaster: macroeconomic and data (measurement error) uncertainty. The former pertains to the volatility over time of the macroeconomic shocks, while the latter is due to the volatility over time of noise component (which comes from data revisions). We have shown in the preceding section the significant effect of noise on the professional forecast error. The question now is how its variability over time (i.e. data uncertainty, DU) does affect forecasters' inattentiveness, and how does this differ from macroeconomic uncertainty (MU). The model will deliver a new non-linear representation of the uncertainty-inattentiveness nexus, while the literature usually assumes that parameters driving inattentiveness are constant over time and across individual forecasters.

Theoretical models focussing on macroeconomic uncertainty and inattentive behaviour are scant with contrary views. Moscarini (2004) argues that if macroeconomic shocks become more volatile and persistent, the price setting task of the firm becomes informationally more demanding and, therefore, firms optimally choose to obtain and to process this information only infrequently. On the other hand, Reis (2006) expresses that more volatile shocks lead to more frequent updating since inattentiveness is costlier in a world that is rapidly changing. The latter view is supported by the empirical outcomes in Coibion and Gorodnichenko (2015) and Mitchell and Pearce (2017). Before proceeding with the empirical assessment, the micro-foundations of professionals' inflation forecasts based on imperfect information must be established.

In the context of the Kalman filter the *a priori* forecast error $x_t - F_{t-1}x_t$ embodies the effects of the innovations υ_t to the inflation gap which crucially affect the predictability of *x*: the higher the variability of υ_t, the higher the variability of the *a priori* forecast error $E(x_t - F_{t-1}x_t)^2$ will be. As far as the *a posteriori* forecast error $x_t - F_t x_t$ is concerned, its variance $E(x_t - F_t x_t)^2$ is minimized by setting an optimal weight *G* by following three steps.[15]

First, the *a posteriori* forecast error $x_t - F_t x_t$ can be redefined by substituting $F_t x_t$ with the average across individuals of equation (4) and by combining it with the average of decomposition (1), i.e. with $(x_t + \overline{\omega}_t) + (1-G)F_{t-1}x_t$:

$$x_t - F_t x_t = x_t - [F_{t-1}x_t + G(x_t + \overline{\omega}_t - F_{t-1}x_t)] = (1-G)(x_t - F_{t-1}x_t) - G\overline{\omega}_t \qquad (15)$$

where, $\overline{\omega}_t$ is the average across individuals of the measurement noise, see equation (10').

Second, given that the covariance between the *a priori* and *a posteriori* errors is zero, using equation (15) we can rewrite the *a posteriori* error variance $E(x_t - F_t x_t)^2$ as the weighted sum:

$$E[(1-G)(x_t - F_{t-1}x_t) - G\overline{\omega}_t]^2 = (1-G)^2 E(x_t - F_{t-1}x_t)^2 + G^2 E(\overline{\omega}_t)^2 \qquad (16)$$

Third, setting the derivative of the *a posteriori* error variance with respect to *G* equal to zero:

$$-2(1-G)E(x_t - F_{t-1}x_t)^2 + 2GE(\overline{\omega}_t)^2 = 0$$

[15] Appendix A5 extends the approach to the general case of *r* unobservable states and *n* measurements in matrix form (in the present context, *r* = *n* = 1). The matrix form is useful to understand the SDM algorithm outlined in Section IV and detailed in Appendix A6.

$E(x_t - F_{t-1}x_t)^2$, and the volatility of the measurement error $E(\varpi_t)^2$, as follows:

$$G = \frac{E(x_t - F_{t-1}x_t)^2}{E(x_t - F_{t-1}x_t)^2 + E(\varpi_t)^2} \qquad (17)$$

Equation (17) shows that G varies inversely with $E(\varpi_t)^2$, which is related to the data uncertainty (DU), while it varies positively with $E(x_t - F_{t-1}x_t)^2$, which is related to the macroeconomic uncertainty (MU). Given that in the three aggregate NI specifications (11)-(13) the forecast revision effect (depicting information rigidities) is measured by the ratio $\frac{1-G}{G}$, we can rearrange the definition (17) as:

$$\frac{1-G}{G} = \frac{E(\varpi_t)^2}{E(x_t - F_{t-1}x_t)^2} \qquad (18)$$

When $E(\varpi_t)^2$ approaches zero (i.e. small data revisions because of low DU) or when $E(x_t - F_{t-1}x_t)^2$ is high (because of large MU, like during the Great Recession), the forecast error approaches the FIRE as $\frac{1-G}{G}$ tends to zero. Conversely, when $E(x_t - F_{t-1}x_t)^2$ approaches zero (e.g. during the Great Moderation) or when $E(\varpi_t)^2$ is large (because data releases substantially revise previous vintages), the inattentiveness effect, captured by $\frac{1-G}{G}$, becomes larger, indicating widening discrepancies between actual forecast errors and FIRE errors.

Events such as the Great Moderation and the Great Recession suggest that the volatility of the macroeconomic shocks (MU) is time-varying and the same applies to DU (see e.g., the stochastic volatility model for inflation in Stock and Watson, 2007). Accordingly, if the variances in equations (17) and (18) vary over time, the corresponding Kalman smoother G (and the related forecasters' inattentiveness) will be no longer constant as in Section II, but a time series G_t driven by DU_t and MU_t fluctuations through the non-linear $\Psi(.)$ function:

$$\beta_t = \frac{1-G_t}{G_t} = \psi\left(\overset{-}{MU}_t, \overset{+}{DU}_t\right) \qquad (19)$$

Given that MU represents the variability of general macroeconomic and political shocks pertaining to a wide range of variables and events (such as the current pandemic), in the application below we use the Economic and Policy Uncertainty (EPU) measure of Baker et al (2016)[16]. Therefore, we assume that, in uncertain times, forecasters use more of the new information and their prediction errors are closer to FIRE. In addition, we measure data uncertainty (DU) with the variance of inflation data revisions by extending the alternative measures listed in Croushore (2011).

In the next section, we will approach the nonlinear relationship (19) by using both the parsimonious multiplicative interactions model (see Brambor et al., 2005), and the general state-dependent framework, that extends the first approach because the non-linear dependency of inattentiveness to uncertainty is modelled without the need of any *ex ante* linearity assumption (see Priestely, 1980).

IV: Macroeconomic Uncertainty versus Data Uncertainty: Empirical Results

The use of the multiplicative interactions model (MIM) is fairly common in the empirical literature when the relationship between inputs (the forecast revisions) and outcomes (the forecast errors) varies depending on the state of another variable (the uncertainty). In fact, if we assume that in equation (18) the numerator is positively related to DU, and the denominator is positively related to MU, the time-varying effect β_t of the forecast revisions on the forecast error is given by a constant (β) and a linear relationship with DU and MU:

$$\beta_t = \frac{1-G_t}{G_t} = \frac{\sigma_{DU}^2\left(1+\gamma_{DU}DU_t\right)}{\sigma_{MU}^2\left(1+\gamma_{MU}MU_t\right)} \approx \beta\left(1+\gamma_{DU}DU_t\right)\left(1-\gamma_{MU}MU_t\right) \qquad (20)$$

where: $\beta = \frac{\sigma_{DU}^2}{\sigma_{MU}^2}$ is the effect corresponding to the zero-uncertainty case; γ_{DU} and $\gamma_{MU} \geq 0$ are parameters measuring the slopes of the linear relationships in equation (20). Therefore, by substituting the assumption (20) in model (13), we obtain:

$$x_{t+h} - F_t x_{t+h} = \beta\left(1+\gamma_{DU}DU_t\right)\left(1-\gamma_{MU}MU_t\right)\left[F_t x_{t+h} - F_{t-1}x_{t+h}\right] + \vartheta c_t + e_{t+h|t}^{FIRE} \qquad (21)$$

The estimated results are found in Table 2 where the OLS estimates of model (21) are in column (1), together with two alternative estimates, where the DU and of MU slopes are restricted to zero (respectively in columns 2 and 3).

[16] EPU is not only a very broad uncertainty measure largely used in empirical works, but also captures subjective (i.e., ex ante, forward looking) components of uncertainty, as recently shown in Bontempi et al. (2021). Details about EPU are in Appendix A3.

Table 2 here

Results are clear: the expected sign of parameters is always consistent with priors. However, the MU parameter is significant, while the DU parameter is not. The estimate outlined in column (2) of the restricted model for the MU slope only (i.e., without the DU effect) is close to the unrestricted model in column (1). This is always consistent with theoretical *a priori*, independently from the presence in the model of the interaction with DU. This relationship is reported in the right panel of Figure 1, where the downwards slope between the actual MU (restricted to lay between 0 and 1, in the x-axis) and the measure of the state-dependent effect of the forecast revisions on the forecast error clearly emerges. The left panel of Figure 1 plots the decreasing pattern of the relationship between forecast revisions and the forecast errors. Note that in the final part of the sample it is close to zero

Figure 1 here

Following this result, our model predicts that at present, when the MU levels are at their historical highs, forecasters should be very close to FIRE prediction errors.

Alternatively, the empirical analysis can be made by using a general class of non-linear time series called State Dependent Models (henceforth SDM), where the coefficients of the model are functions of a "state vector". The principal advantage of SDM is that it allows for a general form of non-linearity and can be fitted without any specific prior assumption about the form of non-linearity. In this part, we apply the SDM approach to equation (14) to capture the effect of uncertainty on inattentiveness in the general non-linear context. It is worth noting that in the SDM context we will have the possibility to investigate whether the non-significant role played by DU in the MIM above is motivated by the linearity assumption there. To simplify the exposition, the state-dependent representation of the aggregate NI model (13) can be simplified as follows:

$$Y_t = \kappa_t + \varphi_t C_t + \psi\left(MU_{t-1}, DU_{t-1}\right) X_t + \varepsilon_t \qquad (22)$$

Following the linear specification outlined in equation (13), *Y* is the forecast error $x_{t+h} - F_t x_{t+h}$, *X* the forecast revision $F_t x_{t+h} - F_{t-1} x_{t+h}$, *C* the noise variable (or, the data revisions c_t), and ε is a sequence of independent zero-mean random shocks corresponding to the FIRE errors $e_{t+h|t}^{FIRE}$.

The parameters of the SDM are estimated recursively through an extended Kalman filter algorithm.[17] The intercept κ (which is zero if the forecast errors are unbiased) and the slope φ (which corresponds to ρ^h) are simple time-varying parameters. The state dependent parameter ψ, instead, is assumed to be locally a linear function of both the measures of uncertainty entering the state vector at time *t-1*, $u_{t-1} = \left(MU_{t-1}, DU_{t-1}\right)'$.

We focus on the estimation of the parameters κ_t, φ_t and $\psi\left(u_{t-1}\right)$, and the estimation problem involves the specification of this non-linear dependency. A simple assumption is that the state dependent ψ is a linear function of the state-vector u_{t-1}: $\psi\left(u_{t-1}\right) = \psi^{(0)} + u'_{t-1}\,\gamma$, where $\psi^{(0)}$ is constant and $\gamma = \left(\gamma_1 \quad \gamma_2\right)'$ is the "gradient" vector. Although this assumption cannot represent all types of non-linear models, it seems reasonable to point out that ψ can be *locally* represented as a linear function of u_{t-1}. Regarding the time-varying parameters κ_t and φ_t, we simply assume that: $\kappa_t = \kappa^{(0)}$ and $\varphi_t = \varphi^{(0)}$, that is without state-dependency.

Based on these assumptions, the parameter "updating" equations may be written as:

$$\psi\left(u_{t+1}\right) = \psi\left(u_t\right) + \Delta\, u_t'\gamma_{t+1}$$
$$\kappa_{t+1} = \kappa_t$$
$$\varphi_{t+1} = \varphi_t$$

where $\Delta\, u_t = u_t - u_{t-1}$ denotes the changes over time of both our uncertainty measures: $\Delta MU_t = MU_t - MU_{t-1}$ and $\Delta DU_t = DU_t - DU_{t-1}$. However, the vector $\gamma_t = \left(\gamma_{1t} \quad \gamma_{2t}\right)'$ is unknown and thus must be estimated. The strategy is to allow it to evolve as a random walk, i.e., $\gamma_{t+1} = \gamma_t + v_{t+1}$, where v_{t+1} is a sequence of independent random shocks such that: $v_{t+1} \sim N(0, \Sigma_v)$.

The estimation procedure determines in each period *t* those values of γ_{1t} and γ_{2t} which minimize the discrepancy between the observed value of Y_{t+1} and its prediction $\hat{Y}_{t+1}$ computed by the model fitted at time *t*. The algorithm is sequential in nature and resembles the recursive

[17] We introduce a recursive method - similar to what was originally used by Priestley (1980) - where the estimation algorithm is detailed in Appendix A6; see also Priestley and Heravi (1986)..

procedure of the Kalman filter algorithm.[18] The starting values for the parameters are obtained by the OLS method using all the available observations. The recursion starts from the second quarter of 1969. In the application of SDM below, we use the same measures of index of data revisions used for the previous MIM.

The time-varying effects of the intercept κ_t (measuring the forecast bias), the noise φ_t (measuring those data revisions which are only part of the NI model), and the forecast revisions (measuring the degree of inattentiveness) are depicted in Figure 2, and suggest the following points.

Figure 2 here

Firstly, the point estimates of bias in the left-hand plot are never significantly different from zero. Secondly, the point estimates of the effect over time of data revisions in the middle plot are significantly different from zero, although accompanied by wide intervals. This outcome is again supportive of the NI model in explaining forecasters' behaviour in the non-linear context. In addition, the estimates of these noise parameters indicate strong inflation persistence over time and are consistent with the unit root process usually found to drive the inflation rate when structural breaks are disregarded. Finally, as outlined in the right-hand plot, the rigidity due to the forecast revisions is almost always significant. This finding indicates the central role played by inattentiveness in explaining the way the professional forecasters update their predictions, which always deviate from FIRE. Specifically, inattentiveness seems larger during the 1970s and the beginning of 1980s when the joint effect of MU and DU results in Kalman gain G estimates around 0.43, with an upper-bound at 0.35, and this is not far from those levels experienced during the Great Moderation. It suggests, despite the troubled oil shocks period, forecasters did not update their forecast regularly. Conversely, towards the end of the sample, during the period of the Great Recession and low data uncertainty results in lower inattentiveness. The lower-bound estimates of G are around 0.7. Coincidentally, the outcome of Coibion and Gorodnichenko (2015) is supported by the evidence here.

The most relevant feature of the SDM model (22) is that it allows the effect of $\hat{\psi}$ on the forecast errors to depend on the state of uncertainty. The panels in Figure 3 plot the state-dependent $\hat{\psi}$ estimates against MU and DU.

Figure 3 here

[18] Appendix A6 outlines model (21) in state-space form and details the recursions to estimate its parameters through the Kalman filter.

The visual inspection of these plots is of interest and suggests a non-linear relationship. A key feature of the SDM approach is that it operates purely on data without any prior knowledge of the underlying functional form. In the model, the pattern of $\hat{\psi}$, when both types of uncertainty (MU and DU) increase, is monotonic, especially for MU. This supports the theoretical predictions outlined in previous section. When MU and DU pass from their respective 5th to the 95th centile,[19] the fluctuations of $\hat{\psi}$ imply the following: when MU increases from minimum levels to maximum levels, *G* estimates rise from about 0.45 to about 0.6. A similar min-max increase in DU has a slightly lower effect (in absolute value). This effect is clearly non-linear. It indicates that the most relevant changes for the inattentiveness effect are when the DU fluctuations are below the average. This could explain the low relevance of the DU effect in the earlier MIM estimates.

V: Concluding Remarks

The purpose of the current paper is to investigate the dynamic nature of professionals' inflation forecasts inattentiveness. We outline and empirically investigate a generalized model of inattentiveness due to informational rigidity. In doing so, we also introduce a novel theoretical model that considers the relationship between inattentiveness and uncertainty and crucially distinguishing between macroeconomic and data uncertainty.

The ensuing empirical investigation uses the Survey of Professional Forecasters. In the first instances, we find that 'noise' significantly affects forecast errors. This result is robust regardless of the definition or measure to capture noise. Subsequently, the paper investigates the role of uncertainty in determining professional forecasters' inattentiveness. We use the EPU measure of Baker et al (2016) to represent macroeconomic uncertainty, while we use the variance of the noise measure to capture data uncertainty. As predicted by our theoretical model, macro and data uncertainty have a distinct effect. Notably, inattentiveness decreases with increasing macro uncertainty, and increases with rising data uncertainty.

Overall, the empirical results – supported by using both multiplicative-interaction and state-dependent models – clearly indicate that professional forecasters are depicted by inattentiveness which is driven over time by (mainly macroeconomic) uncertainty.

[19] To ensure the readability of plots we reported only the 10% trimmed cases of both MU and DU.

This outcome predicts that in recent times, when macroeconomic uncertainty is peaking, the behaviour of professional forecasters is quite close to the FIRE case. In the context of the NI model, this can be interpreted as, in the current pandemic, any update of the statistical information is very important (despite consequence of measurement errors). Indeed, it is the best predictor of what the current the inflation gap is. In the SI model, the probability of forecasters of acquiring no new information is close to zero, as inattentiveness would be more costly.

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Table 1 – Estimates of the forecast error models (11) and (13) [(a)]

	(1)		(2)		(3)	
Intercept	**-0.1003**		**-01036**		**-0.1144**	
s.e.	0.0727		0.0670		0.0702	
Forecast revisions	**1.0523**	***	**1.1407**	***	**1.0623**	***
s.e.	0.2683		0.2700		0.1533	
Noise measures [(4)]	**0.7505**	*	**0.9562**	**	**0.8159**	**
s.e.	0.4250		0.4377		0.4049	
T	205		205		205	
R^2	0.2014		0.2111		0.2732	
SER	1.0137		1.0075		0.9695	

[(a)] OLS estimates in bold and, below, HAC standard errors. Estimation period 1969q1-2020q1.

(1) Model (13) with aggregate backward noise, $\hat{c}_t^B$.

(2) Model (13) with aggregate forward noise, $\hat{c}_t^F$.

(3) Model (11) with average forward individual noise, $\hat{\bar{\omega}}_t$ and unreported 73q3 impulse dummy.

(4) Given that the sample variability of the backward noise measure $\hat{c}_t^B$ in column (1) is about one-third of the forward ones $\hat{c}_t^F$ and $\hat{\bar{\omega}}_t$, we scaled the latter measures by a factor of 3 for comparability.

Table 2 – Estimates of the multiplicative interaction model (a)

	(1)		(2)		(3)	
Intercept (b)	**-0.1501**		**-0.1486**		**-0.1193**	
s.e.	0.1128		0.1120		0.1159	
β (c)	**2.1278**	**	**2.3922**	**	**0.8764**	**
s.e.	0.8529		1.0378			
γ_{MU}	**-1.5738**	***	**-1.6433**	***	**--**	
s.e.	0.3709		0.3317			
γ_{DU}	**0.3885**		**--**		**1.2629**	
s.e.	0.9494				1.5425	
ϑ	**0.8210**	*	**0.8155**	*	**0.7801**	
s.e.	0.5000		0.5005		0.5032	
T	205		205		205	
R2	0.2224		0.2209		0.2107	
SER	1.0053		1.0038		1.0103	

(^{a}) OLS estimates in bold and, below, HAC standard errors of model (21):

$$x_{t+h} - F_t x_{t+h} = \beta\left(1+\gamma_{DU} DU_t\right)\left(1+\gamma_{MU} MU_t\right)\left[F_t x_{t+h} - F_{t-1} x_{t+h}\right] + \vartheta c_t + e^{FIRE}_{t+h|t}$$

where *DU* variable is restricted to lay in the 0-1 range, while *MU* is first restricted to lay in the 0-1 range and then smoothed using a backward moving average of order 4. *c* is the aggregate backward noise.
Estimation period 1969q1-2020q1.

(^{b}) In the theoretical model (21) the intercept should be zero.

(^{c}) β measures the forecast revisions coefficient in the zero-uncertainty cases, i.e., when *MU* and *DU* are at their minimum sample values.

Fig. 1 - The macro uncertainty-driven effects of forecast revisions on forecast errors over time (left) and across states (right)

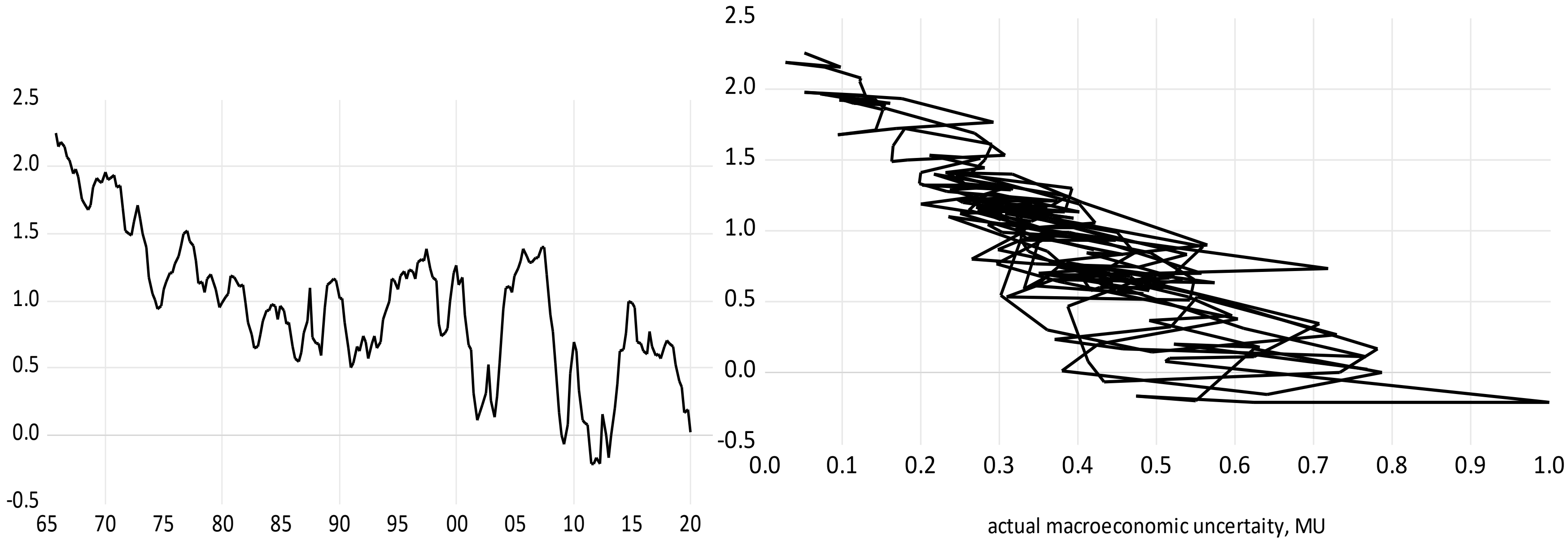
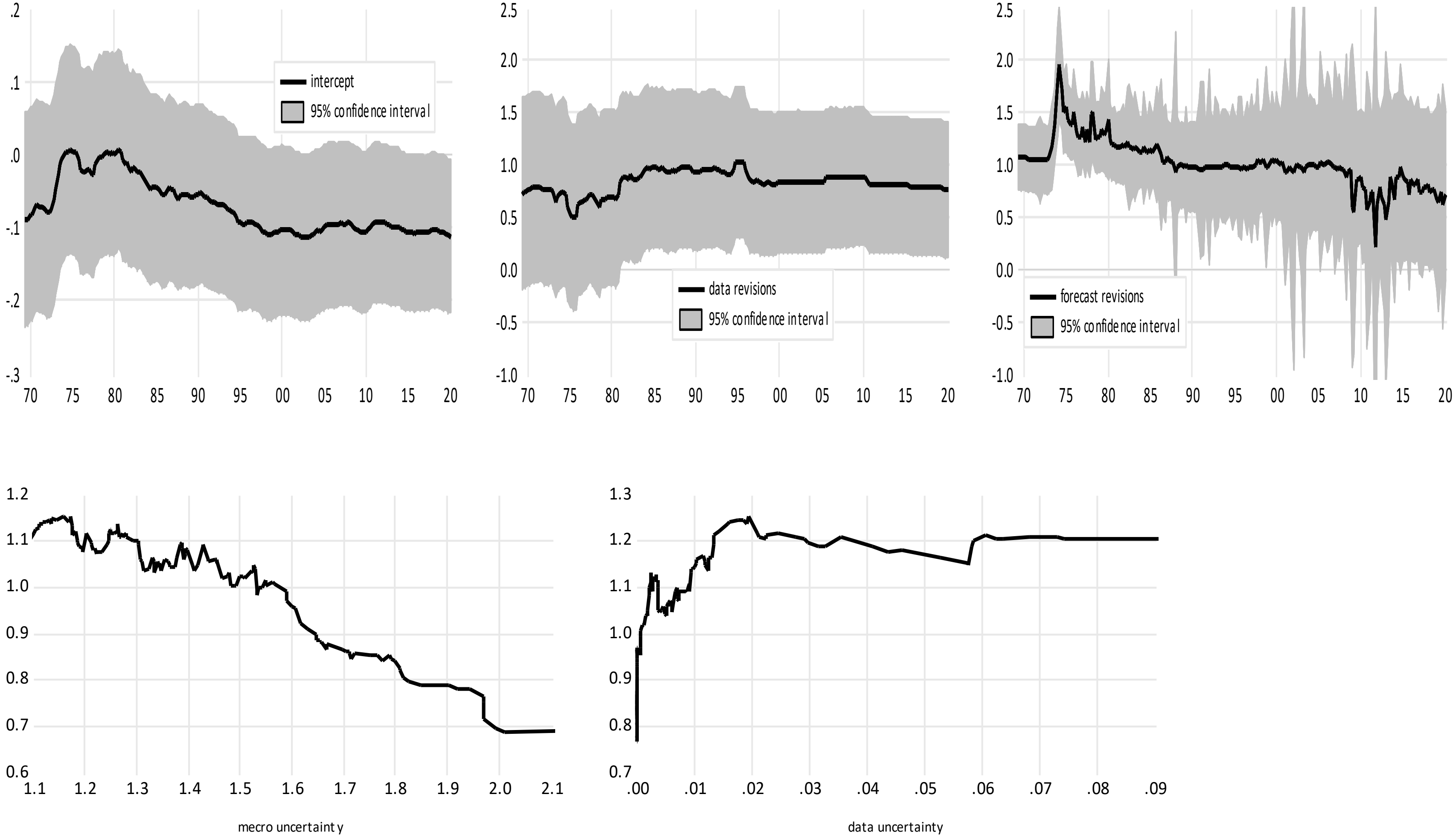


Figure 2 – SDM estimation results: The pattern of the SDM estimation results over time



APPENDICES NOT FOR PUBLICATION

Appendix A1:
FIRE inflation forecast and the Noisy Information Model

The state equation outlining the unobservable inflation gap is

$$\xi_t = \rho \xi_{t-1} + \upsilon_t$$

and the measurement equation for the inflation rate is:

$$y_t = \tau_{m+1} + \xi_t + \omega_t$$

where τ_{m+1} is a deterministic variable and assumed to be known (through the breaking intercept inferences about inflation rate) before modelling and forecasting, while ω_t is a zero-mean stochastic noise affecting inflation. Hence, the unobservable state of inflation is:

$$x_t = y_t - \omega_t = \tau_{m+1} + \xi_t$$

If ω_t is zero, the *observable* inflation model is

$$\left(x_t - \tau_{m+1}\right) = \rho\left(x_{t-1} - \tau_{m+1}\right) + \upsilon_t \qquad \text{(A1.1)}$$

Therefore, from model (A1.1) we can derive the one-step ahead FIRE forecast for actual inflation as follows:

$$F_t^{FIRE} x_{t+1} = \tau_{m+1} + \rho\left(x_t - \tau_{m+1}\right)$$

while the *ex post* realization is: $x_{t+1} = \tau_{m+1} + \rho\left(x_t - \tau_{m+1}\right) + \upsilon_{t+1}$. Consequently, the one-step ahead forecast error is given by the difference:

$$e_{t+1|t}^{FIRE} = x_{t+1} - F_t^{FIRE} x_{t+1} = \upsilon_{t+1}$$

with one forward iteration more, the two-steps ahead forecast becomes:

$$F_t^{FIRE} x_{t+2} = \tau_{m+1} + \rho^2\left(x_t - \tau_{m+1}\right)$$

and the *ex post* realization follows:

$$x_{t+2} = \tau_{m+1} + \rho^2\left(x_t - \tau_{m+1}\right) + \rho\upsilon_{t+1} + \upsilon_{t+2}$$

Subsequently, the 2-steps ahead forecast error can be defined as:

$$e_{t+2|t}^{FIRE} = x_{t+2} - F_t^{FIRE} x_{t+2} = \rho\mu_{t+1} + \mu_{t+2}$$

This can be further generalized to the *h*-steps ahead prediction, where the ex post realization is:

$$x_{t+h} = \tau_{m+1} + \rho^h\left(x_t - \tau_{m+1}\right) + \sum_{j=1}^{h} \rho^{h-j} \upsilon_{t+j} \qquad \text{(A1.2)}$$

and the FIRE forecast is:

$$F_t^{FIRE} x_{t+h} = \tau_{m+1} + \rho^h \left(x_t - \tau_{m+1} \right) = \tau_{m+1} + \rho^h \xi_t \tag{A1.3}$$

the h-steps ahead forecast error in our context is then defined as:

$$e_{t+h|t}^{FIRE} = x_{t+h} - F_t^{FIRE} x_{t+h} = \sum_{j=1}^{h} \rho^{h-j} \upsilon_{t+j} \tag{A1.4}$$

Finally, from the FIRE forecast definition (A1.3) we can estimate the long horizon forecast of the inflation rate which corresponds to the deterministic component of our model:

$$F_t^{FIRE} x_{t+h} \xrightarrow[h \to \infty]{} \tau_{m+1} \tag{A1.5}$$

It is noteworthy that this long horizon forecast is equivalent to "core inflation" definition, see Morley et al (2015) and the literature cited herein.

We know from (A1.4): $E(e_{t+h|t}^{FIRE} \mid I_{t-k}) = 0 \ \forall k \geq 0$, that is the FIRE forecast error $e_{t+h|t}^{FIRE}$ is always unrelated with the full information set available in t or earlier. In addition, we note that the h-steps ahead FIRE forecast error of the simple the AR (1) model outlined in Coibion and Gorodnichenko (2012, 2015) would lead to the same FIRE forecast error as in (A1.4). Therefore, by introducing the notion of inflation rate stationarity around a breaking constant – denoting core inflation - the one advantage of our derivation is relaxing the unreliable assumption (without further qualification) of a stationary inflation rate. Hence, we are consistent with the need for forecasters to handle with caution the issue of modelling the inflation gap, as already noted by Cogley et al (2010), Nason and Smith (2016), and Morley et al (2015).

Appendix A2:
Data sources and definition of the forecast error and the forecast revision

The forecast error, the dependent variable of models (11)-(13) and (14'), is defined as $x_{t+3} - \sum_{i=1}^{N} \frac{F_{it} x_{t+3}}{N}$. It is the difference between the historical inflation rate data x_{t+3} (released three quarters after the SPF survey date) and the average of the one-year ahead individual SPF forecast $F_{it} x_{t+3}$, labelled as "nowcast" because in t (the first quarter of the forecast horizon) some information about the inflation in the year to be forecast is known. The explanatory $\sum_{i=1}^{N} \frac{F_{it} x_{t+3}}{N} - \sum_{i=1}^{N} \frac{F_{it-1} x_{t+3}}{N}$ is the revision of the average forecast, the change from *t-1* to t in the predicted average inflation because of the accrual in t of new information.

Data for the historical GDP deflator inflation rate x_{t+3} are computed using the vintages *v* of the levels of NIPA GDP deflator P^v. The vintages of interest are those available one year later for the same months when the SPF survey was conducted (February, May, August and November), i.e., $v = t+4$, when data for quarter *t+3* are released for first time. The source of P^v vintages is the Real-Time Data Set for Macroeconomists at the Federal Reserve Bank of Philadelphia.[20] Given the vintage *t+4*, we define the historical inflation rate as: $x_{t+3} = 100 \times \left(\frac{P_{t+3}^{t+4}}{P_{t-1}^{t+4}} - 1 \right)$.

Individual nowcast data can be obtained from SPF forecasts of GDP deflator levels as:[21] $F_{it}x_{t+3} = 100 \times \left(\frac{PGDP5_{it}}{PGDP1_{it}} - 1 \right)$. From the same survey in *t,* it is also possible to compute the "pure" forecast, $F_{it}x_{t+4}$ (i.e., the one-year ahead prediction from *t*, the survey date) as: $F_{it}x_{t+4} = 100 \times \left(\frac{PGDP6_{it}}{PGDP2_{it}} - 1 \right)$. If we subtract the pure forecast published in the survey of the previous quarter (in *t-1*) from the nowcast in *t*, we obtain the forecast revision from *t-1* to *t*, i.e., the first explanatory variable of the NI model:

$$F_{it}x_{t+3} - F_{it-1}x_{t+3} = 100 \times \left(\frac{PGDP5_{it}}{PGDP1_{it}} - \frac{PGDP6_{it-1}}{PGDP2_{it-1}} \right).$$

Appendix A3:
Data sources and definition of alternative noise and uncertainty measures

Measures of the individual noise ω_{it} (the main "additional" explanatory variable of the NI model specification) can be proxied using information from the process of data revisions. At the survey date *t*, deflator levels in *t-1* of the vintage *t* (labelled as P_{t-1}^{t}) are the latest available information of the *ith* forecaster which are reported as $PGDP1_{it}$ in the survey. The two figures can be different but, when $P_{t-1}^{t} - PGDP1_{it} = 0$, the forecaster starts the prediction from the latest published figures. While a discrepancy would suggest alternative behaviours: either the forecaster information set is not fully updated, or (in the opposite direction) the forecaster

[20] The first available monthly vintage is that of 1965m11. For further information and data downloads, see the website: **https://www.phil.frb.org/research-and-data/real-time-center/real-time-data**

[21] The definitions in this section are labelled as in the SPF survey. For details, see the Table 3 of the SPF documentation available at the SPF link (see the main text).

estimates the present situation differently than the statistical agency does. However, this potential discrepancy is not the only source of noise in t, as it is well known that statistical agencies make a number of data revisions after the first release to ensure a better measure of inflation. In our context, we define the "final" (better) measure of the GDP deflator the one which is in the denominator of the inflation rate x_{t+3}, i.e., the deflator level for *t-1* available in vintage *t+4*, P_{t-1}^{t+4}. Therefore, the individual noise in t can be estimated as: $\hat{\omega}_{it} = 100 \times \left(\frac{P_{t-1}^{t+4}}{PGDP1_{it}} - 1 \right)$, the percent deviation of the "final" deflator measure for *t-1* from its perceived value by the forecaster at the survey date t, when the forecast is made. As it is a comparison between levels belonging to different vintages (as P_{t-1}^{t+4} belongs to vintage *t+4* and $PGDP1_{it}$ to vintage t), the percent deviation above must be adjusted to account for changes in deflator's base years. This adjustment is accomplished for the four quarters between vintage t and vintage *t+4* when the change in the base year occurs.[22] In the light of the definition above, the $\hat{\omega}_{it}$ average across individuals, $\sum_{i=1}^{N} \frac{\hat{\omega}_{it}}{N} = \hat{\bar{\omega}}_t = \frac{100}{N} \sum_{i=1}^{N} \left(\frac{P_{t-1}^{t+4}}{PGDP1_{it}} - 1 \right)$, can be used as the measure of the aggregate of noise in model (11). We label this first measure as "average forward individual noise".

A second estimate of the aggregate measure of noise does not use individual survey information but only data revisions of NIPA deflator vintages: $\hat{c}_t^F = 100 \times \left(\frac{P_{t-1}^{t+4}}{P_{t-1}^{t}} - 1 \right)$. We label this second measure as "aggregate forward noise". With respect to the average forward individual noise $\hat{\bar{\omega}}_t$, this alternative aggregate forward noise $\hat{c}_t^F$ ignores *PGDP1* information of the SPF survey, as if the individual idiosyncratic components would vanish in averaging $\hat{\omega}_{it}$, as we assume in model (13).[23]

Although both $\hat{\bar{\omega}}_t$ and $\hat{c}_t^F$ estimates are *forward looking* (as they exploit *t+4* vintage information made available only after the survey date t), they should not be related with the FIRE error $e_{t+3|t}^{FIRE}$ because the latter embodies the unforecastable macroeconomic shocks

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[22] The adjustment of $\hat{\omega}_{it}$ is equal to the ratio between the average level of the old base deflator in the new base year and 100 (i.e., the average level of the new base deflator in the new base year).

[23] Also, in this case, we must adjust $\hat{c}_t^F$ data by using the same ratios as those above for $\hat{\omega}_{it}$.

occurring over the forecast horizon (from *t* to *t+3*). The information exploited in estimating $\hat{\bar{\omega}}_t$ and $\hat{c}_t^F$ embodies either future noise-reduction revisions of the first released inflation in *t-1* or the future inclusion of news about inflation data in *t-1*.

However, given the paramount importance of a significant ρ^h coefficient significance to distinguish between NI and SI models, we also estimate the time series of the aggregate noise c_t with a purely *backward-looking* measure. It exploits only the information in those vintages available up to the survey date *t*. In doing so, we assume that c_t is related to the mean of the past inflation revisions corresponding to the four most recent quarters of the data vintage *t*, in symbols: $\hat{c}_t^B = \frac{100}{4}\sum_{j=1}^{4}\left(\frac{P_{t-1-j}^t}{P_{t-5-j}^t} - \frac{P_{t-1-j}^{t-j}}{P_{t-5-j}^{t-j}}\right)$, where inflation revisions are in brackets, the vintage is in the superscript and the calendar date in the subscript. We label this third measure as "aggregate backward noise", and again it refers to the noise in model (13).

If we compare the formulas of the three forward- and backward-looking estimators described above, $\hat{c}_t^B$ is a *mean* of four backward revisions of inflation. The two alternative forward estimates $\hat{\bar{\omega}}_t$ and $\hat{c}_t^F$ focuses *only one* (forward) revision of the level of prices. Therefore, the NI model ϑ estimates coming from the alternative noise measures $\hat{\bar{\omega}}_t$, $\hat{c}_t^F$ and $\hat{c}_t^B$ cannot be compared as the variances of $\hat{\bar{\omega}}_t$, $\hat{c}_t^F$ and $\hat{c}_t^B$ are structurally different. $\hat{\bar{\omega}}_t$ and $\hat{c}_t^F$ are "one-revision" series that come from a distribution whose variability is higher than that of the mean-revision $\hat{c}_t^B$. The latter is smoothed by averaging over four quarters (see also Figure A4.2). Therefore, to make comparable the estimates of ϑ using $\hat{\bar{\omega}}_t$ and $\hat{c}_t^F$ with the one using $\hat{c}_t^B$ we must increase them by a factor equal to the ratio of the sample standard deviation of $\hat{c}_t^B$ to that of $\hat{\bar{\omega}}_t$ and $\hat{c}_t^F$ (very close each other).[24]

In our sample, the backward $\hat{c}_t^B$ $\hat{\bar{\omega}}_t$ and $\hat{c}_t^F$

[24] In addition, as it arises *prices* revisions rather than *inflation* revisions, the ρ^h estimate with $\hat{\bar{\omega}}_t$ and $\hat{c}_t^F$ should also be divided by a factor equal to $(1+x^*)$, where x^* can be proxied by the steady state inflation. However, such adjustment is irrelevant for our estimates as they are historically between the range of 1.10-1.01.

ϑ $\hat{\omega}_t$ and $\hat{c}_t^F$ by a factor of 3 or, equivalently, to divide the levels of $\hat{\omega}_t$ and $\hat{c}_t^F$ by the same amount.

Regarding the unobservable uncertainty series, in this paper we need proxy measures for two distinct uncertainties: the macroeconomic and the data uncertainty.

In the case of data uncertainty (DU), we use the information coming from three estimates. The first one (DU1) is the variability of the professionals' information set coming from SPF individual data. The second one (DU2) comes from : the GARCH error component of the univariate ARMA representation of the classical revision error, i.e. of the difference between the "final"[25] and the first release of the GDP inflation (measured by the growth rate of GDP deflator from *t-4* to *t*). The third one (DU3) is the average of the squared past inflation revisions for the four most recent quarters. These three variability measures are both forward- and backward-looking.[26] Figure A3.1 reports the normalized time pattern of the three estimates of DU. In general, it emerges the tendency of data uncertainty to go down over time.

Figure A3.1 here

In the case of the macroeconomic uncertainty (MU), we used the economic and policy uncertainty index of Baker et al (2016).[27] This indicator is news-based (uncertainty is quantified by the number of times that newspapers report specific terms related with uncertain moods regarding economy and policy events).[28] Figure A3.2 depicts the normalized pattern of BBD.

Figure A3.2 here

[25] In the context of GDP inflation, the final data released for *t* are those reported eight quarters after their first release.

[26] In using them, we lag the forward-looking measures eight quarters to ensure its availability at the time in which the forecast is made. The first measure (DU1) is what we use in the text, but unreported results show the robustness of the estimates using the three alternatives.

[27] Regularly updated statistical information for the economic and policy uncertainty index can be downloaded from the web site: **http://www.policyuncertainty.com/**

[28] For a comparison of the statistical properties of alternative measures of economic uncertainty, see Bontempi et al, (2021).

Appendix A4:
Preliminary evidence about the variables of interest

Figures A4.1 and A4.2 depict the time pattern of the forecast errors and the forecast revisions described in Appendix A2, and of the alternative measures of noise described in Appendix A3.

Figures A4.1 and A4.2 here

In addition to suggesting stationary patterns, the forecast errors and revisions series depicted in Figure A4.1 also show a declining variability over time (from the noisy 1970s to the Great Moderation period), that continues to stay low over the Great Recession, at the end of the sample period.

The forward noise estimates (aggregate $\hat{c}_t^F$ and average of individual data $\hat{\bar{\omega}}_t$) outlined in Figure A4.2 are very similar each other. They only differ in the outlying 1971q3 and 1973q3 surveys. In addition, as anticipated in the discussion in Appendix A3, the fluctuations (in red) of the backward noise $\hat{c}_t^B$ are evidently smaller than those of the forward measures.

The outcomes of the unit root test of Elliott et al (1996) are reported in Table A4.1 and corroborate the diagnosis of stationarity also emerging from the visual inspection of the historical patterns. In fact, for all the variables of interest, the test statistics are always at least 5% significant.

Table A4.1 here

Appendix A5:
The general specification of the state space equation

Using the Kalman approach, the general specification of the state space equation for r unobservable states in matrix form is:

$$\mathbf{x}_t = \mathbf{F}\,\mathbf{x}_{t\text{-}1} + \mathbf{u}_t \tag{A5.1}$$

where $\mathbf{x}$ is a $r \times 1$ vector of states containing the terms of interest for the system, $\mathbf{u}$ is a $r \times 1$ white noise vector of shocks drawn from a zero-mean multivariate normal distribution with $r \times r$ covariance matrix $E\left(u_t u_t'\right)$ = $\mathbf{Q}$; $\mathbf{F}$ is the $r \times r$ matrix of state transition parameters. Measurements of the system are represented by the observation equation:

$$\mathbf{y}_t = \mathbf{H}\;\mathbf{x}_t + \boldsymbol{\omega}_t \tag{A5.2}$$

where $\mathbf{y}$ is a $n \times 1$ vector of measurements, $\boldsymbol{\omega}$ is a $n \times 1$ white noise vector of measurement errors drawn from a zero-mean multivariate normal distribution with $n \times n$ covariance matrix $E\left(\omega_t \omega_t'\right)$

= **R**; **H** is the $n \times r$ transformation matrix that maps the state vector **x** into the measurement **y** domain.

The *a posteriori* forecast $\hat{\mathbf{x}}_{t|t}$ of the unobservable current state is equal to the *a priori* forecast $\hat{\mathbf{x}}_{t|t-1}$ from the previous period of the current state, plus an updating term that depends on innovations in the measurement equation:

$$\hat{\mathbf{x}}_{t|t} = \hat{\mathbf{x}}_{t|t-1} + \mathbf{G}\left(\mathbf{y}_t - \mathbf{H}\ \hat{\mathbf{x}}_{t|t-1}\right)$$

(A5.3)

The updating is driven by the $r \times n$ matrix **G** of the Kalman gain that minimizes the $r \times r$ covariance matrix of the *a posteriori* forecast errors, $\mathbf{P}_{t|t} = E\left[\left(\mathbf{x}_t - \hat{\mathbf{x}}_{t|t}\right)\left(\mathbf{x}_t - \hat{\mathbf{x}}_{t|t}\right)'\right]$. We start with the definition of $\mathbf{P}_{t|t}$ and substituting (15) for $\hat{\mathbf{x}}_{t|t}$ and (16) for $\mathbf{y}_t$ to obtain:

$$\begin{aligned} \mathbf{P}_{t|t} &= var\left[\mathbf{x}_t - \left(\hat{\mathbf{x}}_{t|t-1} + \mathbf{G}\left(\mathbf{y}_t - \mathbf{H}\ \hat{\mathbf{x}}_{t|t-1}\right)\right)\right] \\ &= var\left[\mathbf{x}_t - \left(\hat{\mathbf{x}}_{t|t-1} + \mathbf{G}\left(\mathbf{H}\ \mathbf{x}_t + \boldsymbol{\omega}_t - \mathbf{H}\ \hat{\mathbf{x}}_{t|t-1}\right)\right)\right] \\ &= \left(\mathbf{I} - \mathbf{G}\ \mathbf{H}\right)\mathbf{P}_{t|t-1}\left(\mathbf{I} - \mathbf{G}\ \mathbf{H}\right)' + \mathbf{G}\ \mathbf{R}\ \mathbf{G}' \end{aligned} \qquad \text{(A5.4)}$$

where $\mathbf{P}_{t|t-1} = E\left[\left(\mathbf{x}_t - \hat{\mathbf{x}}_{t|t-1}\right)\left(\mathbf{x}_t - \hat{\mathbf{x}}_{t|t-1}\right)'\right]$ is the $r \times r$ covariance matrix of the *a priori* forecast errors, and $\boldsymbol{\Sigma}_{\boldsymbol{\omega}}$.

The optimal Kalman gain **G**, is derived by solving the FOC $\frac{\partial \mathbf{P}_{t|t}}{\partial \mathbf{G}} = 0$, as follows:

$$\mathbf{G} = \mathbf{P}_{t|t-1}\mathbf{H}'\left(\mathbf{H}\ \mathbf{P}_{t|t-1}\mathbf{H}' + \mathbf{R}\right)^{-1} \qquad \text{(A5.5)}$$

It is evident from the definition (A5.5) that, although we have *n* measurements, the optimal vector of the Kalman gain consists of *r* parameters (that is, the number of states).

Appendix A6:
The State Dependent Model in state space form

The SDM model (19) can be rewritten in a state-space form by outlining the observation and the state equations. The observation equation is

$$Y_t = H_t \theta_t + \varepsilon_t \qquad \text{(A6.1)}$$

where the state-vector θ_t is the model's vector of parameters that measures the respective effect of all the explanatory variables outlined in vector H_t on the forecast error Y_t. Finally, ε_t is a sequence of independent zero-mean random error terms corresponding to the FIRE error, $e^{FIRE}_{t+h|t}$. The state equation is:

$$\theta_t = F_{t-1}\theta_{t-1} + W_t \qquad \text{(A6.2)}$$

where F_{t-1} is the transition matrix, and the vector W_t embodies v_t, which is a sequence of independent random shocks: $v_t \sim N(0, \Sigma_v)$.

We incorporate the observation and the state equations into a single component in model (A5.5) and represent the observation (measurement) equation (A6.1) as follows:

$$Y_t = \begin{pmatrix} 1 & C_t & X_t & 0 & 0 \end{pmatrix} \begin{pmatrix} \kappa_t \\ \varphi_t \\ \psi_{t-1} \\ \gamma_{1t} \\ \gamma_{2t} \end{pmatrix} + \varepsilon_t$$

and the state (transition) equation (A6.2) as:

$$\begin{pmatrix} \kappa_t \\ \varphi_t \\ \psi_{t-1} \\ \gamma_{1t} \\ \gamma_{2t} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & \Delta MU_{t-1} & \Delta DU_{t-1} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \kappa_{t-1} \\ \varphi_{t-1} \\ \psi_{t-2} \\ \gamma_{1t-1} \\ \gamma_{2t-1} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ v_{1t} \\ v_{2t} \end{pmatrix}$$

Applying the Kalman algorithm directly to equations (A6.1) and (A6.2) we obtain the recursion equations:

$$\hat{\theta}_t = F_{t-1}\hat{\theta}_{t-1} + K_t\left[Y_t - \left(H_t F_{t-1}\hat{\theta}_{t-1}\right)\right] \qquad \text{(A6.3)}$$

The recursion equation (A6.3), K_t is the Kalman gain vector given by:

$$K_t = \Phi_t H_t' \sigma_e^{-2}$$

where Φ_t is the variance-covariance matrix of the one-step prediction error of θ_t, that is:

$$\Phi_t = E\left[\left(\theta_t - F_{t-1}\hat{\theta}_{t-1}\right)\left(\theta_t - F_{t-1}\hat{\theta}_{t-1}\right)'\right]$$

and, given that the one-step ahead prediction error of Y_t is defined as:

$$e_t = Y_t - H_t F_{t-1}\hat{\theta}_{t-1} = H_t\left(\theta_t - F_{t-1}\hat{\theta}_{t-1}\right) + \varepsilon_t$$

its variance σ_e^2 can be stated as:

$$\sigma_e^2 = H_t \Phi_t H_t' + \sigma_\varepsilon^2 .$$

If denotes Q_t the variance-covariance matrix of $\left(\theta_t - \hat{\theta}_t\right)$, successive values of $\hat{\theta}_t$ may be estimated by using the standard recursive equations of the Kalman Filter:

$$K_t = \Phi_t H_t' \left(H_t \Phi_t H_t' + \sigma_\varepsilon^2\right)^{-1}$$

$$\Phi_t = F_{t-1} Q_{t-1} F_{t-1}' + \Sigma_W$$

$$Q_t = \Phi_t - K_t\left(H_t \Phi_t H_t' + \sigma_\varepsilon^2\right) K_t'$$

where: $\Sigma_W = \begin{pmatrix} 0 & 0 \\ 0 & \Sigma_v \end{pmatrix}$

In practice, this recursive procedure must start with some value of $t=t_0$ and, hence, initial values are required for $\hat{\theta}_{t_0-1}$ and $\hat{R}_{t_0-1}$. Assuming that equation (19) represents a locally linear model, we apply the OLS estimation procedure in order to find the initial values of $\hat{\kappa}$, $\hat{\varphi}$, $\hat{\psi}$ and of model residuals' variance $\hat{\sigma}_\varepsilon^2$. Therefore, we can start the recursion at t_o by setting the initial vector as $\hat{\theta}_{t_0-1} = \begin{pmatrix}\hat{\kappa} & \hat{\varphi} & \hat{\psi} & 0 & 0\end{pmatrix}'$ and the initial matrix as $\hat{R}_{t_0-1} = \begin{pmatrix} \hat{R}_{\kappa,\varphi,\psi} & 0 \\ 0 & 0 \end{pmatrix}$, where $\hat{R}_{\kappa,\varphi,\psi}$ is the estimated variance-covariance matrix of $\hat{\kappa}$, $\hat{\varphi}$ and $\hat{\psi}$ obtained from the initial OLS model fitting. It also seems reasonable to set all the initial gradients to zero if the initial values are reasonably accurate at t_o.

Given that reasonable values are also required for Σ_v

v_{t+1}, we note that the choice of Σ_v will drive the "smoothness" of the parameters. Hence, in the present case, the diagonal elements of Σ_v are set equal to $\hat{\sigma}_\varepsilon^2$ multiplied by some constant α called the "smoothing factor", and the off-diagonal elements are set equal to zero.[29] In this paper, we follow Haggan et al. (1984), who suggest a smoothing factor in the range of 10^{-1} to 10^{-3}.[30]

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[29] In doing so, we must bear in mind if the elements of Σ_V are set too large the estimated parameters become unstable. But if the elements of Σ_V are made too small it is difficult to detect the non-linearity present in the data.

[30] Alternatively, parameters could have been smoothed by a multi-dimensional form of the non-parametric function fitting technique, see for example Priestley and Chao (1972).

Table A4.1 - Elliott-Rothenberg-Stock DF-GLS test statistic

FORERR (forecast error)	-2.203604**
Lag Length: 0	
FORREV has a unit root (forecast revision)	-3.724952***
Lag Length: 5	
OMEGA_PGDP (aggregate forward noise, $\hat{c}_t^F$)	-4.395977***
Lag Length: 0	
OMEGA_IT (average forward individual noise, $\hat{\bar{\omega}}_t$)	-2.890573***
Lag Length: 5	
RE4AVG (aggregate backward noise, $\hat{c}_t^B$)	-5.180261***
Lag Length: 0	

Deterministic component: only the constant

Automatic Lag Length, based on Modified AIC

Test critical values:	1% level	-2.576753
	5% level	-1.942448
	10% level	-1.615628

*** and ** respectively denotes 5% and 1% significance

Fig. A3.1 – The measure of the data uncertainty, DU

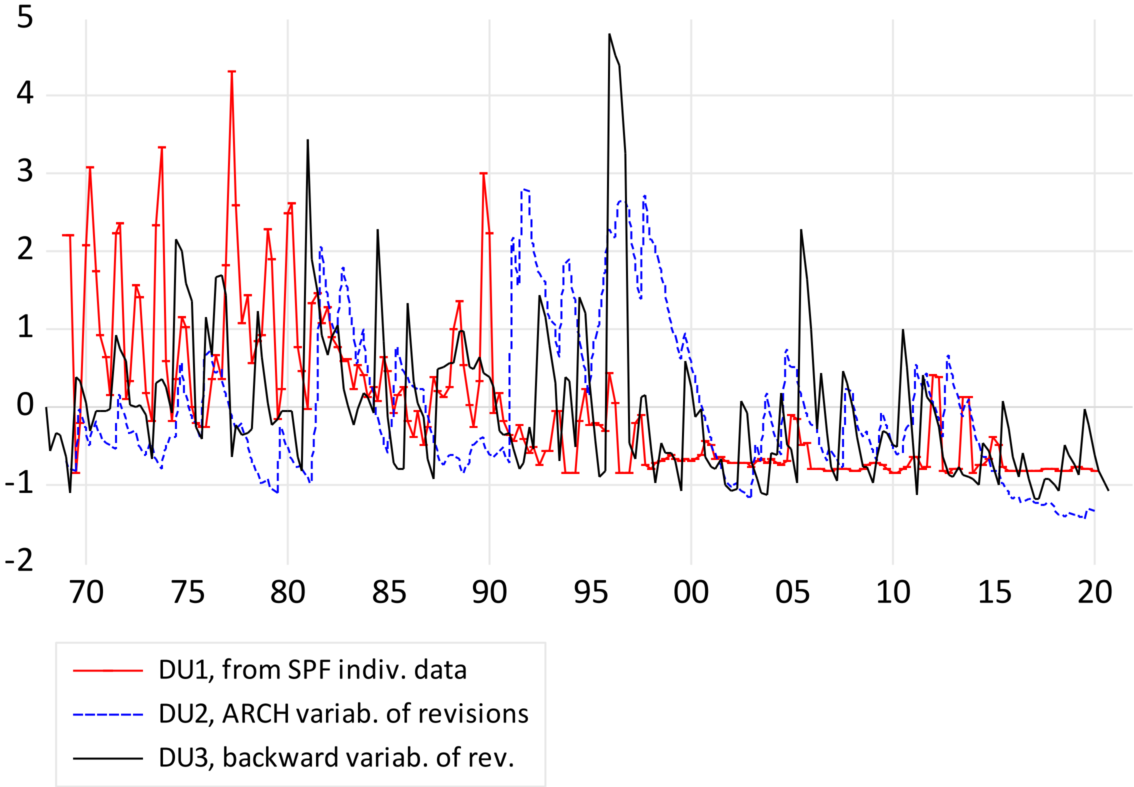

Fig. A3.2 – The macroeconomic uncertainty MU of Baker et al. (2016)

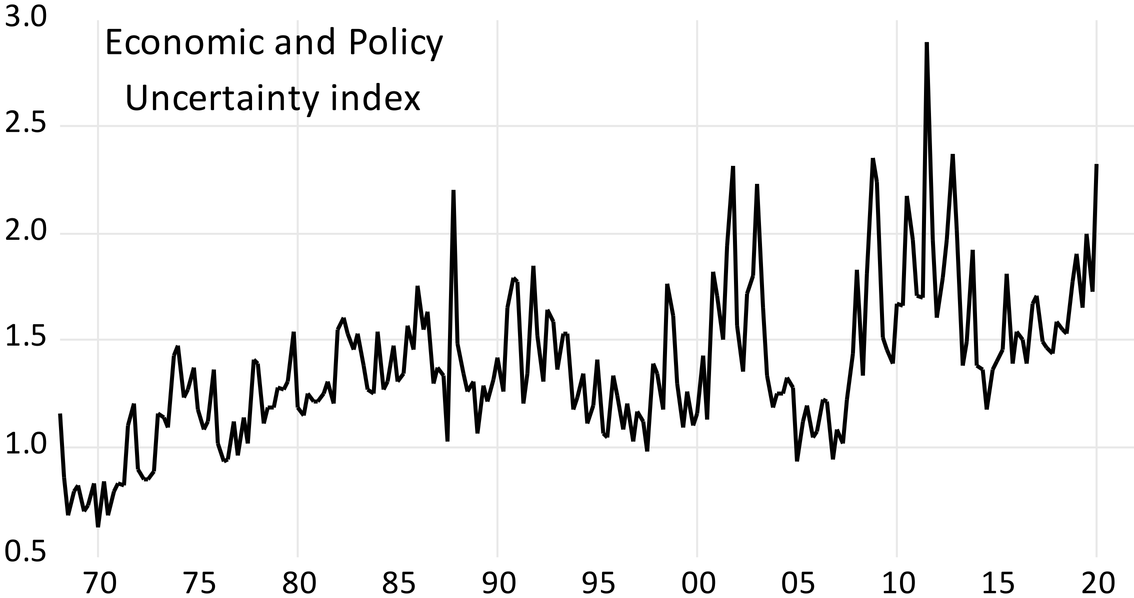

Fig. A4.1 – The dependent forecast error and explanatory forecast revision

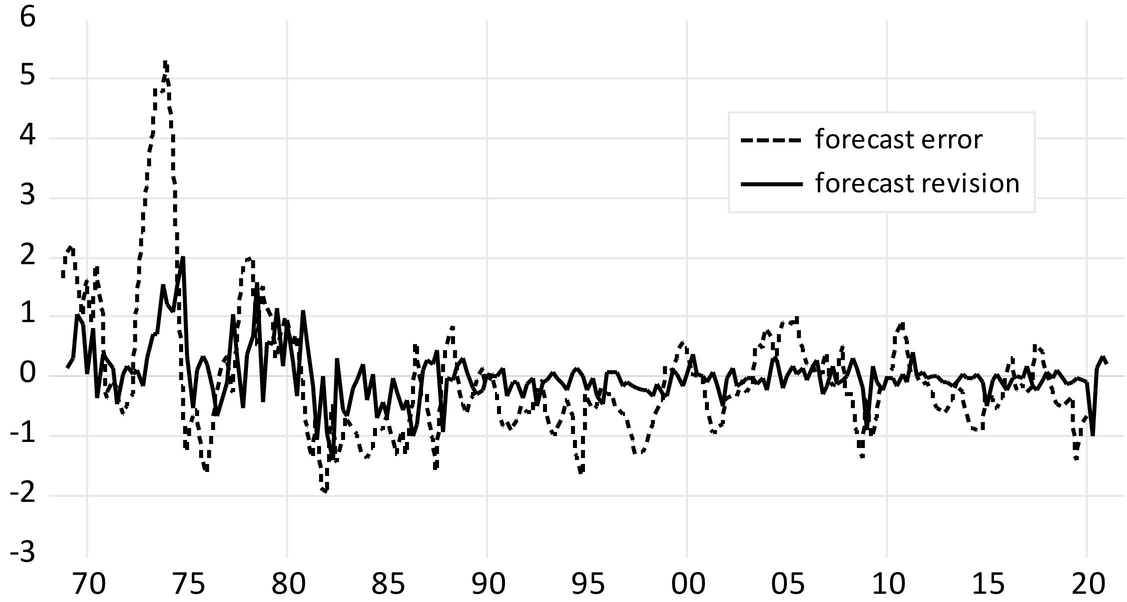

Fig. A4.2 – The alternative measures of forecast noise