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# Should Hong Kong switch to Taylor Rule?—Evidence from DSGE Model

David Meenagh\*      Patrick Minford†      Zhiqi Zhao‡

June 22, 2021

## Abstract

This paper studies the economy of Hong Kong through the lens of a small open economy DSGE model with a currency board exchange rate commitment. It assumes flexible prices and a banking system that provides credit to entrepreneurial household-firms; the money supply is fully backed by reserves under the currency board. We estimate and evaluate the model by Indirect Inference over the sample period of 1994Q1-2018Q3; we find that it matches the data behaviour, as represented by a VAR. We examined the economy's volatility using bootstrapping of the model innovations, under both the estimated currency board model and a standard alternative regime with floating exchange rate and a Taylor rule; we found that Hong Kong welfare is higher in the currency board, which substantially reduces output volatility.

**Keywords:** Currency Board, Monetary Policy, Hong Kong, Indirect Inference

**JEL Codes:** E52, F41, G51

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# 1 Introduction

After 1972 when the Bretton Woods system collapsed, a majority of countries turned to floating exchange rates. Under this mainstream regime of floating exchange rates, monetary policy has been usually modelled as a Taylor rule, a rule setting interest rates to target inflation and real GDP. By the international trilemma, whereby an economy cannot have an independent monetary policy, free capital flows and a fixed exchange rate at the same time, the Taylor rule can work well under floating exchange rates and maintain free capital flows. However, plainly it could not operate under a currency board, the extreme case of pegged exchange rates.

Hong Kong is a typical and conventional currency board system, with the currency strictly linked to the US dollar. Before 1983, the Hong Kong dollar had silver standard and sterling standard, silver standard gives the bank notes issued backed by silver bullion, and notes are backed by UK government securities under sterling standard. In 1982, the Sino-British joint declaration resulted in a sharp depreciation in the Hong Kong dollar, in a sequence of speculative attacks, after a drop in confidence about the future. The Hong Kong dollar experienced a 'Black Saturday' in 1983 as can be seen as in Figure 1. To maintain the stability of the economy, the currency and financial markets, the Hong Kong authority turned to a currency board and fixed exchange rate.

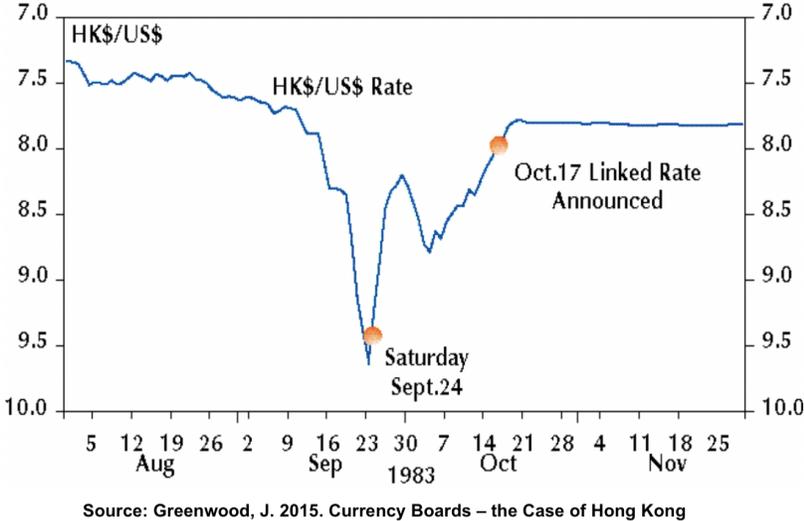


Figure 1: Hong Kong Exchange Rate in 1983

As Figure 2 shows, under this arrangement, the Hong Kong interest rate will typically equal the US interest rate, since the HK dollar is simply a fixed conversion of the US dollar. The only exception was in the 1997 Asian Financial Crisis, when the interest rate rose sharply on fears the Hong Kong dollar would be devalued. After the crisis, during which the rate was held, the normal equality resumed.

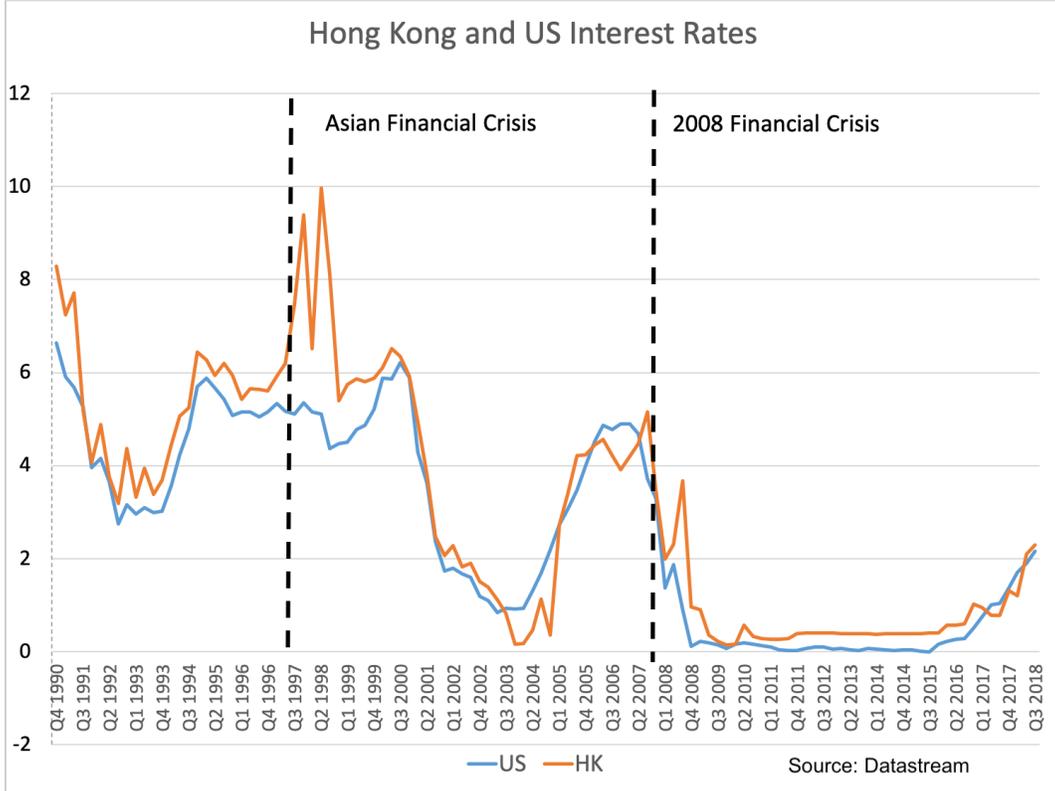


Figure 2: Hong Kong and US Interest Rate

This paper aims to shed light on the following two questions: how the Hong Kong economy works under the currency board; and whether Hong Kong should abandon the currency board for an independent monetary policy with floating exchange rates. To address these questions, we build a DSGE model similar to Le et al. (2016) in which there is both a banking sector and money as cheap collateral. We estimate and test the model by Indirect Inference against the Hong Kong data; the results tell us that this currency board model can fit Hong Kong data well for 1994Q1 to 2018Q3. For the interest rate, the driving force is the foreign interest rate while productivity and consumer preferences are the main sources of other variables' fluctuations. By bootstrapping the model shocks under the alternative Taylor rule regime, we find that the currency board regime generates more stability and less welfare loss.

The rest of paper proceeds as follows. The next section outlines the DSGE model. The third section introduces the indirect inference method, as well as the data and initial parameter calibration. In the fourth section, we estimate and test the model by indirect inference; we also test and reject an alternative model, in which the housing market also acts as collateral for borrowing by impatient consumers. In the fifth section, we discuss the empirical findings from our estimated model, and analyse its behaviour. In the next section, we evaluate whether moving to the alternative floating exchange rate regime would give any welfare gains. The final section concludes.

## 2 Model

The economy is populated by households, entrepreneurs and housing firms. Households buy consumption goods both from the home and the foreign country, work for the entrepreneurs and consume housing. The entrepreneur produces consumption goods by using capital and labour. Housing firms convert investment goods into housing for households. A banking sector lends to entrepreneurs and takes deposits from households as in the Bernanke et al. (1999) financial accelerator model, as modified by Le et al. (2016) who introduce collateral and money into the model. We further assume no price/wage rigidity in the model, given the flexibility of the Hong Kong economy.

### 2.1 Households

The representative patient household maximises the expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta_c^t [\gamma_t^c \log C_t^c + \gamma_t^h \log H_t - \frac{N_t^{1+\eta}}{1+\eta}] \quad (1)$$

Where households' utility is from current consumption  $C_t^c$ , housing  $H_t$  and disutility from working  $N_t$ . Here are inverse elasticity of labour  $\eta$ , consumption shock  $\gamma_t^c$ , housing demand shock  $\gamma_t^h$ . These two shocks follow AR(1) process with i.i.d normal distribution.

This maximisation problem is subject to households' budget constraint:

$$P_t C_t^c + P_t I_t^k + P_t^h [H_t - (1 - \delta^h) H_{t-1}] + B_t + S_t B_t^f = W_t N_t + R_t^k K_{t-1} + (1 + R_{t-1}) B_{t-1} + (1 + R_{t-1}^f) S_t \phi_{t-1} B_{t-1}^f + T_t \quad (2)$$

and capital accumulation function with investment adjustment cost:

$$K_t = (1 - \delta^k) K_{t-1} + [1 - S(\frac{I_t^k}{I_{t-1}^k})] I_t^k \quad (3)$$

$\kappa^k$  is a parameter measures the adjusting investment cost where the cost is  $S(\frac{I_t^k}{I_{t-1}^k}) = \frac{\kappa^k}{2} (\frac{I_t^k}{I_{t-1}^k} - 1)^2$ , while  $S(1) = S'_k(1) = 0$ ,  $S''_k(1) = \kappa^k$ .

For every period, households buy consumption goods, make investment decisions and purchase new housing with a relative housing price  $q_t^h = \frac{P_t^h}{P_t}$ , and purchase domestic and foreign bonds. At the same time, households receive wage  $w_t$  from working, return from physical capital rent, return from, domestic bonds and foreign bonds with their rates  $R_{t-1}$ ,  $R_{t-1}^f$  respectively.  $T_t$  is the lump-sum transfer. To ensure there is a well-defined steady state, this model follows Schmitt-Grohe and Uribe (2003), as well as Adolfson et al. (2007) that there is a risk premium which depends on the ratio of net foreign assets position.  $S$  is the nominal exchange rate and to be set fixed for a currency board.

$$\phi_t = \exp[-\phi_a (Z_t - \bar{Z})] \quad (4)$$

where  $\phi$  is the elasticity of country risk premium,  $Z_t$  is total foreign assets position including the foreign bonds held in the public and those foreign reserve held in the monetary authority, where  $Z_t = B_t^f + F_t$ .

By choosing  $C_t^c, H_t, I_t^k, K_t, N_t, B_t, B_t^f$ , FOCs of households are as following:

$$C_t^c : \lambda_t^c = \frac{\gamma_t^c}{P_t C_t^c} \quad (5)$$

$$I_t^k : q_t^k [1 - S(\frac{I_t^k}{I_{t-1}^k}) - S'(\frac{I_t^k}{I_{t-1}^k}) \frac{I_t^k}{I_{t-1}^k}] + \beta_c E_t [\frac{\lambda_{t+1}^c}{\lambda_t^c} q_{t+1}^k S'(\frac{I_{t+1}^k}{I_t^k}) (\frac{I_{t+1}^k}{I_t^k})^2] = 1 \quad (6)$$

$$K_t : q_t^k = \beta_c E_t \frac{\lambda_{t+1}^c}{\lambda_t^c} [(1 - \delta^k) q_{t+1}^k + R_{t+1}^k] \quad (7)$$

$$H_t : \frac{\gamma_t^h}{H_t} = \lambda_t^c P_t^h - \beta_c E_t \lambda_{t+1}^c P_{t+1}^h (1 - \delta^h) \quad (8)$$

$$N_t : N_t^\eta = \lambda_t^c W_t \quad (9)$$

$$B_t : \lambda_t^c = \beta_c E_t \lambda_{t+1}^c (1 + R_t) \quad (10)$$

$$B_t^f : \lambda_t^c = \beta_c E_t \lambda_{t+1}^c (1 + R_t^f) \phi_t \frac{S_{t+1}}{S_t} \quad (11)$$

The Euler equation for consumption can be given by combining (5) and (10):

$$\frac{\gamma_t^c}{C_t^c} = \beta_c E_t \frac{\gamma_{t+1}^c}{C_{t+1}^c} \frac{(1 + R_t)}{\pi_{t+1}} \quad (12)$$

The optimal condition for housing is from (5) and (8):

$$\frac{\gamma_t^h}{H_t} = \frac{\gamma_t^c}{C_t^c} q_t^h - \beta_c E_t \frac{\gamma_{t+1}^c}{C_{t+1}^c} q_{t+1}^h (1 - \delta^h) \quad (13)$$

Given (5) and (9), the intratemporal condition yields. This condition gives that marginal substitution between consumption and leisure is equal to the real wage.

$$N_t^\eta C_t^c = \frac{W_t}{P_t} \quad (14)$$

The international no arbitrage condition can be taken from (10) and (11):

$$E_t \left( \frac{1 + R_t}{\pi_{t+1}} \right) = E_t \left( \frac{(1 + R_t^f) \phi_t}{\pi_{t+1}} \right) \frac{S_{t+1}}{S_t} \quad (15)$$

$\phi_t$  is the country risk premium discussed in equation (4) which depends on the net foreign assets position. On one hand, it is to explain the fact the lenders would require higher return with those countries in higher debt position. On the other hand, it is to avoid misspecification and singularity problem in closing the model.

The UIP in log-linearised:

$$\hat{r}_t = \hat{r}_t^f + \Delta S_{t+1} - \phi \hat{z}_t$$

As the Hong Kong has fixed exchange rate,  $\Delta S_{t+1} = 0$ , the UIP is:

$$\hat{r}_t = \hat{r}_t^f - \phi \hat{z}_t$$

## 2.2 Entrepreneurs

Entrepreneurs behave as the final goods producer who hire labour and buy capital from households, applying the funds from bank and net worth from themselves to acquire capital. <sup>[1]</sup> Entrepreneurs are risk neutral and have a constant survival rate to the next period, so that entrepreneurs will always need external funds to finance its cost of capital requirement. The set up here has one special state that there exists a perfect competition market in the domestic goods market, which is for the fully flexible economic environment in Hong Kong. This is because Hong Kong is a really small economy that there is no firm is able to set the price. The rest settings of this sector and the external finance premium follow the BGG framework extended by Le et al. (2016) and Gilchrist et al. (2009).

Entrepreneurs maximise the profit from producing goods with the profit function by choosing how much labour to hire and how much capital to operate with cost of capital funds  $R_t^k$ :

$$P_t^d Y_t - W_t N_t - R_t^k K_{t-1} \tag{16}$$

Where  $P_t$  is the general price level,  $N_t$  is labour and  $K_{t-1}$  is capital. The corresponding nominal wage and rental rate are  $W_t$  and  $R_t^k$ .

Subject to the following production technology:

$$Y_t = A_t K_{t-1}^\alpha N_t^{1-\alpha} \tag{17}$$

Here  $A_t$  is the technology process follows ARIMA(1,1,0) process, the log-linearised equation:

$$\hat{a}_t - \hat{a}_{t-1} = \rho_a (\hat{a}_{t-1} - \hat{a}_{t-2}) + \varepsilon_t^a \tag{18}$$

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<sup>1</sup>It may seem puzzling that the household budget constraint contains the whole capital stock, when it sells it to entrepreneurs. However, these entrepreneurs and the bankers who extend credit to them are both subgroups of the whole household sector. Hence the capital stock never moves outside the household sector; it is passed around within it to enable it to be used to produce intermediate output via lending from banking households that embed the credit friction into the cost of capital. They make zero profit, so that household income still consists simply of wage and capital income, paid out of output by entrepreneurs. The credit friction creates an incentive for households to set up as ‘shadow banks’, lending directly (via P2P) to entrepreneurs on an equity basis, cutting out the credit friction. However, in this model this is not permitted.

First order conditions of entrepreneur sector are: Marginal production of labour and labour demand:

$$\frac{W_t}{P_t^d} = (1 - \alpha)A_t K_{t-1}^\alpha N_t^{-\alpha} \quad (19)$$

Marginal production of capital and capital demand:

$$\frac{R_t^k}{P_t^d} = \alpha A_t K_{t-1}^{\alpha-1} N_t^{1-\alpha} \quad (20)$$

Additionally, entrepreneurs need external funds to finance the cost of buying capital. The external finance premium framework is taken from Gilchrist et al. (2009) and Le et al. (2013), with an extension to include money in the external finance premium from Le et al. (2016). Every period, entrepreneurs need to finance their capital costs,  $q_t^k K_t$ , partly with external funds and partly with their net worth  $NW_t$ , via a loan contract with the banks. The loan contract contains a threshold value for an idiosyncratic shock which impacts on the expected return on capital. When the shock hits this threshold value or above, the firm repays the loan while when it comes in below the threshold, the firm defaults. The optimal loan contract ensures that the expected return on bank lending equals to the bank's cost of lending. This implies the following log-linearised condition for the external finance premium and credit rate, as in Bernanke et al. (1999), Gilchrist et al. (2009) and Le et al. (2016) :

Log-linearised external finance premium:

$$E_t c y_{t+1} - (r_t - E_t \pi_{t+1}) = \chi (q_t^k + k_t - n w_t) \quad (21)$$

where the left hand indicates the return of capital equals the real opportunity cost of risk-free deposit with a premium on it,  $c y_t$  is borrowing rate or the credit rate; while the right hand includes the leverage ratio and positive  $\chi$  measures the elasticity of premium to the leverage ratio,  $n w_t$  is entrepreneur net worth given by a fixed survival rate firms' net worth from past plus the total return on capital, minus the expected return or cost on the external financing:

Log-linearised net worth evolution is given by:

$$n w_t = \nu n w_{t-1} + \frac{K}{NW} (c y_t - E_{t-1} c y_t) + E_{t-1} c y_t \quad (22)$$

where  $\nu$  is the survival rate which is assumed to be fixed and  $\frac{K}{NW}$  is the steady state capital to net worth ratio. As those who cannot survive would consume their net worth, the entrepreneur consumption in each period would equals to  $(1 - \nu)$  of the total net worth, which follows log-linearised equation:

$$c_t^e = n w_t \quad (23)$$

Following Le et al. (2016), we here introduce collateral in the loan contract, with money acting as a cheap form of collateral. Firms hold some cash on the balance sheet, which can be recovered

at no value loss and no verification cost. As in that paper, we assume that firms hold non-interest-bearing cash deposits in the bank while households hold savings deposits (yielding a safe interest rate like government bonds); the cash is held by banks on their balance sheets as bank reserves with the Monetary Authority, which in turn holds matching foreign exchange reserves [\[2\]](#). The difference is that in Le et al. (2016), the model contains open market operations in domestic bonds. However, here there are no open market , only foreign exchange intervention through the foreign reserves. Similarly, the modified credit premium equation in this thesis is:

$$E_t c y_{t+1} - (r_t - E_t \pi_{t+1}) = \chi(q_t^k + k_t - n w_t) - \mu m_t^d \quad (24)$$

The money demand is from the firm's balance sheet that firm holds money as collateral to its borrowing to finance the cost of capital, in the form as money to capital demand ratio together with the firm's net worth:

$$m_t^d = (1 + \xi)k_t - \xi n w_t \quad (25)$$

where  $\xi$  is the net worth to money ratio in steady state. As we will see below, this firms' demand for money is supplied automatically at the fixed exchange rate by foreign exchange intervention to keep the currency fixed.

### 2.3 Housing Producer

Housing producer is to maximise its profit by choosing the level of  $I_t^h$ , following the Smets and Wouters (2007), Christiano et al. (2005) the set up for the capital producer, the housing producer behaves similarly and the maximising problem is:

$$\max E_0 \sum_{t=0}^{\infty} \Lambda_{0,t}^e [q_t^h (H_t - (1 - \delta^h) H_{t-1}) - I_t^h] \quad (26)$$

subject to the law of motion in housing:

$$H_t = (1 - \delta^h) H_{t-1} + [1 - \frac{\kappa^h}{2} (\frac{I_t^h}{I_{t-1}^h} - 1)^2] I_t^h \quad (27)$$

This dynamic profit maximisation problem can be solved with the real price of housing  $q_t^h = \frac{P_t^h}{P_t}$ :

$$q_t^h [1 - S(\frac{I_t^h}{I_{t-1}^h}) - S'(\frac{I_t^h}{I_{t-1}^h}) \frac{I_t^h}{I_{t-1}^h}] + \beta_e E_t [\frac{\lambda_{t+1}^e}{\lambda_t^e} S'(\frac{I_t^h}{I_{t-1}^h}) (\frac{I_t^h}{I_{t-1}^h})^2] = 1 \quad (28)$$

$\kappa^h$  is a parameter measures the adjusting investment cost where the cost is  $S(\frac{I_t^h}{I_{t-1}^h}) = \frac{\kappa^h}{2} (\frac{I_t^h}{I_{t-1}^h} - 1)^2$ , while  $S(1) = S'_h(1) = 0, S''_h(1) = \kappa^h$ .

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<sup>2</sup>Le et al. (2016) show the approval in the appendix 1 that with bankruptcy and bank contract decision, the rise in the money would decrease the required return on capital and the credit premium as well

## 2.4 Imports and Exports

With the spirit of small open economy in Armington (1969), Gali and Monacelli (2005), and Minford and Meenagh (2019), the total consumption index  $C_t$  is a CES function of domestic consumption goods  $C_t^d$  and foreign imported consumption goods  $IM_t$

$$C_t = [\omega^{\frac{1}{\theta}} (C_t^d)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} (IM_t)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} \quad (29)$$

and the bundle of the total consumption should satisfy the expenditure constraint of domestic consumption and imported consumption:

$$C_t = \frac{P_t^d}{P_t} C_t^d + Q_t IM_t \quad (30)$$

Where  $\omega$  is the home bias preference towards domestic goods and  $\theta$  measures the elasticity of substitution between domestic and foreign goods, while  $Q_t$  denotes the real exchange rate  $\frac{SP_t^f}{P_t}$ .  $S$  is the nominal exchange rate it is set to be fixed by currency board,  $P_t^d$  is the domestic goods price,  $P_t^f$  is the foreign price. The optimal allocation of the domestic demand for domestic goods and imported goods can be found by the following composite utility index maximisation:

$$\mathcal{L} = [\omega^{\frac{1}{\theta}} (C_t^d)^{\frac{\theta-1}{\theta}} + (1-\omega)^{\frac{1}{\theta}} (IM_t)^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}} + \Lambda_t (C_t - \frac{P_t^d}{P_t} C_t^d + Q_t IM_t) \quad (31)$$

by choosing  $C_t^d, IM_t$ , optimal conditions are:

$$C_t^d = \omega \left( \frac{P_t^d}{P_t} \right)^{-\theta} C_t \quad (32)$$

And domestic demand for foreign goods, which is hence the import demand:

$$IM_t = (1-\omega) \left( \frac{SP_t^f}{P_t} \right)^{-\theta} C_t \quad (33)$$

Consumer price index (CPI):

$$P_t = [\omega (P_t^d)^{1-\theta} + (1-\omega) (SP_t^f)^{1-\theta}]^{\frac{1}{1-\theta}} \quad (34)$$

Symmetrically, the export demand, or the foreign demand for domestic goods can be given as:

$$EX_t = (1-\omega^f) \left( \frac{P_t}{SP_t^f} \right)^{\theta^f} C_t^f \quad (35)$$

$\omega^f, \theta^f$  are home bias preference and elasticity of substitution in foreign economy. By assuming the small open economy, this model treats foreign variables  $\{C_t^f, R_t^f, \pi_t^f\}$  follows AR(1) process, and i.i.d innovation  $\varepsilon_{cf,t}, \varepsilon_{rf,t}, \varepsilon_{\pi^f,t}$  respectively with the definition of foreign policy shock, export demand shock and foreign price shock.

## 2.5 Monetary Operation and Currency Board

As the banking sector and currency board sector are the main to explain the monetary system for currency board in Hong Kong, the Figure 3 is to explain the full mechanism.

Currency Board	
Assets	Liabilities
Reserve	Reserve Certificates
Note Issuing Banks	
Assets	Liabilities
Reserve Certificates	Notes and Coins (M0)
Domestic Banks	
Assets	Liabilities
Loans	Deposits
Non-Bank Public and Government	
Assets	Liabilities
Net Foreign Assets (NFA)	$\sum S$
Deposit	Loans
$K = \sum I$	
M0	
Rest of the World	
Assets	Liabilities
$\sum NX$	NFA
	Reserve

Figure 3: The Balance Sheet of Hong Kong Economy and Currency Board

There are two main channels of overseas monetary transmission. First, as net foreign assets fall with current account deficits, the risk-premium on the HK dollar rises pushing up interest rates. Second, at this interest rate HK entrepreneurs can acquire the money they demand, by borrowing from abroad: equivalently excess money demand creates a slight rise in interest rates, causing money to flow in via private capital flows. This raises the reserves within total net foreign assets, increasing private foreign liabilities; money supply rises in line with reserves, meeting the money demand. Hence money demand in HK is automatically supplied via the balance of payments, in this currency board system just like in any fixed exchange rate regime.

We can summarise this second channel in the following equation:

$$S_t F_t = M_t^s = M_t^d \quad (36)$$

where  $S_t = \bar{S}$  is fixed.

The natural or automatic mechanism in Hong Kong monetary system is not by adjusting the interest rate, but by buying foreign reserve and printing money. It means that there is no Taylor style interest rate and targeting rule in this model. Instead, the monetary authority's foreign exchange intervention supplies any money demanded to hold the exchange rate fixed. Any excess demand for domestic money would cause an increase in the money supply via the foreign exchange reserve by the foreign exchange market.

## 2.6 Government

With no ability to print money in excess of demand, the Hong Kong government must finance its spending by taxes or borrowing, subject to its intertemporal budget constraint:

$$G_t + (1 + r_{t-1})B_{t-1} = B_t + T_t \quad (37)$$

$G_t$  follows AR(1) process and allows government spending shock  $\varepsilon_{g,t}$ .

## 2.7 Balance of Payment

Balance of payment with foreign exchange reserve

$$Z_t = (1 + r_{t-1}^f)Z_{t-1} + \frac{EX}{Q_t} - IM_t \quad (38)$$

$Q_t$  for real exchange rate.

## 2.8 Market Clearing Conditions and Identities

Total foreign assets:

$$Z_t = B_t^f + F_t \quad (39)$$

Goods market:

$$Y_t = C_t^c + C_t^e + I_t^k + I_t^h + G_t + EX_t - IM_t \quad (40)$$

Gross inflation:  $\pi_t = \frac{P_t}{P_{t-1}}$ .

Relative price of house:  $q_t^h = \frac{P_t^h}{P_t}$ ;

## 3 Indirect Inference

Our aim in estimating this model is to obtain a model that can be regarded as consistent with the data, according to powerful probability-based tests, and so reliable for evaluating policy, as we propose to do in comparing the currency board with a floating exchange rate regime. We have no prior beliefs about any parameters that we feel can be regarded as reliable, which rules out Bayesian estimation, widely used though this has been in applied macroeconomics. Le et al. (2016) compare ML and Indirect Inference for this estimation objective when as here small samples are

involved. They show that II gives substantially greater potential power in testing than ML, and offers a much smaller estimation bias.

We now set out to explain Indirect Inference (II), developed by Le et al. (2011) building on Smith (1993). II is based on the idea that if the structural model is true in terms of both specification and parameters, the properties of the actual data should come from the distribution of the properties of the simulated data with some critical minimum probability.

The II method has been in familiar use for many years, in the form of the Simulated Method of Moments, SMM; recent developments have generalised it as Indirect Inference, allowing considerable flexibility in the choice of data features to be matched, known as the 'auxiliary model'. It has been used increasingly widely in applied work- Akcigit and Kerr (2018), Guvenen and Smith (2014), Minford and Peel (2019, chapter 17) surveys its spreading use in applied macro modelling. The approach involves hypothesising that the model being estimated is the true data generating mechanism, DGM; the data is then succinctly described by, for example, moments under SMM. If so then the moments found in the data should come from the model with a probability in excess of the threshold rejection level of 5%, when the usual 95% confidence level is used. To discover the probability distribution of the Moments according to the model, the model is simulated by bootstrapping the random shocks perturbing it many times; the resulting joint distribution of the moments is what the model implies if it is the true DGM. If the data-based moments have a probability less than 5% according to this distribution, the model is rejected. Estimation by II involves searching over model parameters to find the set that is least rejected above the 5% level — this set is the II estimator.

The data properties can be captured by a simple 'auxiliary model' such as a VAR, impulse response functions or the moments as in the SMM. It turns out (Meenagh et al., 2019) that the results are similar in each case. Define the parameters of the structural model and the auxiliary model as  $\theta$  and  $\alpha$  respectively. We first use the actual data to estimate the auxiliary parameters, say  $\alpha$ . Given the null hypothesis  $H_0 : \theta = \theta_0$ , we simulate  $S$  samples using the structural model and estimate the auxiliary parameters using each simulated sample to obtain estimators  $\alpha_s(\theta_0); s = 1, \dots, S$ . To evaluate whether  $\alpha$  comes from the distribution of  $\alpha_s(\theta_0)$ , we compute the Wald statistic:

$$Wald(\alpha) = (\alpha - \alpha_s(\theta_0))'W(\theta_0)^{-1}(\alpha - \alpha_s(\theta_0)) \quad (41)$$

which asymptotically follows a  $\chi^2(k)$  distribution where  $k$  is the number of elements in  $\alpha$  and  $W(\theta_0)$  is the variance-covariance matrix of  $\alpha - \alpha_s(\theta_0)$ . We can check the allocation of  $Wald(\alpha)$  in the distribution of simulated  $Wald(\alpha_s); s = 1, \dots, S$  where  $Wald(\alpha_s)$  is computed when using the  $s^{th}$  simulated sample to estimate  $\beta$ . If  $Wald(\alpha)$  is less than the  $c^{th}$  percentile value of  $Wald_s$  sorted from smallest to largest,  $H_0$  cannot be rejected in a  $c\%$  confidence interval; otherwise the model is false. An alternative way is to compute the transformed Mahalanobis Distance (TMD) and compare it with the critical value of  $t$  distribution on the  $c\%$  confidence interval.

To evaluate the power of II on our model here, we use Monte Carlo experiments to compute the power of the test against parameter mis-estimation. As can be seen in the next section the power

of our test here is considerable, giving us a guarantee that our estimates are reasonably close to the truth.

### Power of the Test for our model of Hong Kong

Here we ask the question: how powerful is the test on this model? How likely is the model to be rejected if the model is somehow falsified?

Following Le et al. (2016), the power of the test is conducted by the following steps:

*Step 1. Generate simulations from true model*

We treat the estimated model, together with its innovations as the 'true' model; we generate 1000 simulations from it and treat these as potential samples of data.

*Step 2. Falsify true model*

We falsify the true model by mis-specifying the estimated parameters of the model and innovations by x% in an alternating way: odd-number parameters reduced, even number parameters increased.

*Step 3. Generate simulations from true model*

Treating the simulations from the 'true' model as the 'true' data, we test the false model on each of the data sets. The power is then measured by the frequency with which the false model is rejected at 95% confidence by these data sets; of course the true model is rejected 5% of the time by construction.

The results are reported in Table 1. It can be seen that indirect inference test of this model is highly reliable as the test is very powerful, more than 50% of the experiments are rejected if the model deviates from the 'true' by 5% and the probability reaches nearly 90% if the false rate increases to 7%. This three variables VAR would hence be an appropriate choice for the auxiliary model, as it can generate a high degree of power without being too impossibly difficult to pass. If we were to increase the number of variables in the VAR, or increase the order of the VAR, the power would be increased. A too powerful VAR would imply that a good close-to-true model would be rejected.

Table 1: Power of Indirect Inference Test

VARX represent: 3 variable-VARX(1) (Y, r, $\pi$ )								
False Rate	True	1%	3%	5%	7%	10%	15%	20%
Power	5%	8%	29.7%	66.2%	89.6%	97%	100%	100%

## 4 Empirical Findings

### 4.1 Indirect Inference Estimation and Test Result.

Indirect Inference estimation finds the structural parameters that minimise the distance between the simulated data and actual data. The process searches randomly from calibrated starting values

taken from earlier work, including Smets and Wouters (2007), Bernanke et al. (1999), Funke and Paetz (2011) and Le et al. (2014). They are shown in Table 2 below, together with the values that emerge from estimation. Key calibrated parameters include the inverse elasticity of labour supply  $\eta$  set at 3; the households discount factor  $\beta_c$  is 0.9929, and the corresponding quarterly steady state interest 0.72%; The output elasticity of capital  $\alpha$  is standard to be 0.3 Capital depreciation rate  $\delta^k$  for 0.025, while the housing depreciation  $\delta^h$  is 0.01. In the bundle of consumption goods, the home bias  $\omega$  for 0.4 and elasticity between domestic goods and imported goods  $\theta$  for 1. Symmetrically, the foreign home bias  $\omega^f$  and foreign elasticity between foreign domestic goods and exported goods from home country  $\theta^f$  are 0.4 and 1 respectively. Capital adjustment cost parameters in physical capital and housing  $\kappa^k$  and  $\kappa^h$  are 6.

A number of ratios are taken from the means of the sample data; and these are held fixed. Thus for the goods market, the consumption to output ratio  $\frac{C}{Y}$  is 0.6367, house investment to output ratio  $\frac{I^h}{Y}$  is 0.1148, the capital investment to output ratio  $\frac{I^k}{Y}$  is 0.1471, the government spending to output ratio  $\frac{G}{Y}$  is 0.1051, the export to output ratio  $\frac{EX}{Y}$  is 1.6803 and import to output ratio  $\frac{IM}{Y}$  is 1.6571. These export and import ratios reflect Hong Kong's role as an international port, with a large re-export business. Pure domestic exports account for about 5% of total exports.

After estimation, it can be seen that the Wald statistic for a VAR of the three central variables, output, interest rate, inflation  $(Y, r, \pi)$  is statistically significant and not rejected by the indirect inference test, with a p-value of 0.12; notice that unsurprisingly the model with the initially calibrated parameter values is strongly rejected.

Table 2: Indirect Inference Estimates of the Structure Model

Parameter	Definition	Calibration	Estimation
$\alpha$	Capital Share in Production	0.3	0.3443
$\eta$	Inverse Elasticity of Labour Supply	3	5.0880
$\delta^k$	Capital Depreciation	0.025	0.0177
$\delta^h$	Housing Depreciation	0.01	0.011
$\omega$	Domestic Home Bias	0.4	0.1822
$\theta$	Elasticity between Domestic and Imported goods in Home	1	1.5340
$\omega^f$	Foreign Home Bias	0.4	0.1809
$\theta^f$	Elasticity between Domestic and Imported goods in Foreign	1	1.2499
$\kappa^k$	Capital Investment Adjustment Cost	6	6.4153
$\kappa^h$	Housing Investment Adjustment Cost	6	11.3376
$\chi$	Feedback from Leverage to Finance Premium	0.05	0.0287
$\mu$	Feedback from Money to Finance Premium	0.7	0.8971
	Variable in the VARX(1)	Trans-W	p-value
Calibration	$Y, r, \pi$ (Output, Interest rate, Inflation)	2.694	0.006
Estimation	$Y, r, \pi$ (Output, Interest rate, Inflation)	1.0924	0.122

### 4.1.1 Residuals and Shocks Property in Estimated Model

Residuals or errors are calculated from the data with the estimated coefficients. In order to determine the time-series process for an error, we need to determine its order of integration. We test for its stationarity via both the ADF test and KPSS test: Table 3 provides the results of these tests which may well conflict, since the ADF test has the null hypothesis of unit root (non-stationarity), while the KPSS test has the null hypothesis of stationarity. Ultimately, the Wald test decides the nature of the error processes; we search for those that can pass the test, using the Table 3 results as a guide to potentially successful error specifications. Figure 4 shows the structural residuals from the estimated model, and Figure 5 the innovations from their chosen time-series process. The successful specification treats all the shocks bar productivity as (trend) stationary; the change in productivity is due to innovation, which suggests it should indeed be a non-stationary process.

Table 3: Residual Stationarity Test and AR(1) Coefficients

Residual	Stationarity Test		Conclusion	AR(1)
	ADF p-value	KPSS stats		
Consumption Preference	0.0364**	0.1431*	Trend Stationary	0.9207
Housing Demand	0.0092	0.281274***	Trend Stationary	0.9256
Productivity	0.9485***	1.1235***	Non-stationary	0.1804†
Government Spending	0.4239***	0.2953	Stationary	0.9852
Foreign Consumption	0.0056	0.1316	Stationary	0.8693
Foreign Inflation	0.001	0.4254*	Stationary	0.6868
Foreign Interest Rate	0.0094	0.0873	Stationary	0.8624

1. KPSS \*, \*\*\* indicates rejection of stationary at 10% and 1% respectively.
2. ADF p-value \*\*, \*\*\* indicates do not reject unit root at 1% and 10% respectively.
3. † The AR(1) coefficient of productivity is for the first order differenced one.

Figure 4: Estimated Model Innovations

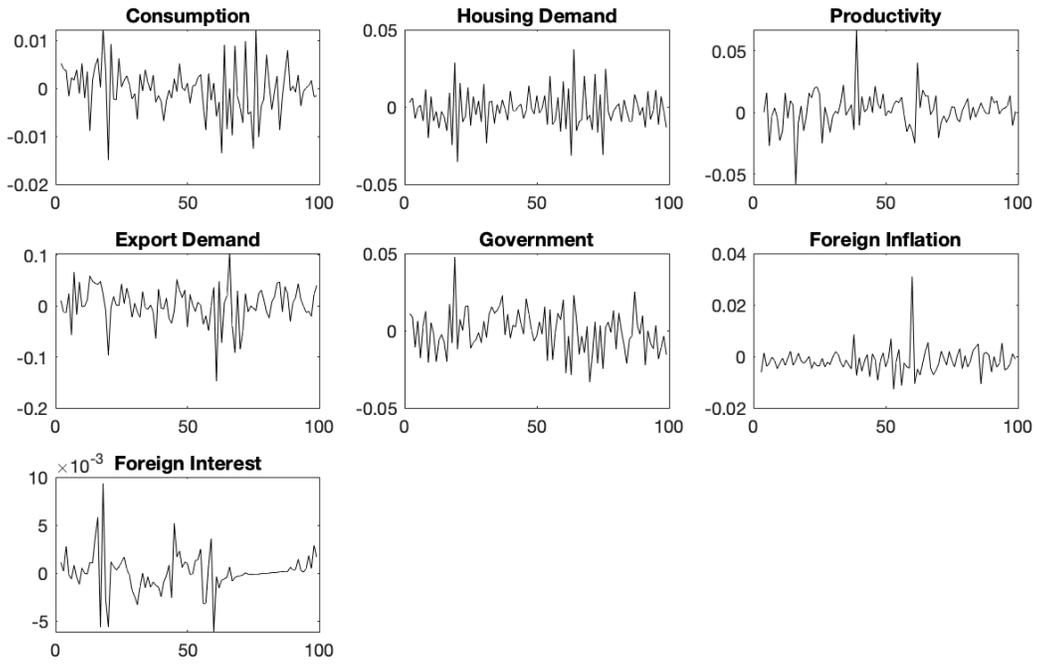
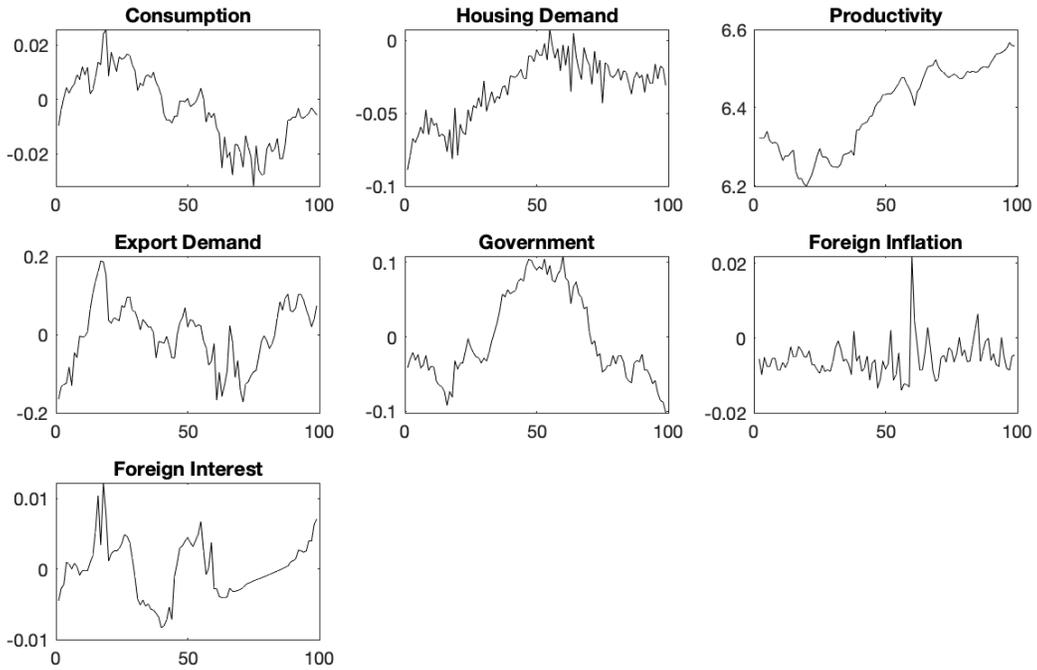


Figure 5: Estimated Model Structure Errors



### 4.1.2 Will a collateral constraint model help to fit the data?

The benchmark model has provided an outline on how the monetary system works with the currency board framework. It passes the indirect inference test to explain the main economic activities, as well as fitting the behaviour in the financial crisis. While our model is fairly rich in monetary transmission mechanisms, including as it does both a banking financial accelerator and money as cheap collateral, it has been argued that the housing market in Hong Kong could also be a source of business cycle transmission via its use as collateral for consumer borrowing, following the model of Iacoviello (2005), Funke and Paetz (2012, 2013). This housing collateral model has not been tested against the Hong Kong data with Indirect inference; in this section we ask if it can contribute to explaining HK experience. In what follows we set out a model augmented with this mechanism, and test it too by indirect inference.

In this model, the households sector is split into a patient and an impatient group. Patient households and other sectors behave as in the benchmark model, while Impatient households borrow to cover the cost of consumption and housing purchase, the borrowing facing an upper bound which cannot exceed a proportion of the housing value:

#### Impatient Households

The representative impatient household maximises the expected utility:

$$E_0 \sum_{t=0}^{\infty} \beta_I^t [\gamma_t^c \log C_{I,t} + \gamma_t^h \log H_{I,t}] \quad (42)$$

Where patient households' utility is from current consumption  $C_{I,t}$ , housing  $H_{I,t}$ . This maximisation problem is subject to households' budget constraint:

$$P_t C_{I,t} + P_t^h [H_{I,t} - (1 - \delta^h) H_{I,t-1}] + (1 + R_{t-1}) L_{t-1} = L_t$$

equivalent to:

$$C_{I,t} + q_t^h [H_{I,t} - (1 - \delta^h) H_{I,t-1}] + \frac{(1 + R_{t-1}) l_{t-1}}{\pi_t} = l_t \quad (43)$$

and borrowing constraint:

$$L_t \leq m E_t \frac{P_{t+1}^h H_{I,t}}{1 + R_t} \quad (44)$$

equivalent to the real borrowing (or loan) constraint:

$$l_t \leq m E_t \frac{q_{t+1}^h \pi_{t+1} H_{I,t}}{1 + R_t}$$

by choosing  $C_{I,t}$ ,  $H_{I,t}$ ,  $L_t$ , the FOCs of the impatient households are:

$$C_{I,t} : \lambda_t^I = \frac{\gamma_t^c}{P_t C_{I,t}} \quad (45)$$

$$L_t : \beta_I E_t \lambda_{t+1}^I (1 + R_t) = \lambda_t^I - \lambda_t^{I'} \quad (46)$$

$$H_{I,t} : \frac{\gamma_t^h}{H_{I,t}} = \lambda_t^I P_t^h - \beta_I E_t \lambda_{t+1}^I P_{t+1}^h (1 - \delta^h) - \lambda_t^{I'} m \frac{E_t P_{t+1}^h}{1 + R_t} \quad (47)$$

Given equation (49) and (50), the housing condition (51) can be:

$$\frac{\gamma_t^h}{H_{I,t}} = \frac{q_t^h}{C_{I,t}} \gamma_t^c - \beta^I (1 - \delta^h) \frac{q_{t+1}^h}{C_{I,t+1}} \gamma_{t+1}^c - \left[ \frac{P_{t+1}}{C_t P_t} \gamma_t^c - \beta^I E_t \frac{1 + R_t}{C_{I,t+1}} \gamma_{t+1}^c \right] m E_t \frac{q_{t+1}^h}{1 + R_t} \quad (48)$$

### Aggregation

Total consumption:

$$C_t = C_{P,t} + C_{I,t}$$

Total housing:

$$H_t = H_{P,t} + H_{I,t}$$

Table 4: Indirect Inference Estimates of the Structure Model with Housing Collateral

	Variable in the VARX(1)	Trans-W	p-value
Benchmark	$Y, r, \pi$ (Output, Interest rate, Inflation)	1.0924	0.122
Collateral	$Y, r, \pi$ (Output, Interest rate, Inflation)	2.2017	0.024

The testing result on collateral model is less than 5% in p-value, which is to reject the collateral model. Recall the testing result from the base model, we can see that the base model is better than the collateral model in matching the behaviour of data. The Indirect Inference do not reject the base model, but reject the model with collateral.

## 4.2 Analysing the estimated model's properties

### 4.2.1 Impulse Response Function

#### The basic workings of the model.

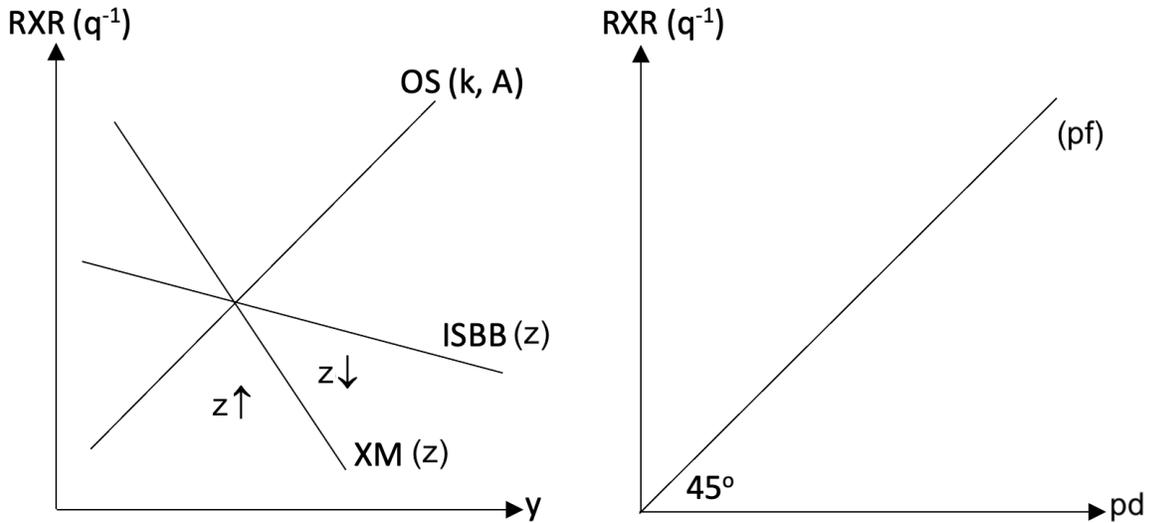
On the demand side the model is driven by consumption, investment and net exports. With domestic output under perfect competition, supply meets this demand at market-clearing home prices. At given prices we can define the demand as like an IS curve, where demand depends on the interest rate and the real exchange rate, RXR (home prices relative to foreign prices). The real rate interest rate is determined via UIP by the foreign rate and the risk-premium governed by net foreign assets ( $z$ , denoted in the Figure); in this 'BB curve' we can substitute out this real rate in

terms of the real exchange rate, to create the ISBB demand curve in RXR,  $y$  (output) space. This curve shifts inwards with falling  $z$  because this raises interest rates.  $z$  in turn falls when to the right of the XM curve defining current balance in RXR,  $y$  space. The model has an equilibrium in RXR,  $y$  and  $z$ , where the OS and XM curves intersect.

On the supply side of the model, output is upward sloping in RXR; this is because RXR drives a wedge between the consumption real wage and the producer real wage; a rising RXR allows the former to rise while the latter falls, driving employment up and with it output. This OS curve shifts up with capital and productivity from the production function.

Home prices are determined by RXR and foreign prices, shown in the right-hand quadrant; since  $RXR = pd - pf$ , effectively RXR and  $pd$  movements coincide for a fixed foreign price level.

Figure 6: Workings of the Model



***Export Demand Shock due to rise in foreign consumption..***

The export demand shock shifts the ISBB curve to the right, raising prices and RXR (lowering  $q$ ), while the XM curve shifts to the right. Hence net foreign assets ( $z$ ) accumulate, pushing the ISBB further out, lowering interest rates. As the shock dies off, these processes are reversed.

Figure 7: Estimated Response to 0.1 Foreign Consumption Shock

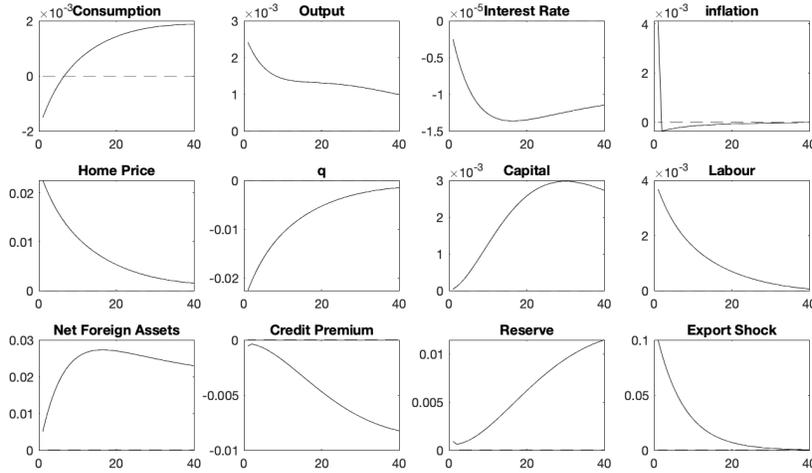
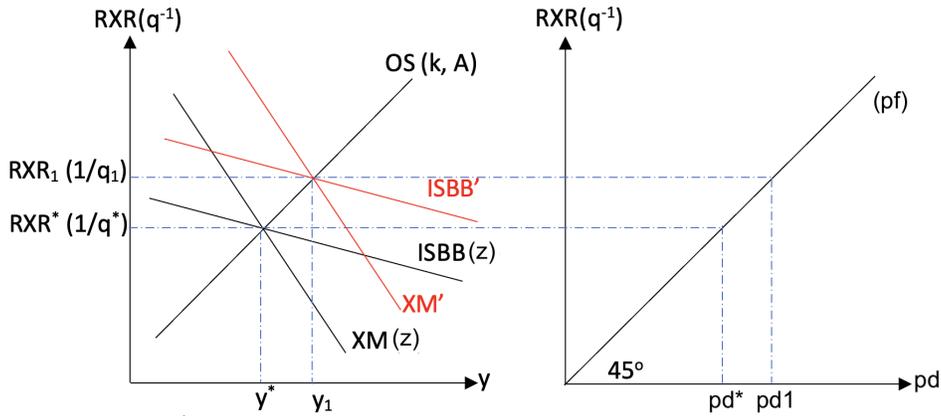


Figure 8: Working of Export Shock



**Foreign Interest Rate Shock.**

The foreign interest shock shifts the ISBB Curve leftwards, raising interest rates and lowering RXR. The domestic economy sees a temporary current account surplus with more accumulated NFA ( $z$ ). This, together with the decline in the shock reverses the process.

Figure 9: Estimated Response to 0.1 Foreign Interest Rate Shock

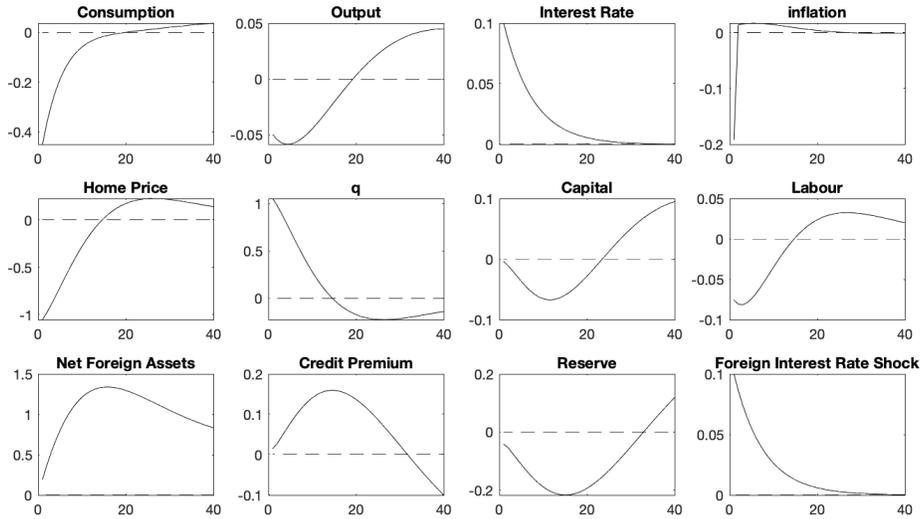
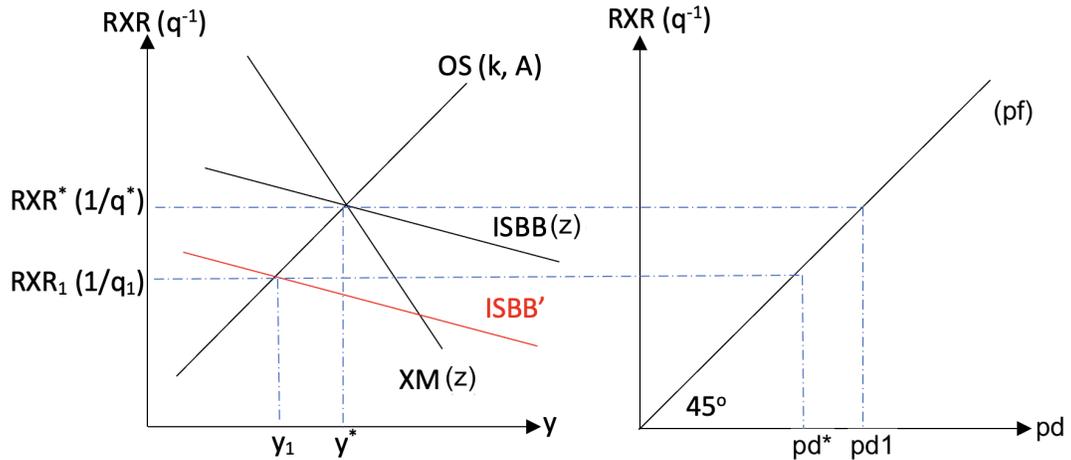


Figure 10: Working of Foreign Interest Rate Shock



## 4.2.2 Variance Decomposition and Historical Decomposition

### *Variance Decomposition.*

Not surprisingly, as seen in Table 4, output is highly influenced by the technology shock with around 90% of the contribution in the fluctuation coming from it over all time scales. Apart from that, the consumption preference shock makes the second largest contribution with around 5% in short run and 8% in long run. Government spending, export demand and foreign interest rate have little impact on output, but foreign inflation contributes modestly to the fluctuation in output, reaching about 3.75% in the long run. Almost all the fluctuation in the domestic interest rate is due to the foreign interest rate shock; plainly, since under the currency board it must strictly follow the interest rate of its anchor economy, with any difference putting pressure on the exchange rate to deviate. When it comes to inflation, all shocks contribute except for government spending, the housing demand shock and export demand. The most important one is the consumption preference shock, contributing 75% in the short run and 62% in the long run.

Table 5: Variance Decomposition

Variable	Preference	Housing	Productivity	Export	Government	Foreign Inflation	Foreign Interest
<b><i>25 Years</i></b>							
Output	7.58%	0.00%	88.31%	0.06%	0.02%	3.75%	0.28%
Interest Rate	0.69%	0.00%	0.03%	0.00%	0.00%	0.09%	99.19%
Inflation	61.99%	0.00%	0.69%	0.83%	0.06%	31.35%	5.09%
<b><i>5 Years</i></b>							
Output	4.58%	0.00%	93.30%	0.15%	0.02%	1.52%	0.42%
Interest Rate	0.11%	0.00%	0.00%	0.00%	0.00%	0.00%	99.89%
Inflation	63.04%	0.00%	0.58%	0.64%	0.05%	30.85%	4.84%
<b><i>1 Year</i></b>							
Output	5.23%	0.00%	93.42%	0.19%	0.03%	0.64%	0.49%
Interest Rate	0.01%	0.00%	0.00%	0.00%	0.00%	0.00%	99.99%
Inflation	75.88%	0.00%	0.41%	0.58%	0.04%	17.15%	5.95%

### *Historical Decomposition.*

As the variance decomposition plainly reveals, Figure 8 shows how output has been largely determined by productivity. The Asian Financial Crisis delivered a big hit to productivity and output. Although output recovered afterwards, it suffered another smaller hit in the 2008 Global Financial Crisis. There were also some declines in 2002 and 2003 due to SARS. Hong Kong had the second largest number of confirmed cases of SARS in the world.

The interest rate was exclusively determined by foreign rate shocks, as we have seen. Hence, it essentially depicts the evolution of US rates, and so no decomposition by shocks is shown for it; Figure 2 at the start shows this in full. During the high-rate cycle 1995-1999, which includes the 1997 Asian Financial Crisis, US Fed kept raising its interest rate to cool down the economy but led to the break of dot-com bubble in 2000. US went to a low-rate cycle from 2000, when the Fed cut down the interest rate a lot. However, the low rate made the real estate markets over-heated, followed by a contractionary monetary policy period from 2004 to 2006. In the 2008 Global Financial Crisis, the expansionary monetary policy in the US drove HK rates down sharply.

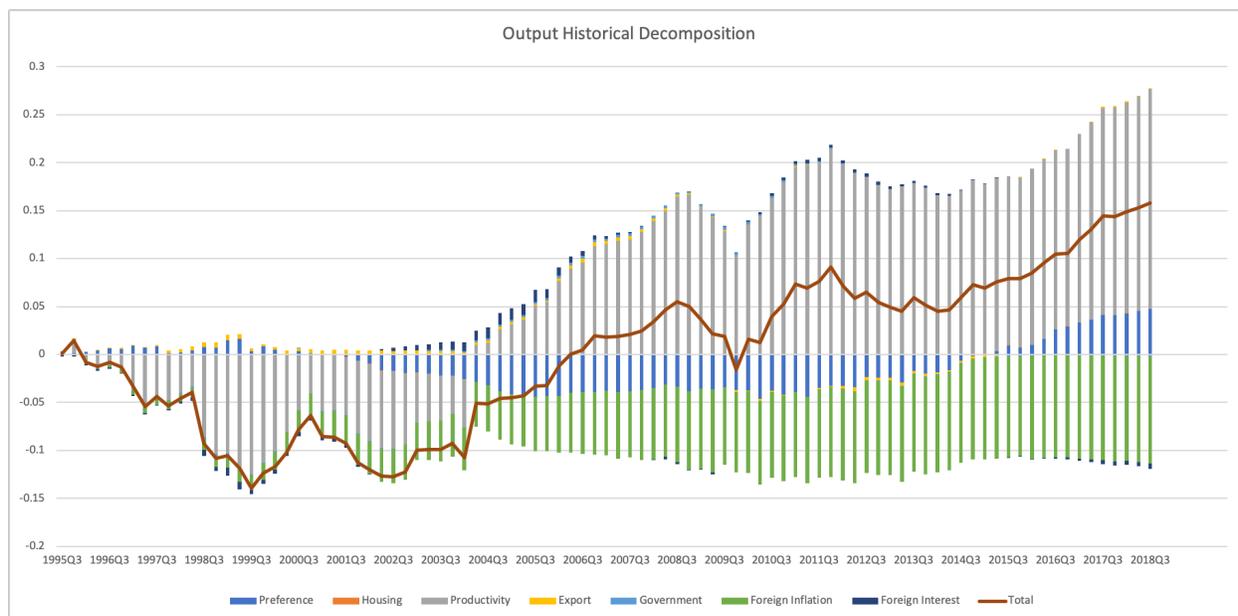


Figure 11: Historical Decomposition of Output

### **4.3 Examining the Policy Regime: would floating exchange rates create more stability?**

Since the linked exchange rate and currency board were founded in 1983, the monetary system in Hong Kong has had many challenges, including several financial crises and speculative attacks. Although this mechanism has been seen as a success for Hong Kong, there are still some discussions and arguments on that if it could better to switch to another regime. One alternative is to abandon the currency board and have a floating exchange rate; this is what we examine here, by simulating the economy with repeated bootstrapping under the two regimes, current and floating. For the

Taylor rule error, we cannot use Hong Kong data since it has no Taylor rule. Instead, we use the error from US data as a proxy, where the standard error of the US Taylor rule is 0.0002.

In the floating exchange regime, the Taylor rule and UIP are set as:

$$r_t = \rho_\pi \pi_t + \rho_y y_t + \gamma_t^r \quad (49)$$

$$r_t = r_t^f + s_{t+1} - s_t \quad (50)$$

$q_t = p_t^f - p_t^d$  in currency board, now it is:

$$q_t = s_t + p_t^f - p_t^d \quad (51)$$

Where  $r$  is interest rate,  $\pi$  is inflation and  $y$  is output;  $\rho_\pi$  is the feedback from inflation,  $\rho_y$  is the feedback from output;  $\gamma_t^r$  is the Taylor rule shock, following the AR(1) process:

$$\gamma_t^r = \rho^r \gamma_{t-1}^r + \epsilon_t^r \quad (52)$$

Figure 9 displays the impulse responses to a monetary shock in the alternative floating exchange rate model. A positive Taylor Rule shock acts as a tightening monetary policy, raising the interest rate. A higher interest rate lowers the consumption in the Euler equation and also decreases investment. This downward shift in the demand side then goes to the supply side, output and inflation decrease. The lower domestic price further results in a real depreciation and domestic goods are relatively more competitive, we can see export increases with more accumulated net foreign assets.

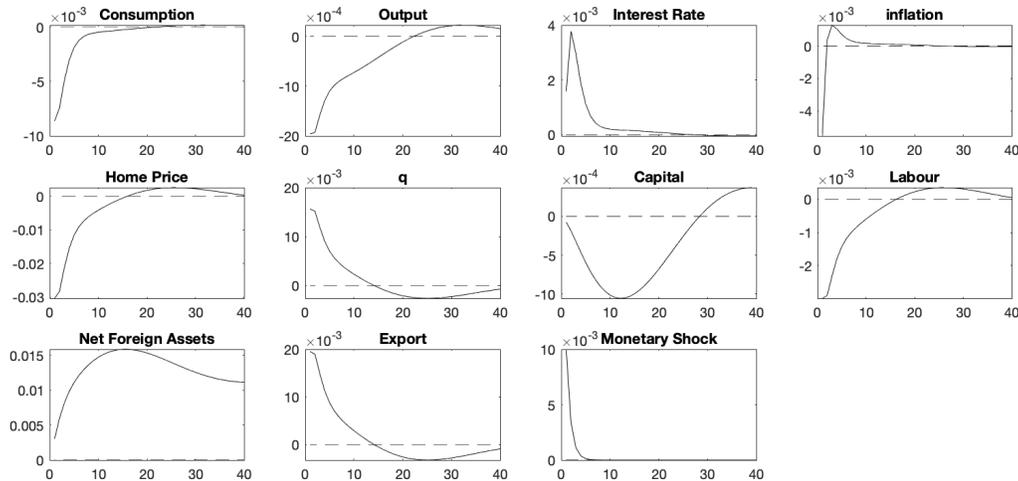


Figure 12: IRFs to 0.01 Monetary Shock in Floating Exchange Rate

The floating exchange rate model behaves like those in the literature, we then ask the question

which regime is better, fixed rate or floating rate? To answer this research question, we calculate the variance of output and inflation, together with the welfare cost measure which follows Gali and Monacelli (2005). In order to capture the variance, we bootstrap both models by their actual shocks from data 1000 times, get the variance of output and inflation in each simulation and then take the average.

Table 6: Stability and Welfare Loss Under Two Exchange Rate Regimes

	Fixed	Floating		
		$\rho_\pi : 1.5, \rho_y : 0.06$	$\rho_\pi : 1.7, \rho_y : 0.06$	$\rho_\pi : 1.5, \rho_y : 0.08$
Output Variance	0.0075	0.0116	0.0125	0.0082
Inflation Variance	0.0003	0.0001	0.0001	0.0001
Welfare Loss	0.0081	0.0117	0.0138	0.0083

From Table 5, we can see that output is more stable under the currency board than under floating exchange rates, with all three policy rule settings of the Taylor Rule. Inflation is more stable, but only by a slight amount; the Taylor Rule can slightly dampen the dollar inflation coming from abroad. But the gain this brings is smaller than the loss from the substantially greater output volatility; essentially this reflects the considerable stability of dollar inflation in this period, as had US inflation been unstable, the returns to setting an independent inflation could have been substantial. Overall, Hong Kong therefore appears to benefit from the currency board.

## 5 Conclusion

This paper sets out a DSGE model of Hong Kong’s economy under its currency board system. The model is estimated and tested by indirect inference; it matches the data behaviour well. We establish by Monte Carlo experiment that the power of the test is high, such that there is virtually no chance the estimated model can be more than 7% false. We also consider a rival model including a housing collateral constraint; this is rejected by the data. Using the estimated model, we investigate an independent monetary policy under floating as alternative to the currency board regime. We find that there is a gain in inflation stability but that this is slight compared with a substantial loss of output stability; the currency board gives overall superior welfare. Future work could look at a model with price/wage rigidity in place of our flex price framework. It could also consider whether linking the currency board to the Chinese RMB would be an improvement in the policy regime.

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## Appendix A. Log-linearisation equations

### Benchmark Model

#### i. Households

Euler Equation:

$$\hat{c}_t^c = E_t c_{t+1}^c - (\hat{r}_t - E_t \pi_{t+1}) + \hat{\gamma}_t^c$$

Investment:

$$\hat{i}_t^k = \frac{1}{1 + \beta_c} \hat{i}_{t-1}^k + \frac{\beta_c}{1 + \beta_c} E_t \hat{i}_{t+1}^k + \frac{1}{\kappa^k (1 + \beta_c)} \hat{q}_t^k$$

Capital tobin's q:

$$\hat{q}_t^k = \beta_c (1 - \delta^k) E_t \hat{q}_{t+1}^k + [1 - \beta_c (1 - \delta^k)] E_t \hat{r}_{t+1}^k - (\hat{r}_t - E_t \pi_{t+1})$$

House Demand:

$$[1 - \beta_c (1 - \delta^h)] (\hat{\gamma}_t^h - \hat{h}_t) = \hat{q}_t^h - \hat{c}_t^c - \beta_c (1 - \delta^h) E_t (\hat{q}_{t+1}^h - \hat{c}_{t+1}^c) + \beta_c (1 - \delta^h) \hat{\gamma}_t^c$$

Labour Supply and real wage

$$\eta \hat{m}_t + \hat{c}_t^c + \hat{\gamma}_t^c = \hat{w}_t - \hat{p}_t$$

UIP with Risk Premium:

$$\hat{r}_t = \hat{r}_t^f - \phi_a \hat{z}_t$$

Capital Accumulation:

$$\hat{k}_t = (1 - \delta^k) \hat{k}_{t-1} + \delta^k \hat{i}_t^k$$

#### ii. Entrepreneurs

Production Function:

$$\hat{Y}_t = \hat{A}_t + \alpha \hat{k}_{t-1} + (1 - \alpha) \hat{n}_t$$

Labour Demand:

$$\hat{A}_t - \alpha \hat{n}_t + \alpha \hat{k}_{t-1} = \hat{w}_t - \hat{p}_t^d$$

Capital Demand and real capital rental rate:

$$\hat{A}_t + (1 - \alpha) \hat{n}_t + (\alpha - 1) \hat{k}_{t-1} = \hat{r}_t^k - \hat{p}_t^d$$

Credit premium:

$$E_t c_{y_{t+1}} - (r_t - E_t \pi_{t+1}) = \chi (\hat{q}_t^k + \hat{k}_t - \hat{n} \hat{w}_t) - \mu \hat{m}_t^d$$

Net worth evolution

$$\hat{n} \hat{w}_t = \frac{\bar{K}}{\bar{NW}} (c_{y_t} - E_{t-1} c_{y_t}) + E_{t-1} c_{y_t} + \nu n \hat{w}_{t-1}$$

Money demand from entrepreneur:

$$\hat{m}_t^d = (1 + \xi)\hat{k}_t - \xi n\hat{w}_t$$

Entrepreneur consumption:

$$\hat{c}_t^e = n\hat{w}_t$$

### iii. Housing Producer

Housing Price, Housing Supply:

$$\hat{i}_t^h = \frac{1}{\kappa^h(1 + \beta_E)}\hat{q}_t^h + \frac{\beta_E}{1 + \beta_E}E_t\hat{i}_{t+1}^h + \frac{1}{1 + \beta_E}\hat{i}_{t-1}^h$$

Housing Accumulation:

$$\hat{h}_t = (1 - \delta^h)h_{t-1} + \delta^h\hat{i}_t^h$$

### v. Monetary Operation

Foreign Reserve intervention and Currency board balance sheet:

$$\hat{f}_t = \hat{m}_t^s$$

### vii. Marketing Clearing

Goods Market:

$$\hat{Y}_t = \frac{\bar{C}}{\bar{Y}}\hat{c}_t + \frac{\bar{I}^k}{\bar{Y}}\hat{i}_t^k + \frac{\bar{I}^h}{\bar{Y}}\hat{i}_t^h + \frac{\bar{G}}{\bar{Y}}\hat{g}_t + \frac{\bar{E}X}{\bar{Y}}e\hat{x}_t - \frac{\bar{I}\bar{M}}{\bar{Y}}i\hat{m}_t$$

Money Market:

$$\hat{m}_t^d = \hat{m}_t^s$$

### viii. Trade

Balance of payment with foreign reserve:

$$\hat{z}_t = \bar{r}r_{t-1}^f + (1 + \bar{r})z_{t-1} + \frac{\bar{E}X}{\bar{Z}}(e\hat{x}_t - \hat{q}_t) - \frac{\bar{I}\bar{M}}{\bar{Z}}(i\hat{m}_t)$$

$$i\hat{m}_t = -\theta\hat{q}_t + \hat{c}_t \text{ (Import Demand)}$$

$$e\hat{x}_t = \theta^f\hat{q}_t + \hat{c}_t^f \text{ (Export Demand)}$$

Real exchange rate:

$$\hat{q} = p_t^f - p_t^d$$

### ix. Some Identity

CPI and CPI inflation:

$$\hat{p}_t = \omega\hat{p}_t^d + (1 - \omega)\hat{p}_t^f$$

$$\pi_t = p_t - p_{t-1}$$

$$\pi_t^f = p_t^f - p_{t-1}^f$$

$$z_t = \frac{\bar{B}^f}{\bar{Z}} \hat{b}_t^f + \frac{\bar{F}}{\bar{Z}} \hat{f}_t$$

## x. Structure Shocks Process

Preference shock to consumption:

$$\gamma_t^c = \rho_c \gamma_{t-1}^c + \varepsilon_{c,t}$$

House demand shock:

$$\gamma_t^h = \rho_h \gamma_{t-1}^h + \varepsilon_{h,t}$$

Technology shock:

$$A_t - A_{t-1} = \rho_a (A_{t-1} - A_{t-2}) + \varepsilon_{A,t}$$

Hong Kong is a small open economy and can be treated as no effect to the rest of the world, world shock  $\varepsilon_{rf,t}, \varepsilon_{\pi^f,t}$

$$r_t^f = \rho_{rf} r_{t-1}^f + \varepsilon_{rf,t}$$

$$\pi_t^f = \rho_{pf} \pi_{t-1}^f + \varepsilon_{\pi^f,t}$$

Foreign consumption innovation and export demand shock  $\varepsilon_{cf,t}$

$$c_t^f = \rho_{cf} c_{t-1}^f + \varepsilon_{cf,t}$$

Government spending shock  $\varepsilon_{g,t}$ :

$$g_t = \rho_g g_{t-1} + \varepsilon_{g,t}$$

## Collateral Model

### ii. Impatient Households

Housing Demand:

$$[1 - \beta_I(1 - \delta^h) - m + \beta_I m](\hat{\gamma}_t^h - \hat{h}_{I,t}) = \hat{q}_t^h + (1 - m)(\hat{\gamma}_t^c - \hat{c}_{I,t}) + (\beta_I \delta^h - m) E_t(\hat{q}_{t+1}^h - \hat{c}_{I,t+1}) + \beta_I \delta^h \hat{\gamma}_{t+1}^c - m(\hat{R}_t - E_t \pi_{t+1})$$

Consumption:

$$\frac{\bar{C}_I}{\bar{Y}} \hat{c}_{I,t} + \frac{\bar{q}_h \bar{H}_I}{\bar{Y}} [\delta^h \hat{q}_t^h + \hat{h}_{I,t} - (1 - \delta^h) \hat{h}_{I,t-1}] + \frac{\bar{L}(1 + \bar{r})}{\bar{Y}} (r_{t-1} + l_{t-1}) = \frac{\bar{L}}{\bar{Y}} \hat{l}_t$$

Borrowing:

$$\hat{l}_t = E_t \hat{q}_{t+1}^h + \hat{h}_{I,t} - (\hat{R}_t - E_t \pi_{t+1})$$

Total consumption:

$$\hat{c}_t = \frac{\bar{C}_P}{\bar{C}} \hat{c}_{p,t} + \frac{\bar{C}_I}{\bar{C}} \hat{c}_{I,t}$$

Total housing:

$$\hat{h}_t = \frac{\bar{H}_P}{\bar{H}} \hat{h}_{p,t} + \frac{\bar{H}_I}{\bar{H}} \hat{h}_{I,t}$$