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Targeting moments for calibration compared with indirect inference

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Abstract

A common practice in estimating parameters in DSGE models is to find a set that when simulated gets close to an average of certain data moments; the model's simulated performance for other moments is then compared to the data for these as an informal test of the model. We call this procedure informal Indirect Inference, III. By contrast what we call Formal Indirect Inference, FII, chooses a set of moments as the auxiliary model and computes the Wald statistic for the joint distribution of these moments according to the structural DSGE model; it tests the model according to the probability of obtaining the data moments. The FII estimator then chooses structural parameters that maximise this probability. We show in this note via Monte Carlo experiments that the FII estimator has low bias in small samples, whereas the III estimator has much higher bias. It follows that models estimated by III will typically also be rejected by formal indirect inference tests.

JEL Codes: C12, C32, C52

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1 Introduction

A popular way to calibrate dynamic stochastic general equilibrium (DSGE) models is to calibrate them with parameter values chosen to 'target' (i.e. exactly replicate) a set of moments. The model is then asked how well it can match some other moments when simulated; this match is informally carried out, in the hope that the simulated and data moments are 'similar'. An early example of this method is Chari et al. (2002); two recent examples are Baslandze (2022), and Khan and Thomas (2013)[1]. This methodology — which we call 'informal indirect inference', III — is presented as a way of finding a model version sufficiently 'close to' the data that it can be treated as the true model. In this short paper we evaluate this methodology via Monte Carlo (MC) experiments. What we find is that it leads to highly biased 'estimates' of the model parameters. By contrast we know from previous MC experiments that formal indirect inference (FII) using moments as the auxiliary model produce estimates with very low bias.

Under FII a set of around 9 moments are chosen from among those available, this number being sufficient to generate high, but not excessively high, power against parameter inaccuracy. The joint distribution of the model-simulated moments is then calculated. Which particular moments are chosen makes little difference; the key to 'goldilocks' power lies in the number used. This is because all the moments are nonlinear functions of the structural parameters; hence any set of the data-based moments as a group will in all cases only have a high probability of occurring in the model-simulated joint distribution if the model is the true one. The estimated parameters are those that maximise this probability.

By contrast under III, the joint distribution of the model-simulated moments is not calculated, and so neither is the probability of the data-based moments, including the untargeted ones. Hence in general the parameter set chosen does not maximise this probability. It might be thought that in practice it would come close because of the exact targeting carried out on most of the moments. However, we find in our MC experiments that this is not so — presumably due to the untargeted moments whose matching behaviour is not formally included in the process. Yet the joint distribution involves the joint behaviour of all the moments.

2 Indirect Inference on a DSGE model

DSGE models (possibly after linearization) have the general form:

$$\begin{aligned} A_0 E_t y_{t+1} &= A_1 y_t + B z_t \\ z_t &= R z_{t-1} + \varepsilon_t \end{aligned} \tag{1}$$

where y_t contains the endogenous variables and z_t the exogenous variables. The exogenous variables may be observable or unobservable. For example, they may be structural disturbances. We assume that z_t may be represented by an autoregressive process with disturbances ε_t that are $NID(0, \Sigma)$. Assuming that the conditions of Fernandez-Villaverde et al. (2007) are satisfied, the solution to this model can be represented by a VAR of form

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = F \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + G \begin{bmatrix} \xi_t \\ \varepsilon_t \end{bmatrix}. \tag{2}$$

where ξ_t are innovations.

[1] Chari et al. (2002) choose a set of parameters for a sticky-price benchmark model that approximately fits about a dozen moments; they then test the model against a real exchange rate-cross-country-consumption correlation, showing that it badly fails to replicate the absence of this correlation in the data.

Baslandze (2022) sets out a model of innovation by firms, both regular and spinouts. She derives the steady state growth equilibrium for the model outcomes. The moments of these when simulated with heterogeneous firm shocks are compared with the data moments; a subset of the model parameters is chosen to minimise the distance between a weighted average of several moments, mostly with unit weights but one with a double weight. Various relationships in the data are then compared with those implied by the model and found to be similar — e.g. the share of spinouts in states with different non-compete laws.

Khan and Thomas (2013) set out a DSGE model of an economy with credit constraints. They calibrate the parameters to match a series of individual data. Later they compare its behaviour in various aspects with the data behaviour, suggesting it is broadly similar.

A special case of the DSGE model is where all of the exogenous variables are unobservable and may be regarded as structural shocks. An example is the Smets and Wouters (2007) US model to be examined below. This case, and its solution, can be represented as above for the complete DSGE model.

2.1 FII Estimation

The FII criterion is based on the difference between features of the auxiliary model (such as coefficients estimates, impulse response functions, moments or scores) obtained using data simulated from an estimated (or calibrated) DSGE model and those obtained using actual data; these differences are then represented by a Wald statistic; we call it the IIW (Indirect Inference Wald) statistic. The specification of the auxiliary model reflects the choice of descriptor variables.

If the DSGE model is correct (the null hypothesis) then, whatever the descriptors chosen, the features of the auxiliary model on which the test is based will not be significantly different whether based on simulated or actual data. The simulated data from the DSGE model are obtained by bootstrapping the model using the structural shocks implied by the given (or previously estimated) model and computed from the historical data. We estimate the auxiliary model — a VAR(1) — using both the actual data and the N samples of bootstrapped data to obtain estimates a_T and $a_S(\theta_0)$ of the vector α. We then use a Wald statistic (WS) based on the difference between a_T, the estimates of the data descriptors derived from actual data, and $\overline{a_S(\theta_0)}$, the mean of their distribution based on the simulated data, which is given by:

$$WS = (a_T - \overline{a_S(\theta_0)})' W^{-1}(\theta_0)(a_T - \overline{a_S(\theta_0)})$$

where θ_0 is the vector of parameters of the DSGE model on the null hypothesis that it is true and $W(\theta_0)$ is the weighting matrix. Following Guerron-Quintana et al. (2017) and Le et al. (2011, 2016), $W(\theta_0)$ can be obtained from the variance-covariance matrix of the distribution of simulated estimates a_S

$$W(\theta_0) = \frac{1}{N}\Sigma_{s=1}^{N}(a_s - \overline{a_s})'(a_s - \overline{a_s}) \tag{3}$$

where $\overline{a_s} = \frac{1}{N}\Sigma_{s=1}^{N} a_s$. WS is asymptotically a $\chi^2(r)$ distribution, with the number of restrictions, r, equal to the number of elements in a_T. A detailed steps of involved in finding the Wald statistic can be found in Le et al. (2016) and Minford et al. (2016).

Estimation based on indirect inference focuses on extracting estimates of the structural parameters from estimates of the coefficients of the auxiliary model by choosing parameter values that minimise the distance between estimates of the auxiliary model based on simulated and actual data. A scalar measure of the distance may be obtained using a Wald statistic. This can be minimised using any suitable algorithm.

The II estimation may be expressed as

$$\hat{\theta} = argmin\ WS(\theta) \tag{4}$$

Under the null hypothesis of full encompassing and some regularity conditions, Dridi et al. (2007) show the asymptotic normality of II estimator $\widehat{\boldsymbol{\theta}}$,

$$\sqrt{T}(\widehat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \backsim N(\mathbf{0}, \Xi(N, W) \tag{5}$$

with

$$\Xi(N, W) = \left\{ \frac{\partial'(a)}{\partial(\theta_0)} W(\theta_0)^{-1} \frac{\partial'(a)}{\partial(\theta_0)} \right\}^{-1}. \tag{6}$$

$W(\theta_0)$ is the weighting matrix, which can obtained from bootstrap samples as in (3).

2.2 The Auxiliary Models

Le et al. (2017) show that the particular DSGE models we are examining are over-identified, so that the addition of more VAR coefficients (e.g. by raising the order of the VAR) increases the power of the test, because more nonlinear combinations of the DSGE structural coefficients need to be matched. Le et al. (2016) note that increasing the power in this way also reduces the chances of finding a tractable model that would pass the test, so that there is a trade-off for users between power and tractability. Le et al. (2016) and others (for example Minford et al, 2018; Meenagh et al, 2019; Meenagh et al, 2022) suggest the use of a three variable VAR (1) as auxiliary model. In this case, there are 9 VAR coefficients to match in the Wald statistics. Minford et al. (2016) also consider the IRF and simulated moments, which all have 9 elements to match in the Wald statistics, as the auxiliary model, and show that the power of the II tests when using the different auxiliary models are similar. Considering the covariance matrix and using its lower triangular elements, there are 3(3+1)/2=6 elements to compare in a three variable case.

In the Monte Carlo experiments below, we consider estimation with three different auxiliary models: 1) II using 9 VAR coefficients; 2) II using 9 moments, consisting of 6 covariance elements and 3 first order autocorrelation; 3) II using 6 moments, only including 6 covariance elements. The first two of these carry out formal II estimation, while the last is considered as informal II estimation.

3 Monte Carlo Experiments

We now perform some experiments comparing the formal and informal II estimation in small samples. The sample size is chosen as 200, which is typical for macro data. We take the Smets-Wouters (2007) model, with their estimated parameters to be the true model and generate 1000 samples of data from it. These are treated as the observed data in the II estimation. We design Monte Carlo simulation following the same approach as Le et al. (2016), Meenagh et al. (2019) and Meenagh et al. (2022).

The true parameter values are from Smets and Wouters (2007), Table 4. In estimation, we start the initial parameter values by falsifying them by 10% in both directions (+/− alternately). We then estimate each sample and report the absolute bias and standard deviation of the II estimators. The results are reported in the Table 1, where y: real GDP, pi : inflation rate, r : real interest rate.

Table 1: Bias of II estimates by using different data descriptors

Parameter	True Values	Formal II estimation: 9 VAR coefficients jointly as auxiliary model		Formal II estimation: 9 Moments jointly as auxiliary model		Informal II estimation: Average of 6 Moments as single auxiliary model	
		Bias%	Std dev	Bias%	Std dev	Bias%	Std dev
α : Income share of capital	0.19	0.64	0.020	0.74	0.021	6.53	0.018
h : External habit formation	0.71	5.02	0.065	7.22	0.069	9.09	0.049
ι_p : Degree of price indexation	0.22	0.90	0.023	2.64	0.026	6.17	0.021
ι_w : Degree of wage indexation	0.59	1.38	0.060	1.92	0.068	10.17	0.058
ξ_p : Degree of price stickiness	0.65	4.16	0.068	2.70	0.074	8.78	0.063
ξ_w : Degree of wage stickiness	0.73	1.42	0.075	0.34	0.083	4.50	0.073
φ : Elasticity of the capital adjustment cost function	5.48	1.86	0.557	2.72	0.608	7.20	0.524
Φ : 1+the share of fixed costs in production	1.61	0.32	0.166	3.06	0.182	3.92	0.140
ψ : Elasticity of the capital utilization adjustment cost	0.54	0.00	0.057	1.14	0.062	6.80	0.051
$r_{\Delta y}$: Taylor Rule response to change in output	0.22	2.10	0.023	2.18	0.024	5.63	0.020
ρ : Taylor rule coefficient (interest rate smoothing)	0.81	5.60	0.047	9.90	0.060	3.60	0.058
r_π : Taylor Rule response to inflation	2.03	2.24	0.188	3.72	0.204	5.76	0.188
r_y : Taylor Rule response to output	0.08	2.50	0.009	2.76	0.009	6.08	0.008
σ_c : Elasticity of intertemporal substitution for labour	1.39	4.58	0.137	1.90	0.155	10.58	0.132
σ_l : Elasticity of labour supply to real wage	1.92	1.08	0.208	2.48	0.211	6.53	0.191
Average		2.26	0.113	3.02	0.124	6.75	0.106

Notes: The true parameter values are from Smets and Wouters (2007) table 4. Three variables used in VAR are (y, pi, r), as in Le et al. (2016). Bias denotes the bias of II estimates. Std dev denotes the standard deviation.

We find that the FII estimator has a very small bias. The average absolute biases of the FII estimator based on using VAR coefficients and the 9 moments as auxiliary models are 2.26% and 3.02% respectively. Le et al. (2016) and Meenagh et al (2022) find that the comparable FIML estimates are heavily biased in small samples. FII estimation is, by contrast, found to be almost unbiased, which is clearly a very useful property for those using DSGE models in practice. The average bias of the III estimator, based on 6 moments as the auxiliary model, is twice to three times as large at 6.75%. The informal II estimator thus has a much higher bias than the two formal II estimators. The standard deviation of the three II estimators are more or less similar.

The three variables we choose follow Le et al. (2016). To check if our results are stable across different variables, we redo the Monte Carlo experiment by using three principal components of the 7 endogenous variables in Smets and Wouters (2007)'s model. The results, available on request, are similar.

4 Conclusions

A common practice in estimating parameters in DSGE models is to find a set that when simulated gets close to an average of certain data moments; the model's simulated performance for other moments is then compared to the data for these as an informal test of the model. We call this procedure informal Indirect Inference, III. By contrast what we call Formal Indirect Inference, FII, chooses a set of moments as the auxiliary model and computes the Wald statistic for the joint distribution of these moments according to the structural DSGE model; it tests the model according to the probability of obtaining the data moments. The FII estimator then chooses structural parameters that maximise this probability. We show in this note via Monte Carlo experiments that the FII estimator has low bias in small samples, whereas the III estimator has much higher bias. It follows that models estimated by III will frequently be substantially different from the true model and hence rejected by formal indirect inference tests.

References

[1] Baslandze, S. (2022) "Entrepreneurship through Employee Mobility, Innovation, and Growth," paper given at 2022 Konstanz seminar, https://konstanzseminar.org/wp-content/uploads/2022/05/Baslandze.pdf

[2] Chari, V., Kehoe, P., and McGrattan, E. (2002). Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates? Review of Economic Studies, 69 (3), 533-563.

[3] Dridi, R., Guay, A. and Renault, E., 2007. Indirect inference and calibration of dynamic stochastic general equilibrium models. Journal of Econometrics, 136(2), 397-430.

[4] Fernandez-Villaverde, J., Rubio-Ramirez, F., Sargent, T, and Watson, M. ,2007.'ABCs (and Ds) of Understanding VARs'. American Economic Review, 1021-1026.

[5] Guerron-Quintana, P., Inoue, A. and Kilian,L., 2017. Impulse Response Matching Estimators for DSGE Models. Journal of Econometrics, 196, (1), 144-155.

[6] Juillard, M. 2001. Dynare: A program for the simulation of rational expectation models (No. 213). Society for Computational Economics.

[7] Khan, A., and Thomas, J. (2013)"Credit Shocks and Aggregate Fluctuations in an Economy with Production Heterogeneity", Journal of Political Economy, Vol. 121, No. 6 (December 2013), pp. 1055-1107.

[8] Le, V.P.M., Meenagh, D., Minford, P., Wickens, M., 2011. How much nominal rigidity is there in the US economy — testing a New Keynesian model using indirect inference. Journal of Economic Dynamics and Control 35(12), 2078–2104.

[9] Le, V.P.M., Meenagh, D., Minford, P.,Wickens, M., and Xu, Y., 2016. Testing macro models by indirect inference: a survey for users. Open Economies Review 27, 1-38.

[10] Le, V.P.M, Meenagh, D., Minford, P., Wickens, 2017. A Monte Carlo procedure for checking identification in DSGE models. Journal of Economic Dynamics and Control, 76, (C), 202-210

[11] Meenagh, D., Minford, P., Wickens, M. and Xu, Y. (2019) 'Testing Macro Models by Indirect Inference: A Survey of recent findings', Open Economies Review, 2019, vol. 30, issue 3, No 8, 593-620.

[12] Meenagh, D., Minford, P. and Xu, Y. (2022). "Why does Indirect Inference estimation produce less small sample bias than maximum likelihood? A note," Cardiff Economics Working Papers E2022/10, Cardiff University, Cardiff Business School, Economics Section.

[13] Minford, P.,Ou, Z. and Zhu, Z. (2021) "Can a small New Keynesian model of the world economy with risk-pooling match the facts?"International Journal of Finance & Economics, 2021, vol. 26, issue 2, 1993-2021.

[14] Minford, P., Wickens, M., and Xu, Y, 2016. Comparing different data descriptors in Indirect Inference tests on DSGE models, Economics Letters, 145, (C), 157-161.

[15] Minford, P., Wickens, M., and Xu, Y, 2018. Testing Part of a DSGE Model by Indirect Inference, Oxford Bulletin of Economics and Statistics, Oxford Bulletin of Economics and Statistics 81(1), pp. 178-194.

[16] Smets, F., Wouters, R., 2007. Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach. American Economic Review, 97, 586–606.