Exchange Rate Risk and Deviations from Purchasing Power Parity

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Abstract
Firstly, we show that domestic prices of net importer countries incorporate a risk premium, driven by higher moments of future nominal exchange rate returns and secondly, using US dollar exchange rates against three currencies of major net exporting countries to the US such as Canada, Japan and the European Union, we find that the skewness of the future nominal exchange rate is the major and statistically robust moment-based factor of the deviations from purchasing power parity (PPP). Our estimates further suggest that only low and moderate exchange rate risks induce risk premia that drive deviations from PPP.

Keywords: Purchasing Power Parity, risk-aversion, exchange rate, downside risk

JEL classification: G15, F31, F41

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1. Introduction

The finance literature has long emphasized investors’ higher-order risk attitude and corresponding moment-based measures of risk such as asset-related skewness and kurtosis as important factors which determine asset returns. Studies by Jondeaua and Rockinger (2003) and Brunnermeier et al. (2010), also document the association of higher moments of the nominal exchange rate with stock indices and interest rate differentials, respectively. Arghyrou et al. (2011) - AGP hereafter- further establish that there is a link between the variance of future floating exchange rates and deviations from the law of one price. Motivated by those findings and the large volume of foreign exchange (forex) trading, this paper extends the AGP framework and shows that in the absence of complete hedging against exchange rate risk, the skewness and kurtosis of future nominal exchange rates - exchange rate skewness and kurtosis hereafter- are factors which drive deviations from the direct generalization of the law of one price, the purchasing power parity (PPP).¹ Focusing on the pre-Brexit-pre-Covid period, our empirical results suggest that the exchange rate skewness is a dominant factor of the latter.

The main theoretical explanations offered for PPP’s empirical failure, defined in the literature as the first PPP puzzle (Sarno 2005), include: shifts in the demand of domestic goods relative to foreign, e.g. changes in consumers’ preferences or fiscal shocks (Alesina and Perotti, 1995); shifts in relative productivity, as originally suggested by Balassa (1964) and Samuelson (1964); and market imperfections (e.g. transaction costs and other barriers to trade) resulting into deviations from the cornerstone assumption of PPP, the law of one price (LOOP).² Market imperfections are also the main explanation offered for the second PPP puzzle, that is the excessive persistence of deviations from PPP (Rogoff, 1996). Dumas (1992) and Sercu et al. (1995) show that deviations from the LOOP are non-mean reverting as long as transaction costs

¹ Approximately $5 trillion ($220 billion) of foreign exchange transactions take place per day (hour). Most of the trading involves the US dollar, the Euro and the Yen.
are small enough relative to arbitrage-trading costs but become quickly mean-reverting when arbitrage profits exceed the costs of arbitrage trading imposing the LOOP. Empirical studies, reviewed by Sarno (2005), provide substantial evidence for non-linear PPP adjustment, justifying the aforementioned failure of standard linear time-series techniques to validate long-run PPP. A common characteristic of the explanations for the two PPP puzzles discussed above is that they refer to movements of the first moment of nominal or real exchange rate. To the best of our knowledge, except from AGP, no study has so-far explained deviations from PPP on the basis of higher moments of the exchange rate distribution.

The finance literature suggests that assets with return distributions that are characterized by positive skewness and low kurtosis are attractive to risk-averse investors because they periodically pay large returns, and the downside risk is low. Several studies show that risk measures such as systematic and idiosyncratic skewness and kurtosis induce risk-premia which drive asset prices and returns. In this paper, we argue that the asymmetry of the distribution of the nominal exchange rate coupled with the aversion of consumers and investors toward exchange rate risk induces premia that drive wedges between domestic and foreign prices, and thus deviations from PPP. Exchange rate risk at date $t$ arises from the fact that risk-averse buyers purchase goods and services in foreign prices with payments completed or cleared in...
date \( t + n \) with \( n > 0 \). Extending the AGP framework, we show that risk-averse buyers pay risk premia to domestic importers and/or domestic producers of homogeneous goods which depend on the skewness and kurtosis of the floating nominal exchange rate. We argue that exchange rate risk-premia incorporated in relative domestic prices of net importer countries drive permanent deviations from the LOOP, thus explaining the first PPP puzzle. Persistent shocks to the higher moments of the expected exchange rate distribution induce persistent deviations from the LOOP, explaining the second PPP puzzle. Finally, under a perfectly credible fixed exchange rate regime, the premium reduces to zero. This is consistent with the findings of previous studies supporting PPP under fixed exchange rate regimes.6

Changes in the distribution of the nominal exchange rate correspond to changes of different levels of risk as they are perceived by risk averse agents. The substantial use of various forex hedging instruments, constitutes evidence that exchange rate risk is an important factor for traders. Bodnar et al. (1998) report that 83% of the largest firms and 45% of medium size firms hedge exchange rate risk using derivatives. The empirical analysis of Allayannis and Weston (2001) shows that firms hedge about 22% of their exchange rate exposure and that the use of foreign currency derivatives increases total firm value by 4.87% on average.7 Survey data from the Foreign Exchange Committee (FXC) covering the months of October and April for the years 2004 to 2021 suggests that 55-60% of forex transactions involve outright forwards, forex swaps and currency options and only 40-45% of the total volume are spot transactions.8 In theory, forex traders can fully insure against any exchange rate risk using derivatives such as options, forward and futures contracts. In practice however, complete hedging of forex risk can be unachievable due to high insurance fees. Stulz (1996) and Cooper and Melo (1999) note that

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6 See, for example, Gaillot (1970) and Taylor and McMahon (1988).
7 Guay and Kothari (2002) argue that the percentage of the hedged exchange rate exposure reported by Allayannis and Weston (2001) is overstated because foreign sales used to scale the amount of foreign currency derivatives understates exchange rate exposure. However, they do not report any numbers for the fraction of sales in their long sample.
8 The survey covers all transactions from trades in United States, Canada, and Mexico.
the cost of hedging may rise with the probability of default, limiting the ability to hedge, while Brown (2001) presents evidence that the cost of hedging increases as the volatility in forex markets increases which then limits the degree of hedging.⁹ Hence, it is reasonable to assert, not only that exchange rate risk is a factor of PPP but also that forex traders, hedge only against the most painful outcomes that is, extreme risk. Our empirical analysis is a direct test of the former and an indirect test of the latter.

We test the empirical validity of our theory using a sample that excludes recent turbulent periods such as most of the Brexit post-referendum period as well as the period of the Covid pandemic that caused major disruptions to exchange rates. We do so by employing a model with time-varying coefficients using US dollar real exchange rate rates against three major international currencies of countries that the US is a major trade partner and a net importer, namely the Canadian dollar, the Euro and the Japanese Yen. We thus treat the US as being the domestic market. Specifically, our analysis covers the period 1973-2016 using monthly data. Our benchmark series of real exchange rates use CPI series while PPI-based real exchange rates are also utilized for robustness. Our findings indicate that exchange rate skewness is a significant and statistically robust moment-based risk determinant which drives deviations from PPP. Our estimates further suggest that only low and moderate risks influence PPP, as measured by the asymmetry of the exchange rate distribution. Specifically, an increase in skewness, which is equivalent to an increase of extreme exchange rate risk and a decrease in low and moderate risk, always increases the real exchange rate against all three currencies. Although it appears as if extreme exchange rate risk is not priced by risk averse agents, we argue that this is due to the fact that extreme risk is a priori hedged while the agents price only future unhedged risk. That is, if extreme appreciations of the exchange rate are hedged,

⁹ Similar findings are reported by Adam et al. (2017), while Nguyen et al. (2007) also highlight size along with leverage as determinants of the use of derivatives.
domestic agents only price low and moderate risks which decrease as skewness increases, inducing positive deviations from PPP. Hence, in response to lower moderate and low risks, domestic agents tend to reduce the risk-premium which is incorporated in domestic prices over foreign ones. According to our estimates, a unit increase in skewness leads to an average deviation from PPP, of about one percent, mainly for transaction clearing horizons of one or two months ahead. Such deviation is substantial given that the standard deviation of the measure of skewness in most cases is significantly higher than unity. As suggested previously, the high cost of insurance against exchange rate risk may act as deterrent of complete hedging, leaving only lower-level risk unhedged. This is consistent with findings in the literature suggesting that the use of derivatives for hedging risk is only moderate. We therefore propose exchange rate skewness as a solution to the first two PPP puzzles.

According to our estimates, the variance of future nominal exchange rate returns is found to be robustly statistically insignificant in explaining deviations from PPP. Although most of the coefficients on expected kurtosis are statistically insignificant, in the few cases where it is statistically significant, the signs of the coefficients support the underlying argument that only changes in lower-level risks drive deviations from PPP. Our main results also hold when the real exchange rates are constructed using the producer price index (PPI). To address concerns of possible bias in the confidence intervals, we estimate rolling samples of fixed length which demonstrate the stability and robustness of the confidence intervals.

The remainder of this paper is structured as follows: Section 2 documents our argument that higher moments of nominal exchange rates drive deviations from PPP. Section 3 discusses our data and presents estimates of the higher moments of the distribution of future nominal exchange rates. Section 4 presents estimates of the time-varying effects of the higher moments on real exchange rates. Section 5 evaluates the robustness of the confidence intervals and discusses hedging of extreme exchange rate risk. Finally, section 6 offers concluding remarks.
2. Higher moments of the nominal exchange rate and PPP

Consider a net importer (home) country and a foreign country which consume homogeneous good $x$ in a perfectly competitive market; $x$ may represent a basket of goods rather than a single good. Without loss of generality, we normalise $x$ to unity and assume zero transportation and transaction costs since the idea is to demonstrate that there are deviations from PPP even in the absence of such costs. There is a risk-averse representative agent that purchases $x = 1$, either from the domestic market or directly from the foreign market. In the first case, the agent’s ex-post net real wealth per unit of the domestic currency is given by $w_t - 1$, whereas in the second case it is given by $w_t - S_{t+d} P_t^f P_t^{-1}$, where $P_t$ and $P_t^f$ denote the domestic and foreign price indices, respectively, and $S_{t+d}$ denotes the nominal exchange rate at time $t + d$ when the transaction is cleared. In the first case, there is no uncertainty regarding wealth after the purchase whereas in second case, the agent’s ex-post net wealth is subject to some degree of uncertainty since $S_{t+d}$ is not known at the time of the purchase.\(^{10}\) The representative agent has expectations about $S_{t+d}$, given information up to period $t$. The growth rate of the nominal exchange rate between periods $t$ and $d$ is random with a time-varying variance:

$$\frac{S_{t+d} - S_t}{S_t} \approx \ln(S_{t+d}) - \ln(S_t) = \epsilon_{t+d},$$

where $E_t \epsilon_{t+d} = 0$ and $E_t \epsilon_{t+d}^2 = \sigma_{t+d}^2$.\(^{11}\) Although the exact value of $S_{t+d}$ is not known in period $t$, the agent has full information regarding the distribution of $S_{t+d}$. The underlying

\(^{10}\) A few examples related to individual consumers would include online purchases where the price is expressed only in foreign currency and the transaction clears at a later stage and appears on bank statements several days after; Payments in foreign currency by travellers, using credit cards issued in the traveller’s country of origin; Payments made by checks, payable at foreign banks. Such checks usually take several weeks to clear (6 weeks or more).

\(^{11}\) In most of the cases in our sample (discussed in the following section) the random walk hypothesis, for various $d$’s, cannot be rejected by the data. Even for the rest of the cases where the random walk is rejected, the processes are nearly random walks.
assumption is that the representative agent dislikes any level of transaction risk of high future appreciations of the foreign currency and thereby uses hedging instruments to reduce it. Hedging is incorporated in a simplified way, allowing the representative agent to hold insurance instruments of total cost \( H(S_t) \) that ensure compensation if certain high level appreciations of the foreign currency occur in period \( t + d \), where the first and second derivatives of \( H \) are given by \( H'(S_t) > 0 \) and \( H''(S_t) < 0 \), respectively. The representative agent buys insurance against high level exchange rate risk at the beginning of the period, no matter whether the purchase of \( x \) will be from the domestic or the foreign market. To keep the notation simple, wealth, \( w \), is always net of the real insurance fee \( H/P \).

The utility function of the representative agent residing in the home country is defined over wealth \( w \) and it is given by \( u(w) \) which is continuously differentiable, increasing in \( w \), and strictly concave. Following AGP, in equilibrium, prices must be such that the domestic agent is indifferent between buying \( x \) from the domestic market, via either a domestic importer or a domestic producer, and buying it directly from a foreign supplier that is,

\[
-u(w_t - 1) = E_t u(w_t - S_t P^f_t P^{-1}_t),
\]

where the left-hand-side of (1) corresponds to the utility of the domestic agent from buying a unit of \( x \) domestically and the right-hand-side corresponds to the expected utility of the domestic agent if he buys a unit of \( x \) directly from a foreign supplier and the transaction clears in period \( t + d \). Following AGP, we approximate (1) around \( \bar{w}_t = w_t - S_t P^f_t P^{-1}_t \):

\[
-u'(\bar{w}_t) \pi_t + \frac{u''(\bar{w}_t)}{2} \pi_t^2 - \frac{u'''(\bar{w}_t)}{3!} \pi_t^3 + \frac{u''''(\bar{w}_t)}{4!} \pi_t^4 + o(\pi_t) \approx \frac{u''(\bar{w}_t)}{2} \left( \frac{S_t P^f_t}{P_t} \right)^2 \sigma_{t+d}^2 -
\]

\[
\frac{u''''(\bar{w}_t)}{3!} \left( \frac{S_t P^f_t}{P_t} \sigma_{t+d} \right)^3 \frac{E_t \varepsilon_{t+d}^3}{\sigma_{t+d}^3} + \frac{u''''(\bar{w}_t)}{4!} \left( \frac{S_t P^f_t}{P_t} \sigma_{t+d} \right)^4 \frac{E_t \varepsilon_{t+d}^4}{\sigma_{t+d}^4} + o \left( \frac{E_t \varepsilon_{t+d}^n}{\sigma_{t+d}^n} \right),
\]

\(1\) It is reasonable to assume that domestic buyers (consumers or firms) focus only on hedging extreme exchange rate risk as low and moderate risk might be too costly to insure against.

\(13\) Although we do not model the decision of domestic importers/producers, we implicitly assume that if they attempt to reduce the price in order to dominate the market, then the foreign producers will respond by decreasing the foreign price as well, and vice-versa.
where $\pi_t = (P_t - S_t P^f_t)/P_t$ is the exchange rate induced risk-premium, per unit of the domestic price that the risk-averse domestic agent is willing to pay to the domestic importer or producer due to the uncertainty about the floating nominal exchange rate. Assuming that $\pi^2_t \approx 0$, $o(\pi_t) \approx 0$ and $o(\frac{\hat{\epsilon}_t\epsilon^3_{t+d}}{\sigma^3_{t+d}}) \approx 0$, equation (2) reduces to

$$\pi_t \approx \alpha_{1t}\sigma^2_{t+d} + \alpha_{2t}Skewness_{t+d} + \alpha_{3t}Kurtosis_{t+d},$$

where $skewness_{t+d} = E_t \hat{\epsilon}^3_{t+d}/\sigma^3_{t+d}$, $kurtosis_{t+d} = E_t \hat{\epsilon}^4_{t+d}/\sigma^4_{t+d}$,

$$\alpha_{1t} = \frac{-u_2(\bar{\omega}_t)}{2} \left( \frac{s^f_t}{p_t} \right)^2, \alpha_{2t} = \frac{u_3(\bar{\omega}_t)}{3!} \left( \frac{s^f_t}{p_t} \right) \sigma_{t+d}^3, \text{ and } \alpha_{3t} = \frac{-u_4(\bar{\omega}_t)}{4!} \left( \frac{s^f_t}{p_t} \sigma_{t+d} \right)^4,$$

with $u_2(\bar{\omega}_t) = u''(\bar{\omega}_t)/u'(\bar{\omega}_t)$, $u_3(\bar{\omega}_t) = u'''(\bar{\omega}_t)/u'(\bar{\omega}_t)$ and $u_4(\bar{\omega}_t) = u''''(\bar{\omega}_t)/u'(\bar{\omega}_t)$.

AGP assume that agents are concerned only about the overall exchange rate risk, as captured by $\sigma^2_{t+d}$. In this paper, we argue that agents may also behave differently against high level exchange rate risk as captured by skewness and kurtosis of the nominal exchange rate. Equation (3) can also be rearranged in terms of the logarithm of the real exchange rate, $Q_t = S_t P^f_t / P_t$, as follows:

$$q_t \equiv lnQ_t \approx -\alpha_{1t}\sigma^2_{t+d} - \alpha_{2t}Skewness_{t+d} - \alpha_{3t}Kurtosis.$$  

Approximations (3) and (4) suggest that the risk-premium as well as the real exchange rate are driven by the agent’s second-order risk attitude, $u''$, as well as her higher-order risk attitude, $u'''$ and $u''''$. The underlying assumption is that the agent does not insure against exchange rate risk by omitting to insure against high level risk or extreme risk. Since the cost of insurance is high, part of the risk remains unhedged. To simplify the exposition of our analysis we separate exchange rate risk into two categories, extreme risk and moderate risk which incorporates both moderate risk and low-level risk. It is reasonable to assume that if the agent buys insurance against exchange rate risk, given the high cost of insurance, she insures

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14 AGP did not consider terms that involve third and fourth powers, and thus did not examine the role of expected skewness and kurtosis.
only against the most painful outcomes that is, extreme risk. These assumptions are consistent with findings in the literature according to which firms hedge only a small percentage of their exchange rate exposure (Allayannis and Weston, 2001) while hedging is limited due its high cost (e.g. Stulz, 1996, Cooper and Melo, 1999, Brown, 2001). Given our assumptions, the higher-order properties of the agent’s utility function depend on whether the agent is hedged against extreme exchange rate risk \( H(S_t) > 0 \) or not \( (H(S_t) = 0) \), and can be summarized as follows:

\[
\begin{align*}
    u(w_t) &= \begin{cases} 
    \bar{u}(w_t) & \text{if } H(S_t) = 0 \\
    \underline{u}(w_t) & \text{if } H(S_t) > 0 
    \end{cases} \\
    &\text{such that } u''' \geq 0 \text{ if } H = 0 \\
    &\text{and } u''' \leq 0 \text{ if } H > 0.
\end{align*}
\]

As noted by Courbage and Rey (2010), within an expected utility framework, a common assumption is “complete properness”, i.e. the feature where successive derivatives of the utility function alternate in signs.\(^{15}\) The utility function of a prudent agent is characterized by \( u'''(w_t) > 0 \) if \( H(S_t) = 0 \) and \( u'''(w_t) < 0 \) if \( H(S_t) > 0 \), while the utility function of a temperate agent is characterized by \( u''''(w_t) < 0 \) if \( H(S_t) = 0 \) and \( u''''(w_t) > 0 \) if \( H(S_t) > 0 \). As skewness increases, extreme exchange rate appreciations become more likely while moderate and small appreciations become less likely. In other words, as skewness increases, extreme risk increases while moderate risk reduces because mass from the ‘middle’ of the distribution flows into the left and extreme right. A prudent forex trader would be willing to pay a risk premium, incorporated in domestic prices, against the highest unhedged risk since the degree of ‘pain’ (utility loss) involved by adding risk increases as wealth \( w_t \) decreases (Eeckhoudt and Schlesinger, 2013). The premium is paid to domestic importers of foreign-produced \( x \) or to domestic producers of \( x \) that deliver a certain level of net wealth for the agent at the end of a \( d \)-period horizon.\(^{16}\) It follows that if extreme exchange rate risk is a unhedged

\(^{15}\) The term “complete properness” is used by Pratt and Zeckhauser (1987), while Caballé and Pomansky (1996) refer to it as “mixed risk aversion”. This is the case with the commonly used constant relative risk aversion (CRRA) utility function, which implies that both \( \alpha_{2t} \) and \( \alpha_{3t} \) are strictly positive.

\(^{16}\) As skewness increases, large positive shocks in the value of the future exchange rate, \( S_{t+d} \), become more likely. If the agent is not hedged against those shocks, they are translated into large negative shocks on the agent’s


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risk from excessively fat right tails (extreme appreciations of the foreign currency) is eliminated via hedging \( H(S_t) > 0 \), the utility function of the temperate agent will be characterized by \( u'''(\tilde{w}_t) > 0 \) and a negative risk premium response \( \alpha_{3e} < 0 \) to an increase in kurtosis. That is, as kurtosis increases, low and moderate risk is reduced while depreciations of the foreign currency become more likely, inducing a negative effect on the risk premium.

### 3. Data and the distribution of future nominal exchange rate returns

#### 3.1. Data

The empirical analysis that follows employs a dataset of nominal exchange rates which includes end of month closing bilateral nominal exchange rates for Canadian dollars, Euro and Japanese Yen per US dollars, from January 1973 to October 2016, except for the Euro which starts from January 1982.\(^{19}\) To test our hypothesis, we have chosen to end our sample in October 2016 in order to avoid most of the recent turbulent periods. Specifically, we avoid the period after the Brexit referendum as well as the period of the Covid pandemic.\(^{20}\) We define the net importer country as the country which has trade deficits in goods. The data on trade balances are obtained from the United States Census Bureau. We do not consider trade in services as it includes an element of financial investment income. Since the US has trade deficit in goods against all three countries in most months of the sample, we consider the US as the home country and the net importer. The US has a bilateral trade surplus with Canada only in 5 months of our sample (May 2015 and March 2016 to June 2016) and with the EU only in 1

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\(^{19}\) Prior to January 1999, the Euro/USA exchange rate refers to a synthetic euro exchange rate which is calculated by weighting the bilateral exchange rates of the (then) eleven euro-area countries using weights based on country trade shares. When we run our main regression in the following section, we start the sample from January 1997.

\(^{20}\) Although our sample does not include exchange rates with the British pound, the Brexit vote was an international event with evidence suggesting significant side effects on currencies such as the Japanese Yen and the Euro (e.g. Dao et al., 2019, Janjusevic and Chegeni, 2020). The Covid pandemic, beginning 2020, also caused major disruptions for exchange rates with global capital flows, unusual policy interventions and changes of exchange rate determinants (e.g. Konstantakis et al., 2021, Beckmann and Czudaj, 2022). Given that most of the Brexit post-referendum and Covid period would also be on the right tail of our sample, we have chosen to exclude most of it from our empirical analysis.
month of sample (February 1997) and none with Japan. Consumer price indexes for Canada, the Euro area, Japan and the US are included to calculate the CPI-based real exchange rate, $Q_t$ and Producer price indexes are included to calculate the PPI-based exchange rate as a robustness test. Due to data availability, the CPI based real exchange rate for euro area starts from 1983M10 while PPI based real exchange rate for euro area starts from 1996M01. The data source is the IMF’s International Financial Statistics databank.

### 3.2. Estimating higher moments of nominal exchange rate distributions

For notational convenience, let $\theta_t(d) \equiv E_t \varepsilon_{t+d}^2 = \sigma_{t+d}^2$, with the corresponding skewness and kurtosis, defined in section 2, as $Sk_t(d)$ and $Ku_t(d)$, respectively. We estimate $\theta_t(d)$, $Sk_t(d)$ and $Ku_t(d)$, following the methodology of León et al. (2005), by considering the following system of equations:

\begin{align*}
\theta_t(d) &= \mu_{\theta_0} + \mu_{\theta_1} \varepsilon_{t+d}^2 + \mu_{\theta_2} \theta_{t-1}(d) \quad (5) \\
Sk_t(d) &= \mu_{\varsigma_0} + \mu_{\varsigma_1} \eta_{t+d}^3 + \mu_{\varsigma_2} Sk_{t-1}(d) \quad (6) \\
Ku_t(d) &= \mu_{k_0} + \mu_{k_1} \eta_{t+d}^4 + \mu_{k_2} Ku_{t-1}(d) \quad (7)
\end{align*}

for $d \geq 1$, where

$$
\varepsilon_{t+d}(d)|I_t \sim (0, \theta_t(d)), \eta_{t+d} = \varepsilon_{t+d}/\sqrt{E_t \varepsilon_{t+d}^2} \equiv \varepsilon_{t+d} \theta_t^{-\frac{1}{2}}(d),
$$

with $\eta_{t+d} \sim (0,1)$ and $I_t$ is a set that contains all information up to date $t$. Following León et al. (2005), we abstract from unnecessary constants so that the logarithm of the likelihood function of the conditional distribution of $\varepsilon_{t+d} = \theta_t^{-\frac{1}{2}}(d) \eta_{t+d}$ is given by

$$
L_t = -\frac{1}{2} \ln \theta_t(d) - \frac{1}{2} \eta_{t+d}^2 + \ln \psi^2(\eta_{t+d}) - \ln I_t(d),
$$

where,

\[ \text{León et al. (2005), extends Harvey and Siddique (1999), by assuming time-varying kurtosis. León et al. (2005) also assumes that } d = 1. \]
\[ \psi(\eta_{t+d}) = \left[ 1 + \frac{Sk_t(d)}{3!} (\eta_{t+d}^3 - 3\eta_{t+d}) + \frac{Ku_t(d) - 3}{4!} (\eta_{t+d}^4 - 6\eta_{t+d}^2 + 3) \right], \]

and

\[ \Gamma_t(d) = 1 + \frac{Sk_t^2(d)}{3!} + \frac{(Ku_t(d) - 3)^2}{4!}. \]

The estimates of (5)-(7) are provided in the appendix.\textsuperscript{23} Figures 1 to 3, display the estimated time series for the higher moments of the future exchange rate distribution for different transaction clearing periods, as defined by \( d \). Our theory is expected to hold for relatively short transaction clearing periods as it is rather unlikely that sellers will agree to be paid several periods ahead with an unspecified exchange rate. Therefore, in the analysis that follows we limit the transaction clearing period to be rather short by choosing the maximum value of \( d \) to be 3 months, thereby we present estimates for \( d = 1, 2 \) and 3.

The volatilities of the variance, skewness and kurtosis increase with \( d \) in most of the cases. That is, as the transaction clearing period is lengthened, the uncertainty about the future nominal exchange rate increases, the distribution of the nominal exchange becomes more asymmetric, and the tails of the distribution become heavier. The only exceptions are the skewness and the kurtosis of the EU/USA bilateral exchange rate which are, on average, more volatile when the transaction clearing period is short, i.e. \( d = 1 \). This implies that extreme values of the EU/USA exchange rate are, on average, less likely if the transaction clearing period is long, i.e. \( d = 3 \), apart from the momentous event of the oil price collapse which occurred in the mid 2014.\textsuperscript{24} All moments capture the event while \( d = 3 \) exhibits a much larger swing than \( d = 2 \) and \( d = 1 \), not only for the variance but also for skewness and kurtosis, with skewness being negative in almost all periods from mid 2014 until the end of 2015. Negative

\textsuperscript{22} Refer to León et al. (2005) for the details regarding the derivation of the p.d.f.

\textsuperscript{23} The standard errors of the estimated coefficients correspond to the Bollerslev-Wooldridge robust standard errors and are calculated through \( Var(\hat{\beta}) = J^{-1}(\beta_0)I(\beta_0)J^{-1}(\beta_0) \), where \( J(\beta_0) \) is the negative of the expected Hessian matrix and \( I(\beta_0) \) is the information matrix which can be calculated by the outer product of the score vector.

\textsuperscript{24} The oil price collapse of mid 2014 was one of the three biggest declines in oil prices since World War II.
skewness suggests that the markets expect that appreciations of the Euro, the Canadian dollar and the Yen against US dollar are more likely, while the probability of extreme appreciations of the US dollar are non-zero. The fact that \( d = 3 \) exhibits larger swings than \( d = 1 \) and \( d = 2 \) for all moments, suggests that in the event of a significant oil price decline, not only exchange rate uncertainty increases as the transaction clearing horizon extends but also markets expect that most of the probability mass will be even more concentrated to the right tail of the exchange rate distribution while extreme depreciations and appreciations of the foreign currencies (Euro, Canadian dollar and Yen) are even more likely, especially extreme depreciations. The large negative skewness and the fatter tails of the distribution as a consequence of the oil price collapse are due to the fact that the US is relatively more exposed to oil price fluctuations than the EU. The variances of all exchange rates exhibit a sharp upswing in September of 2008, and in the case of EU/USA \( d = 3 \) a month before, following the Lehman Brothers episode. The skewness reaches very negative values in September 2008 for CA/USA and EU/USA and significantly positive values for JP/USA. This implies that following the episode, markets expected a high probability of US dollar depreciations relative to both the Canadian dollar and the Euro without ruling out the possibility of extreme depreciations of both the Euro and Canadian dollar. On the contrary, following the Lehman episode, the markets expected a high probability of US dollar appreciations relative to the Yen, without ruling out the possibility extreme appreciations of the latter. As in the cases of the other two moments, kurtosis for all exchange rates and transaction clearing horizons, exhibit increases in September, following the Lehman episode, with more evident the increases in the exchange rates of CA/USA and JP/USA. The EU/USA exchange rate increased in September, but the increase was not as sharp as the upswings of the other two exchange rates because the
former began increasing from July. Across the sample, exchange rates exhibit positive skewness about 59% of the time and excess kurtosis about 58% of the time.  

Figure 1: Estimated Higher Moments of Bilateral Exchange Rate

<table>
<thead>
<tr>
<th>Country Pair</th>
<th>skewness</th>
<th>kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Canada/USA</td>
<td>57%</td>
<td>0.27%</td>
</tr>
<tr>
<td>EU/USA</td>
<td>59%</td>
<td>0.91%</td>
</tr>
<tr>
<td>Japan/USA</td>
<td>62%</td>
<td>0.45%</td>
</tr>
</tbody>
</table>

4. Time-varying effects of higher moments

Stationarity tests suggest that the bilateral real exchange rates for all three countries are unit root processes. Therefore, we a priori impose the unit root on the real exchange rates, using the first difference of the real exchange rate as our dependent variable. The latter allows us to have a regression which consists of four stationary variables as follows:  

\[ \Delta \ln Q_t = \alpha_0 + \alpha_1 \sigma_{t+d}^2 + \alpha_2 Sk_{t+d} + \alpha_3 Ku_{t+d} + u_t, \]  

25 In particular, the skewness of the CA/USA exchange rate is positive 57%, 55% and 57% of the time at \( d = 1 \), \( d = 2 \) and \( d = 3 \), respectively. The skewness of the EU/USA exchange rate is positive 59%, 52% and 56% of the time at \( d = 1 \), \( d = 2 \) and \( d = 3 \), respectively, while the skewness of the JP/USA exchange rate is positive 62%, 61% and 54% of the time at \( d = 1 \), \( d = 2 \) and \( d = 3 \), respectively. Our estimates also indicate that the exchange rate series exhibit excess kurtosis 0.27%, 0.69% and 0.73% of the time for CA/USA, 0.91%, 0.37% and 0.39% of the time for EU/USA, and 0.45%, 0.46% and 0.92% for JP/USA, respectively at \( d = 1 \), \( d = 2 \) and \( d = 3 \).

26 Unit roots cannot be rejected for either country at the 5% significance level. The unit root results are available upon request.
where $\Delta \ln Q_t$ is the first difference of the logarithm of the real exchange rate and $u_t$ is a random error term.\(^{27}\) Since all variables in (8) are stationary, the time-varying coefficients are estimated using the semi-parametric approach of Robinson (1989). Parameter $\alpha_{it}$ is defined as $\alpha_{it} = \alpha_i(z_t)$ for $i = 0, 1, 2, 3$, where $z_t$ is a smoothing variable such that $z_t = \tau = t/T$ and $\alpha_i(z_t) = f(\tau)$ for $t = 1, 2, ... T$. The time-varying coefficients are obtained by combining ordinary least squares and a local polynomial kernel estimator, known as the Nadaraya-Watson approach, as described in Casas and Fernandez-Casal (2019).\(^{28}\) Following Cifarelli and Paladino (2015), we argue that any potential measurement errors in our regressors are more than compensated by the greater accuracy of the estimates of the conditional higher moments provided by the GARCH-type model.\(^{29}\)

Figures 2 to 4 display the estimated time-varying coefficients for different transaction clearing horizons, along with 95% confidence intervals. The intercept coefficient is estimated to be about zero, thereby it is not reported to save on space. The time-varying confidence intervals for the variance cover zero in all exchange rates and transaction clearing horizons. The only case where the confidence interval is marginally statistically insignificant is the case of the JP/USA where the lower limit is close to zero and the positive region where the upper

\(^{27}\) The percentage of months where the US deficit becomes surplus is tiny. Even if intercept dummies are included, effectively taking those observations off the sample, the results remain the same.

\(^{28}\) Assuming that $\alpha_i(z_t)$ is twice differentiable, it can be linearly approximated around $z$, using the Taylor Rule as $\alpha_i(z_t) \approx \alpha_i(z) + \alpha'_i(z)[z_t - z]$, where $\alpha'_i(z)$ is the first derivative with respect to $z$. Then, estimates for $\alpha_i(z)$ and $\alpha'_i(z)$ are obtained by solving the minimization problem $\hat{\alpha}_i = \min \sum_{t=1}^{T} [\Delta \ln Q_t - x_t^i \theta_0 - (z_t - z)x_t^i \theta_1]^2 K_b(z_t - z)$, where $x_t^i = \{1, \sigma_{t+d}^2, \text{Skewness}_{t+d}, \text{Kurtosis}_{t+d}\}$ and $K_b(z_t - z)$ is the Epanechnikov kernel with bandwidth $b$. The latter is selected by leave-one-out cross-validation apart from a few cases where such procedure implies relatively large bandwidths, inconsistent with most of the other cases and oversmoothed (constant) estimates. These are the cases of CPI-based EU/US $d = 3$, the three PPI-based EU/US cases and the two PPI-based cases of JP/US $d = 1$ and $d = 2$. For the first case, we choose the bandwidth to be equal to unity so that it is roughly equal to the other two CPI-based $d = 3$ cases and the other two CPI-based EU/US cases. For the PPI-based cases, we choose bandwidths which are equal to the corresponding values of the CPI-based cases. In this way, we avoid oversmoothed estimates. Further details on the estimation procedure can be found in Casas and Fernandez-Casal (2019).

\(^{29}\) The use of GARCH measures of variance, derived from preliminary estimates, as regressors is not uncommon in the literature, e.g. French et al. (1987), Ramachandran (2004) and Ramachandran and Srinivasan (2007).
limit lies. The estimates on the variance effect essentially suggest that first-order uncertainty is not statistically significant in driving the real exchange rate and deviations from PPP.

Figure 2: Estimated time-varying coefficients: Canada – CPI

θ_t(d)  
Sk_t(d)  
Ku_t(d)  

The coefficients on skewness on the other hand, are positive and statistically significant, as they never cover zero, for all three exchange rates and transaction clearing horizons. This result indicates that increases in skewness tend to increase the gap between the foreign price measured in US dollars and the domestic price. It provides support to the theoretical argument that US agents tend to pay a lower risk premium to domestic importers/producers in response to reduced low or moderate risk, under the assumption that extreme appreciations of the foreign currencies are a priori hedged. The estimates suggest that the positive effect of skewness on the real exchange rate is rising over time in the case of CA/USA and decreasing over time in the cases of EU/USA and JP/USA. The coefficient on skewness in the case of EU/USA exhibits a sharp decrease in the early 2002, in response to the sharp decrease in inflation in the EU area which began in April, relative to the April increase in US inflation. The fact that EU products became relatively cheaper decreased the
impact of moderate exchange rate risk reduction. As table 1 suggests, a unit increase in skewness implies an average increase of the logarithm of the real exchange rate, or deviation from PPP, of about one percent, mainly for transaction clearing horizons of one or two months ahead. This is a non-negligible increase as the standard deviation of the measure of skewness in most cases is higher than unity. When the mean coefficient is significantly lower than unity, the standard deviation of skewness is significantly higher than unity (e.g. CA/USA and JP/USA for $d = 3$ and JP/USA for $d = 2$). The means of the 95% confidence intervals demonstrate that the coefficients are precisely estimated.

Figure 3: Estimated time-varying coefficients: EU – CPI

Most of the confidence intervals of the coefficients on kurtosis cover zero, apart from a few cases where the coefficients are positive and marginally statistically significant in the sense that the lower limits are positive but very close to zero. These cases include EU/USA after May 2002 for all transaction clearing horizons and the case of JP/USA for $d = 2$, 80% of the months the period between 1/7/1986 and 1/5/1997, and for $d = 3$ in almost all the months.
of the period between 1/1/1985 and 1/3/2007. These results are consistent with the hedging argument of extreme exchange rate risk and the negative impact of reduced low exchange rate risk on the risk-premium that US agents are willing to pay to domestic importers and producers. Specifically, the left tale of the exchange rate distribution is irrelevant under the assumption that risk from extreme appreciations of the foreign currency is hedged which means that extreme risk is priced already. It follows that as kurtosis increases and the left tail of the distribution becomes longer and fatter, the corresponding decrease in low or moderate exchange rate risk for US agents implies a lower risk premium and consequently, lower domestic prices relative to foreign ones and positive deviations from PPP.

\(\theta_t(d)\)  \(\text{Sk}_t(d)\)  \(\text{Ku}_t(d)\)

\[d = 1: \text{Pseudo } R^2 = 0.40\]

\[d = 2: \text{Pseudo } R^2 = 0.54\]

\[d = 3: \text{Pseudo } R^2 = 0.43\]

---

30 In the case of EU/USA after May 2002, the mean coefficients on skewness (%) and corresponding mean of 95% confidence intervals (%) are 0.16 (0.06, 0.26) for \(d = 1\), 0.17 (0.05, 0.3) for \(d = 2\) and 0.17 (0.08, 0.33) for \(d = 3\). The standard deviations of kurtosis are 2.37, 2.85 and 3.79 for the cases of \(d = 1\), \(d = 2\) and \(d = 3\), respectively. While the estimated coefficients do not cover zero and are positive, the confidence intervals are wider than those of skewness coefficients.
Table 1: Means of coefficients on skewness and standard deviations of skewness - CPI

<table>
<thead>
<tr>
<th></th>
<th>d = 1</th>
<th>d = 2</th>
<th>d = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Canada/USA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>coeff. on skewness %</td>
<td>1.1</td>
<td>0.8</td>
<td>0.3</td>
</tr>
<tr>
<td>conf. interval %</td>
<td>(0.8, 1.4)</td>
<td>(0.68, 1)</td>
<td>(0.2, 0.4)</td>
</tr>
<tr>
<td>s.d. of skewness</td>
<td>0.83</td>
<td>1.34</td>
<td>3.35</td>
</tr>
<tr>
<td></td>
<td>European Union/USA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>coeff. on skewness %</td>
<td>0.65</td>
<td>1.3</td>
<td>0.9</td>
</tr>
<tr>
<td>conf. interval %</td>
<td>(0.5, 0.8)</td>
<td>(1.04, 1.54)</td>
<td>(0.68, 1.2)</td>
</tr>
<tr>
<td>s.d. of skewness</td>
<td>2.16</td>
<td>1.28</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>Japan/USA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>coeff. on skewness %</td>
<td>1.3</td>
<td>0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>conf. interval %</td>
<td>(1, 1.5)</td>
<td>(0.4, 0.6)</td>
<td>(0.21, 0.29)</td>
</tr>
<tr>
<td>s.d. of skewness</td>
<td>1.28</td>
<td>4.16</td>
<td>7.08</td>
</tr>
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</table>

*The table reports the mean of the time-varying coefficients ($\alpha_{2t}$) in percentages and standard deviations of estimated skewness across the sample using CPI based real exchange rates. In parenthesis, are the means of the upper and the lower bounds of the 95% confidence intervals, displayed in figures 2-4.

Finally, the measure of the goodness of fit of the regressions, suggests that in all cases the transaction clearing horizon of two months fits the data the best. The main results from CPI-based real exchange rates are also confirmed when the real exchange rate is computed using the Producer Price Index (PPI). Although there a few cases where the estimated coefficients on the variance become positive without covering zero, the confidence intervals are extremely wide. There are also a few cases where the coefficients on kurtosis become negative for certain periods without covering zeros. However, these results provide very weak and limited evidence that kurtosis contributes positively to the risk-premium as the coefficients are not only very small but also cover zero in most periods. Table 2 displays the means of the coefficients on skewness, along with means of the 95% confidence intervals using PPI based real exchange rates, and the standard deviation of skewness. Figures displaying estimated coefficients from PPI based real exchange rates are reported in the appendix.
Table 2: Means of coefficients on skewness and standard deviations of skewness - PPI*

<table>
<thead>
<tr>
<th></th>
<th>( d = 1 )</th>
<th>( d = 2 )</th>
<th>( d = 3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Canada/USA</td>
<td>European Union/USA</td>
<td>Japan/USA</td>
</tr>
<tr>
<td>coef. on skewness %</td>
<td>0.8</td>
<td>0.6</td>
<td>0.2</td>
</tr>
<tr>
<td>conf. interval %</td>
<td>(0.62, 0.93)</td>
<td>(0.5, 0.8)</td>
<td>(0.15, 0.3)</td>
</tr>
<tr>
<td>s.d. of skewness</td>
<td>0.83</td>
<td>1.34</td>
<td>3.35</td>
</tr>
</tbody>
</table>

*The table reports the mean of the time-varying coefficients (\( \alpha_{2t} \)) in percentages and standard deviations of estimated skewness across the sample using PPI based real exchange rates. In parenthesis, are the means of the upper and the lower bounds of the 95% confidence intervals, displayed in figures 2-4.

5. Robustness of confidence intervals and hedging hypothesis

As noted by Cifarelli and Paladino (2015), any potential errors-in-variables distortion is more than compensated by the greater accuracy of estimates provided by GARCH-type procedures, while the use of such estimates as regressors is not uncommon in the literature. Our main results implied by the time-varying estimates are reinforced by the fact that they hold for both CPI-based and PPI-based real exchange rates. Evaluating potential generated regressor bias in models with time-varying coefficients is currently an unknown territory. Such bias may affect the confidence intervals of the estimated coefficients of equation (8), as the argument is that the uncertainty of estimating the regressors of (8) is not taken into account when estimating the coefficients in (8). To address this issue, we assess the stability of the confidence intervals of the estimated coefficients by estimating a rolling sample of fixed length and reporting the 97.5\(^{th}\) percentile of the upper bound and the 2.5\(^{th}\) percentile of the lower bound of a set period. By utilizing those percentiles, we get rid of the combined 5% of potential upper and lower extreme values. We focus specifically on the confidence intervals of CPI-based skewness coefficients which are the highlight of our main results.
Figure 5: Percentiles of Confidence Intervals of Skewness Coefficients from Rolling Samples

$\text{d} = 1$

$d = 2$

$d = 3$

Canada

EU

Japan

*The focus period is set to 1995:1-2005:12 for CA/USA and JP/USA and to 2002:1-2010:12 for EU/USA. The rolling samples for CA/USA and JP/USA is fixed to 252 observations that start from 1/1/1985, rolling over for one year until and including 1/1/1994. The rolling samples for EU/USA is fixed to 168 observations that start from 1/1/1997, rolling over for one year until and including 1/1/2002. The percentiles reported on the figures are the 97.5th for the upper bound of the confidence interval and the 2.5th for the lower bound of the confidence interval. The real exchange rates are computed using CPI.

Since CPI-based results are indicative and to save on space, we do not report results from PPI-based exchange rates which are also available upon request. Figure 5 demonstrates that the confidence intervals for all bilateral real exchange rates at all transaction clearing horizons do not cover zero and always lie within the positive region. This result provides strong support regarding the stability of the confidence intervals and the validity of the result that US agents respond to moderate and low exchange rate risks by increasing (decreasing) the risk-premium incorporated in domestic prices when such risks increase (decrease).

Hedging in Forex trading is deemed to be illegal in the US in the sense of buying and selling the same currency pair at the same or different strike prices. However, holding forex options which enable recouping the difference between the market price and the strike price when extreme appreciations of the forex occur or using other derivatives when extreme
appreciations of foreign currencies are expected is a perfectly legitimate strategy.\textsuperscript{31} We argue that US agents, especially large corporations that purchase foreign goods and services denominated in foreign currencies, hold derivative portfolios to hedge the exchange rate risk but only on a limited basis, focusing on extreme appreciations of foreign currencies. Operational hedging is another option that agents can adopt when extreme appreciations of foreign currencies are expected.\textsuperscript{32} Even though, direct testing of this hypothesis is challenging, existing empirical evidence from corporate derivatives is supportive to the hypothesis. Guay and Kothari (2002) find that the use of corporate derivatives, not inclusive to exchange rate risk, appears to be moderate by taking into account firm size, operating and investing cash flows.\textsuperscript{33} Although increased use of derivatives among more geographically diverse firms is observed, the magnitudes of the derivatives positions are found to be quite small. Furthermore, Brown (2001), provides evidence that a large US based corporation would be more concerned with downside exchange rate risk than upside risk. As Brown’s case study suggests, the cost of initiating and maintaining a derivatives program is not trivial. Therefore, the assumption of hedging extreme downside risk only, is justified by the fact that large-scale hedging would be too expensive and detrimental for either businesses and/or consumers.

\textsuperscript{31} The evidence suggests that option derivatives is a significant component of derivatives portfolios. For instance, when the foreign currency significantly appreciates, foreign call options enable domestic (US) agents to recoup the price difference and thus fully hedge extreme exchange rate risk, while moderate and low risks remain unhedged. Specifically, domestic agents can purchase foreign exchange at the exercise price which is lower than the market price and then sell it at the market price. Futures and forward contracts also enable domestic agents to eliminate extreme appreciations of the foreign exchange when the latter are expected. In principle, low and moderate exchange rate risks can also be hedged, however this would imply higher overall insurance fees than any premium paid to domestic importers or producers. For instance, domestic agents can use put options and sell foreign exchange at the exercise price which is higher than the market price and then purchase at the market price.\textsuperscript{32} For instance, an agent may postpone or cancel a purchase when an extreme appreciation of foreign exchange is expected to occur (e.g. see Boyabathi and Toktay, 2004).\textsuperscript{33} Specifically, Guay and Kothari (2002), present detailed evidence on the cash flow and market value sensitivities of financial derivatives portfolios to extreme simultaneous changes in interest rates, currency exchange rates, and commodity prices using an extended random sample of large non-financial corporations.
6. **Concluding Remarks**

We extend the theoretical framework of Arghyrou et al. (2011), where transactions clear in future dates and induce deviations from the law of one price due to uncertainty about future nominal exchange rates. We show that the latter induces deviations from PPP, not only because of first-order uncertainty, as in Arghyrou et al., but also because of higher order uncertainty. We further argue that due to high insurance fees, only extreme exchange rate risks can realistically be fully hedged via options markets. If extreme risk is hedged, then our framework suggests that only low and moderate risks induce risk premia incorporated in domestic prices which then drive deviations from PPP. To test our theory, we employ a model with time-varying coefficients using data for bilateral exchange rates between the US, and three of its main net exporting partners, Canada, EU and Japan. Firstly, our estimates suggest that higher-order uncertainty measured by skewness is a major factor driving deviations from PPP. Secondly, our estimates are consistent with the hypothesis that US agents hedge a priori against extreme exchange rate risk and are willing to pay risk premia only for unhedged low and moderate risk. Our estimates demonstrate that the skewness of future nominal exchange rates constitutes an important factor of the deviations from PPP.

**References**


Appendix


<table>
<thead>
<tr>
<th></th>
<th>Canada/USA</th>
<th>European Union/USA</th>
<th>Japan/USA</th>
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<tbody>
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<td>(32.985)</td>
<td>(20.287)</td>
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<td>-0.714</td>
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<td>(0.001)</td>
<td>(0.001)</td>
<td>(1.879)</td>
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</tbody>
</table>

*For the European Union, the sample begins from 1983M10 for the CPI, while for the PPI it begins from 1996M01.

Figure A1: Estimated time-varying coefficients: Canada – PPI
Figure A2: Estimated time-varying coefficients: EU – PPI

\[ \text{Variance}_{t+d/t} \]
\[ \text{Skewness}_{t+d/t} \]
\[ \text{Kurtosis}_{t+d/t} \]
\[ d = 1: \text{Pseudo } R^2 = 0.31 \]
\[ d = 2: \text{Pseudo } R^2 = 0.32 \]
\[ d = 3: \text{Pseudo } R^2 = 0.26 \]

Figure A3: Estimated time-varying coefficients: Japan – PPI

\[ \text{Variance}_{t+d/t} \]
\[ \text{Skewness}_{t+d/t} \]
\[ \text{Kurtosis}_{t+d/t} \]
\[ d = 1: \text{Pseudo } R^2 = 0.38 \]
\[ d = 2: \text{Pseudo } R^2 = 0.51 \]
\[ d = 3: \text{Pseudo } R^2 = 0.42 \]