Testing competing world trade models against the facts of world trade

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Abstract

We carry out an indirect inference test of two versions of a computable general equilibrium (CGE) model of world trade. One of these, the ‘classical’ model, is well-known as the Heckscher-Ohlin-Samuelson model of world trade, in which countries trade homogeneous products in world markets and produce according to their comparative advantage as determined by their resource endowments. The other, the ‘gravity’ model, assumes products are differentiated by geographical origin, so that trade is determined largely by demand and relative prices differing according to distance; trade in turn affects productivity through technology transfer. These two CGE models of world trade behave in very different ways and predict quite different effects for trade policy, underlining the importance of discovering which best fits the facts of international trade. Our findings here are that the classical model fits these facts fairly well in general, while the gravity model is largely strongly rejected by them.

Keywords: Bootstrap, indirect inference, gravity model, classical trade model, trade

\textit{JEL classification:} F10-14, F16-17

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1. Introduction

Recent years have seen vigorous controversies over Brexit, the US-China tariff wars and the continuing discussions to strengthen the Comprehensive and Progressive Agreement for Trans-Pacific Partnership (CPTPP). To evaluate the welfare effects of these trade policies, one needs both a general equilibrium model of trade relationships between countries and a means to assess its quantitative accuracy. A variety of computable general equilibrium (CGE) models of trade have been used for welfare evaluation, such as the Global Trade Analysis Project (GTAP) model (Corong et al., 2017) and the CE-SiFo model (Felbermayr et al., 2020). These and other CGE models have attempted to mediate in these policy debates by providing relevant welfare measures.

At the heart of these debates however there is a fundamental disagreement about how trade works and affects the economy. In the last few years for example debate has raged over whether EU trade arrangements are beneficial, in particular to the UK. The EU is a customs union and so erects trade barriers around its Single Market where economic activity is regulated according to EU rules. The welfare effects of a customs union have always been controversial. According to classical trade theory global welfare is reduced compared with free trade as is the average welfare of citizens inside the customs union; however one country’s citizens may gain from the union if it is a net exporter to others in the union, as then its terms of trade gain may offset the losses experienced by its consumers (Meade, 1955). However in recent times a new line of reasoning has become popular among trade economists: this ‘gravity model’ (eg. Costinot and Rodriguez-Clare, 2014) regards trade as an outcrop of internal trade, the only difference being that it crosses borders. Otherwise it grows naturally due to the specialisation and division of labour within neighbouring markets. Viewed through the lens of the gravity model a
customs union merely makes official what is already a fact of neighbourly inter-trade. Other sorts of trade, with more distant markets, grows analogously but more weakly, the greater the distance; size of distant markets may make up for their distance to some extent, because they are a ‘neighbourhood’ that naturally leads to inter-trade. As part of this view of trade as dominated by inter-trade, substitutability between heterogeneous goods and services of different origins is treated as fairly low. ‘Gravity’ in trade creation can be thought of as a function of distance and size. In this view of trade it makes no sense to put obstacles in the way of trade with close neighbours such as the EU in the hope of boosting trade with distant markets via new trade agreements that lower trade costs. The disruption from the former will reduce welfare while the gains from the latter will be small, simply because the reduced trade costs will have little effect in switching demand from existing products in the presence of weak and imperfect competition. Furthermore, protection is seen in a fairly positive light in the gravity model, because low substitutability between countries’ goods implies that there is scope for protection to improve the terms of trade - the ‘optimal tariff’ mechanism; He et al. (2017) show that it pushes optimal tariff rates before and after retaliation above 100% - clearly a worrying policy implication, which in itself casts doubt on the model’s realism. In a similar finding, Chen et al (2021) showed that the welfare losses from tariffs were much greater in the classical model than in the gravity model, with the latter contributing large terms of trade gains. This highlights the need for a way of testing these CGE models against the data, to establish whether the classical or the gravity trade model is the true one driving the data.

The process of testing general equilibrium trade models, including the today widely-used gravity models, has taken several forms. One dominant recent approach has been to find micro relationships across countries, in which trade is found to be related to dis-
tance and GDP, as well as other variables regarded as ‘cultural gravity’ such as colonial
ties. Thus in his recent presidential Royal Economic Society Presidential speech and
the associated Economic Journal article (Carrere et al., 2020) Peter Neary pointed to a
general equilibrium model that could generate these cross-section relationships. Others
have argued in a similar way from panel data relationships that include time as well as
cross-section variation, and also include price variables (Costinot and Rodriguez-Clare,
2014).

However these are reduced form relationships between solved-out values of endoge-
nous variables, since trade prices and GDP are all determined by the underlying struc-
tural CGE model. While one can reverse engineer a CGE model that generates them,
this does not establish identification. Other structural CGE trade models can also gener-
ate them. To test the different CGE structural models requires an empirical comparison
to be made in terms of the different models’ ability to match these regressions on en-
dogenous variables.

Minford and Xu (2018)- hereafter MX- suggested a way forward, setting up two
rival CGE models, ‘classical’ and ‘gravity’, designed to capture the trade time-series
developments for major trading countries or country blocs across major product cat-
egories. The classical CGE model consists of the Hecksher-Ohlin-Samuelson model
of goods and factor markets under perfect competition; the gravity model adopts the
same general market structure but imposes gravity assumptions on it, including lim-
ited substitutability across country-source of products, and a link from trade intensity
to productivity. MX tested this model by indirect inference on UK data, treating world
prices and other country behaviour as exogenous, an appropriate assumption given the
small size of the UK economy relative to the world (about 4% of world GDP). Chen
et al. (2021) extended this test to other countries/country blocs, which are too large for
this assumption to be appropriate; for this they used a new ‘part-of-model test’ (Minford et al., 2019) in which the country or bloc model is simulated together with simulations of the world and other country/bloc variables from a VAR model representing the reduced form of the full unknown true world model. These tests found the classical model to be universally accepted, while the gravity model was strongly rejected in two countries, and accepted in only two. In this paper our contribution is to extend the indirect inference test to a full Global Model of world trade, testing it on data from the world as a whole and in a more powerful test.

These tests on time series are the ones relevant to policymakers who are changing policies over time. Our structural trade models, to be useful to them, should predict the effects of variables’ change over time in such a way that the parameters are constant across regime change, so satisfying Lucas’ critique (Lucas, 1976). Our test therefore establishes whether the models can be used reliably to assess policy changes.

For this purpose we use a small World Trade Model of a few country-groups and commodity groups: 4 products × 5 country - groups × 4 factors of production. This is still a highly complex construction, especially when we embed in it gravity elements, to be discussed below. The model is a comparative static one, in which exogenous shocks have an immediate effect on the endogenous variables. Consequently the observed shocks in this model are to be interpreted as accumulated effects of current and lagged real-time shocks, capturing the process of adjustment over time; they are therefore autocorrelated, and their autocorrelation processes are included in the model as parameters. This interpretation allows us to retain the CGE structure of the trade model on which we are primarily focused in our test.

We test the rival models by indirect inference. There is by now a substantial body of work using this method to test macroeconomic models (Le et al., 2016; Meenagh et al.,
2019). It involves first estimating an ‘auxiliary’ model whose role is to describe the data behaviour; this can take the form of moments or Impulse Response Functions or, as here, regression equations, to be described shortly. In the second stage, the structural model being tested is simulated by bootstrapping its shock innovations to generate a large number of parallel ‘histories’, on which the same auxiliary model is estimated. This creates a distribution of the auxiliary model parameters with which the data-based values can be compared; if these reach a sufficient likelihood level, the model is not rejected.

We look at five countries/groups, the UK, US, the Euro Area, China, and ROW; here we test them together in a full Global Model. For our auxiliary model of a country or bloc we compute a series of regressions relating different data series of the country or bloc related to trade. These series are all non-stationary, so they are related to each other via common trends in cointegrating regressions. We can think of these as being formed from reduced form relationships between endogenous and exogenous variables that are cointegrated because the latter cause the former. We thus use a series of cointegrating relationships between key trade variables as our auxiliary model.

To test a model’s simulation performance against the data behaviour requires careful selection of the data features to be matched. Indirect inference tests tend towards unlimited power as the number of features is increased: as one tries to match all features of behaviour one ultimately requires to have the real world itself as the model. Hence to give the test a reasonable level of power, that on the one hand will reject tractable models of some moderate falsity but on the other will not reject all models that are even slightly false, a small number of relevant data behaviour features need to be selected; this number differs according to the modelling context and we establish it through Monte Carlo experiment in each context.
The main data movement we want to explain is in output shares by sector and trade (export+import or total trade) shares by country bloc. These two sets of shares summarise the economy’s output structure and direction of trade. Accompanying these trends are:

a) world relative prices and relative productivity of manufactures and services, treating raw materials as the numeraire.

b) five countries/groups relative factor supplies of land, unskilled and skilled labour.

To construct these relationships we relate the trade shares and the output shares and these other elements in a series of cointegrating regressions; these constitute the auxiliary model. We would hope to find a suitable number of key coefficients from this to use as elements of the Wald statistic matching the data behaviour to the simulated behaviour from the structural model.

These can be used to summarise the relationships found in the data for a country whose trade behaviour we wish to explain. To create the world behaviour, we combine all these countries’ behaviour, weighted by GDP.

In what follows, in section 2 we recapitulate, in the briefest of outlines, the features of the two CGE models (for more details see MX and Chen et al, 2021); we also set out the auxiliary model, while the full Global Model is listed in the Model Appendix, and the full data set in the Data Appendix. In section 3, we also briefly explain the operation of indirect inference. In section 4 we describe our testing results for the world economy. We end by drawing some conclusions.
2. The rival classical and gravity models of trade - a brief overview

2.1. The classical model of trade

We begin with the ‘classical’ model of world trade, whose intellectual origins lie in the work of Ricardo (1817), Heckscher (1919), Ohlin (1933), Stolper and Samuelson (1944) and Rybczynski (1955). In this model output is determined by factor supplies and sectoral productivity. Outputs here are defined as intermediate products, which will be used as inputs into final goods for consumption; they are divided into primary (agriculture and raw materials), manufactures, traded services and nontraded output. Capital is freely available from the rest of the world at the world’s exogenous cost of capital. To this set-up we add a model of Retail Consumption, which follows the model of intermediate trade.

2.1.1. The model of intermediate trade

The model of intermediate trade is as in Minford et al. (2015), a CGE model of trade, output, factor supply and demand; with four products, four factors and here five ‘countries’ (or country blocs), of which the UK is one, and the others are the Euro Area, the US, China, and the Rest of the World. Capital is mobile. The products are manufactures, other goods (agriculture and raw materials), traded services and non-traded; all supplies of each product are assumed to be perfectly substitutable, as if defined commodities in a supply chain.

We will describe the models from the viewpoint of a home country, which we will take to be the UK, for illustration.
These products are considered as intermediates which are supplied at the border or the factory gate in country markets to country retail distribution industries that operate under imperfect competition as set out below.

This intermediate model follows the one Minford et al. (1997) developed for assessing the effects of globalisation on the world economy. This model performed well empirically in accounting for the trade trends of the 1970-1990 period; it identified a group of major causal ‘shocks’ during this period which between them gave a good fit to the salient features of the period- including terms of trade, production shares, sectoral trade balances, relative wage movements and employment/unemployment trends.

The model adopts the key assumptions of the Heckscher-Ohlin-Samuelson set-up. Production functions are assumed to be Cobb-Douglas and identical across countries, up to a differing productivity multiplier factor; thus factor shares are constant, enabling us to calibrate the model parsimoniously from detailed UK data that we were able to gather. There are four sectors: non-traded and three traded ones, viz. primary, basic (unskilled-labour-intensive) manufacturing, and services and other (skilled-labour-intensive) manufacturing. Three immobile factors of production are identified: unskilled and skilled labour and land. Capital is mobile. All sectors are competitive and prices of traded goods of each sector are equalised across borders.

This set-up gives rise to a well-known set of equations:

1. given world prices of traded goods, price=average costs determine the prices of immobile factors of productions.

2. these factor prices induce domestic supplies of these factors.

3. outputs of each sector are determined by these immobile factor supplies; non-traded sector output is fixed by demand, the traded sector outputs by the supplies of immobile factors not used in the non-traded sector.
4. demands for traded goods are set by the resulting level of total GDP.

5. world prices are set by world demand = world supply.

The world is divided into five blocs: the UK, the Euro Area, the US, China, ROW (rest of world).

We treat primary sector output (agriculture mainly) as politically controlled and essentially fixed exogenously because of interventionist planning systems. The supply of land is adjusted (via planning and other controls) to enforce this output requirement but otherwise to satisfy land demands from other sectors.

2.1.2. Model of retail consumption

Consumers can choose consumption by product origin for each sector. The idea is that distribution is a perfectly competitive industry that bundles up intermediate output sold in perfectly competitive world markets. Retail products are thus bundles of intermediate supply-chain products. These bundles are ‘branded’ to create distinct products that consumers will not easily switch from owing to shortage of time, habit etc. However bundlers will buy inputs that are commoditised to yield best value.

We make the assumption that in the major consumer markets of the UK, the Euro Area, the US and China, these retail brands are differentiated by geographical origin because country suppliers own retailer groups as their marketing agents and also face country-specific trade frictions such as differential tariffs and transport costs. However in the rest of the world countries retailers are independent of country suppliers, and buy intermediate inputs from any supply source without preference, with the typical country imposing manufacture tariffs and facing the same transport costs from all origins. These assumptions imply that any excess supply of an intermediate product by any country’s industry in the Euro Area and the US can be sold in the rest of the world’s markets at
the going world price; hence there can be no imbalance of trade by geographical origin requiring relative country trade prices to change. Thus in this default classical version of the model origin heterogeneity has no effect, as it crucially does in the gravity version.

2.2. The Gravity Model Variant- modifying the retail consumption sector

Our aim here is to create a gravity version of a full CGE model, with the complete set of goods and factor markets. What then are the essential gravity components to be included? We suggest two main key features:

1. that on the demand side there is highly imperfect substitutability between products: it is this feature that makes geography so dominant, since once demanded a product is difficult to dislodge; similarly, selling into distant markets is hard because it has to be broken into by large price cuts.

2. that trade itself stimulates productivity in the growing traded sector. A popular channel for this is FDI but the idea is more general: the bigger trade, the bigger the market size and hence the profits to investment and knowledge transfer (Feyrer, 2009, 2011; Pain and Young, 2004; Dhingra et al., 2016; Cai et al., 2019). These ideas on the transmission of productivity via trade have given rise to much econometric estimation, often in large panel data sets, of micro relationships at low SIC levels between productivity, FDI, patents, trade and other variables. Gravity theorists have interpreted these as showing that trade, as an exogenous factor determined by demand and distance, determines FDI, patents etc and so productivity. However, in a classical model the exogenous variables are countries’ factor supplies and policies determining productivity; the identification is entirely different, usefully distinguishing the gravity CGE version.
from the classical version.

In our Gravity version of the CGE model, we embed these two features by first removing the differential branding behaviour in the rest of the world markets: instead of assuming that they are indifferent to origin as in the main Classical model version, we assume that they too brand by origin, so that each country or bloc now needs to find a means by which total demands for its exports by origin equal its total demands for imports. This current account equilibrium by origin is achieved by its real exchange rate. Relative country prices must change to create trade balance by geographical origin.

The second Gravity feature of the trade-productivity transmission is added by including the change in the total country trade share of GDP into the equations for productivity growth.

3. Indirect Inference testing

Indirect Inference is a relatively unfamiliar method of estimation and testing; until recently it has mainly been used in the form of the Simulated Method of Moments. We use it here because we need a method that will powerfully reject a mis-specified model in the small samples that we have (in these trade models limited years of annual data). The two main alternatives today are Bayesian estimation with strong priors or Maximum Likelihood (equivalent to Bayesian estimation with flat priors).

The former is an appropriate method when much is already known about the issue at hand, so that priors can be set out that command general assent; often the case in the physical sciences and indeed in some parts of the social sciences. However, this condition does not apply here: the macroeconomics of trade in the world economy remains controversial.
Maximum Likelihood estimation is based on minimising the model’s now-casting prediction errors and its associated test is based on the likelihood implied by these errors. The two main difficulties of this method are first that it exhibits high estimation bias in small samples and second that the power of the test in small samples is also rather limited and in particular its power to reject a mis-specified model is close to zero, because such a model including its error processes can be fitted closely to the data, so creating small errors. Le et al. (2016) carried out a Monte Carlo comparison of this method with Indirect Inference, treating the widely used Smets and Wouters (2007) model of the US as the true model, and concluded that, while indeed ML methods suffered from these problems, by contrast Indirect Inference offered very low bias and potentially large power. The method involves first describing the data behaviour in the sample by an ‘auxiliary model’, for which we use an appropriate description of the data behaviour; and then simulating the structural model by bootstrapping its innovations to create many parallel samples (or histories) from each of which implied auxiliary model coefficients are estimated, generating a distribution of these coefficients according to the structural model. We then ask whether the auxiliary model coefficients found in the actual data sample (actual history) came from this distribution with a high enough probability to pass the Wald test (where we put the test threshold at 5%) - the detailed steps in creating the Indirect Inference Wald statistic are shown in Appendix D.

3.1. The Auxiliary Model

For each country we specify an auxiliary model to be matched by the simulated Global Model. As an example here we show the auxiliary model for the US. The variables in the auxiliary model are: the US share of trade with the Euro Area (EA), $TS_{EA} =$
\[
\begin{align*}
M_{EA} + X_{EA} \quad & \text{with China, } TS_{CHINA} = \frac{M_{CHINA} + X_{CHINA}}{GDP_{US}}; \text{ with the Rest of of the World, } TS_{ROW} = \frac{M_{ROW} + X_{ROW}}{GDP_{US}}; \\
\text{the US output ratio, output in manufacturing divided by output in services,} \\
\text{denoted by } OS_{US} = \frac{y_M}{y_S}. \text{ These we put on the left hand side for convenience; and on the} \\
\text{right hand side we have the variables: the US productivity residual relative to that of} \\
\text{services, denoted by } \frac{\pi_M}{\pi_S}; \text{ the relative US factor share, which is skilled labour divided by} \\
\text{unskilled labour, } H \quad N; \text{ and the wages of unskilled relative to those of skilled workers, } \frac{w}{h}; \\
\text{finally, the Euro Area GDP and China GDP. The auxiliary model equations are:} \\

TS_{EA} = \gamma_1 + a_{11} \frac{\pi_M}{\pi_S} + a_{12} \frac{N}{H} + a_{13} \log(GDP_{EA}) + a_{14} \log(GDP_{CHINA}) + a_{15} \frac{w}{h} + \epsilon_1 \quad (1) \\

TS_{CHINA} = \gamma_2 + a_{21} \frac{\pi_M}{\pi_S} + a_{22} \frac{N}{H} + a_{23} \log(GDP_{EA}) + a_{24} \log(GDP_{CHINA}) + a_{25} \frac{w}{h} + \epsilon_2 \quad (2) \\

OS_{US} = \gamma_3 + a_{31} \frac{\pi_M}{\pi_S} + a_{32} \frac{N}{H} + a_{33} \log(GDP_{EA}) + a_{34} \log(GDP_{CHINA}) + a_{35} \frac{w}{h} + \epsilon_3 \quad (3) \\

TS_{ROW} = \gamma_4 + a_{41} \frac{\pi_M}{\pi_S} + a_{42} \frac{N}{H} + a_{43} \log(GDP_{EA}) + a_{44} \log(GDP_{CHINA}) + a_{45} \frac{w}{h} + \epsilon_4 \quad (4) \\
\end{align*}
\]

These four cointegrating regressions yield 20 coefficients, \(a_{11} - a_{45}\), describing the data behaviour of US traded variables. They are used in testing the Global Model on US data. The equivalent coefficients are used for the other countries/blocs we focus on: the UK, China and the Euro Area. For the world as a whole these coefficients are all used,
weighted by GDP; thus 20 weighted coefficients from all four countries/blocs.

### 3.2. Testing the Global Model on world data by Indirect Inference

Our main aim is to test the two full models against all countries’ auxiliary model. This answers the question whether the full world model is consistent with average world behaviour. We also ask whether each country’s individual behaviour is consistent with the two world models.

In principle if either world model is true, it should not be rejected by any of these tests.

### 3.3. Model bootstrapping and Monte Carlo experiments to check test power

The model of each country’s trade, as set out above, is nonlinear; it is solved here in Matlab- for details see Appendix C. When all countries are combined in the full World Model, the model is solved subject to non-negativity limits for production sectors. Model bootstraps with high volatility are eliminated.

It is important to assess the power of this exact test procedure, since that is a guide to the accuracy of any model passing the test. Here we carried out several Monte Carlo experiments to assess the power of our various potential tests, ranging from using some of the four regressions in the auxiliary model to using all; the more that are used the higher the test power but we need to be careful the power does not get so large that no tractable model within moderate distance of the truth could pass the test. In fact we find that using all four equations (1)-(4), the power is substantial but not excessive. We began by assessing the power of a single country model test, where world prices and
Table 1: Power of Indirect Inference Wald test

<table>
<thead>
<tr>
<th>Percent Misspecified</th>
<th>Rejection Rates at 95% Confidence Level</th>
<th>UK</th>
<th>Full World Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>True</td>
<td>5.00%</td>
<td>5.00%</td>
<td></td>
</tr>
<tr>
<td>1%</td>
<td>40.5%</td>
<td>5.5%</td>
<td></td>
</tr>
<tr>
<td>3%</td>
<td>99.9%</td>
<td>13.8%</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>100.0%</td>
<td>31.6%</td>
<td></td>
</tr>
<tr>
<td>7%</td>
<td>100.0%</td>
<td>57.4%</td>
<td></td>
</tr>
<tr>
<td>10%</td>
<td>100.0%</td>
<td>67.7%</td>
<td></td>
</tr>
<tr>
<td>15%</td>
<td>100.0%</td>
<td>68.3%</td>
<td></td>
</tr>
<tr>
<td>20%</td>
<td>100.0%</td>
<td>82.1%</td>
<td></td>
</tr>
</tbody>
</table>

other countries’ behaviour could be treated as exogenous- as is the case for the UK, being a small open economy. Our Monte Carlo experiment for the UK model alone on this basis is shown in the middle column of Table 1, where we see how frequently the true trade model is rejected as it becomes more false: we falsify all parameters by x% alternately odd and even; we create 1000 samples from the classical model, treated as true, and test the model, true and falsified on these samples to check the frequency of rejection. To get an idea of how many equations to use in the auxiliary model, we used the UK model with exogenous rest of world data (as done in Minford and Xu, 2018). The test rejects models whose parameters are only 3% falsified, virtually all the time. Thus if not rejected by this test, a model must be very close to the truth. Any model with 3% or more inaccuracy is rejected virtually 100% of the time.

This experiment suggested that the auxiliary model with four equations as shown above has considerable but not excessive power.

Next, we carried out a confirmatory experiment on the full World Model, solved and bootstrapped as explained above, to check its power in a test using this 4-equation auxiliary model on all countries’ data, weighted by GDP. The last column of Table 1 shows that the power is somewhat weaker in this full test across all countries than it is
Table 2: Test results of the full world global model

<table>
<thead>
<tr>
<th>Country</th>
<th>Classical Model</th>
<th>Gravity Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>UK</td>
<td>0.2429</td>
<td>0.0412*</td>
</tr>
<tr>
<td>US</td>
<td>0.0337*</td>
<td>0.0078*</td>
</tr>
<tr>
<td>Euro Area</td>
<td>0.0936</td>
<td>0.0114*</td>
</tr>
<tr>
<td>CH</td>
<td>0.0829</td>
<td>0.0142*</td>
</tr>
<tr>
<td>World</td>
<td>0.3095</td>
<td>0.026*</td>
</tr>
</tbody>
</table>

Note: p-value with * indicates a rejection of the model at 5% significance level.

for just one small country; but it is still considerable. The test will reject nearly 60% of the time with only 7% model falsity.

4. Results of the tests

In this section we test the full global Model against each country in turn; and then against the weighted average of all countries’ auxiliary model parameters, our main focus. What we can see from all these results is a pattern in which across all countries, as well as at the world level the Gravity model is strongly rejected. The classical model is comfortably accepted with good p-values for all countries except the US, as well as at the world level. For the US it appears that the classical model is marginally rejected. This result is however somewhat anomalous, given that the classical model fits the world as a whole as well as all other countries; it is also at variance with the findings of Chen et al (2021) for the US, using the part-of-model test (Minford et al, 2019) in which the US model is simulated together with reduced form simulations of world variables. In that test the classical model was easily accepted by the US data, as was also the gravity version.

Our overall conclusion from these tests is therefore that the classical model version
broadly fits the data while the gravity version is broadly rejected by it.

5. Conclusions

In this paper we have carried out an indirect inference test of two versions of a CGE model of world trade. One of these, the ‘classical’ model, is well-known as the Heckscher-Ohlin-Samuelson model of world trade, in which countries trade homogeneous products in world markets and produce according to their comparative advantage as determined by their resource endowments. The other, the ‘gravity’ model, assumes products are differentiated by geographical origin, so that trade is determined largely by demand and relative prices differing according to distance; trade in turn affects productivity through technology transfer. These two CGE models of world trade behave in very different ways and predict quite different effects for trade policy, underlining the importance of discovering which best fits the facts of international trade. Our findings here are that the classical model fits these facts fairly well in general, while the gravity model is largely strongly rejected by them.
Appendix A: Data Appendix

We use annual data over the period 1970-2019. All variables are expressed in constant prices based on 2015 US price index. All values are converted into US dollar.

Figure 1: UK Data from 1970 to 2019
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition and Description</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y^j$</td>
<td>Real Output (GDP) by sector $i =$ Agriculture, Industry, Service, Nontraded in country $j$; (constant 2015, billion US dollars)</td>
<td>World Bank</td>
</tr>
<tr>
<td>$G^j$</td>
<td>Government Spending of country $j$; (constant 2015, billion US dollars)</td>
<td>World Bank and AWM</td>
</tr>
<tr>
<td>$PoP^j$</td>
<td>Population age 15 and over in country $j$; (millions)</td>
<td>AWM, UN, ONS</td>
</tr>
<tr>
<td>$K^j$</td>
<td>Capital of country $j$; (constant 2015, billion dollars)</td>
<td>World Bank</td>
</tr>
<tr>
<td>$L^j$</td>
<td>Agriculture land of country $j$ (million hectares)</td>
<td>World Bank, OECD, Eurostat</td>
</tr>
<tr>
<td>$H^j$</td>
<td>Skilled labour of country $j$ (millions); $PoP^j \times tertiary_rate^j$</td>
<td>ONS, Higher Education Statistics Agency, World Bank</td>
</tr>
<tr>
<td>$N^j$</td>
<td>Unskilled labour of country $j$ (millions);</td>
<td>Calculated $N^j = PoP^j - H^j$</td>
</tr>
<tr>
<td>$r^j$</td>
<td>Interest rate of country $j$</td>
<td>BOE, FRED, World Bank</td>
</tr>
<tr>
<td>$l^j$</td>
<td>Return on land index of country $j$</td>
<td>OECD, FRED, World Bank, Eurostat</td>
</tr>
<tr>
<td>$w^j$</td>
<td>Wages (of unskilled labour) of country $j$ (constant 2015=100 dollar index);</td>
<td>ONS, AWM, OECD, Trading Economics</td>
</tr>
<tr>
<td>$h^j$</td>
<td>Skilled wage of country $j$ (dollar index); $h^j = w^j \cdot decile95$; where decile95 is the Interdecile ratio of top 10% income and average income</td>
<td>WID</td>
</tr>
<tr>
<td>$X^j_{jj}$</td>
<td>Export of country $j$ to country $jj$ (constant 2015, billion US dollars)</td>
<td>IMF, World Bank</td>
</tr>
<tr>
<td>$M^j_{jj}$</td>
<td>Import of country $j$ from country $jj$ (constant 2015, billion US dollars)</td>
<td>IMF, World Bank</td>
</tr>
<tr>
<td>$RER^j$</td>
<td>Real effective exchange rate of country $j$ (2015=100)</td>
<td>FRED, AWM</td>
</tr>
<tr>
<td>$p_i$</td>
<td>World prices of tradable goods by sector $i =$ Agriculture, Industry, Service (constant 2015=100 dollar index);</td>
<td>World Bank Commodity Price Data (The Pink Sheet); OECD Statistics, ONS</td>
</tr>
</tbody>
</table>

Table 3: Data Description

1. where $j =$ UK, US, China, Euro Area and the Rest of the World

2. FRED denotes Federal Reserve Bank of St. Louis; OECD stands for Organisation for Economic Co-operation and Development, data website https://data.oecd.org/; WID is World Inequality Database; AWM is the Area-wide Model database; BoE is Bank of England; ONS stands for Office for National Statistics.
Figure 2: US Data from 1970 to 2019

Figure 3: China Data from 1970 to 2019

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Figure 4: Euro Area Data from 1970 to 2019
Appendix B: Full Listing of the Global Model

\( j = UK, US, Euro\ Area, China, ROW; \)

B.1 Prices of factors:

\[
\ln(w^j) = \left( \frac{1}{0.52234} \right) \cdot \left\{ \ln(p_M \cdot \pi_M^j) - 0.14366 \cdot \ln(h^j) - 0.035 \cdot \ln(l^j) - 0.299 \cdot \ln(p_M \cdot r^j) \right\}
\] (B.1.1)

\[
\ln(h^j) = \left( \frac{1}{0.51832} \right) \cdot \left\{ \ln(p_S \cdot \pi_S^j) - 0.21168 \cdot \ln(w^j) - 0.033 \cdot \ln(l^j) - 0.237 \cdot \ln(p_M \cdot r^j) \right\}
\] (B.1.2)

\[
\ln(l^j) = \left( \frac{1}{0.079} \right) \cdot \left\{ \ln(p_A \cdot \pi_A^j) - 0.147 \cdot \ln(w^j) - 0.132 \cdot \ln(h^j) - 0.642 \cdot \ln(p_M \cdot r^j) \right\}
\] (B.1.3)

\[
\ln(r^j) = \left( \frac{1}{0.331} \right) \cdot \left\{ \ln(p_D \cdot \pi_D^j) - 0.38024 \cdot \ln(w^j) - 0.17576 \cdot \ln(h^j) - 0.113 \cdot \ln(l^j) - 0.331 \cdot \ln(r^j) \right\}
\] (B.1.4)

B.2 Supplies of factors:

\[
N^j = e_N^j \cdot \left( \frac{w^j}{b} \right)^{0.1} \cdot POP^{0.5} \cdot G^{0.5}
\] (B.2.1)

\( e_N^j \) is error process.

\[
H^j = e_H^j \cdot \left( \frac{h^j}{w^j} \right)^{0.1} \cdot G^{0.5}
\] (B.2.2)

\( e_H^j \) is error process.

\[
L^j = l^{j^{-1}} \cdot \left( 0.113 \cdot p_D^j \cdot y_D^j + 0.035 \cdot y_M^j \cdot p_M + 0.033 \cdot p_S \cdot y_S^j + 0.079 \cdot p_A \cdot y_A^j \right) \cdot e_A^j
\] (B.2.3)

\[
K^j = \frac{1}{(p_M \cdot r^j)} \cdot \left( 0.331 \cdot p_D^j \cdot y_D^j + 0.299 \cdot y_M^j \cdot p_M + 0.237 \cdot p_S \cdot y_S^j + 0.642 \cdot p_A \cdot y_A^j \right) \cdot e_K^j
\] (B.2.4)

B.3 Output of each sector:
\[ y_M^j = \left( \frac{1}{0.52234 \cdot p_M} \right) \cdot \{ N^j \cdot w^j \cdot e_M^j - 0.38024 \cdot p_D^j \cdot y_D^j - 0.21168 \cdot p_S^j \cdot y_S^j - 0.147 \cdot p_A^j \cdot y_A^j \} \]  
(B.3.1)

\[ y_S^j = \left( \frac{1}{0.51832 \cdot p_S} \right) \cdot \{ H^j \cdot h^j \cdot e_S^j - 0.168 \cdot p_D^j \cdot y_D^j - 0.14366 \cdot p_M^j \cdot y_M^j - 0.132 \cdot p_A^j \cdot y_A^j \} \]  
(B.3.2)

\[ y_D^j = \frac{r a^j}{1 - r a^j} \cdot (y_A^j + y_M^j + y_S^j) \]  
(B.3.3)

\[ y^j = y_A^j + y_M^j + y_S^j + y_D^j \]  
(B.3.4)

\( y_A^j \) is the exogenous variable. \( r a^j \) is the average ratio between \( y_D^j \) and \( y^j \) in each country.

B.4 Error Process

We assume the log (errors) in the model follow a AR(1) process with intercept and trend, i.e.,

\[ \ln(\pi_{i,i}) = c_{1,i} + \rho_{1,i} \ln(\pi_{i,i-1}) + \phi_{1,i} t + \epsilon_{i,i}, \quad i = M, S, A, d \]  
(B.4.1)

\[ \ln(e_{i,i}) = c_{2,i} + \rho_{2,i} \ln(e_{i,i-1}) + \phi_{2,i} t + \eta_{i,i}, \quad i = A, N, H, K \]  
(B.4.2)

\( t = 1, 2, ... \) denotes time period. We estimate \( c_{1,i}, c_{2,i}, \rho_{1,i}, \rho_{2,i}, \phi_{1,i}, \phi_{2,i} \) by OLS. \( \epsilon_{i,i} \) and \( \eta_{i,i} \) are iid innovations.

B.5 Demand of each sector:

\[ E_T^j = y_A^j + y_M^j + y_S^j - (N^j) \]  
(B.5.1)

\[ E^j = E_T^j + y_D^j \]  
(B.5.2)

\[ E_S^j = a_S^j + 0.9 \cdot E_T^j - 12.0 \cdot (p_S - p_T^j) \]  
(B.5.3)
\[ E_A^j = a_A^j + 0.05 \cdot E_T^j - 5.0 \cdot (p_A - p_T^j) \quad (B.5.4) \]

We estimate \( a_S^j, a_A^j \) by OLS; \( NX^j \) is the total net exports of country \( j \) in tradable sectors.

B.6 Prices of goods:

\[ p_T^j = p_M \cdot \left( \frac{E_M^j}{E_T^j} \right) + p_S \cdot \left( \frac{E_S^j}{E_T^j} \right) + p_A \cdot \left( \frac{E_A^j}{E_T^j} \right) \quad (B.6.1) \]

\[ p_D^j = \left( \frac{(p_T^j)^{E_T}}{E_T^j} \right)^{\frac{1}{y_D^j}} \quad (B.6.2) \]

\[ y_{i,world}^j = \sum_j y_{i}^j \quad (B.6.3) \]

\[ E_{i,world}^j = \sum_j E_i^j \quad (B.6.4) \]

where \( i = A, M, S \).

B.7 Market Clearing condition:

\[ y_{i,world}^j - E_{i,world}^j = 0 \quad (B.7.1) \]

It has been used to solve for the world price in each sector.

B.8

Exports and Imports in Classical Model:

\[ \ln(M_{jj}^j) = a_{jj}^j + b_{jj}^j \ln(E_T^j) + e_{ij}^j, \quad j \neq j \quad (B.8.1) \]

\[ \ln(X_{jj}^j) = c_{jj}^j + d_{jj}^j \ln(E_{jj}^j) + e_{ij}^j, \quad j \neq j \quad (B.8.2) \]

Exports and Imports in Gravity Model:

\[ \ln(M_{jj}^j / E_T^j) = c_{ij}^j + \psi_{i}^j \text{RXR}^j + e_{ij}^j \quad (B.8.3) \]
\[
\ln(X^j_{jj}/E^{jj}) = cx^j_{jj} + \psi_2RXj + ex^j_{jj}
\] (B.8.4)

\[jj = UK, US, Euro Area, China, and ROW; M^j_{jj} \text{ is the country } j \text{'s imports from country } jj, X^j_{jj} \text{ is the country } j \text{'s exports to country } jj; E^{jj} \text{ is the aggregate demand in country } jj. em^j_{jj} \text{ and } ex^j_{jj} \text{ are trade share error process. We estimate } a^j_{jj}, b^j_{jj}, c^j_{jj}, d^j_{jj}, cm^j_{jj}, cx^j_{jj} \text{ by OLS. } \psi_1, \psi_2 \text{ are the elasticities of demand to the real exchange rate at (import) } \psi_1 \text{ and (export) } \psi_2 \text{ in each country } j. RXR^j \text{ is the real exchange rate in country } j.\]

B.9 Current account balance:

\[M^j = \sum_{jj} M^j_{jj}, \quad j \neq jj\] (B.9.1)

\[X^j = \sum_{jj} X^j_{jj}, \quad j \neq jj\] (B.9.2)

\[NX^j = X^j - M^j\] (B.9.3)

\[0 = \sum_j NX^j\] (B.9.4)

where \(NX^j\) is the total net exports of country \(j\) in tradable sectors.

Appendix C: The Solution Procedure for the Global Trade Model

The endogenous variables in the structural model are solved as functions of the exogenous variables and errors. In the global trade model, we assume the \(POP^j, G^j, r^j, BoP^j, y^j_A\) are exogenous variables. Note that the agriculture output \(y^j_A\) as treated politically controlled and essentially fixed exogenously because of the highly interventionist planning system. All the other variables are endogenous variables.

In the UK trade model, Minford and Xu (2018) show how to solve the model when the world prices are exogenous. However, in the global trade model, the world prices are endogenous variables and determined by world market clear conditions. Hence, we
need to solve the world price firstly.

The following step summarise the steps to solve the world price.

**Step 1:** Given the world price \((p_M, p_S, p_A)\) and the productivity errors \((\ln(\pi_M^j), \ln(\pi_S^j), \ln(\pi_A^j))\) solve for \(w^j, h^j, l^j\) from factor price equations (See Full Model Appendix Section B.1).

**Step 2:** Given \(POP^j, G^j, w^j, b^j, h^j, l^j\) and the factor supply errors \((e_N^j, e_H^j)\), solve for labour supply \(N^j\) and \(H^j\) from factor supply equations (See Full Model Appendix Section B.2).

**Step 3:** Given the price \((p_M, p_S, p_A, p_D)\), agriculture output \((y_A^j)\), factor supply \((N^j, H^j)\), factor cost \((w^j, h^j)\) and the factor demand errors \((e_M^j, e_S^j, e_A^j)\), solve for \(y_M^j, y_S^j, y_D^j\) from output equations (See Full Model Appendix Section B.3).

**Step 4:** Given the price \((p_M, p_S, p_A, p_D)\) and output \(y_M^j, y_S^j, y_D^j\), solve for \(E_A^j, E_S^j\) from demand equations (See Full Model Appendix Section B.5).

**Step 5:** Given \(y_A^j, y_S^j, E_A^j, E_S^j\) for the five blocks, solve the world prices \((p_S, p_A)\) from world demand and supply equations,

\[
\sum_j y_A^j = \sum_j E_A^j
\]
\[
\sum_j y_S^j = \sum_j E_S^j
\]

where \(y_A^j, y_S^j, E_A^j, E_S^j\) are both functions of \(p_A\) and \(p_S\), which are derived in step 3 and 4. Note that \(p_M\) is normalised to be 1, so that \(p_A\) and \(p_M\) are relative prices to \(p_M\). If there are tariffs, the domestic price of each block equals the world price times the tariffs. \(p_S^i = (1 + T_S^i p_M^{\text{World}})\) for \(i = \text{UK, EU, US, China, ROW}\) \(p_A^i = (1 + T_A^i p_M^{\text{World}})\) for \(i = \text{UK, EU, US, China, ROW}\). In the base model, we assume \(T_S^i = 0\) and \(T_A^i = 0\).

**Step 6:** Given the world price, solve the other endogenous variables.

### Appendix D: Deriving the Indirect Inference Wald statistic

The following steps summarise our implementation of the Wald test by bootstrapping.

**Step 1:** Estimate the errors of the economic model conditional on the observed data
and parameters in the structural model. In the global trade model, the structural errors $\varepsilon_t$ includes productivity errors ($\ln(\pi_k^M), \ln(\pi_{fj}^L), \ln(\pi_{j}^A)$), factor supply errors ($e_{fM}^j, e_{fH}^j$) and factor demand errors ($e_{fM}^j, e_{fS}^j, e_{fA}^j$) for the five blocks, which can be derived from the output, factor supply and factor demand equations of the five blocks.

Step 2: Derive the simulated data Under the null hypothesis the $\{\varepsilon_t\}_{t=1}^T$ are the structural errors, the simulated disturbances are drawn from these errors. Many of the structural errors are assumed to be generated by autoregressive processes rather than being serially independent. If they are, then under our method we need to estimate them. We derive the simulated data by drawing the bootstrapped disturbances by time vector to preserve any simultaneity between them. Then from the bootstrapped errors and fixed exogenous variables, we derive the endogenous variables from the solved model (See Appendix B for the way of how to solve the model). To obtain the $N$ bootstrapped simulations we repeat this, drawing each sample independently.

Step 3: Compute the Wald statistic We estimate the auxiliary model — using both the actual data and the $N$ samples of simulated data to obtain estimates $a_T$ and $a_S(\theta_0)$ of the vector $a$. The distribution of $a_T - a_S(\theta_0)$ and its covariance matrix $W(\theta_0)^{-1}$ are estimated by bootstrapping $a_S(\theta_0)$. The bootstrapping proceeds by drawing $N$ bootstrap samples of the structural model, and estimating the auxiliary VAR on each, thus obtaining $N$ values of $a_S(\theta_0)$; we obtain the covariance of the simulated variables directly from the bootstrap samples. The resulting set of $a_k$ vectors ($k = 1, \ldots, N$) represents the sampling variation implied by the structural model from which estimates of its mean, covariance matrix and confidence bounds may be calculated directly. Thus, the estimate of $W(\theta_0)$ is

$$W(\theta_0) = \frac{1}{N} \sum_{k=1}^{N} (a_k - \bar{a}_k)'(a_k - \bar{a}_k) \quad (D.1)$$

where $\bar{a}_k = \frac{1}{N} \sum_{k=1}^{N} a_k$. We then calculate the Wald statistic for the data sample; we estimate the bootstrap distribution of the Wald from the $N$ bootstrap samples. The Wald statistics are given by

$$WS = (a_T - \bar{a}_S(\theta_0))' W(a_S(\theta_0))^{-1} (a_T - \bar{a}_S(\theta_0)) \quad (D.2)$$
References


