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## Fertility versus Productivity: A Model of Growth with Evolutionary Equilibria (Supplementary Materials)

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## Supplementary Materials for

### Fertility versus Productivity: A Model of Growth with Evolutionary Equilibria

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#### Mathematical Derivation of Propositions

The representative (sexless) agent born in period  $t - 1$  (Age 0) makes decisions in period  $t$  (Age 1) by maximizing a CES utility function under given generalized prices  $(\pi_n, \pi_q, \pi_z)$ , where  $\pi_z$  is normalised to 1:

$$\max_{n_t, q_t, z_t} U(n_t, q_t, z_t) = \left[ \alpha^{\frac{1}{s_t}} \cdot n_t^{\frac{s_t-1}{s_t}} + \beta^{\frac{1}{s_t}} \cdot q_t^{\frac{s_t-1}{s_t}} + \gamma^{\frac{1}{s_t}} \cdot \left( \frac{z_t}{z_{t-1}} \right)^{\frac{s_t-1}{s_t}} \right]^{\frac{s_t}{s_t-1}}$$

The three choice variables,  $n_t, q_t, z_t$ , are respectively target number of children per agent (so there are  $2n_t$  surviving children per household and per mother), target quality of children relative to parent generation (so if parent generation has a human capital level of  $Q_{t-2}$ , children generation is expected to have a human capital level of  $Q_{t-2}q_t$ ) and self-oriented consumption (excluding childcare and eldercare). The optimization problem faces the following constraints, where  $z1_t, z2_t, z3_t$  are the consumption flows under three possible states of pre-mature death during working ages and  $b_t \equiv \frac{n_t}{(1-m0_t)(1-m1_t)}$  is the crude births to achieve  $n_t$ .

$$z_t \equiv m2_t \times z1_t + (1 - m2_t)m3_t \times z2_t + (1 - m2_t)(1 - m3_t) \times z3_t \quad [\text{H}]$$

$$(1 + ADR_t)z1_t = w_t \text{ if die after Age 1, where } ADR_t \text{ is the 60+ dependency ratio} \quad [\text{Ha}]$$

$$\sum_{i=0}^1 (1 + ADR_{t+i}) z2_t + \pi_{n,t+1} b_t + \pi_{q,t+1} q_t b_t = \sum_{i=0}^1 w_{t+i} \text{ if die after Age 2} \quad [\text{Hb}]$$

$$\sum_{i=0}^2 (1 + ADR_{t+i}) z3_t + \pi_{n,t+1} b_t + \pi_{q,t+1} q_t b_t = \sum_{i=0}^2 w_{t+i} \text{ if die after Age 3} \quad [\text{Hc}]$$

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Instead of setting up a Lagrangian, we substitute all the constraints into the objective function to remove  $z_t$  and take partial derivatives with respect to the two controls  $(n_t, q_t)$ , resulting in two marginal conditions:

$$z_{t-1} \left( \frac{\alpha z_t / z_{t-1}}{\gamma n_t} \right)^{\frac{1}{s_t}} = (1 - m2_t) m3_t \times \frac{\pi_{n,t+1} + \pi_{q,t+1} q_t}{\sum_{i=0}^1 (1 + ADR_{t+i})} + (1 - m2_t) (1 - m3_t) \times \frac{\pi_{n,t+1} + \pi_{q,t+1} q_t}{\sum_{i=0}^2 (1 + ADR_{t+i})} \quad [M1]$$

$$z_{t-1} \left( \frac{\beta z_t / z_{t-1}}{\gamma q_t} \right)^{\frac{1}{s_t}} = (1 - m2_t) m3_t \times \frac{(\pi_{q,t+1} n_t)}{\sum_{i=0}^1 (1 + ADR_{t+i})} + (1 - m2_t) (1 - m3_t) \times \frac{(\pi_{q,t+1} n_t)}{\sum_{i=0}^2 (1 + ADR_{t+i})} \quad [M2]$$

Equations [M1] and [M2] are standard marginal conditions in probabilistic settings: the left-hand sides of equations are the marginal rates of substitution (MRS) of the mean or representative agent in each generation and the right-hand sides are the expected ratios of effective costs<sup>1</sup>. The effective cost of  $z_t$  includes the eldercare burden ( $ADR$  terms in the denominators), while effective cost of child quantity/quality includes the premature burden (mortality rates in the numerators). In fact, a more complete right-hand side should also have a term  $m2_t \times \frac{0}{1 + ADR_t}$  to explicitly include the case where the agent dies at the end of Age 1 or at the beginning of Age 2 before any children is born. The probability weight is  $m2_t$ , the effective cost of child quantity/quality is obviously zero, and the effective cost of  $z_t$  is just for one-period  $1 + ADR_t$ .

Now consider the production side of the economy. A production unit (a farm or a firm) per capita faces a standard maximization problem with constant-return-to-scale technology:

$$\max \Pi_t = Y_t - w_t \hat{L}_t, \text{ subject to:}$$

$$Y_t = \exp(\epsilon_t^Y) \hat{L}_t^{\theta_1} H_t^{\theta_2} (\bar{F} / P_{t-1})^{1-\theta_1-\theta_2}, \text{ where } \epsilon_t^Y \sim N(0, \sigma_Y^2) \quad [F]$$

$Y_t$  is output per capita,  $\hat{L}_t$  is the labor force ratio ( $\hat{L}_t \equiv L_t / P_{t-1}$ ),  $H_t$  is the human capital per capita of the labor force, and  $\bar{F}$  is fixed natural resources proportional to land and not reproducible, such as agricultural land, mines and oil. If we multiply both sides of [F] with the population stock  $P_{t-1}$ , then we arrive at an aggregate production function, which is constant return to scale with respect to aggregate labor force  $L_t$ , aggregate

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<sup>1</sup>The optimality condition holds for every individual with any value of  $s$ , therefore, strictly speaking, we should integrate individuals over the range of  $s$ . Here we use the average of sum for the sum of average, an approximation. The two are not the same because of nonlinearity, but they are equivalent when the equation system is solved by linear approximation as ours is (p16 s1.4).

human capital  $H_t P_{t-1}$  and aggregate natural capital  $\bar{F}$ . Without loss of generality, we can normalize this fixed quantity at  $\bar{F} = 1$ . The marginal condition of the firm is:

$$MPL \equiv \frac{\theta_1 Y_t}{\hat{L}_t} = w_t \quad [M3]$$

Combine [M1], [M2] and [M3] with [H], [Ha]-[Hc], [F], [A1]-[A14] and definitions of short-hand variables (see [D1]-[D3] below) to form a dynamic stochastic general equilibrium (DSGE) system. To stationarize this system, we need to identify the trends. From equation [F], we can derive the output growth rate along the balanced growth path:  $g_Y = \theta_2 g_H - (1 - \theta_1 - \theta_2) g_P$ . If we stationarize the two underlying drivers of growth by their own past values:

$$\hat{P}_t \equiv \frac{P_t}{P_{t-1}} = 1 + g_{P_t}$$

$$\hat{H}_t \equiv \frac{H_t}{H_{t-1}} = 1 + g_{H_t}$$

Therefore,  $Y_t$  can be stationarized by the two drivers in the light of [F]:

$$\hat{Y}_t \equiv \frac{Y_t}{X_t}, \text{ where } X_t \equiv H_t^{\theta_2} P_{t-1}^{\theta_1 + \theta_2 - 1}$$

Based on the equations, we can classify the non-stationary variables in terms of trends:

$$\text{Deflated by } P: \hat{P}_t = \frac{P_t}{P_{t-1}}, \hat{B}_t = \frac{B_t}{P_{t-1}} \equiv CBR_t, \hat{D}_t = \frac{D_t}{P_{t-1}} \equiv CDR_t, \hat{L}_t = \frac{L_t}{P_{t-1}}, \hat{P}1_t, \hat{P}2_t, \hat{P}3_t$$

$$\text{Deflated by } H: \hat{H}_t = \frac{H_t}{H_{t-1}}, \hat{Q}_t = \frac{Q_t}{H_t}$$

$$\text{Deflated by } X: \hat{X}_t = \frac{X_t}{X_{t-1}}, \hat{Y}_t = \frac{Y_t}{X_t}, \hat{w}_t, \hat{\pi}_{nt}, \hat{\pi}_{qt}, \hat{z}_t, \hat{z}1_t, \hat{z}2_t, \hat{z}3_t$$

There are in total 25 endogenous variables  $\mathbf{y}_t$ , of which 6 are observable (but only two are finally used for estimation: population and wage). There are 6 exogenous shocks  $\mathbf{u}_t$  (but only two are finally kept:  $\epsilon_t^{\pi n}$  and  $\epsilon_t^{\pi q}$  to match the number of target observables). The parameter vector  $\boldsymbol{\theta}$  includes: (1) fixed parameters which are either calibrated or estimated  $\bar{\boldsymbol{\theta}}$ ; (2) time-varying parameters which are exogenously evolving  $\boldsymbol{\theta}_t$  such as  $s_t, WP_t, FL_t, SCH_t, URB_t$ . These parameters evolve according to some law of motion (e.g.  $s_t$ ) that is external to and not anticipated by the agents. Given that some parameters are changing over time, the steady state of the stationarized model is also time varying.

To completely solve this (stationarized) dynamic equation system, we can use the perturbation method, i.e. obtain the steady state for each period, and then add on the complementary functions to capture the deviation from the steady state. Given the evolutionary feature of the model, we will only focus on the time-varying steady states. The model can be estimated by minimizing the gap between the steady states predicted by the model and the trend observed in the data.

### List of Stationarized Equations

$$[M1] \frac{\hat{z}_{t-1}}{\hat{x}_{t+1}\hat{x}_t} \left( \frac{\alpha \hat{x}_t \hat{z}_t / \hat{z}_{t-1}}{n_t} \right)^{\frac{1}{s_t}} = (1 - m2_t)m3_t \times \frac{\hat{\pi}_{n,t+1} + \hat{\pi}_{q,t+1}q_t}{\frac{(1-m0_t)(1-m1_t)}{\sum_{i=0}^1(1+ADR_{t+i})}} + (1 - m2_t)(1 - m3_t) \times \frac{\hat{\pi}_{n,t+1} + \hat{\pi}_{q,t+1}q_t}{\frac{(1-m0_t)(1-m1_t)}{\sum_{i=0}^2(1+ADR_{t+i})}}$$

$$[M2] \frac{\hat{z}_{t-1}}{\hat{x}_{t+1}\hat{x}_t} \left( \frac{\beta \hat{x}_t \hat{z}_t / \hat{z}_{t-1}}{q_t} \right)^{\frac{1}{s_t}} = (1 - m2_t)m3_t \times \frac{\hat{\pi}_{q,t+1}n_t}{\frac{(1-m0_t)(1-m1_t)}{\sum_{i=0}^1(1+ADR_{t+i})}} + (1 - m2_t)(1 - m3_t) \times \frac{\hat{\pi}_{q,t+1}n_t}{\frac{(1-m0_t)(1-m1_t)}{\sum_{i=0}^2(1+ADR_{t+i})}}$$

$$[M3] \hat{w}_t = \frac{\theta_1 \hat{Y}_t}{\hat{L}_t}$$

$$[H] \hat{z}_t \equiv m2_t \times \hat{z}1_t + (1 - m2_t)m3_t \times \hat{z}2_t + (1 - m2_t)(1 - m3_t) \times \hat{z}3_t$$

$$[Ha] (1 + ADR_t)\hat{z}1_t = \hat{w}_t$$

$$[Hb] \sum_{i=0}^1(1 + ADR_{t+i}) \hat{z}2_t + (\hat{\pi}_{n,t+1}b_t + \hat{\pi}_{q,t+1}q_t b_t)\hat{X}_{t+1} = \hat{w}_t + \hat{w}_{t+1}\hat{X}_{t+1}$$

$$[Hc] \sum_{i=0}^2(1 + ADR_{t+i}) \hat{z}3_t + (\pi_{n,t+1}b_t + \pi_{q,t+1}q_t b_t)\hat{X}_{t+1} = \hat{w}_t + \hat{w}_{t+1}\hat{X}_{t+1} + \hat{w}_{t+2}\hat{X}_{t+2}\hat{X}_{t+1}$$

$$[F] \hat{Y}_t = \exp(\epsilon_t^Y) \hat{L}_t^{\theta_1}, \text{ where } \epsilon_t^Y \sim N(0, \sigma_Y^2)$$

$$[A1] \hat{P}_t \equiv 1 - \hat{D}_t + \hat{B}_t$$

$$[A2] \hat{D}_t \equiv m0_t \hat{B}_t + m1_t(1 - m0_{t-1}) \frac{\hat{B}_{t-1}}{\hat{P}_{t-1}} + m2_t \frac{\hat{P}_{1t-1}}{\hat{P}_{t-1}} + m3_t \frac{\hat{P}_{2t-1}}{\hat{P}_{t-1}} + \frac{1}{4} \frac{\hat{P}_{3t-1}}{\hat{P}_{t-1}} + \frac{1}{4} \frac{\hat{P}_{3t-2}}{\hat{P}_{t-1}\hat{P}_{t-2}} + \frac{2}{4} \frac{\hat{P}_{3t-3}}{\hat{P}_{t-1}\hat{P}_{t-2}\hat{P}_{t-3}}$$

$$[A3] \hat{B}_t = (1 - \mu_t) \times \frac{\hat{P}_{1t} + \hat{P}_{2t}}{2} \times b_{t-1} \equiv CBR_t$$

$$[A4] \hat{P}_{1t} \equiv (1 - m1_t)(1 - m0_{t-1}) \frac{\hat{B}_{t-1}}{\hat{P}_{t-1}}$$

$$[A5] \hat{P}_{2t} \equiv (1 - m2_t) \frac{\hat{P}_{1t-1}}{\hat{P}_{t-1}}$$

$$[A6] \hat{P}_{3t} \equiv (1 - m3_t) \frac{\hat{P}_{2t-1}}{\hat{P}_{t-1}}$$

$$[A7] ADR_t \equiv \left( \frac{3}{4} \frac{\hat{P}_{3t-1}}{\hat{P}_{t-1}} + \frac{2}{4} \frac{\hat{P}_{3t-2}}{\hat{P}_{t-1}\hat{P}_{t-2}} + \frac{1}{4} \frac{\hat{P}_{3t-3}}{\hat{P}_{t-1}\hat{P}_{t-2}\hat{P}_{t-3}} \right) / \hat{L}_t$$

$$[A8] \hat{L}_t \equiv \hat{P}_{1t} + \hat{P}_{2t} + \hat{P}_{3t}$$

$$[A9] \hat{H}_t \equiv \frac{\hat{P}_{1t}}{\hat{L}_t} \hat{Q}_{t-1} + \frac{\hat{P}_{2t}}{\hat{L}_t} \frac{\hat{Q}_{t-2}}{\hat{H}_{t-1}} + \frac{\hat{P}_{3t}}{\hat{L}_t} \frac{\hat{Q}_{t-3}}{\hat{H}_{t-1}\hat{H}_{t-2}}$$

$$[A10] \hat{\pi}_{nt} = \exp(\epsilon_t^{\pi n}) \bar{\Phi}_{nt} \widehat{W}_t$$

$$[A11] \hat{\pi}_{qt} = \exp(\epsilon_t^{qn}) \bar{\Phi}_{qt} \widehat{W}_t$$

$$[A12] \mu_t = \tau_0 + \tau_\mu \times \mu_{t-1} + \tau_A \times \ln A_t + \tau_w \times \ln \widehat{W}_t + \epsilon_t^\mu, \text{ where } \epsilon_t^\mu \sim N(0, \sigma_\mu^2)$$

$$[A13] \ln A_t = a_0 + a_A \ln A_{t-1} + a_b b_t + \epsilon_t^A, \text{ where } \epsilon_t^A \sim N(0, \sigma_A^2)$$

$$[A14] \widehat{Q}_t = \exp(\epsilon_t^Q) \left( \frac{\widehat{Q}_{t-2}}{\widehat{H}_t \widehat{H}_{t-1}} q_{t-1} \right)^{1-\varepsilon}, \text{ where } \epsilon_t^Q \sim N(0, \sigma_Q^2)$$

$$[D1] b_t \equiv \frac{n_t}{(1-m_0)(1-m_1)}$$

$$[D2] \widehat{W}_t \equiv \frac{w_t}{w_{t-1}} = \frac{\widehat{w}_t \widehat{X}_t}{\widehat{w}_{t-1}}$$

$$[D3] \widehat{X}_t \equiv \widehat{H}_t^{\theta_2} \widehat{P}_{t-1}^{\theta_1 + \theta_2 - 1}$$

**[Proposition 1] Demand function for child quality**

The most important two equations in the system are the two marginal conditions [M1] and [M2]. If we focus on the steady state, all the shocks are set to zero and time subscripts can be ignored. Multiply [M1] by  $n$  and [M2] by  $q$ , then divide the two sides of the two equations (also make use of the definition of  $b$  in [D1]), we have:

$$\left(\frac{\alpha q}{\beta n}\right)^{\frac{1}{s}} \left(\frac{n}{q}\right) = \frac{(1-m_2)m_3^{\frac{\hat{\pi}_n b + \hat{\pi}_q q b}{2(1+ADR)} + (1-m_2)(1-m_3)^{\frac{\hat{\pi}_n b + \hat{\pi}_q q b}{3(1+ADR)}}}{(1-m_2)m_3^{\frac{\hat{\pi}_q q b}{2(1+ADR)} + (1-m_2)(1-m_3)^{\frac{\hat{\pi}_q q b}{3(1+ADR)}}}}$$

Factor out the common terms on the denominator and the numerator, resulting in:

$$\left(\frac{\alpha}{\beta}\right)^{\frac{1}{s}} \left(\frac{q}{n}\right)^{\frac{1-s}{s}} = 1 + \frac{\hat{\pi}_n}{\hat{\pi}_q} \frac{1}{q} \rightarrow \frac{q}{n} \propto \frac{\hat{\pi}_n}{\hat{\pi}_q}$$

This is a very informative equation, suggesting that the ratio of quality and quantity is inversely dependent of the ratio between the costs of quality and quantity. In the special case of Cobb-Douglas where  $s = 1$  as assumed by many papers, it results in a quadratic equation and an analytical solution is available:

$$q = \frac{\beta}{\alpha - \beta} \frac{\hat{\pi}_n}{\hat{\pi}_q}, \text{ if } s = 1 \text{ and } \alpha > \beta.$$

Mortality has no effect on child quality (but this is only on the demand side. See proposition 2 for the supply side where it does have an effect).

**[Proposition 2] Supply function of child quality**

Make use of equation [A14] in steady state:

$$\hat{Q} = \left(\frac{\hat{Q}}{\hat{H}^2} q\right)^{1-\varepsilon}$$

Use definitions of  $\hat{P}i$  and  $\hat{L}$  to express  $\hat{Q}$  in terms of  $\hat{H}$ :

$$\hat{Q} = \frac{\hat{H}}{\frac{\frac{(1-m_0)(1-m_1)}{\hat{p}}}{CCC} + \frac{\frac{(1-m_0)(1-m_1)(1-m_2)}{\hat{p}^2}}{CCC} + \frac{1}{\hat{H}} + \frac{\frac{(1-m_0)(1-m_1)(1-m_2)(1-m_3)}{\hat{p}^3}}{CCC} \frac{1}{\hat{H}^2}}, \text{ where:}$$

$$CCC \equiv \frac{(1-m_0)(1-m_1)}{\hat{p}} + \frac{(1-m_0)(1-m_1)(1-m_2)}{\hat{p}^2} + \frac{(1-m_0)(1-m_1)(1-m_2)(1-m_3)}{\hat{p}^3}.$$

Combine this expression of  $\hat{Q}$  with equation [A14], we can solve for  $q$ :

$$q = \left(1 + \frac{(1-m_2)}{\hat{p}} + \frac{(1-m_2)(1-m_3)}{\hat{p}^2}\right)^{\frac{\varepsilon}{1-\varepsilon}} \left(1 + \frac{(1-m_2)}{\hat{p}} \frac{1}{\hat{H}} + \frac{(1-m_2)(1-m_3)}{\hat{p}^2} \frac{1}{\hat{H}^2}\right)^{\frac{\varepsilon}{\varepsilon-1}} \hat{H}^{2-\varepsilon} \quad [X]$$

This is another mechanism determining  $q$ . It is positively related to the overall human capital growth rate ( $\hat{H}$ ), which also defines the technological progress growth rate ( $\hat{X} =$

$\widehat{H}^{\theta_2} \widehat{P}^{\theta_1 + \theta_2 - 1}$ )—see the original production function equation [F]. There are two reinforcing sources of this positive effect:

- The first term describes the contribution from family education;
- The second term describes the contribution from nonfamily education.

To see this, consider the special case where there is no external effect of nonfamily education ( $\varepsilon = 0$ ). Now  $q$  is a simple quadratic function of the overall human capital growth rate:  $q = \widehat{H}^2$ . In other words, the overall human capital growth only comes from family education. It is quadratic because there are two generations away between the parents and their children.

[Corollary 1]  $q$  rises when  $m_2$  drops.

To simplify symbols, we focus on the terms in the bracket of equation [X] and define:

$$a \equiv \frac{1-m_2}{\widehat{p}} \in (0,1), b \equiv \frac{1-m_3}{\widehat{p}} \in (0,1), \text{ and } x \equiv \frac{1}{\widehat{H}} \in (0,1)$$

So, it becomes:

$$\blacksquare \equiv \frac{1}{1+a+ab} + \frac{ax}{1+a+ab} + \frac{abx^2}{1+a+ab}$$

Take partial derivative of this term with respect to  $a$ :

$$\frac{\partial \blacksquare}{\partial a} = -\frac{1+b}{(1+a+ab)^2} - \frac{ax(1+b)}{(1+a+ab)^2} + \frac{x}{1+a+ab} - \frac{abx^2(1+b)}{(1+a+ab)^2} + \frac{bx^2}{1+a+ab} = \frac{-(1+ax+abx^2)(1+b)+(1+a+ab)(x+bx^2)}{(1+a+ab)^2}$$

We know that  $0 < x < 1$ , so:

$$1 + ax + abx^2 < 1 + a + ab$$

Therefore, the derivative is negative:

$$\frac{\partial \blacksquare}{\partial a} < \frac{-(1+ax+abx^2)(1+b)+(1+ax+abx^2)(x+bx^2)}{(1+a+ab)^2} = -\frac{(1+ax+abx^2)(1-x)(1+b(1+x))}{(1+a+ab)^2} < 0$$

To summarize, when  $m_2$  drops,  $a \equiv \frac{1-m_2}{\widehat{p}}$  rises,  $\blacksquare$  drops,  $\blacksquare^{\frac{\varepsilon}{\varepsilon-1}}$  rises (because  $\frac{\varepsilon}{\varepsilon-1} < 0$ ), so  $q$  rises.

[Corollary 2]  $q$  rises when  $m_3$  drops.

Now take derivative of  $\blacksquare$  with respect to  $b$ :

$$\frac{\partial \blacksquare}{\partial b} = -\frac{a}{(1+a+ab)^2} - \frac{a^2x}{(1+a+ab)^2} - \frac{a^2bx^2}{(1+a+ab)^2} + \frac{ax^2}{1+a+ab} = \frac{-a(1+ax+abx^2)+ax^2(1+a+ab)}{(1+a+ab)^2}$$

Similarly, we know that



$$1 + ax + abx^2 < 1 + a + ab$$

The derivative is also negative:

$$\frac{\partial \blacksquare}{\partial b} < \frac{-a(1+ax+abx^2)+ax^2(1+ax+abx^2)}{(1+a+ab)^2} = -\frac{a(1+ax+abx^2)(1-x^2)}{(1+a+ab)^2} < 0$$

To summarize, when  $m_3$  drops,  $b \equiv \frac{1-m_3}{\hat{p}}$  rises,  $\blacksquare$  drops,  $\blacksquare^{\frac{\varepsilon}{\varepsilon-1}}$  rises (because  $\frac{\varepsilon}{\varepsilon-1} < 0$ ), so  $q$  rises.

### [Proposition 3] Demand function for child numbers

Define the expected disposable income as:

$$\hat{\omega} \equiv \frac{m_2}{1+ADR} \times \hat{w} + \frac{(1-m_2)m_3}{2(1+ADR)} \times (\hat{w} + \hat{w}\hat{X}) + \frac{(1-m_2)(1-m_3)}{3(1+ADR)} \times (\hat{w} + \hat{w}\hat{X} + \hat{w}\hat{X}^2)$$

Then combining equation [H] and [Ha]-[Hc] gives:

$$b(\hat{\pi}_n + \hat{\pi}_q q) = \frac{\hat{\omega} - \hat{z}}{\hat{X} \left[ \frac{(1-m_2)m_3}{2(1+ADR)} + \frac{(1-m_2)(1-m_3)}{3(1+ADR)} \right]}$$

In the special case of  $s = 1$ , substitute the above into equation [1], we can solve for  $\hat{z}$ :

$$\hat{z} = \frac{\gamma}{\alpha + \gamma} \hat{\omega}$$

Combine with equation [2] and [Proposition 1], we can solve for  $n$  ( $q$  is already solved in [Proposition 1]):

$$n = (1 - m_0)(1 - m_1) \frac{\alpha - \beta}{\alpha + \gamma} \frac{\hat{\omega}}{\hat{\pi}_n \left( \frac{(1-m_2)m_3}{2(1+ADR)} + \frac{(1-m_2)(1-m_3)}{3(1+ADR)} \right) \hat{X}}$$

Note that crude birth per capita is defined as  $b = \frac{n}{(1-m_0)(1-m_1)}$ , so there is no effect of child mortality rates on  $b$ . A smaller elasticity of substitution changes the response so that the elasticity of births with respect to mortality is positive.

We can define an effective price of  $n$  (with respect to the price of  $z$ , the numeraire):

$$\hat{\Pi}_n \equiv \hat{\pi}_n \frac{\left( \frac{(1-m_2)m_3}{2(1+ADR)} + \frac{(1-m_2)(1-m_3)}{3(1+ADR)} \right)}{(1-m_0)(1-m_1)}$$

The terms in the numerator adjust for probability of premature death in childcare and burden in eldercare and infant mortalities.

- The probability of a person dying before she has any child in her Age 2 is  $m_2$  and the childcare expense is simply 0. The consumption price needs to include the resources spent on eldercare during her Age 1 only, so the effective price of consumption during Age 1 is  $1(1 + ADR)$ .

- If the person dies after she has children in her Age 2 but before Age 3, the probability is  $(1 - m_2)m_3$  and the childcare price is  $\hat{\pi}_n$ . The consumption price needs to include the ‘pension’ paid for eldercare during her Ages 1 and 2, so the effective price of consumption during Ages 1 and 2 is  $2(1 + ADR)$ .
- If the person dies after Age 3, the probability is  $(1 - m_2)(1 - m_3)$  and the childcare price is also  $\hat{\pi}_n$ . The consumption price needs to include the ‘pension’ paid for eldercare during her Ages 1, 2 and 3, so the effective price of consumption during Ages 1, 2 and 3 is  $3(1 + ADR)$ .

Therefore, the solution of  $n$  can also be rewritten as:

$$n = \frac{\alpha - \beta}{\alpha + \gamma} \cdot \frac{\hat{\omega}}{\hat{\pi}_n \hat{X}}$$

If there is no economic growth  $\hat{X} = 1$ , then this solution is similar to equation (1) in Foreman-Peck (2011) Appendix. If there is economic growth, then  $n$  falls unless there are other changes such as falls in child mortality or ADR lowering child price (eventual demographic transition).

When  $m_0$  and  $m_1$  rise, there are equal negative effects on  $n$ , but no effect on  $b$ . There are two *direct* effects of  $m_2$  and  $m_3$  on  $n$ : (1) the **income effect** through the numerator ( $\hat{\omega}$ ), and (2) the **substitution effect** through the denominator ( $\hat{\pi}_n$ ). (NB: There are also *indirect* effects of  $m_2$  and  $m_3$  through wage and human capital, which can only be obtained after solving for the general equilibrium.)

Starting with  $m_2$  we can prove that the wealth effect of  $m_2$  is negative  $\frac{\partial \hat{\omega}}{\partial m_2} \leq 0$ :

$$\frac{\partial \hat{\omega}}{\partial m_2} = \frac{1}{1+ADR} \times \hat{\omega} - \frac{m_3}{2(1+ADR)} \times (\hat{\omega} + \hat{\omega}\hat{X}) - \frac{(1-m_3)}{3(1+ADR)} \times (\hat{\omega} + \hat{\omega}\hat{X} + \hat{\omega}\hat{X}^2)$$

In balanced growth path, there is a non-negative growth rate, so  $\hat{X} \geq 1$ , therefore, the above is:

$$\frac{\partial \hat{\omega}}{\partial m_2} \leq \frac{1}{1+ADR} \times \hat{\omega} - \frac{m_3}{2(1+ADR)} \times (\hat{\omega} + \hat{\omega}) - \frac{(1-m_3)}{3(1+ADR)} \times (\hat{\omega} + \hat{\omega} + \hat{\omega}) = 0$$

Therefore, the total effect of  $m_2$  on  $n$  is ambiguous:

$$\frac{\partial n}{\partial m_2} = \underbrace{\frac{\partial n}{\partial \hat{\omega}}}_{>0} \underbrace{\frac{\partial \hat{\omega}}{\partial m_2}}_{<0} + \underbrace{\frac{\partial n}{\partial \hat{\pi}_n}}_{<0} \underbrace{\frac{\partial \hat{\pi}_n}{\partial m_2}}_{<0} \geq 0$$

The intuition behind this ambiguous effect is that when mortality rate  $m_2$  rises, the lifetime expected wage drops, leading to a negative income effect. At the same time, a rise in  $m_2$  also means a relatively cheaper price of  $n$ , leading to a positive substitution effect.

Now turn to the effect of  $m_3$ . We can prove that the wealth effect of  $m_3$  is also negative, i.e.  $\frac{\partial \hat{\omega}}{\partial m_3} \leq 0$ .

$$\frac{\partial \hat{\omega}}{\partial m_3} = \frac{1-m_2}{2(1+ADR)} \times (\hat{\omega} + \hat{\omega}\hat{X}) - \frac{(1-m_2)}{3(1+ADR)} \times (\hat{\omega} + \hat{\omega}\hat{X} + \hat{\omega}\hat{X}^2)$$

In balanced growth path, there is a non-negative growth rate, so  $\hat{X} \geq 1$  and:

$$\frac{\partial \hat{\omega}}{\partial m_3} = \frac{\hat{\omega}(1-m_2)}{6(1+ADR)} (1 + 2\hat{X})(1 - \hat{X}) \leq 0$$

Therefore, the total effect of  $m_3$  on  $n$  is negative:

$$\frac{\partial n}{\partial m_3} = \frac{\partial n}{\partial \hat{\omega}} \frac{\partial \hat{\omega}}{\partial m_3} + \frac{\partial n}{\partial \hat{\Pi}_n} \frac{\partial \hat{\Pi}_n}{\partial m_3} < 0$$

$\begin{matrix} >0 & <0 & & <0 & & >0 \end{matrix}$

The difference from  $m_2$  is that when  $m_3$  rises, the effect on  $\hat{\Pi}_n$  is positive, so the substitution effect reinforces the income effect.

*Human capital growth and the demand for children:*

$$n^D = \text{constant} \times \frac{\frac{m_2}{1+ADR} + \frac{(1-m_2)m_3}{2(1+ADR)} \times (1 + \hat{X}) + \frac{(1-m_2)(1-m_3)}{3(1+ADR)} \times (1 + \hat{X} + \hat{X}^2)}{\hat{X}}$$

Take partial derivative of  $n^D$  with respect to  $\hat{X}$ :

$$\begin{aligned} \frac{\partial n^D}{\partial \hat{X}} &= \text{constant} \times \left[ \frac{\left[ \frac{(1-m_2)m_3}{2(1+ADR)} + \frac{(1-m_2)(1-m_3)}{3(1+ADR)} \right] \times (1 + \hat{X})}{\hat{X}} \right. \\ &\quad \left. - \frac{\frac{m_2}{1+ADR} + \frac{(1-m_2)m_3}{2(1+ADR)} \times (1 + \hat{X}) + \frac{(1-m_2)(1-m_3)}{3(1+ADR)} \times (1 + \hat{X} + \hat{X}^2)}{\hat{X}^2} \right] \\ &= \text{constant} \times \frac{\frac{m_2}{1+ADR} - \frac{(1-m_2)m_3}{2(1+ADR)} - \frac{(1-m_2)(1-m_3)}{3(1+ADR)}}{\hat{X}^2} \end{aligned}$$

The sign of the partial derivative hinges on the sign of the numerator:

$$\frac{m_2}{1+ADR} - \frac{(1-m_2)m_3}{2(1+ADR)} - \frac{(1-m_2)(1-m_3)}{3(1+ADR)}$$

We know that the inequality below follows as long as  $m_2 < 0.5$  (which is empirically true):

$$\begin{aligned} \frac{m_2}{1+ADR} - \frac{(1-m_2)m_3}{2(1+ADR)} - \frac{(1-m_2)(1-m_3)}{3(1+ADR)} &< \frac{m_2}{1+ADR} - \frac{(1-m_2)m_3}{1+ADR} - \frac{(1-m_2)(1-m_3)}{1+ADR} \\ &= \frac{2 \times (m_2 - 0.5)}{1+ADR} < 0 \end{aligned}$$

Therefore,  $\frac{\partial n^D}{\partial \hat{X}} < 0$ .

*Wages and the demand for children:*

The wage effect through the generalized child price cancels with the wage effect on expected disposable income. So, there is no effect on the demand for children from wage growth (contrary to Malthus) in this special case when  $s = 1$ . With  $s < 1$  as it was throughout the income effect of a wage increase dominated the substitution effect—demand for children increased with wage increases but by less so as the elasticity of substitution rose.

## Data Sources and Methods

The table at the end of this Appendix lists all data sources of this paper with the numbered identifiers of this text.

### [Population]

Broadberry et al (2015) in Bank of England, A Millennium of Macroeconomic Data, <https://www.bankofengland.co.uk/statistics/research-datasets> (Table A2) provide annual data for England's population 1086-1870. Table A18 gives English population from official Census sources from 1841 to 2016.

### [Earnings]

The real earnings (per capita) series is mainly based on a combination of Clark (2007) for 1209-1869, Clark (2005) for 1870-1962 and ONS official publications for 1963-2016 summarized by Clark (2018) at MW. At the same time, with weights from Horrell and Humphries (1995) and Levi (1867) given the role of female workers in household income, we make use of HW (2015) female wage data to enhance the gender dimension in Clark (2018) before 1850.

- **1851-2016:** MW ([5] and [6]) are used without adjustment.
- **1209-1850:** A weighted average between male earnings ( $E_M^{Clark}$ ) and female earnings ( $E_F^{HW}$ ) is used, with weights throughout this period,  $\bar{E} = 0.75E_M^{Clark} + 0.25E_F^{HW}$ .
  - $E_M^{Clark}$ : According to the assumption from Levi (1867) in Clark (2007), we can calculate the male earnings ( $E_M^{Clark}$ ) as [4] divided by 0.625.<sup>4</sup>
  - $E_F^{HW}$ : The female earnings is based on HW (2015):  $E_F^{HW} = w_F^{HW} N_F^{HW}$ , where both female wage ( $w_F^{HW}$ ) and number of female working hours ( $N_F^{HW}$ ) are themselves weighted averages between annual contract (mainly taken by single women, 30%) and casual contract (mainly taken by married women, 70%). The annual contract is assumed to be 300 working days, while the casual contract is assumed to be 6 workdays a week and 17 work weeks a year (102 workdays a year). Finally, the casual contracted female workers are also subject to a lower participation rate, which is based on the average of series [20]-[23].

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<sup>4</sup> In Clark (2007) and in the MW website documentation, it is assumed that female income accounts for about 25% of a two-person household income. In other words,  $E = \frac{E_M + 0.25E_M}{2} = \frac{1.25E_M}{2} \rightarrow E_M = \frac{E}{0.625}$ .

#### [Crude Birth Rate]

The Crude Birth Rate (CBR) is defined as the total birth flow divided by the population stock in the beginning of each period. To be consistent with the 15-year time interval in the model, the length of period in data is also 15 years. The annual birth flows in [24] and [25] are spliced using England [1] as the benchmark ([2] covers England and Wales) before summed over 15-year intervals. We splice the annual birth flows in [26] and [27] and sum over 15-year intervals.

#### [Crude Death Rate]

The Crude Death Rate (CDR) is defined as the total death flow divided by the population stock in the beginning of each period.

#### [Female Age of First-Time Marriage]

There are fragmentary data on female marriage age ([35]-[38]). We first splice [35] and [38] using the later series [35] as benchmark. Then, combine this spliced series with [36] and [37] by linear interpolation to construct a series of female marriage age.

#### [Childless/Celibacy Rate]

There are three series used for computing childless rate, where [40] overlaps with [41] substantially. We first adjust [41] by multiplying the average ratio between [40] and [41], treating the later series [40] as benchmark. Then, use the adjusted series to fill in the missing values in [40]. Combine the result with [42] and apply linear interpolation to fill the rest of missing values.

#### [Mortality Rates]

The calculation of mortality rates is based on data series [28]-[34], which are from four sources: HMD, WS (1989), Russell (1948) and Hatcher et al (2006). They cover different periods of time and the overlapped parts are averaged.

- **1841-2015:** HMD mortality data are age specific, so it is straightforward to calculate the mortality rates in different age/generation.
- **1541-1871:** [32] is a table of 10 different levels of age-specific mortality rates in WS (1989, Table A14.5) corresponding to different life expectancies, it is not a time series of mortality rates directly. We therefore work out the two neighbouring levels in [32] according to the values of life expectancy for each date [29] and derive the weights to give the target life expectancy. The weights are

then used to average the two corresponding levels of age-specific mortality rates. For example, in 1541, the observed life expectancy (from [29]) is 33.75. We look at [31] to search for the two neighbouring levels that give the closest life expectancy to 33.75. It turns out to be level 7 (33.31) and level 8 (35.77). To obtain 33.75, we need to solve the weights from the identity:  $33.31x + 35.77(1 - x) = 33.75$ , resulting in  $x = 82.1\%$  to be used as the weight for level 7 mortality rates (in [32]) and  $1 - x = 17.9\%$  to be used as the weight for level 8 mortality rates (in [32]) to compute the estimates of mortality rates in 1541.

- **1086-1450:** [33] has a low 25-year frequency in Russell (1948), but the calculation of mortality rates is straightforward.
- **1395-1529:** [34] is a low-frequency time series of life expectancy in Hatcher et al (2006). We apply the same method as in dealing with WS (1989) by matching the levels in [32] and then work out the implied mortality rates.

We illustrate the construction of generation-specific mortality rates by an example in Figure A1, where we show the four underlying sources of  $m_0$  (childhood: aged between 0 and 15) and the 15-year moving average trend of the combined series.

#### [Female Literacy]

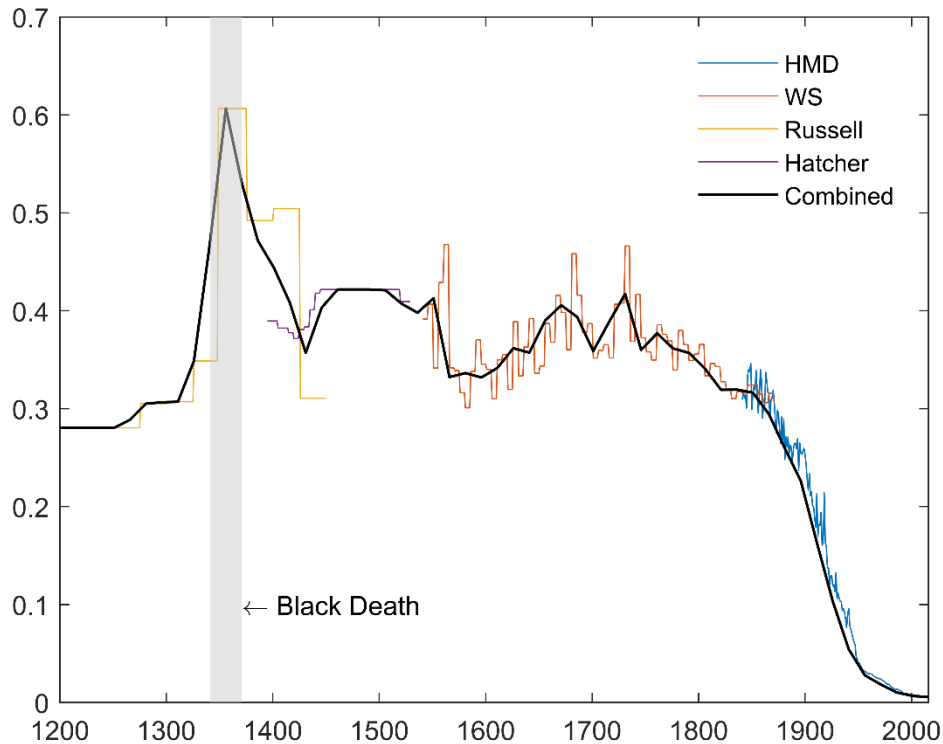
The two series on female literacy rate [44] and [45] are interpolated before they are combined by simple average. The missing values at the two ends are extrapolated linearly with a lower bound of 0 and an upper bound of 1.

Similarly, we can calculate the male literacy rate. The male and female literacy rate can then be averaged to obtain the overall literacy rate, which is used below to derive the school enrolment rate.

#### [Urbanization Rate]

The urbanization rate is based on Madsen and Murin (2017). Observations beyond the sample period of the original data (1270-2011) are linearly extrapolated.

**Figure A2.1** Childhood Mortality Rate ( $m_0$ )



Notes: The vertical axis is the mortality rate for the childhood generation (0-15) defined over 15-year intervals. Data sources: HMD, human mortality database website; WS, Wrigley and Schofield (1989); Russell (1948); Hatcher et al (2006).

#### [Wage Premium]

The (male) wage premium or gender pay gap ( $WP$ ) is defined as the logarithm of the ratio between male wage and female wage. The male wage is calculated as an average of [13]-[19] with necessary splicing and interpolation.

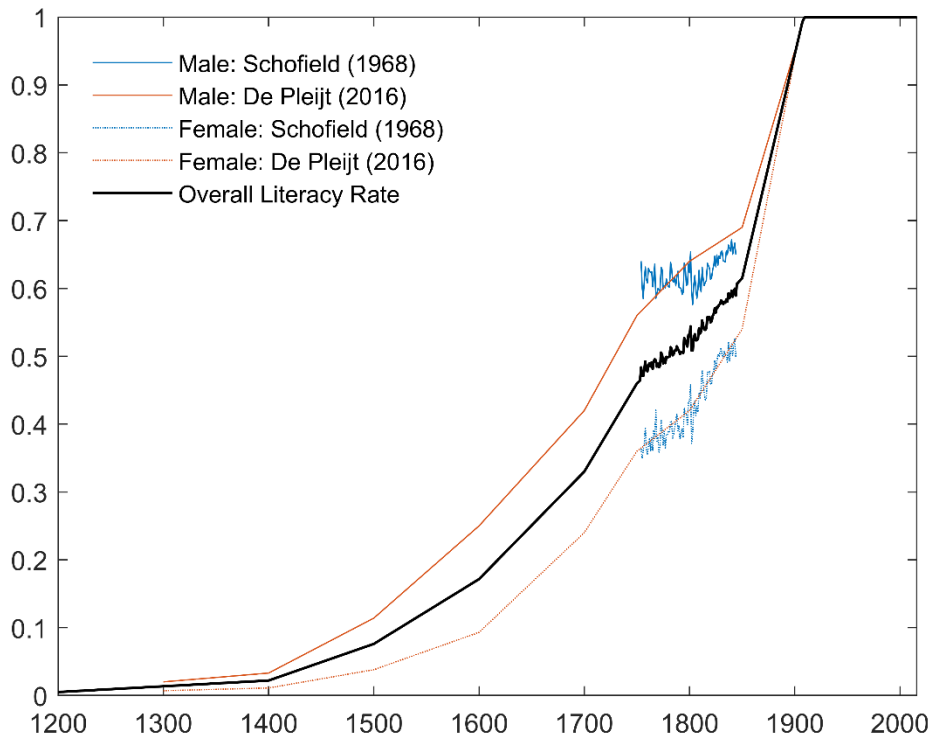
#### [Food Price]

We use oat price [50] and wheat price [51] in Clark (2006) to construct an index for food price during the early period (1209-1800). More comprehensive food items are included in Mitchell (1962) based on Rousseaux (1938) for the period 1800-1910. Feinstein (1972) [53] is used for the period 1900-1965, after which the ONS data<sup>5</sup> [54] are used.

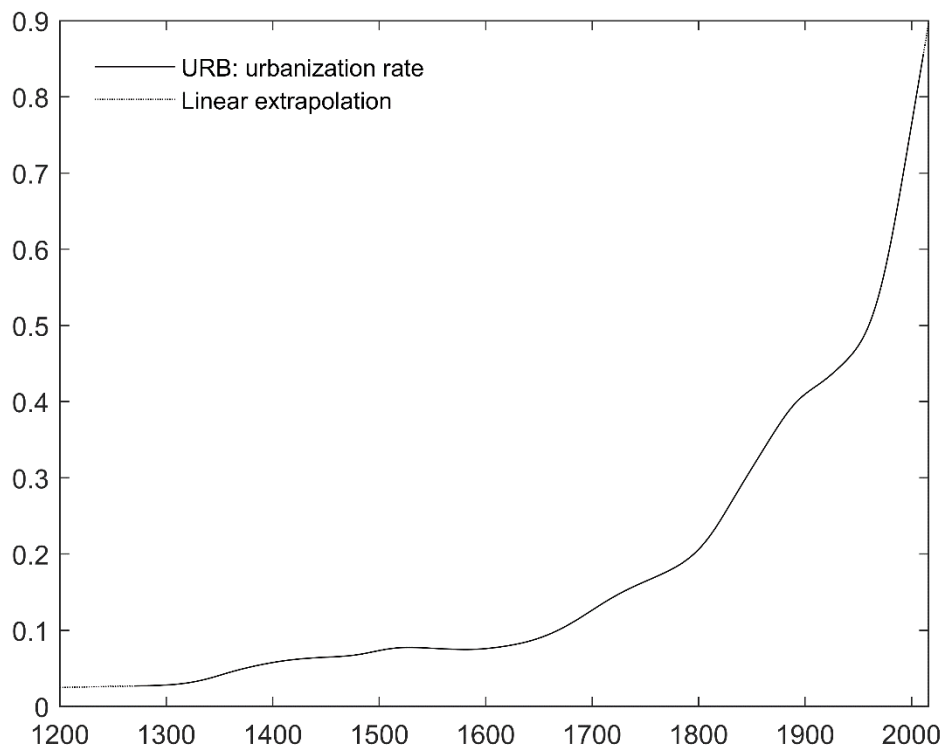
<sup>5</sup> <http://www.ons.gov.uk/ons/guide-method/user-guidance/prices/cpi-and-rpi/cpi-and-rpi-basket-of-goods-and-services/index.html>



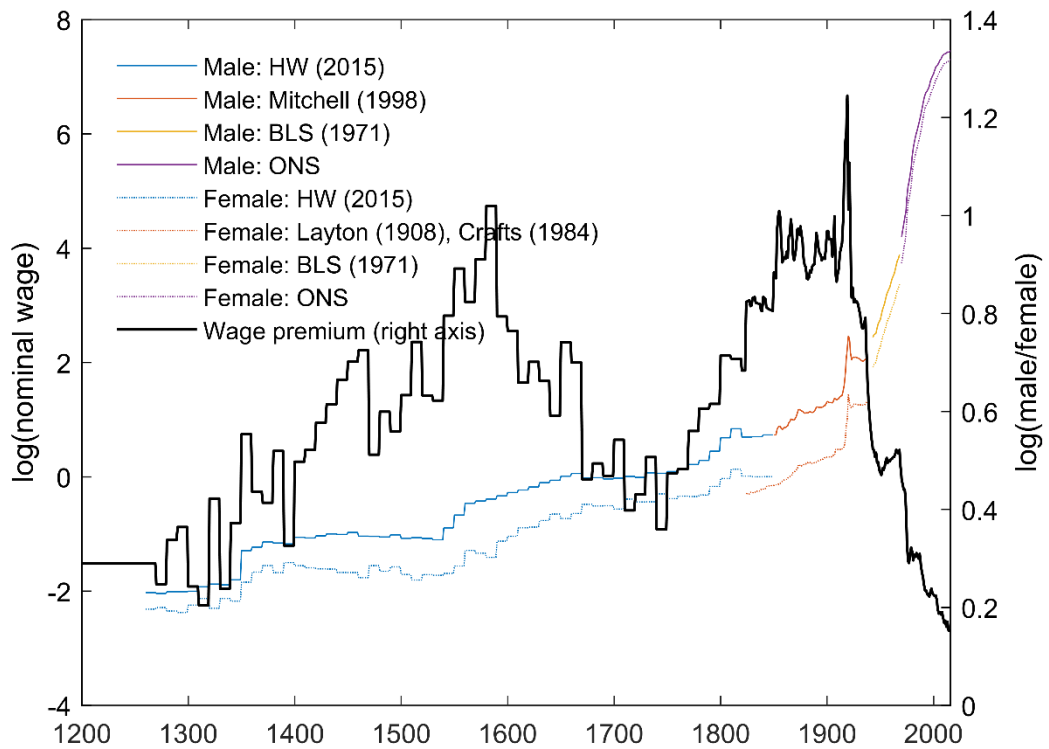
**Figure A2.2** Literacy by Gender



**Figure A2.3** Urbanization Rate in England (*URB*)

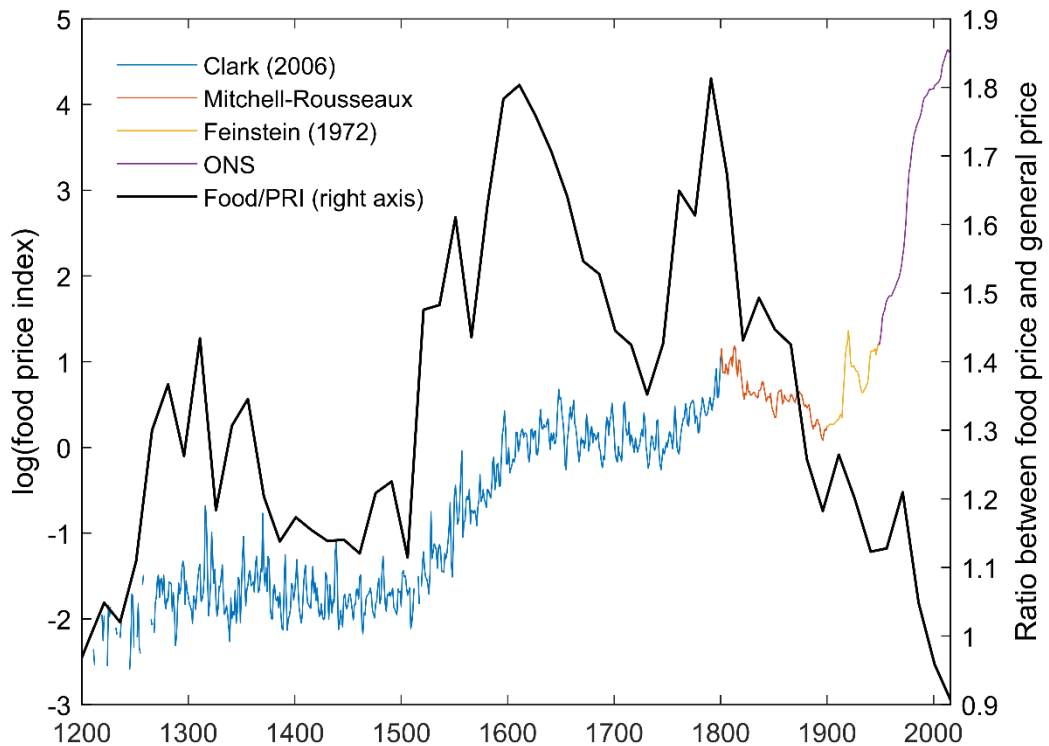


**Figure A2.4 Construction of Male-Female Wage Premium (*WP*)**



Notes: The left axis is the logarithmic level of nominal wages by gender from different sources. The right axis is the (male) wage premium defined as  $\log(\text{male}/\text{female}) \approx (\text{male}-\text{female})/\text{female}$ .

**Figure A2.5 Food Price and the Ratio to RPI (*FPR*)**

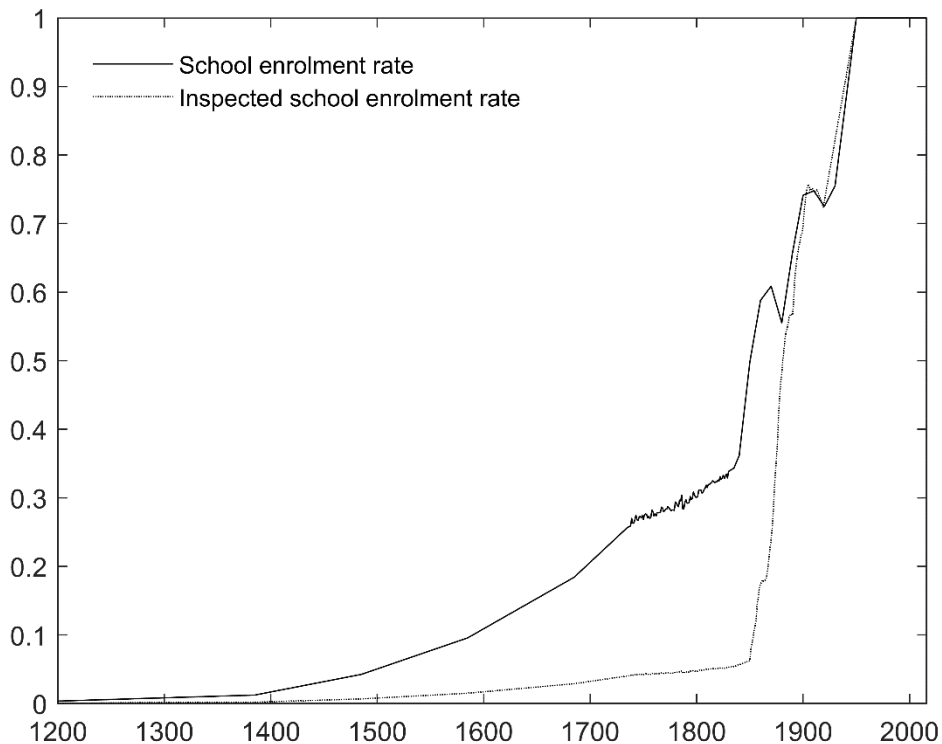


[School Enrolment Rate]

The crude rate combines two approaches to calculating the school enrolment rate: the interpolated school enrolment rate and the implied school enrolment rate. The first approach, the interpolated school enrolment rate, is a linear interpolation/extrapolation of the ratio of [46] divided by [48], with a lower bound of 0 and an upper bound of 1. This is used for the period after 1830 when the total number of pupils is available in [46]. The second approach, the implied school enrolment rate, is based on the positive relationship between school enrolment rate and overall literacy rate. The average ratio between the two series can be used to fill in the missing values of schooling enrolment rates (the overall literacy rate is continuously available, see the method in [Female Literacy Rate] above). This approach is used for the period before 1830 when no data on schooling is available at all.

There is an alternative measure of school provision, the inspected school enrolment. We apply the same method to [47] instead of [46]. It is argued that inspected school provides a higher quality of education, while the crude enrolment rate is more a measure of foregone child labour.

**Figure A2.6** School Enrolment Rates ( $S$  and  $\tilde{S}$ )



Here are some general procedures for the data management:

- **Low-frequency data**, such as decadal wages in HW (2015), are converted to annual data assuming that each year within the period share the same value.
- **Overlapped data**, such as female nominal wages, are combined by simple average.
- **Missing data**, such as female participation rates, are filled by linear interpolation and linear extrapolation.
- **Nominal data**, such as female wages, are converted to real counterparts using the RPI series in MW website [55].
- **Different geographic coverages**, such as births and deaths, are adjusted to England territory assuming that England is a fixed proportion of the UK, while average rates, such as wages and participation rates, are assumed the same across the UK.

### List of Data Sources

ID	Symbol	Definition	Year	Geography	Frequency	Direct Source	Root Source
[1]	pop	population	1086-1870	England/GB	annual	BoE: A2.B	Broadberry et al (2015)
[2]			1541-1841	England	annual	BoE: A18.K	Wrigley et al (1997)
[3]			1842-2016	England	annual	BoE: A18.K	Census
[4]	rearnings	real annual earnings	1209-1869	England	annual	MW	Clark (2007)
[5]			1870-1962	England	annual	MW	Clark (2005)
[6]			1963-2016	GB	annual	MW	ONS
[7]	fnwage1	female nominal wage	1875-1938	UK = GB + NI	annual	Crafts (1984)	
[8]	fnwage2	female nominal wage	1970-1996	UK = GB + NI	annual	ONS: NES	
[9]			1997-2016	UK = GB + NI	annual	ONS: ASHE	
[10]	fnwage3	female nominal wage (domestic)	1823-1907	England	annual	Layton (1908)	
[11]	fnwage4	female nominal wage (industrial)	1824-1900	England	benchmarks	Layton (1908)	
[12]	mnwage1	male nominal wage	1851-1968	London	annual	BLS (1971)	
[13]	mnwage2	male nominal wage	1970-1996	UK = GB + NI	annual	ONS: NES	
[14]			1997-2016	UK = GB + NI	annual	ONS: ASHE	
[15]	mnwageM_A	male nominal wage	1790-1860	UK = GB + NI	annual	Mitchell (1998)	Deane and Cole (1962)
[16]	mnwageM_B	male nominal wage	1850-1910	UK = GB + NI	annual	Mitchell (1998)	Wood (1909)
[17]	mnwageM_C	male nominal wage	1860-1936	UK = GB + NI	annual	Mitchell (1998)	Bowley (1937)
[18]	[m/f]nwage_level	nominal wage by gender	1938-1968	UK = GB + NI	biannual	BLS (1971)	
[19]	[m/f]nwageHW_[A/C]	nominal wage by gender and contract	1260-1850	England	decadal	HW (2015, 2019)	
[20]	fpart1	female participation rate	1500-1770	England	benchmarks	Allen (2013)	
[21]			1787-1865	England	benchmarks	HH (1995)	
[22]			1971-2016	UK = GB + NI	annual	ONS	
[23]	fpart2	female participation rate	1840-1966	GB	benchmarks	BLS (1971)	
[24]	birth1	births	1539-1870	England	annual	WS (1997)	
[25]	birth2	births	1841-2016	England + Wales	annual	HMD	Registrar General, ONS
[26]	death1	deaths	1539-1870	England	annual	WS (1989)	
[27]	death2	deaths	1841-2016	England + Wales	annual	HMD	Registrar General, ONS
[28]	lexp1	life expectancy	1841-2016	England + Wales	annual	HMD	Registrar General, ONS

[29]	<b>lexp2</b>	life expectancy	1541-1871	England + Wales	5-year	WS (1989)	
[30]	<b>lexp3</b>	life expectancy	1395-1529	England (3 cities)	annual	Hatcher et al (2006)	
[31]	<b>mortality.HMD</b>	age-specific death and pop	1841-2016	England + Wales	annual	HMD	Registrar General, ONS
[32]	<b>mortality.WS</b>	life expectancy and mortality	1541-1871	England + Wales	5-year	WS (1989)	
[33]	<b>mortality.R</b>	age-specific mortality	1086-1450	England	25-year	Russell (1948)	
[34]	<b>mortality.H</b>	life expectancy	1395-1529	England	decadal	Hatcher et al (2006)	
[35]	<b>A_marriage1</b>	female age at marriage	1252-1478	England (5 villages)	benchmarks	Hallam (1985)	
[36]			1556-1814	England	benchmarks	Schofield (1985)	
[37]			1846-2011	England + Wales	annual	ONS	
[38]	<b>A_marriage2</b>	female age at marriage	1272-1509	England	benchmarks	Russell (1948)	
[39]	<b>A_birth</b>	female age at first birth	1938-2013	England + Wales	annual	ONS	
[40]	<b>psingle</b>	Proportion of single woman	1851-1911	England	benchmarks	WS (1989)	
[41]	<b>childless</b>	celibacy/childless rate at age 40-45	1596-1861	England + Wales	5-year	WS (1989)	
[42]			1950-2001	England + Wales	annual	ONS	
[43]	<b>illegit</b>	illegitimacy birth rate	1838-1938	England + Wales	annual	Mitchell (1998)	
[44]	<b>[m/f]lit1</b>	[m/f] literacy rate	1300-1900	England	benchmarks	De Pleijt (2016)	
[45]	<b>[m/f]lit2</b>	[m/f] literacy rate	1754-1844	England	annual	Schofield (1968)	
[46]	<b>pupil</b>	number of pupil (all school)	1830-1930	England + Wales	decadal	Lindert (1986)	
[47]	<b>pupil_insp</b>	number of pupil (inspected)	1850-1919	England + Wales	annual	Mitchell (1998)	
[48]	<b>[m/f]pop_[age group]</b>	population by age and gender	1840-1930	England + Wales	decadal	Mitchell (1998)	
[49]	<b>URB</b>	urbanization rate	1270-2011	UK = GB + NI	annual	Madsen and Murtin (2017)	
[50]	<b>poClark</b>	price of oat	1209-1914	England	annual	Clark (2006)	
[51]	<b>pwClark</b>	price of wheat	1209-1914	England	annual	Clark (2006)	
[52]	<b>pfoodMR</b>	food price index	1800-1913	England	annual	Mitchell (1998)	Rousseaux (1938)
[53]	<b>pfoodF</b>	food price index	1900-1965	England	annual	Feinstein (1972)	
[54]	<b>pfoodONS</b>	food price inflation	1949-2016	UK = GB + NI	annual	ONS	
[55]	<b>price</b>	RPI	1209-2016	England/UK	annual	MW	Clark (2005)

- **Notes:** [m/f] indicates either “m” (male) or “f” (female), [A/C] indicates either “A” (annual contract) or “C” (casual contract), and [age group] indicates either age group “5-9” or “10-14”.
- **Abbreviations:** MW, [www.measuringworth.com](http://www.measuringworth.com); HMD, human mortality database, [www.mortality.org](http://www.mortality.org); BoE, Bank of England, “a millennium of macroeconomic data”; ONS, Office for National Statistics; BLS, Bulletin of Labour Statistics; NES, New Earnings Survey; ASHE, Annual Survey of Hours and Earnings; WS, Wrigley and Schofield ; HW, Humphries and Weisdorf ; HH, Horrell and Humphries.