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The Exponential HEAVY Model: An Improved Approach to Volatility Modeling and Forecasting

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Abstract

This paper proposes an Exponential HEAVY (EHEAVY) model, which specifies the dynamics of returns and realized measures of volatility in an exponential form. The model ensures positivity of volatility and allows for asymmetric effects without restrictions on parameters, hence is more flexible. A joint quasi-maximum likelihood estimation and closed-form multi-step ahead forecasting is derived. The EHEAVY model is applied to 31 assets from the Oxford-Man Institute's realized library, and the empirical results demonstrate that return volatility dynamics are driven by the realized measure, while the asymmetric effect is captured by the return shock. The out-of-sample forecast results show that the EHEAVY model has superior forecasting performance compared the HEAVY, AHEAVY, and realized EGARCH models. The portfolio exercise further confirms the superior economic value of the EHEAVY model, as measured by the certain equivalent return and expected utility.

Keywords: HEAVY model, High-frequency data, Asymmetric effects, Realized variance, Portfolio

JEL Classification: C32, C53, G11, G17

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1 Introduction

The modeling and forecasting of return volatility have significant implications in asset pricing, portfolio selection, and risk management practices. Many studies have introduced non-parametric estimators of realized volatility using intra-day data (Andersen et al., 2001; Barndorff-Nielsen and Shephard, 2002; Barndorff-Nielsen et al., 2008, 2009) and voluminous empirical evidence on modelling and forecasting the realized measure of volatility has been developed, for example, the ARFIMA model in the original or logarithmic form (see Andersen et al., 2003; Chiriac and Voev, 2011; Koopman et al., 2005; Asai et al., 2012; Allen et al., 2014) and the Heterogeneous Autoregressive (HAR-RV) model by Corsi (2009).

Recently, the intra-daily estimators of volatility - known as realized measures — have been used to improve the conditional volatility of daily return models. One of the popular models is the so-called "High-rEquency-bAsed VolatilitY"(HEAVY) model, initially proposed by Shephard and Sheppard (2010). The two-equation system, the HEAVY-r and the HEAVY-RM, jointly estimates conditional variances of return and the realized measures of volatility based on daily and intra-daily data. The HEAVY model adopts to information arrival more rapidly than the classic daily GARCH process and hence it provides more reliable forecasts. Various extensions of HEAVY models have been developed. Hansen et al. (2012) introduced the Realized GARCH model that corresponds most closely to the HEAVY framework. The Realized GARCH model is based on measurement equations that tie the realized measure to the latent conditional variance of return. An exponential type of realized GARCH model is developed by Hansen and Huang (2016). Cipollini et al. (2013) refer to the HEAVY model by constraining the bivariate vector multiplicative error representation for squared returns and realized variance. Borovkova and Mahakena (2015) apply the HEAVY models with different error distributions, such as student-t and skewed-t, and extend the HEAVY-r equation with a news sentiment proxy and a time to maturity variable. Karanasos et al. (2020) enrich the HEAVY model with long memory features and asymmetric effects. Yfanti et al. (2020) add a range-based Garman-Klass volatility to the HEAVY framework. The multivariate specification of the HEAVY model is developed by Noureldin et al. (2012) and extended by Opschoor et al. (2017), Creal et al. (2013), Sheppard and Xu (2019), and Bauwens and Xu (2022).

However, HEAVY models proposed so far are linear. The parameters in the HEAVY-r and

HEAVY-RM equations are constrained to be positive to guarantee the positivity of volatility, which can be too restrictive. Furthermore, the models do not fully address the asymmetric effect, where variances react differently to positive and negative shocks. In comparison to GARCH models, HEAVY models have more possibilities to represent shocks using either return shock or realized return shock, but it is unclear which one is better suited to capture the asymmetric effects. In the HEAVY model by Shephard and Sheppard (2010), the dynamics of volatility are driven solely by lagged realized measure, making it easy to use lagged realized measures to capture the asymmetric effect. On the other hand, in the Realized GARCH model by Hansen et al. (2012, 2016), the return shocks capture asymmetric effects in both the return and realized measure equations. In Karanasos et al. (2020)'s HEAVY model, both return shocks and realized return shocks are utilized to model the asymmetric effects.

In this paper, we introduce an EHEAVY model, which extends the linear HEAVY model by using an exponential form for both the return and realized measure equations. The EHEAVY model inherits the benefits of the EGARCH model, which guarantees that the conditional variance of return and realized measure is positive without imposing any restrictions on the parameter set, and enables incorporation of asymmetric effects naturally. We propose a joint quasi-maximum likelihood (QML) estimation approach for the EHEAVY model, and our Monte Carlo simulations demonstrate that the QML estimator has desirable small sample properties.

Using the EHEAVY framework, we empirically investigate the presence of asymmetric effects in the volatility of 31 stocks from the Oxford Man institution of realized library, without imposing any positivity restrictions on the parameters. Our findings reveal that the asymmetric effects in the conditional variance of the return equation are captured by the return shock, rather than the realized return shock.

It is worth noting that the EHEAVY model is closely related to Hansen and Huang's (2016) realized EGARCH model, but there are some differences. Firstly, we adopt an EGARCH specification and excludes the absolute standardized return in the return variance equation. This is because empirical results suggest that the return variance dynamics are primarily driven by the realized measure, not absolute standardized return. This aligns with previous findings in the HEAVY literature. In contrast, Hansen and Huang (2016) include the absolute standardized return in return variance equation[1]. Secondly, Hansen and Huang (2016) use a measurement

[1] Hansen and Huang use a squared standardized return. Actually, the effect between the absolute and squared

equation where the log of the realized measure of volatility is a function of the conditional volatility of return in the same period. In our model, we use a HEAVY-RM structure where the realized measure of volatility is a lagged function of realized measure. Therefore, the realized EGARCH model is used to estimate/predict the conditional variance of return, while the EHEAVY model can also model and predict realized variance as in HEAVY-type models. Thirdly, we derive a joint quasi-maximum likelihood estimation approach for the EHEAVY model and a closed-form multi-step ahead forecasting procedure.

To assess the EHEAVY model's performance, we conduct an out-of-sample forecasting exercise and compare it with the HEAVY, asymmetric HEAVY (AHEAVY) of Shephard and Sheppard (2010), and realized EGARCH model at the daily, weekly, and monthly horizons. The results suggest that the EHEAVY model outperforms the benchmark HEAVY and AHEAVY models when forecasting the conditional variance of return. It performs similarly to the realized EGARCH model statistically, but the EHEAVY strategy leads to a portfolio with a higher certain equivalent return and expected utility than the realized EGARCH model. Overall, the out-of-sample forecasting exercise indicates the benefits of using the EHEAVY model, both in statistical and economic terms.

The remainder of the paper is organized as follows. Section 2 introduces the EHEAVY models. Section 3 presents the multiplicative error representation and the multi-step forecast formulas. Section 4 describes the quasi-maximum likelihood estimation procedure. Section 5 presents the empirical application. Section 6 summarizes the findings and concludes the paper. The supplementary appendix (SA) contains additional empirical results.

2 The Exponential HEAVY Models

The benchmark HEAVY specification of Shephard and Sheppard (2010) use two variables: daily financial returns (r_t) and a corresponding sequence of intraday realized measures of volatility, RM_t. Realized measures are theoretically high-frequency, nonparametric-based estimators of the variation of open-to-close returns. We form the signed square rooted realized measures as follows: $\widetilde{RM_t} = sign(r_t)\sqrt{RM_t}$, where the $sign(r_t) = 1$, if $r_t \geqslant 0$ and $sign(r_t) = -1$, if $r_t < 0$. $\widetilde{RM_t}$ is also known as the realized return. Then, the return and realized measures are

standardized return is rather close. We adopt the absolute standardized return in line with the EGARCH model.

characterized by the following relation:

$$\begin{aligned} r_t^2 &= h_t \varepsilon_{rt} \\ RM_t &= m_t \varepsilon_{Rt} \end{aligned} \quad \text{or} \quad \begin{aligned} r_t &= \sqrt{h_t} e_{rt} \\ \widetilde{RM}_t &= \sqrt{m_t} e_{Rt} \end{aligned} \tag{1}$$

The first representation is multiplicative error specification, where the stochastic term ε_{it} $(i = r, R)$ is independent and identically distributed, which is positively defined and has a unit mean. This implies that $\mathbb{E}(r_t^2 | \mathcal{F}_{t-1}) = h_t$. The second representation is a GARCH type model, where e_{it} is independent and identically distributed, which has zero mean and unit variance. This implies that $\mathbb{V}ar(r_t | \mathcal{F}_{t-1}) = h_t$. In other words, the GARCH model for the conditional variance of the returns (or the realized returns), is similar to the multiplicative error model[2] for the conditional mean of the squared returns (or the realized measures).

The EHEAVY model consists of the following two equations:

$$\begin{aligned} \log h_t &= \omega_r + \beta_r \log h_{t-1} + \alpha_{rR} |e_{Rt-1}| + \gamma_{rr} e_{rt-1}, \\ \log m_t &= \omega_R + \beta_R \log m_{t-1} + \alpha_{RR} |e_{Rt-1}| + \gamma_{Rr} e_{rt-1} \end{aligned} \tag{2}$$

where $corr(e_{rt}, e_{Rt}) = \rho$.

The first equation is the EHEAVY-r equation and the second equation is EHEAVY-RM equation. In the EHEAVY-r equation, the parameter β_r summarizes the persistence of volatility, whereas α_{rR} represents how informative the realized measures are about the future volatility of return. The asymmetric effect is represented by $\gamma_{rr} e_{rt-1}$. In the EHEAVY-RM equation, the parameter β_R summarizes the persistence of realized measure volatility and the asymmetric effect is represented by $\gamma_{Rr} e_{rt-1}$. EHEAVY model is stationary if $\beta_r < 1$ and $\beta_R < 1$. One advantage of the EHEAVY versus HEAVY model is that the positivity of variance is guaranteed without any restrictions on the parameter set.

It is notable that the realized EGARCH model of Hansen and Huang (2016) also includes $\alpha_{rr} |e_{rt-1}|$ in the return equation[3]. Their representation is more like an EGARCH-X model, where both $\alpha_{rr} |e_{rt-1}|$ and $\alpha_{rR} |e_{Rt-1}|$ are included. Consistent with the evidence in the HEAVY

[2]Engle (2002) first proposed the multiplicative error model using the various GARCH family specifications to estimate the volatility, which is a non-negative process.

[3]Hansen and Huang (2016) use a quadratic form $\alpha_{rr}(e_{rt-1})^2$

literature, we find that the estimated α_{rr} is very small or insignificant[4], which implies that the informative absolute (or squared) return about future volatility is small. So $\alpha_{rr}|e_{rt-1}|$ is excluded in the EHEAVY model. The EHEAVY-r equation has the same number of parameters as the EGARCH model.

The EHEAVY-RM equation is closer to the HEAVY-RM of Shepard and Shepperd (2010) with the exponential representation. Shepard and Shepperd (2010) suggested an AHEAVY model, where the asymmetric effect is captured by the binary lagged realized measure in the HEAVY-r and HEAVY-RM equation. Our empirical evidence shows that the asymmetric effects are mostly captured by the return shock, not the realized measure. So, the EHEAVY model includes $\gamma_{rr}e_{rt-1}$ and $\gamma_{rR}e_{rt-1}$ terms to capture the asymmetric effects.

3 Representation and Forecasting

In this section, the EHEAVY models are represented as vector multiplicative error representation, from which the closed-form formulas for multi-step ahead forecasts and a Quasi-maximum likelihood estimation procedure can be derived.

3.1 Vector Multiplicative Error Representation

Defining $x_t = [r_t^2, RM_t]'$, $\widetilde{x}_t = [r_t, \widetilde{RM_t}]'$, $\mu_t = [h_t, m_t]'$ and $e_t = [e_{rt}, e_{Rt}]'$, the vector multiplicative representation of EHEAVY model is

$$\begin{aligned} \widetilde{x}_t &= \sqrt{\mu_t} \odot e_t, \quad e_t|\mathcal{F}_{t-1} \sim D(0, P) \\ \log \mu_t &= \omega + B \log \mu_{t-1} + A e_{t-1} + \Gamma |e_{t-1}|, \end{aligned} \tag{3}$$

where e_t are a sequence of independent and identically distributed variables with mean 0 and time-invariant positive definite covariance matrix P with ones on the main diagonal so that $E(x_t|\mathcal{F}_{t-1}) = \mu_t$, and

$$\omega = \begin{bmatrix} \omega_r \\ \omega_R \end{bmatrix}, \ A = \begin{bmatrix} 0 & \alpha_{rR} \\ 0 & \alpha_{RR} \end{bmatrix}, \ \Gamma = \begin{bmatrix} \gamma_{rr} & 0 \\ \gamma_{Rr} & 0 \end{bmatrix}, \ B = \begin{bmatrix} \beta_r & 0 \\ 0 & \beta_R \end{bmatrix}. \tag{4}$$

[4]See Table ?? or appendix A for detailed estimation

It is notable that if

$$A= \begin{bmatrix} \alpha_{rr} & 0 \\ 0 & \alpha_{RR} \end{bmatrix}, \ \Gamma= \begin{bmatrix} \gamma_{rr} & 0 \\ 0 & \gamma_{RR} \end{bmatrix}, \tag{5}$$

the top part of (**??**) becomes the EGARCH model.

3.2 Multiple-step Ahead Forecasting

The HEAVY and EHEAVY model can be used to predict both the conditional variance of return and the realized measure of volatility. The latter has been the subject of very active literature (see, for example, Andersen et al., 2001, 2003 ; Corsi, 2009 ; Bollerslev et al., 2016; Taylor, 2017).

Suppose the forecaster models x_t and obtains s-step-ahead forecasts $E(x_{t+s}|\mathcal{F}_t)$, or in the shorthand notation $E_t(x_{t+s})$, where $\mathcal{F}_t$ is the forecaster's information set available at time t. Let $\mu_{t+s|t} = E_t(x_{t+s})$. Now let's move steps ahead, x_{t+s}, $s > 0$ is not known and needs to be substituted with its corresponding conditional expectation μ_{t+s}. The multi-step ahead forecasts of the EHEAVY model are not straightforward, as the conditional expectation of log function is not equal to the log function of the conditional expectation. To do so, we denote $\phi_t = \log(\mu_t)$ and

$$\begin{aligned} \phi_{t+1|t} &= \omega + B\phi_t + \Gamma e_t + A|e_t|, \\ \phi_{t+2|t} &= \omega + A\bar{e} + B\phi_{t+1|t} \\ &= \bar{\omega} + B\phi_{t+1|t}, \end{aligned} \tag{6}$$

where $\bar{e} = E(|e_t|)$ and $\bar{\omega} = \omega + A\bar{e}$. If e_t is symmetric normally distributed, $E(|e_t|) = \sqrt{2/\pi}$. More wisely, $E(|e_t|)$ can be estimated by the unconditional mean of $|e_t|$.

And then, for $s > 2$,

$$\phi_{t+s|t} = \bar{\omega} + B\phi_{t+s-1|t}, \tag{7}$$

which can be solved recursively for any horizon s. A closed form forecasts for $\phi_{t+s|t}$ can also be derived as:

$$\phi_{t+s|t} = \tilde{\omega} + B^{s-1}\phi_{t+1|t} \tag{8}$$

where $\tilde{\omega} = \frac{(1-B^{s-1})\bar{\omega}}{1-B}$.

We then derive a formula for $\mu_{t+s|t} = E_t(x_{t+s})$. With the log specification one would have to account for distributional aspects of $\log(\mu_{t+s|t})$ in order to produce an unbiased forecast of $\mu_{t+s|t}$. Using the second-order approximation [5]

$$\mu_{t+s|t} \approx \exp(\phi_{t+s|t})(1 + \frac{\sigma^2_{\phi,t+s|t}}{2}) \tag{9}$$

where $\sigma^2_{\phi,t+s|t}$ is the s-step-ahead conditional second moment $\phi_{t+s|t}$. The conditional second moments are estimated using their unconditional sample counterparts.

The EHEAVY model s-step ahead forecasts $\mu_{t+s|t}$ are derived by setting A, B, Γ to the matrices defined in (**??**). Then, the s-step ahead forecast of the conditional variance of return ($h_{t+s|t}$) corresponds to the first element of $\mu_{t+s|t}$ and the s-step ahead forecast of the realized measure of volatility ($m_{t+s|t}$) corresponds to the second element of $\mu_{t+s|t}$.

4 Estimation

The parameters in return and realized measure equation are not variation free in the EHEAVY models, hence a joint estimation method is required. Below we derive a Quasi-maximum likelihood estimation approach.

We estimate the EHEAVY model using the vector multiplicative error representation shown in (**??**). More generally, let $\widetilde{x_t}$ be a $k-$dimensional process, and let $\theta' = [\theta'_1, \theta'_2]$, where $\theta'_1 = vech(P)$, and the operator $vech$ stacks the lower triangular elements of a symmetric $(k \times k)$ matrix into a $k \times (k+1)/2$ vector, and θ'_2 contains the parameters in μ_t. We assume that e_t follows a multivariate normal distribution, $e_t|\mathcal{F}_{t-1} \sim N(0, P)$, where $\mathcal{F}_{t-1}$ represents the information set available up to time $t-1$. The likelihood function is equivalent to the one in the Constant Conditional Correlation (CCC)-GARCH model (Bollerslev,1990; Jeantheau,1998). The log-likelihood function for the observation at time t is given by:

[5] The full approximation is given by $\exp\left(\phi_{t+s|t}\right)\left(1 + \sum_{k=1}^{\infty} \frac{1}{k!}\phi_{k,t+s|t}\right)$ where $\phi_{k,t+s|t}$ is the s-step-ahead kth conditional moment about the conditional mean. See Taylor (2017) for the details of a full approximation.

$$
\begin{aligned}
l_t(\theta) &= -\frac{k}{2}\log(2\pi) - \frac{1}{2}\log|M_tPM_t| - \frac{1}{2}\widetilde{x_t}'\left(M_tPM_t\right)^{-1}\widetilde{x_t} \\
&= -\frac{k}{2}\log(2\pi) - \log|M_t| - \frac{1}{2}\log|P| - \frac{1}{2}\widetilde{x_t}'M_t^{-1}P^{-1}M_t^{-1}\widetilde{x_t}
\end{aligned} \tag{10}
$$

where $M_t = diag(\sqrt{\mu_t}) = diag(\sqrt{\mu_{1t}}, \sqrt{\mu_{2t}}, ..., \sqrt{\mu_{kt}})$.

Let $l(\theta) = \sum_{t=1}^{T} l_t(\theta)$, the QMLE for $\hat{\theta}$ equals

$$
\hat{\theta} = \arg\max_{\theta} l(\theta).
$$

Explicit expressions for the score vector and the Hessian matrix of the log-likelihood function can be derived following the CCC-GARCH literature; see Nakatani and Teräsvirta (2009) lemma 3.1 and 3.2 for example.

The asymptotic distribution of QML estimators for the EHEAVY model is similarly complicated to the EGARCH and realized EGARCH models[6]. Therefore, it is currently beyond the scope of this article to fully derive the asymptotic theory for the estimators. However, it is worth mentioning that Hansen and Huang (2016) did provide the asymptotic distribution of the QML estimator for the realized EGARCH model, but they did not establish the conditions under which this distribution holds. As a result, the asymptotic distribution cannot be verified.

In the absence of the asymptotic distribution theory, some results from a simulation study can provide insight into the performance of the QML estimators. The data generation processes in the simulation are as follows. The return and realized return data are simulated according to (**??**), assuming normality distribution. The parameters of the process from which the data are generated are taken from the empirical results reported in Table **??** in the next section. A sample of T observations is generated and used for estimation; we set $T = 2000$ and 5000 to illustrate the impact of increasing the sample size. The largest sample size of 5000 is chosen close to the sample size of the application in Section **??**. The data simulation and the parameter estimation is repeated $S = 1000$ times.

The simulation study focused on the bias, root mean squared error, and normality of the

[6] The consistency and asymptotic properties of QML estimators for the multivariate EGARCH are not available under general conditions (see for example, Nakatani and Teräsvirta, 2009 and Francq et al., 2013). A limitation in the development of the asymptotic properties for the EGARCH is the lack of an invertibility condition (See Wintenberger 2013, Martinet and McAleer 2018, and Demos and Kyriakopoulou 2019 for a discussion)

sampling distribution of the QML estimator. These results can help to provide some preliminary evidence of the estimator's performance. The relative bias (RB) and root mean squared error (RMSE) are defined as

$$RB(\hat{\phi}) = 100 \times \frac{1}{S}\sum_{s=1}^{S}\frac{(\hat{\theta}_s - \theta_0)}{\theta_0},\ RMSE(\hat{\theta}) = 100\sqrt{\frac{1}{S}\sum_{s=1}^{S}(\hat{\theta}_s - \theta_0)^2}, \tag{11}$$

for the estimator $\hat{\theta}$ of the parameter θ (an element of θ) having the true value θ_0 and estimated by $\hat{\theta}_s$ for the s-th simulated data set.

Table **??** provides a synthetic view of the simulation results. The relative bias from the QML estimator is small, even at a small sample size ($T = 2000$). When the sample size is increased to 5000, both the RMSE and the relative biases decrease. These results indicate the likely consistency of the QML estimator. The Jarque-Bera statistics of the sampling distributions are reported as a test of the normality. The result shows that normality of QML estimators are rejected at sample size $T = 2000$, but not at the sample size $T = 5000$. In brief, the simulation results indicate that the consistency and the asymptotic normality are likely properties of the QML estimator. It is also notable that the QML estimator is actually a ML estimator since the estimated model is correctly specified in the simulation study.

Insert Table **??** *here*

5 Empirical Application

5.1 Data

We use daily data for 31 assets from the Oxford-Man Institute's (OMI) realized library for the period between 03/01/2000 and 31/5/2021. The Symbol Names of 31 assets are presented in Table A1. The OMI's realized library provides daily stock market returns and various realized volatility measures calculated from high-frequency data sourced from the Reuters DataScope Tick History database. The data cleaning procedure and realized measure calculation are described in Shephard and Sheppard (2010). We use the realized kernel as the realized measure, estimated using the Parzen kernel function. This estimator is similar to the well-known realized variance, but is more robust to market microstructure noise and provides a more accurate estimate of the quadratic variation. The realized kernel is calculated as follows: $RK_t = \sum_{k=-H}^{H} k(h/(H+1))\gamma_h$, where $k(x)$ is the Parzen kernel function with $\gamma_h = \sum_{j=|h|+1}^{n} x_{jt}x_{j-|h|,t}$; $x_{jt} = X_{t_{j,t}} - X_{t_{j-1,t}}$ are the 5-minute intra-daily returns where $X_{t_{j,t}}$ are the intra-daily prices and $t_{j,t}$ are the times of trades on the t-th day. Shephard and Sheppard (2010) state that they choose the bandwidth H as in Barndorff-Nielsen et al. (2009).

The realized measure is directly linked to the volatility of open-to-close returns, but only captures a portion of the volatility of close-to-close returns. In our estimation, we use both open-to-close returns and close-to-close returns.

Insert Table **??** *here*

Table **??** presents 31 assets extracted from the database and provides volatility estimates for their squared returns and realized kernel time series for the respective sample period. We calculate the mean and standard deviation (StDev) of the annualized volatility, which is the square root of 252 times the squared return or the realized kernel. The mean column shows that the assets have annualized realized measure of volatilities between 9% and 30%, with corresponding results for the squared close-to-close returns between 14% and 40%. On average, the realized measure is about 63% of the squared return. The realized measure misses out on the overnight return, which accounts for its lower level. On the other hand, the annualized volatility of open-to-close returns is similar to the annualized realized measure of volatility. It is typically slightly higher than the realized measure, but the difference is very small. The StDev column shows much higher standard deviations for the squared return than the realized measure. The standard deviations of squared close-to-close returns are usually twice as high as the standard deviations of the realized measure. The squared open-to-close returns also have much higher standard deviation than the realized measure. This shows that the realized measure is a more stable measurement of volatility than the squared returns.

5.2 Estimation results

We estimate the EHEAVY model using both the open-to-close returns and close-to-close returns. Table **??** presents summary statistics (median, minimum, maximum) of the parameter estimates of the 31 assets of EGARCH, EGARCHX, EHEAVY models. The detailed estimates for each of the assets are presented in Table A2 and A3 in the Appendix.

Insert Table **??** *here*

Based on the estimation using open-to-close returns, the empirical results can be summarized as follows. The EGARCH estimates are in line with expectations. The persistence parameter β is high and close to one, while the leverage parameter γ is negative and significant. In the EGARCH-X model, the estimated α_{rr} is significant in 13 out of 31 cases, with a small median value of 0.02 and ranging from -0.032 to 0.142. The estimated α_{rR} is significant in all 31 cases, with a much larger median value of 0.366 and ranging from 0.211 to 0.602. This is consistent with the findings in the HEAVY literature, indicating that the future volatility of returns is mainly driven by the information from the realized measure. The estimated γ_{rr} is relatively large and significant in 28 out of 31 cases, while the estimated γ_{rR} is significant only in 4 out of 31 cases and with a much smaller size. This suggests that the leverage effects in the return volatility equation are mainly driven by the return shock, not the realized return shock. In the EHEAVY model, all coefficients are significant in almost all 31 cases. In particular, the coefficients for asymmetric effect γ_{rr} and γ_{Rr} are negative and significant, indicating that the asymmetric effect is a common stylized fact in volatility modeling. The estimates of ρ are around 0.8 and very similar across assets, showing a high correlation between returns and realized returns. This is evidence of joint estimation of the return and realized return equations, as proposed in section 4.

Based on the information criteria used, the EGARCH-X model has the highest log-likelihood values. However, the log-likelihood gain of EGARCH-X over EHEAVY is only 17 (based on the median value) for the 4 additional parameters, indicating that the improvement is minor. On the other hand, the gains of both EGARCH-X and EHEAVY over EGARCH are substantial (median value 131 and 124, respectively). Additionally, in 19 out of 31 cases, EGARCH-X has the highest log-likelihood values, while in 11 out of 31 cases, EHEAVY has the highest log-likelihood values. When considering the BIC criteria, the EHEAVY model shows a clear better fit than the other models, achieving the best BIC criteria in 21 out of 31 cases. It should be noticed that EGARCH and EHEAVY are not nested, but they have the same number of parameters, so choosing between them using their log-likelihood values is equivalent to a choice based on model choice criteria.

The results for close-to-close returns are similar to those obtained for open-to-close returns.

The EHEAVY model outperforms the conventional EGARCH models. The lagged realized measure is found to be the main driver of return volatility dynamics, and asymmetric effect is captured by previous period's return shocks. Overall, these findings provide further support for the use of the EHEAVY model in volatility modeling.

5.3 News impact curve

Additional insights about the value of the EHEAVY structure are evident from the news impact curve. This curve, introduced by Engle and Ng (1993), illustrates the impact that return shocks having on volatility. The news impact curve measures the impact that e_{rt} has on h_{t+1} in percentages, as defined by $\mathrm{E}(\log h_{t+1} \mid e_{rt} = e_r) - \mathrm{E}(\log h_{t+1})$. We plot the impact curve of the EHEAVY model, the EGARCH model, and the realized EGARCH model of Hansen and Huang (2016). As return shocks are contemporaneously correlated with realized return shocks, one unit return shock will also incur ρ unit realized return shock. The news impact curve for the EHEAVY model is given by $\alpha_{rR}|e_{Rt-1}| + \gamma_{rr}e_{rt-1} = \rho\alpha_{rR}|e_{rt-1}| + \gamma_{rr}e_{rt-1}$. For the EGARCH and realized EGARCH models, the news impact curve is simply given by $\alpha_{rr}|e_{rt-1}| + \gamma_{rr}e_{rt-1}$.

Insert Figure **??** *here*

Taking EURO50 close-to-close return for example, the news impact curve is plotted in Figure 1. As is evident from Figure 1, the generalized structure of the EHEAVY model has a more profound effect on the news impact curve than the EGARCH and realized EGARCH model. The news impact curve of realized EGARCH model is very close to that of the EGARCH model, and it does not show an increasing news impact curve when news is positive. The EHEAVY model allows good news and bad news to have a different impact on volatility. It also allows big news to have a greater impact on volatility than the EGARCH and realized EGARCH model in both directions.

Insert Figure **??** *here*

Figure 2 gives another example: SPX close-to-close return news impact curve. As in the previous example, the EHEAVY model shows the highest variation in both directions, indicating that it allows for big news to have a greater impact on volatility compared to the EGARCH and realized EGARCH models. This is particularly evident in the positive news region, where the news impact curve for EHEAVY is significantly steeper than the other two models. Overall, the results suggest that the EHEAVY model provides a more accurate representation of the relationship between news and volatility, especially in capturing the asymmetry between positive and negative news impacts.

5.4 Forecasting comparison

Next, we will conduct an out-of-sample forecasting comparison. In this application, we will focus on forecasting the volatility of close-to-close returns, which is more suitable for most applications in portfolio allocation or risk management.

We will compare the EHEAVY model with the following three popular models: 1) the benchmark HEAVY model; 2) the asymmetric HEAVY (AHEAVY) model of Shepard and Shepperd (2010); and 3) the realized EGARCH of Hansen and Huang (2016). The out-of-sample period will comprise the last 1000 observations of the full-sample period for each asset. The four models will be re-estimated every observation based on a rolling sample window of sample size $T - 1000$. As shown in Appendix table A1, the full sample size is around $T = 5000$ for most of the assets, which leaves the estimated sample around 4000 observations. We will report $s = 1$, 5, and 22 for horizons of 1-day, 5-day, and 22-day ahead out-of-sample forecasts.

We use the following two loss functions for the volatility of the close-to-close return

$$MSE^a_{t,s}(r^2_{t+s}, h^a_{t+s|t}) = \sum_{t=T-1000+s}^{T} (r^2_{t+s} - h^a_{t+s|t})^2 \tag{12}$$

$$QMLIK^a_{t,s}(r^2_{t+s}, h^a_{t+s|t}) = \sum_{t=T-1000+s}^{T} \left(\frac{r^2_{t+s}}{h^a_{t+s|t}} - \log\left(\frac{r^2_{t+s}}{h^a_{t+s}}\right) - 1 \right). \tag{13}$$

where $h^a_{t+s|t}$ denotes the $s-$step forecast using model a conditional on time t information.

In order to formally determine whether the quality of the forecasts differ significantly across the different models, we employ the Model Confidence Set (MCS) method developed by Hansen, Lunde, and Nason (2011). This method identifies the subset of models that includes the best forecasting model with 95% confidence. For each of the two loss functions and three forecast horizons, we can determine the MCS subset of models.

Insert Table **??** *here*

Table ?? compares the out-of-sample forecasts of different models by reporting the ratio of losses incurred relative to the benchmark HEAVY model. The table is divided into two panels, and panel A reports the average ratios calculated across the 31 assets. The results show that the EHEAVY model outperforms the HEAVY model, incurring significantly lower average losses for all three forecasting horizons, particularly when using the QLIK loss. Moreover, the realized EGARCH model and the EHEAVY model have comparable average losses across the three forecasting horizons, which are lower than the average losses incurred by the two HEAVY models. Notably, the AHEAVY model shows slightly better forecasting performance compared to the HEAVY model. Overall, the results suggest that the EHEAVY model provides the most accurate and reliable out-of-sample forecasts for the return volatility of the 31 assets considered in this study.

Panel B in Table ?? summarizes the MCS of the out-of-sample forecasts by reporting the numbers of the asset for which each model is part of the 95% MCS. When the MSE loss function is used, it is observed that the four models are included in the 95% MCS for almost all assets, indicating that there is no clear winner in terms of forecast accuracy. However, for the EHEAVY and realized EGARCH models, a slightly higher number of assets are included in the MCS when the forecasting horizon is 22. When the QLIK loss function is used, the EHEAVY and realized EGARCH models are still included in the 95% MCS for almost all assets across the three forecast horizons, suggesting their superior forecast accuracy compared to the HEAVY and AHEAVY model. Only a few assets have the HEAVY model included in the 95% MCS for QML loss, while the AHEAVY model has a slightly higher number of assets included than the HEAVY model. These results suggest that the EHEAVY and realized EGARCH models are consistently among the best-performing models for most assets and loss functions considered in this study.

To summarize, the out-of-sample forecasting comparisons suggest that the EHEAVY model performs similarly well to the realized EGARCH model in forecasting the variance of returns, with both models exhibiting superior performance compared to the HEAVY and AHEAVY models.

5.5 Portfolio exercise

In addition to statistical gains in volatility predictability, market investors prioritize economic significance in assessing the quality of volatility forecasts. Specifically, they are interested in how well these forecasts perform in asset allocation. To evaluate the economic value of volatility forecasts, we adopt a mean-variance utility framework, where the investor allocates assets between stock and a risk-free asset. This approach is consistent with the portfolio allocation literature (e.g., Campbell and Thompson, 2008; Neely et al., 2014; Rapach et al., 2010; Wang et al., 2016).

The mean-variance utility function can be expressed as:

$$U_t\left(r_{t+s}\right) = E_t\left[w_t(r_{t+s} - r_{t+s,f}) + r_{t+s,f}\right] - \frac{1}{2}\gamma \operatorname{Var}_t\left[w_t(r_{t+s} - r_{t+s,f}) + r_{t+s,f}\right] \tag{14}$$

where w_t is the weight of stock in this portfolio, r_{t+s} is the stock return, $r_{t+s,f}$ is the risk-free rate and γ is the risk aversion coefficient. Dropping constant terms and expressing the excess return as $r^e_{t+s} = r_{t+s} - r_{t+s,f}$, the expected utility is

$$U_t\left(w_t\right) = w_t E_t(r^e_{t+s}) - \frac{1}{2}\gamma w_t^2 E_t(h_{t+s}). \tag{15}$$

Maximizing $U_t(r_{t+s})$ respect to w_t yield the ex-ante optimal weight of stock index at day $t+s$

$$w_t^* = \frac{1}{\gamma}\frac{E_t(r^e_{t+s})}{E_t(h_{t+s})}. \tag{16}$$

The volatility forecasts utilized in our analysis are obtained from the four models introduced in the previous section. In terms of return forecasting, we employ a simple approach based on the historical average excess return over the estimation window, as suggested by Welch and Goyal (2008). In order to ensure realistic portfolio weights, we follow previous literature (e.g., Rapach et al., 2010; Neely et al., 2014) and constrain the optimal weight w_t^* to the range of 0 to 1.5, which effectively precludes short sales and limits the leverage to no more than 50%.

The portfolio's performance is assessed based on three metrics: portfolio excess return, Sharpe Ratio, and certainty equivalent return (CER). The CER is calculated based on the mean-variance utility function specified in (**??**), given the optimal weight w_t^*. The three metrics are estimated empirically by averaging the respective expressions across the same out-of-sample forecasts discussed in Section 5.4.

One issue with the three metrics is the potential mis-forecasting of returns. In other words, even if volatility forecasting is accurate, if the returns are not correctly predicted, the portfolio's performance may be poor. Therefore, to avoid the misspecification of return forecasting, an expected utility-based approach following Bollerslev et al. (2018) has also been employed, which specifically focuses on volatility forecasting. This approach assumes a constant conditional Sharpe Ratio, defined as $SR \equiv E_t\left(r^e_{t+1}\right)/\sqrt{E_t\left(h_{t+s}\right)}$. Under this assumption, the expected utility is

$$U_t\left(w_t\right) = w_t SR\sqrt{E_t\left(h_{t+s}\right)} - \frac{\gamma}{2}w_t^2 E_t\left(h_{t+s}\right), \tag{17}$$

which simply depends on the position w_t, together with the expected volatility $E_t\left(h_{t+s}\right)$.

The optimal portfolio that maximizes utility is obtained by investing the fraction of wealth $w_t^* = \frac{SR/\gamma}{\sqrt{E_t(h_{t+s})}}$ in the risky asset. This "volatility timing" behavior mimics the trading behavior of many hedge funds with explicit volatility targets and "risk parity investors". We use $SR = 0.4$ and $\gamma = 2$ as the annualized Sharpe Ratio and coefficient of risk aversion, respectively, as suggested by Bollerslev et al. (2018) [7].

To explicitly quantify the utility gains from different volatility models, let $E_t^a(\cdot)$ denote the expectations from model a. Also, let $E_t(\cdot)$ denote the expectations from the true (unknown) risk model. Assuming that the investor uses model a, to choose the position $w_t^a = 20\%/\sqrt{E_t^a\left(h_{t+s}\right)}$, the expected utility , $U_t^a \equiv U_t\left(w_t^a\right)$, may be expressed as

$$U_t^a = 8\%\frac{\sqrt{E_t\left(h_{t+s}\right)}}{\sqrt{E_t^a\left(h_{t+s}\right)}} - 4\%\frac{E_t\left(h_{t+s}\right)}{E_t^a\left(h_{t+s}\right)}.$$

Importantly, the expected returns do not enter this expression. We then evaluate this expected utility empirically by averaging the corresponding realized expressions over the same out-of-sample forecasts as discussed in Section 5.4.

$$U_t^a = \sum_{t=T-1000+s}^{T}\left(8\%\frac{\sqrt{r^2_{t+s}}}{\sqrt{h^a_{t+s|t}}} - 4\%\frac{r^2_{t+s}}{h^a_{t+s|t}}\right)$$

[7] However, we also explore the sensitivity of our results to different values of SR and γ. Nevertheless, we find that the overall conclusions are robust to these parameter choices.

where the true volatility is proxied by r_{t+s}^2, as in (12) and (13) [8].

To formally determine whether the portfolio excess return, CER, and expected utility significantly differ across the different risk models, we apply the MCS test. This approach identifies the subset of models that contain the models that imply the best criteria with 95% confidence. It is worth noting that the MCS test cannot be applied to Sharp ratio, as Sharp ratio is a scalar for each asset.

Insert Table **??** *here*

Table **??** presents the results of the portfolio analysis for the four models. The analysis is conducted for three investment horizons: 1-day, 5-day, and 22-day. The ratios of the economic values incurred by different models relative to those of the benchmark HEAVY model are reported.

Panel A presents the average ratios calculated across the 31 assets. The results show that the EHEAVY model outperforms the other three models in terms of return, Sharpe Ratio, CER, and expected utility across all three investment horizons. Specifically, for the 1-day horizon, the EHEAVY strategy generates a portfolio with a return of approximately 8%, a Sharpe Ratio of about 58%, a CER about 105%, and an expected utility about 77% higher than the HEAVY strategy. The realized EGARCH and AHEAVY models also show improved performance compared to the HEAVY model, but the magnitudes of improvement are not as large as that of the EHEAVY model. Similar results can be found for the 5-day and 22-day ahead forecasts.

Panel B summarizes the MCS of the out-of-sample forecasts by reporting the numbers of the asset in which each model is part of the 95% MCS. The analysis reveals that for portfolio return, there are no significant differences among the four models, as they have similar numbers of assets included in the 95% MCS at the three forecasting horizons. However, when the CER and expected utility are considered, the EHEAVY model exhibits superior forecasting performance, with a significantly larger number of assets included in the 95% MCS compared to the other three models. The realized EGARCH model also performs well, ranking second in terms of number of assets included in the 95% MCS. On the other hand, the HEAVY and AHEAVY models have a few number of assets included in the 95% MCS.

[8] We also tried to use rv_{t+s} as true volatility, the results does not change significantly

In summary, the portfolio analysis reveals that the EHEAVY model outperforms the other models in terms of economic value, as measured by the CER and expected utility. This suggests that the EHEAVY model can significantly improve the economic value of volatility forecasts. This is particularly useful for investors, traders, and risk managers who need to make informed decisions based on their expectations of future market volatility.

6 Conclusions

This paper introduces the EHEAVY model as an extension of Shephard and Sheppard's (2010) HEAVY model with several improvements. Firstly, the EHEAVY model maintains variance positivity without restricting the parameter set, making it more flexible. Secondly, it includes asymmetric effects which are crucial in volatility modelling. Thirdly, it provides a joint quasi-maximum likelihood estimation and closed-form multi-step ahead forecast procedure. Empirical findings indicate that the return volatility dynamic is primarily influenced by the realized measure, and the asymmetric effect is due to return shocks rather than the realized measure. The out-of-sample forecasting comparisons demonstrate that the EHEAVY model outperforms other models in forecasting volatility statistically, which is further confirmed by a portfolio analysis. Overall, the results suggest that the EHEAVY model exhibits excellent forecasting performance compared to HEAVY and other competing models.

The EHEAVY model has a simple structure, a straightforward estimation and inference procedure. It can significantly improve the economic value of volatility forecasts. Thus, the market practitioners can readily use the model to forecast volatility. With more accurate forecasts, these market participants can better assess and manage their risks, and make more informed investment decisions. For future research, the EHEAVY model can be extended by adding other realized measures, additional exogenous variables, jump components in return or realized measure equations.

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Table 1 – Simulation results for QMLE of the EHEAVY parameters

	$T = 2000$				$T = 5000$			
	True	RB	RMSE	N.test	True	RB	RMSE	N.test
w_r	-0.30	-1.347	15.265	0.000	-0.30	-0.491	1.490	0.990
w_R	-0.30	-0.878	10.842	0.000	-0.30	-0.166	1.704	0.882
α_{rR}	0.30	0.236	6.055	0.000	0.30	0.127	1.648	0.535
α_{RR}	0.40	0.513	6.681	0.000	0.40	-0.271	2.282	0.968
β_r	0.96	-0.392	6.252	0.000	0.96	-0.041	0.408	0.013
β_R	0.95	-0.414	6.259	0.000	0.95	-0.054	0.572	0.014
γ_{rr}	-0.10	1.317	2.706	0.098	-0.10	0.650	1.111	0.032
γ_{Rr}	-0.10	0.631	3.451	0.278	-0.10	-0.078	1.770	0.798

RB: relative bias (in percentages); RMSE: root mean squared error; see the definitions in (**??**). N. test: p-value of the Jarque–Bera normality test of the sampling distribution. The reported values are obtained from 1000 simulated samples of the data generating process defined in Section 4.3.

Table 2 – Data descriptive statistics

	$r^2_{t,oc}$		$r^2_{t,cc}$		rk_t	
Symbol	Mean	StDev	Mean	StDev	Mean	StDev
Symbol	Mean	StDev	Mean	StDev	Mean	StDev
AEX	19.14	58.66	30.20	90.48	18.94	41.00
AORD	11.37	33.18	14.18	40.78	11.43	33.62
BFX	16.91	53.84	24.97	82.66	15.56	33.11
BSESN	26.52	73.62	34.11	113.67	21.74	55.93
BVLG	10.33	27.44	20.37	63.63	9.61	18.94
BVSP	32.46	84.07	52.22	154.24	30.25	59.08
DJI	19.23	62.21	22.57	87.40	17.04	40.12
FCHI	20.99	55.16	31.85	89.56	21.01	40.85
FTMIB	21.79	66.42	39.34	140.98	18.84	28.39
FTSE	21.33	64.90	21.73	65.81	20.27	51.21
GDAXI	24.64	66.65	33.99	98.28	24.44	48.37
GSPTSE	15.27	59.91	17.63	76.25	12.04	42.55
HSI	17.00	60.45	33.57	104.67	15.53	32.19
IBEX	23.02	65.02	33.73	101.83	23.17	41.90
IXIC	27.46	84.76	40.05	118.11	23.86	57.06
KS11	20.55	62.85	34.64	106.81	17.60	33.17
KSE	24.59	58.84	29.44	73.34	19.17	33.58
MXX	23.59	64.79	25.83	70.04	17.69	32.61
N225	19.72	71.59	35.04	100.69	17.60	49.25
NSEI	22.07	70.98	33.46	115.24	18.65	52.81
OMXC20	21.14	63.40	26.12	74.09	19.12	48.68
OMXHPI	21.90	61.17	28.30	77.33	20.08	44.57
OMXSPI	21.60	67.17	27.65	85.35	19.03	53.95
OSEAX	27.16	77.85	30.13	89.10	21.63	60.62
RUT	23.10	68.98	38.58	117.01	18.65	45.35
SMSI	22.77	69.16	32.52	107.83	21.71	47.22
SPX	19.90	64.16	24.46	86.94	16.68	40.08
SSEC	30.76	74.33	37.52	99.60	26.41	43.12
SSMI	14.76	52.42	21.43	68.79	13.74	35.12
STI	10.50	24.38	18.40	78.33	9.05	15.51
STOXX50E	26.55	75.92	32.56	90.84	25.59	57.26
Average	21.23	62.72	29.89	92.57	18.91	42.49

This table provides descriptive statistics for the dataset of 31 assets. Columns 2-3 report the corresponding time-series averages (Mean) and standard deviations (StDev) of the squared open-to-close returns ($r^2_{t,oc}$). Columns 4-5 report the corresponding Mean and StDev of the squared close-to-close returns ($r^2_{t,cc}$). Columns 6-7 report the corresponding Mean and StDev of the realized kernel (rk_t).

Table 3 – EHEAVY models estimated parameters

	Open-to-Close Return				Close-to-Close Return			
	Penal A: EGARCH model estimation results							
	Min	Med	Max	No Sig.	Min	Med	Max	No Sig.
w_r	-0.111	-0.059	-0.022	**28**	-0.087	-0.043	0.005	**24**
α_{rr}	0.118	0.160	0.227	**31**	0.103	0.158	0.214	**31**
β_r	0.960	0.978	0.988	**31**	0.956	0.976	0.985	**31**
γ_{rr}	-0.143	-0.09	-0.023	**31**	-0.152	-0.106	-0.025	**31**
LL	-24723	-21025	-8199	**1**	-22191	-18853	-6740	**0**
BIC	16430	41983	49480	**1**	13510	37739	44416	**0**
	Penal B: EGARCH-X model estimation results							
	Min	Med	Max	No Sig.	Min	Med	Max	No Sig.
w_r	-0.414	-0.273	-0.115	**30**	-0.468	-0.316	-0.038	**30**
α_{rr}	-0.032	0.021	0.142	**13**	-0.013	0.044	0.205	**23**
α_{rR}	0.221	0.366	0.602	**31**	0.051	0.195	0.619	**30**
β_r	0.919	0.965	0.980	**31**	0.939	0.949	0.998	**31**
γ_{rr}	-0.158	-0.106	0.001	**28**	-0.169	-0.138	-0.013	**29**
γ_{rR}	-0.072	0.009	0.059	**4**	-0.068	-0.028	0.087	**2**
LL	-24663	-20894	-8130	**19**	-22078	-18673	-6631	**20**
BIC	16307	41840	49378	**8**	13308	37397	44207	**6**
	Penal C: EHEAVY model estimation results							
w_r	-0.432	-0.282	-0.158	**31**	-0.478	-0.268	-0.104	**31**
α_{rR}	0.275	0.406	0.588	**31**	0.277	0.394	0.675	**31**
β_r	0.921	0.966	0.983	**31**	0.935	0.967	0.990	**30**
γ_{rr}	-0.150	-0.093	-0.023	**30**	-0.154	-0.108	-0.052	**31**
w_R	-0.504	-0.327	-0.204	**31**	-0.523	-0.305	-0.186	**31**
α_{RR}	0.301	0.448	0.696	**31**	0.298	0.430	0.685	**31**
β_R	0.887	0.962	0.981	**31**	0.919	0.962	0.989	**31**
γ_{Rr}	-0.149	-0.093	-0.026	**30**	-0.146	-0.105	-0.033	**31**
ρ	0.818	0.845	0.890	**31**	0.773	0.815	0.886	**31**
LL	-24674	-20901	-8136	**11**	-22071	-18679	-6636	**10**
BIC	16303	41832	49372	**21**	13302	37393	44176	**24**

Minimum (Min), median (Med), and maximum (Max) are the summary statistics of the estimates for the 31 assets. All estimates are provided the supplementary appendix. No Sig. denotes the numbers of significance for the corresponding parameters. The total number of assets is 31. LL denotes partial log-likelihood value defined by Hansen et al. (2012), $LL_h = -\frac{1}{2}\sum_{t=1}^{T}\left(\log(2\pi) + \log(h_t) + (r_t^2/h_t)\right)$. BIC denotes the Bayesian information criteria.

Table 4 – Out of sample forecasts of volatility of close-to-close return

Panel A: Ratio of the losses						
	s=1		s=5		s=22	
(Average)	MSE	QLIK	MSE	QLIK	MSE	QLIK
HEAVY	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
AHEAVY	0.9651	0.8811	0.9916	0.9813	1.0003	0.9857
REGARCH	0.9440	0.8249	0.9635	0.9288	0.9620	0.9182
EHEAVY	0.9486	0.8266	0.9660	0.9359	0.9630	0.9118

Panel B: MCS test						
	s=1		s=5		s=22	
MSC	MSE	QLIK	MSE	QLIK	MSE	QLIK
HEAVY	31	2	30	10	28	5
AHEAVY	30	8	31	13	26	11
REGARCH	31	29	31	30	31	25
EHEAVY	31	28	31	30	31	27

This table reports the ratio of the losses for the different models relative to the losses of the benchmark HEAVY model. The average ratios across the 31 assets are reported. REGARCH denotes the realized EGARCH model. Panel B reports the numbers whereby each model is part of the 95% model confidence set. The total number of assets is 31.

Table 5 – Performances of portfolios formed by volatility of close-to-close return forecasts

Panel A: Ratio of portfolio performances

	s=1				s=5				s=22			
(Average)	R	SR	CER	EU	R	SR	CER	EU	R	SR	CER	EU
HEAVY	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
AHEAVY	0.997	0.981	1.054	1.044	1.021	1.024	1.181	1.049	1.094	1.012	1.152	1.020
REGARCH	1.011	1.202	1.554	1.673	1.094	1.120	1.276	1.221	1.155	1.029	1.220	1.167
EHEAVY	1.081	1.579	2.052	1.773	1.189	1.441	1.652	1.315	1.291	1.045	1.458	1.179

Panel B: MCS test

	s=1				s=5				s=22			
	R	SR	CER	EU	R	SR	CER	EU	R	SR	CER	EU
HEAVY	24		0	5	23		1	8	25		2	20
AHEAVY	23		0	5	29		1	9	27		5	22
REGARCH	25		20	28	30		16	29	30		13	30
EHEAVY	30		26	29	31		22	31	31		22	31

This table shows the performances of portfolios formed by the different volatility forecasts. It gives the mean excess return (R), Sharpe Ratio and certainty equivalent return (CER) and expected utility (EU) of each portfolio. The ratios of the performance for the different models relative to the that of HEAVY model are calculated. Panel B reports the numbers whereby each model is part of the 95% model confidence set. The total number of assets is 31.

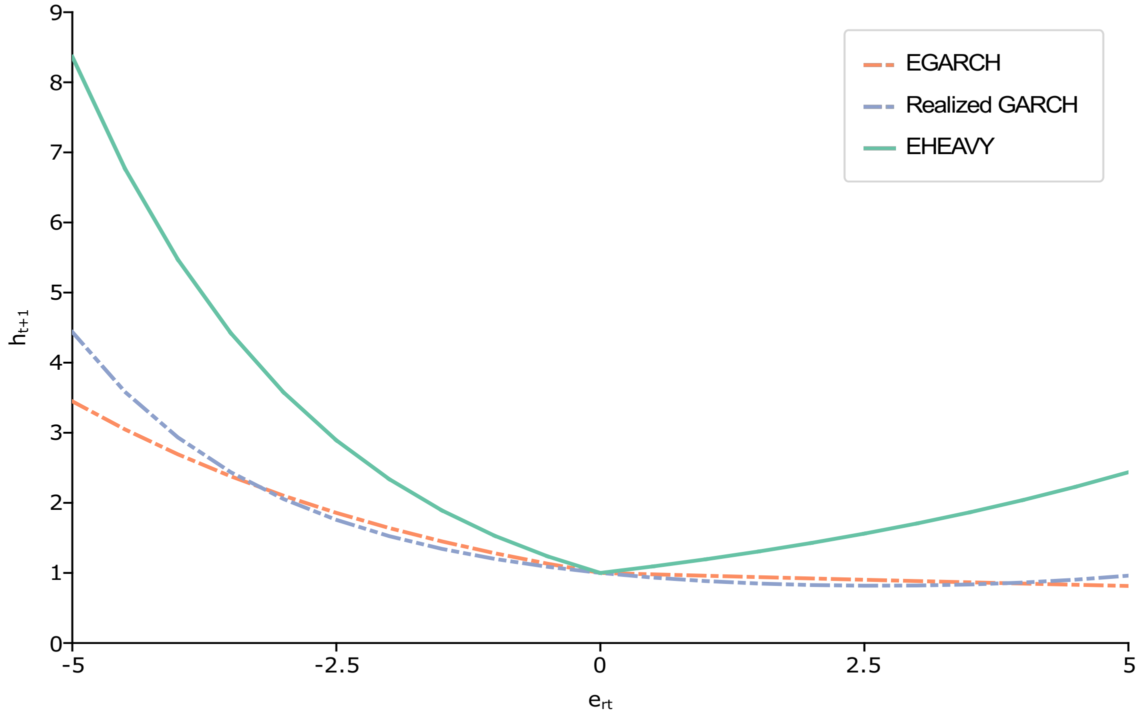

Figure 1 – News impact curve for the EGARCH, realized EGARCH and EHEAVY model: EURO50 close-to-close return.

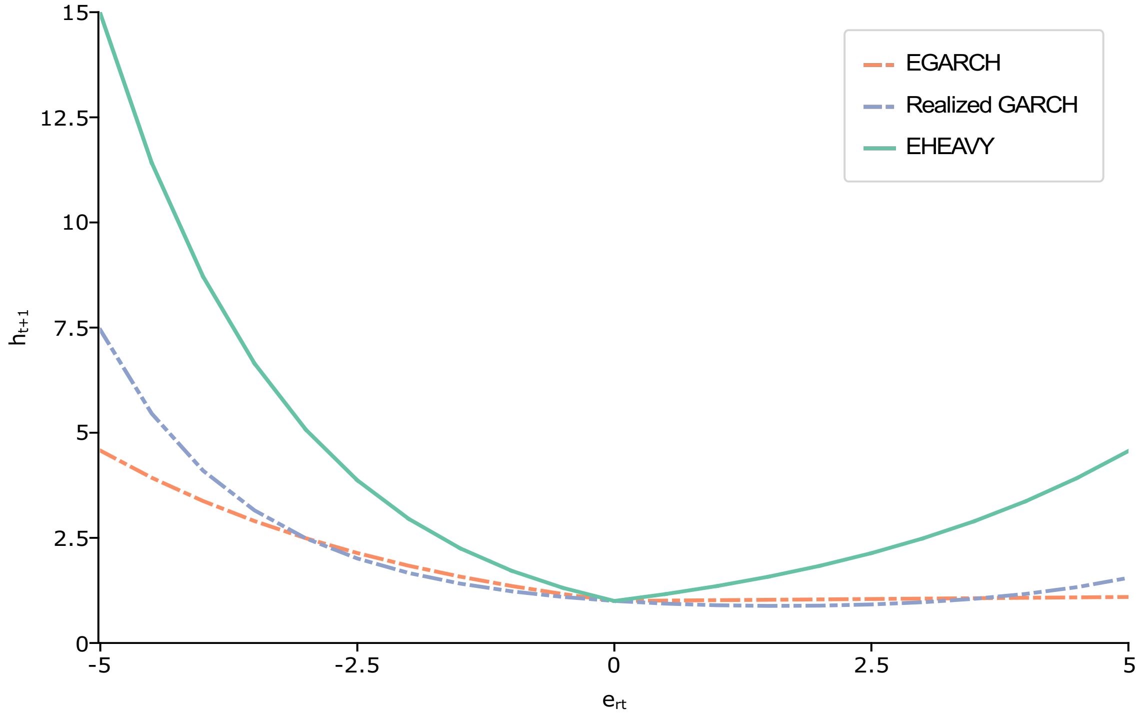

Figure 2 – News impact curve for the EGARCH, EHEAVY and realized EGARCH model: SPX close-to-close return

Supplementary Appendix

A Additional Tables

Table A1: Symbol Names

Symbol	Name	Start Date	Obs.(T)
.AEX	AEX	January 03, 2000	5473
.AORD	All Ordinaries	January 04, 2000	5422
.BFX	Bell 20	January 03, 2000	5471
.BSESN	S&P BSE Sensex	January 03, 2000	5322
.BVLG	PSI All-Share	October 15, 2012	2207
.BVSP	BVSP BOVESPA	January 03, 2000	5282
.DJI	Dow Jones Industrials	January 03, 2000	5380
.FCHI	CAC 40	January 03, 2000	5475
.FTMIB	FTSE MIB	June 01, 2009	3056
.FTSE	FTSE 100	January 04, 2000	5414
.GDAXI	DAX	January 03, 2000	5440
.GSPTSE	S&P/TSX Composite	May 02, 2002	4786
.HSI	HANG SENG	January 03, 2000	5258
.IBEX	IBEX 35	January 03, 2000	5440
.IXIC	Nasdaq 100	January 03, 2000	5384
.KS11	Korea Composite	January 04, 2000	5283
.KSE	Karachi SE 100	January 03, 2000	5229
.MXX	IPC Mexico	January 03, 2000	5384
.N225	Nikkei 225	February 02, 2000	5219
.NSEI	NIFTY 50	January 03, 2000	5313
.OMXC20	OMX Copenhagen 20	October 03, 2005	3902
.OMXHPI	OMX Helsinki All Share	October 03, 2005	3943
.OMXSPI	OMX Stockholm All Share	October 03, 2005	3943
.OSEAX	Oslo Exchange All-share	September 03, 2001	4931
.RUT	Russel 2000	January 03, 2000	5381
.SMSI	Madrid General	July 04, 2005	4069
.SPX	S&P 500	January 03, 2000	5383
.SSEC	Shanghai Composite	January 04, 2000	5184
.SSMI	Swiss Stock Market	January 04, 2000	5377
.STI	Straits Times	January 03, 2000	3439
.STOXX50E	EURO STOXX 50	January 03, 2000	5472

This table provides descriptive statistics for the dataset of 31 assets. The sample ending date is June 24, 2021. Column 2 show the asset names. Column 3 show the sample starting date. Column 4 is in the number of observations (or the sample size T) of the asset.

Table A2: EHEAVY models Estimated parameters: Open-to-Close Return

	EHEAVY-r					EHEAVY-RM					
Symbol	w	α_{RR}	β_R	γ_{Rr}	LL	w	α_{RR}	β_R	γ_{Rr}	ρ	LL
AEX	-0.287	0.378	0.971	-0.120	-18627	-0.310	0.412	0.967	-0.120	0.833	-18636
AORD	-0.187	0.275	0.970	-0.104	-16490	-0.208	0.301	0.968	-0.110	0.890	-16233
BFX	-0.323	0.432	0.964	-0.107	-18377	-0.337	0.445	0.964	-0.101	0.837	-17974
BSESN	-0.422	0.588	0.952	-0.064	-20360	-0.504	0.696	0.941	-0.059	0.841	-19099
BVLG	-0.407	0.497	0.970	-0.064	-6636	-0.424	0.519	0.967	-0.064	0.845	-6511
BVSP	-0.236	0.384	0.962	-0.071	-22071	-0.294	0.473	0.953	-0.080	0.864	-21470
DJI	-0.328	0.463	0.957	-0.146	-18132	-0.329	0.462	0.956	-0.137	0.833	-17603
FCHI	-0.245	0.352	0.967	-0.115	-19767	-0.248	0.360	0.966	-0.112	0.838	-19774
FTMIB	-0.358	0.516	0.951	-0.115	-11535	-0.354	0.501	0.952	-0.111	0.818	-11081
FTSE	-0.300	0.405	0.969	-0.093	-19310	-0.341	0.456	0.966	-0.081	0.836	-19093
GDAXI	-0.234	0.326	0.972	-0.109	-20444	-0.252	0.346	0.971	-0.114	0.828	-20174
GSPTSE	-0.260	0.337	0.975	-0.096	-14759	-0.264	0.344	0.972	-0.108	0.859	-13711
HSI	-0.244	0.338	0.971	-0.023	-18324	-0.299	0.420	0.961	-0.025	0.846	-17965
IBEX	-0.312	0.411	0.971	-0.088	-20435	-0.339	0.448	0.969	-0.083	0.831	-20496
IXIC	-0.327	0.447	0.965	-0.109	-20077	-0.362	0.488	0.962	-0.110	0.836	-19240
KS11	-0.326	0.393	0.983	-0.047	-18679	-0.341	0.412	0.981	-0.049	0.846	-18044
KSE	-0.158	0.424	0.921	-0.066	-20175	-0.212	0.560	0.887	-0.063	0.866	-18901
MXX	-0.252	0.410	0.955	-0.078	-20552	-0.327	0.512	0.942	-0.077	0.837	-19042
N225	-0.269	0.401	0.963	-0.057	-18818	-0.293	0.436	0.958	-0.070	0.863	-18157
NSEI	-0.432	0.551	0.966	-0.073	-19105	-0.464	0.598	0.959	-0.070	0.834	-18192
OMXC20	-0.262	0.437	0.948	-0.076	-14503	-0.355	0.565	0.935	-0.079	0.846	-13957
OMXHPI	-0.231	0.335	0.970	-0.100	-14382	-0.282	0.408	0.962	-0.112	0.863	-13890
OMXSPI	-0.273	0.376	0.972	-0.135	-13966	-0.293	0.402	0.968	-0.149	0.864	-13343
OSEAX	-0.237	0.364	0.966	-0.093	-19049	-0.274	0.407	0.962	-0.100	0.857	-17775
RUT	-0.192	0.345	0.954	-0.116	-20173	-0.204	0.344	0.955	-0.120	0.855	-18928
SMSI	-0.321	0.431	0.971	-0.092	-15299	-0.349	0.465	0.968	-0.093	0.855	-15042
SPX	-0.282	0.406	0.960	-0.150	-18218	-0.283	0.404	0.959	-0.146	0.840	-17358
SSEC	-0.387	0.511	0.972	-0.040	-21147	-0.466	0.612	0.964	-0.043	0.836	-20196
SSMI	-0.422	0.559	0.950	-0.087	-17316	-0.441	0.565	0.954	-0.072	0.825	-16734
STI	-0.406	0.513	0.962	-0.047	-10360	-0.438	0.550	0.958	-0.043	0.846	-9884
EURO50	-0.207	0.320	0.967	-0.138	-20799	-0.223	0.344	0.964	-0.133	0.847	-20619

This table provides QML estimates of EHEAVY model for the dataset of 31 assets. All parameters are statistically significant at 5% level.

Table A3: EHEAVY models Estimated parameters: Close-to-Close Return

	EHEAVY-r					EHEAVY-RM					
Symbol	w	α_{RR}	β_R	γ_{Rr}	LL	w	α_{RR}	β_R	γ_{Rr}	ρ	LL
AEX	-0.235	0.341	0.970	-0.154	-20928	-0.259	0.356	0.969	-0.137	0.807	-18617
AORD	-0.181	0.278	0.970	-0.107	-17547	-0.200	0.298	0.966	-0.113	0.886	-16222
BFX	-0.338	0.450	0.970	-0.121	-20266	-0.337	0.430	0.972	-0.105	0.815	-17962
BSESN	-0.362	0.536	0.955	-0.075	-21524	-0.451	0.643	0.943	-0.064	0.804	-19085
BVLG	-0.366	0.526	0.954	-0.106	-8136	-0.388	0.517	0.952	-0.099	0.809	-6501
BVSP	-0.130	0.296	0.961	-0.091	-24674	-0.268	0.439	0.955	-0.080	0.815	-21449
DJI	-0.343	0.478	0.959	-0.132	-18813	-0.312	0.428	0.963	-0.133	0.824	-17586
FCHI	-0.268	0.394	0.967	-0.141	-21842	-0.290	0.401	0.968	-0.113	0.811	-19761
FTMIB	-0.389	0.626	0.938	-0.140	-13167	-0.416	0.599	0.942	-0.109	0.779	-11070
FTSE	-0.278	0.371	0.973	-0.103	-19409	-0.332	0.436	0.969	-0.091	0.835	-19079
GDAXI	-0.240	0.335	0.975	-0.129	-22124	-0.272	0.354	0.977	-0.118	0.810	-20152
GSPTSE	-0.214	0.307	0.970	-0.124	-15426	-0.223	0.305	0.970	-0.113	0.850	-13693
HSI	-0.189	0.295	0.972	-0.074	-21655	-0.222	0.319	0.969	-0.057	0.785	-17947
IBEX	-0.317	0.458	0.963	-0.108	-22331	-0.368	0.500	0.962	-0.086	0.806	-20478
IXIC	-0.299	0.427	0.967	-0.128	-21883	-0.305	0.418	0.967	-0.123	0.797	-19215
KS11	-0.308	0.412	0.975	-0.082	-21169	-0.326	0.420	0.973	-0.064	0.787	-18026
KSE	-0.104	0.277	0.954	-0.056	-21268	-0.186	0.445	0.919	-0.068	0.841	-18884
MXX	-0.231	0.383	0.958	-0.081	-20975	-0.323	0.506	0.943	-0.070	0.831	-19037
N225	-0.124	0.362	0.937	-0.104	-22153	-0.228	0.420	0.939	-0.098	0.824	-18140
NSEI	-0.271	0.327	0.990	-0.078	-22000	-0.422	0.483	0.989	-0.063	0.773	-18200
OMXC20	-0.230	0.459	0.935	-0.093	-15350	-0.289	0.495	0.936	-0.092	0.834	-13948
OMXHPI	-0.177	0.302	0.967	-0.115	-15525	-0.268	0.404	0.961	-0.109	0.838	-13882
OMXSPI	-0.260	0.395	0.965	-0.151	-15115	-0.338	0.472	0.962	-0.146	0.835	-13334
OSEAX	-0.244	0.360	0.971	-0.087	-19406	-0.276	0.397	0.966	-0.098	0.853	-17771
RUT	-0.172	0.295	0.968	-0.119	-22618	-0.190	0.308	0.962	-0.128	0.810	-18899
SMSI	-0.335	0.454	0.972	-0.118	-16590	-0.401	0.519	0.970	-0.107	0.815	-15026
SPX	-0.284	0.422	0.959	-0.141	-19172	-0.257	0.373	0.961	-0.145	0.830	-17342
SSEC	-0.410	0.571	0.962	-0.052	-22109	-0.478	0.651	0.955	-0.054	0.813	-20200
SSMI	-0.375	0.539	0.947	-0.140	-19147	-0.408	0.536	0.952	-0.111	0.812	-16706
STI	-0.478	0.675	0.940	-0.071	-12173	-0.523	0.685	0.938	-0.033	0.792	-9890
EURO50	-0.192	0.301	0.970	-0.147	-21887	-0.224	0.337	0.967	-0.136	0.841	-20602

This table provides QML estimates of EHEAVY model for the dataset of 31 assets. All parameters are statistically significant at 5% level.

B Other Models

In this appendix, we present the specification of asymmetric HEAVY of Shepard and Shepperd (2010) and realized EGARCH Hansen and Huang (2016) model.

The AHEAVY model of Shepard and Shepperd (2010) is

$$\begin{aligned} h_t &= \omega_r + (\alpha_{rR} + \gamma_{rR} s_{t-1}) RM_{t-1} + \beta_r h_{t-1}, \\ m_t &= \omega_R + (\alpha_{RR} + \gamma_{RR} s_{t-1}) RM_{t-1} + \beta_R m_{t-1}. \end{aligned} \tag{18}$$

where $s_t = 0.5[1 - sign(r_t)]$, that is, $s_t = 1$ if $r_t < 0$ and 0 otherwise; γ_{ii}, γ_{ij} $(i \neq j)$ are the own and cross leverage parameters, respectively[9]; positive γ_{ii}, γ_{ij} means a larger contribution of negative 'shocks' in the volatility process.

[9]This type of asymmetry was introduced by Glosten et. al., (1993).

The realized EGARCH model of Hansen and Huang (2016) is

$$\log h_t = \omega_r + \beta_r \log h_{t-1} + \alpha_{rr} e_{rt-1}^2 + \gamma_{rr} e_{rt-1} + \alpha_{rR} u_{Rt-1}, \tag{19}$$
$$\log RM_t = \omega_R + \beta_R \log h_t + \alpha_{Rr} e_{rt-1}^2 + \gamma_{Rr} e_{Rt-1} + u_{Rt}.$$

where $u_{Rt} \sim N(0, \sigma_u)$. The first equation is referred to GARCH equation, and the second one is referred to measurement equation. It can be seen that the GARCH equation is close to our EHEAVY-r equation, where the measurement equation has a different specification as EHEAVY-RM equation.