DCC-HEAVY model: a multivariate GARCH model utilising the high-frequency data
Additional Appendix

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A Appendix: ADCC-HEAVY Model

In this appendix, we derive the multiple-step ahead forecasts of ADCC-HEAVY model and present the empirical results from ADCC-HEAVY model.

Define D_t=$di\alpha g[d_t] = di\alpha g(d_{1t}^h, d_{2t}^h, ..., d_{kt}^h)$, where $d_{jt}^h = 1$ if $r_{jt} < 0$ and $d_{jt}^h = 0$, if $r_{it} \succeq 0$ for $i = 1, .., k$, and $D_t^r = D_t\ D_t'$. The ADCC-HEAVY model consists of the conditional and realized covariance equations:

1) The ADCC-HEAVE model for the conditional covariance equations

$$\begin{aligned} H_t &= di\alpha g[h_t^{1/2}]R_t di\alpha g[h_t^{1/2}] \quad (1) \\ h_t &= \omega_h + A_h v_{t-1} + B_h h_{t-1} + \gamma_h D_{t-1} v_{t-1} \\ R_t &= \tilde{R} + \alpha_r RL_{t-1} + \beta_r R_{t-1} + \gamma_r D_{t-1}^r RL_{t-1} \end{aligned}$$

where $\tilde{R} = (1 - \beta_r)\ \bar{R} - (\alpha_r + \gamma_r \bar{D}^r)\bar{P}$ is a $k \times k$ matrix and $\bar{D}^r$ equals the unconditional mean D_t^r .

If asymmetric effects are significant, then γ_h and γ_r should be positive.

2) The ADCC-HEAVE model for the realized covariance equations

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$$
\begin{aligned}
M_t &= di\alpha g[m_t^{1/2}]P_t di\alpha g[m_t^{1/2}]. \\
m_t &= \omega_m + A_m v_{t-1} + B_m m_{t-1} + \gamma_m D_{t-1} v_{t-1} \\
P_t &= \tilde{P} + \alpha_p RL_t + \beta_p P_{t-1} + \gamma_r D_{t-1}^r RL_{t-1}
\end{aligned}
$$

Furthermore, the Heterogeneous Auto Regressive (HAR) model of Corsi (2009) has arguably emerged as the most widely used univariate realized volatility-based forecasting model. The model was extended to a multivariate setting by Chiriac and Voev (2010). We add the HAR effects to realized covariance equations[1]:

$$
\begin{aligned}
M_t &= di\alpha g[m_t^{1/2}]P_t di\alpha g[m_t^{1/2}]. \qquad (2) \\
m_t &= \omega_m + A_m v_{t-1} + B_m m_{t-1} + \gamma_m D_{t-1} v_{t-1} + \alpha_m^w v_{t-1}^w + \alpha_m^m v_{t-1}^m \\
P_t &= \tilde{P} + \alpha_p RL_t + \beta_p P_{t-1} + \gamma_p D_{t-1}^r RL_{t-1} + \alpha_p^w RL_{t-1}^w + \alpha_p^m RL_{t-1}^m
\end{aligned}
$$

where $v_{t-1}^w = \frac{1}{5}\sum_{j=1}^{5} v_{t-j}$ and $v_{t-1}^m = \frac{1}{22}\sum_{j=1}^{22} v_{t-j}$ and $RL_{t-1}^w = \frac{1}{5}\sum_{j=1}^{5} RL_{t-j}$ and $RL_{t-1}^m = \frac{1}{22}\sum_{j=1}^{22} RL_{t-j}$. So $\tilde{P} = (1 - \alpha_p - \beta_p - \gamma_r \bar{D}^r - \alpha_p^w - \alpha_p^m)\bar{P}$.

We then express the ADCC-HEAVY model in multiplicative error form. Define $x_t = [r_t^2, v_t]'$ and $\mu_t = [h_t, m_t]'$, where x_t and μ_t is a $(2k \times 1)$ the vector. The vector multiplicative representation for conditional and realized variance equation is

$$
\begin{aligned}
E(x_t|\mathcal{F}_{t-1}) &: = \mu_t \\
\mu_t &= \omega + A x_{t-1} + B\mu_{t-1} + \Gamma D_{t-1} x_{t-1} + A_w x_{t-1}^w + A_m x_{t-1}^m \qquad (3)
\end{aligned}
$$

where

$$
\begin{aligned}
\omega &= \begin{bmatrix} \omega_h \\ \omega_m \end{bmatrix}, A = \begin{bmatrix} 0 & A_h \\ 0 & A_m \end{bmatrix}, B = \begin{bmatrix} B_h & \\ & B_m \end{bmatrix}. \\
\Gamma &= \begin{bmatrix} 0 & \Gamma_h \\ 0 & \Gamma_m \end{bmatrix}, A_w = \begin{bmatrix} 0 & A_h^w \\ 0 & A_m^w \end{bmatrix}, A_m = \begin{bmatrix} 0 & A_h^m \\ 0 & A_m^m \end{bmatrix}
\end{aligned}
$$

The matrix multiplicative representation for conditional and realized corre-

[1] Note we also test the HAR effect in conditional covariance equations, but the effects are insignificant, so we only add HAR effect in realized covariance equations.

lation equation is

$$
\begin{aligned}
E(\mathbf{Y}_t|\mathcal{F}_{t-1}) &: = \mathbf{\Phi}_t \\
\mathbf{\Phi}_t &= \boldsymbol{W} + \alpha \mathbf{Y}_{t-1} + \beta \mathbf{\Phi}_{t-1} + \gamma D_{t-1} \mathbf{Y}_{t-1} + \alpha_w \mathbf{Y}^w_{t-1} + \alpha_m \mathbf{Y}^m_{t-1} \qquad (4)
\end{aligned}
$$

where

$$
\begin{aligned}
\boldsymbol{W} &= \begin{bmatrix} \tilde{R} \\ \tilde{P} \end{bmatrix}, \alpha = \begin{bmatrix} & \alpha_r \\ 0 & \alpha_p \end{bmatrix}, \ \beta = \begin{bmatrix} \beta_r & \\ & \beta_p \end{bmatrix} \\
\gamma &= \begin{bmatrix} & \gamma_r \\ 0 & \gamma_p \end{bmatrix}, \ \alpha_w = \begin{bmatrix} & \alpha^w_r \\ 0 & \alpha^w_p \end{bmatrix}, \ \alpha_m = \begin{bmatrix} & \alpha^m_r \\ 0 & \alpha^m_p \end{bmatrix}
\end{aligned}
$$

We are primarily interested in forecasting the conditional covariance of daily returns, H_t. The s-step forecasts of $H_{t+s|t}$ is:

$$
\begin{aligned}
E_t[H_{t+s}] &= E_t[di\alpha g(h^{1/2}_{t+s}) R_{t+s} di\alpha g(h^{1/2}_{t+s})] \\
&= E_t[di\alpha g(h^{1/2}_{t+s})] E_t[R_{t+s}] E_t[di\alpha g(h^{1/2}_{t+s})]
\end{aligned}
$$

We need forecasting of $E_t[h_{t+s}]$ and and $E_t[R_{t+s}]$.

Let's start with eq.(3) and forecast $E_t(\mu_{t+s})$.We move steps ahead, x_{t+s}, $s > 0$ is not known and needs to be substituted with its corresponding conditional expectation μ_{t+s}, hence

$$
\begin{aligned}
\mu_{t+1|t} &= \omega + A x_t + B \mu_t + \Gamma D_t x_t + A_w x^w_t + A_m x^m_t \\
\mu_{t+s|t} &= \omega + (A + B + 0.5\Gamma)\mu_{t+s-1|t} + A_w x^w_{t+s-1|t} + A_m x^m_{t+s-1|t} \quad \text{for } 2 \preceq s \prec 22 \\
\mu_{t+s|t} &= \omega + (A + B + 0.5\Gamma + A_w + A_m)\mu_{t+s-1|t} \quad \text{for } s \succeq 22 \qquad (5)
\end{aligned}
$$

where $x^w_{t+s-1|t} = \frac{1}{5}\sum_{j=1}^{5} x_{t+s-j|t}$, $x^m_{t+s-1|t} = \frac{1}{22}\sum_{j=1}^{22} x_{t+s-j|t}$ and $x_{t+s-j|t} = \mu_{t+s-j|t}$ if $s > j$. Then, $\mu_{t+s|t}$ can be solved recursively for any horizon s.

We then derive a similar result for $E_t(\mathbf{\Phi}_{t+s})$ from eq.(4) as following:

$$
\begin{aligned}
\mathbf{\Phi}_{t+1|t} &= \boldsymbol{W} + \alpha \mathbf{Y}_t + \beta \mathbf{\Phi}_t + \gamma D_t \mathbf{Y}_t + \alpha_w \mathbf{Y}^w_t + \alpha_m \mathbf{Y}^m_t \\
\mathbf{\Phi}_{t+s|t} &= \boldsymbol{W} + (\alpha + \beta + 0.5\gamma)\mathbf{\Phi}_{t+s-1|t} + \alpha_w \mathbf{Y}^w_{t+s-1|t} + \alpha_m \mathbf{Y}^m_{t+s-1|t} \quad \text{for } 2 \preceq s \prec 22 \\
\mathbf{\Phi}_{t+s|t} &= \boldsymbol{W} + (\alpha + \beta + 0.5\gamma + \alpha_w + \alpha_m)\mathbf{\Phi}_{t+s-1|t} \quad \text{for } s \succeq 22. \qquad (6)
\end{aligned}
$$

where $\mathbf{Y}^w_{t+s-1|t} = \frac{1}{5}\sum_{j=1}^{5} \mathbf{Y}_{t+s-j|t}$, $\mathbf{Y}^m_{t+s-1|t} = \frac{1}{22}\sum_{j=1}^{22} \mathbf{Y}_{t+s-j|t}$ and $\mathbf{Y}_{t+s-j|t} =$

$\mathbf{\Phi}_{t+s-j|t}$ if $s > j$. Then, $\mathbf{\Phi}_{t+1|t}$ can be solved recursively for any horizon s.

Next, the DCC-HEAVY model multiple-step ahead forecast can be derived from (5) and (6) by setting $A=\begin{bmatrix} 0 & A_h \\ 0 & A_m \end{bmatrix}$, $B=\begin{bmatrix} B_h & \\ & B_m \end{bmatrix}$, $\alpha=\begin{bmatrix} & \alpha_r \\ 0 & \alpha_p \end{bmatrix}$, $\beta=\begin{bmatrix} \beta_r & \\ & \beta_p \end{bmatrix}$.

B Appendix: More Empirical Results

In this appendix, we report more empirical results. In appendix B1, we report and analysis the ADCC-HEAVY model estimation results. In appendix B2, we report more results from DCC-HEAVY model with different stock pairs.

1) ADCC-HEAVY Estimation Results

Table 1: Step 1 parameter estimates in the variance equation

	AHEAVY-R			AHEAVY-RM				
	α_{rr}	β_r	γ_r	α_m	β_m	γ_r	α_m^w	α_m^m
BAC	0.655	0.549	0.081	0.347	0.365	0.217	0.070	0.101
	(6.38)	(2.11)	(0.50)	(9.12)	(4.07)	(5.66)	(3.69)	(3.69)
JPM	0.800	0.225	0.157	0.409	0.207	0.155	0.170	0.122
	(14.27)	(2.58)	(2.56)	(10.81)	(3.12)	(5.62)	(4.40)	(4.40)
IBM	0.562	0.530	0.297	0.276	0.441	0.147	0.083	0.100
	(4.51)	(2.70)	(2.32)	(7.49)	(4.55)	(6.17)	(3.53)	(3.53)
DD	0.329	1.084	0.000	0.272	0.365	0.108	0.169	0.113
	(0.65)	(1.33)	(.)	(8.41)	(5.27)	(4.90)	(3.75)	(3.75)
XOM	0.718	0.261	0.114	0.278	0.223	0.128	0.314	0.077
	(11.97)	(3.07)	(1.70)	(7.03)	(2.35)	(4.96)	(2.80)	(2.80)
AA	0.915	0.098	0.056	0.257	0.472	0.113	0.065	0.115
	(7.77)	(0.54)	(1.39)	(7.95)	(2.44)	(4.45)	(2.20)	(2.20)
AXP	0.744	0.348	0.140	0.341	0.482	0.159	-0.020	0.115
	(13.58)	(3.05)	(1.37)	(10.17)	(7.25)	(5.87)	(3.50)	(3.50)
DD	0.653	0.414	0.086	0.294	0.464	0.118	0.067	0.088
	(8.16)	(3.89)	(1.05)	(8.18)	(4.23)	(4.80)	(3.24)	(3.24)
GE	0.847	0.132	0.214	0.305	0.606	0.145	-0.060	0.072
	(11.15)	(1.10)	(3.26)	(10.60)	(6.42)	(5.91)	(2.72)	(2.72)
KO	0.495	0.635	0.034	0.324	0.481	0.083	0.058	0.076
	(4.43)	(3.61)	(0.33)	(10.12)	(3.96)	(3.84)	(2.14)	(2.14)

Note: robust t statistics are reported in the bracket.

In Table 1, in the conditional variance (AHEAVY-R) equation, the asymmetric effect is positive and significant in most cases. In the realized variance (AHEAVY-RM) equation, the asymmetric effect is positive and significant in most cases. The HAR effects are significant in all the cases.

Table 2: Step 2 parameter estimates in the correlation equation

ADCC-R			ADCC-RC			
β_r	α_{rR}	γ_{rR}	α_m	β_m	γ_m	α_m^w
0.721	0.143	0.019	0.089	0.757	0.010	0.082
(16.86)	(6.95)	(1.24)	(21.18)	(11.69)	(4.84)	(2.12)

Panel B: Log-likelihood decomposition (DCC-HEAVY vs DCC-GARCH)

	DCC-GARCH	ADCC-HEAVY	ADCC-HEAVY Gains
Variance	-4316	-4247	69
Correlation	-2714	-2705	9
Joint distribution	-7030	-6955	78

Note: robust t statistics are reported in the bracket.

In Table 2, in the conditional correlation (ADCC-R) equation, the asymmetric effect is positive but insignificant. In the realized correlation (ADCC-RC) equation, the asymmetric effect is positive and significant. The HAR effect is also significant.

Table 3 reports the ratio of the losses for the ADCC-HEAVY model relative to the losses of the ADCC-GARCH model at the different forecast horizon. The results show that ADCC-HEAVY outperforms DCC-GARCH. This is true for the whole covariance matrix forecast as well as its decomposition into variance and correlation components, which provides further insight into the source of forecast gains. Based on RMSE as loss function, it can be seen that the overall forecast gain is large. The ADCC-HEAVY model reduces the forecast error by 40% compared with the DCC-GARCH model. Consistent with the in sample log-likelihood statistics, most of the forecasting gains are coming from variance equation.

2) More Estimation Results from DCC-HEAVY model

For robustness, we estimate the DCC-HEAVY model including 4 stocks, 3 stocks and 2 stocks. The first step variance estimation is exactly the same asTable 1 in the main paper. We report the second step correlation estimation in Table 4A, 5A and 6A. The out of sample comparison results are reported in Table 4B, 5B, 6B and 6C.

Table 3: Ratio of the losses(ADCC-HEAVY vs DCC-GARCH)

		RMSE	QLIK
Covariance	s=1	0.6391	0.7841
	s=5	0.6092	0.8143
	s=22	0.6751	0.9703
Variance	s=1	0.8336	0.6534
	s=5	0.7760	0.7296
	s=22	0.8399	0.9360
Correlation	s=1	0.9355	0.9902
	s=5	0.9645	0.9810
	s=22	1.0084	0.9825

Note: Ratio of the losses for the ADCC-HEAVY models relative to the losses of the DCC-GARCH model

The results are consistent with 10 stocks estimates. It can be seen that the conditional correlation is mainly driven by the lagged realized correlation. The DCC-HEAVY model outperforms DCC-GARCH model both in and out of the sample. However, most of the gains are coming from the variance equation.

Table 4A: Step 2 correlation estimation results (4 stocks portfolio)

Stocks	DCC-GARCH		DCC-HEAVY		DCCX-GARCH		
AA&	0.027	0.948	0.670	0.192	0.035	0.770	0.108
1 2 3	(2.71)	(42.58)	(7.37)	(3.93)	(3.16)	(14.31)	(2.86)
AA&	0.010	0.981	0.747	0.168	0.007	0.775	0.147
4 5 7	(4.43)	(163.01)	(6.34)	(2.53)	(0.88)	(8.08)	(2.46)
AA&	0.011	0.978	0.500	0.251	0.011	0.516	0.238
8 9 10	(4.07)	(139.24)	(3.68)	(3.91)	(1.45)	(3.65)	(3.50)

Table 4B: Ratio of the losses (DCC-HEAVY vs DCC-GARCH)

		AA&1 2 3		AA&4 5 7		AA&8 9 10	
		RMSE	QLIK	RMSE	QLIK	RMSE	QLIK
Covariance	s=1	0.6371	0.7652	0.7855	0.7400	0.7777	0.6749
	s=5	0.6843	0.7918	0.7966	0.7852	0.8007	0.7754
	s=22	0.7190	1.1097	0.7560	0.9295	0.7922	0.9465
Variance	s=1	0.8116	0.6572	0.8849	0.7270	0.8325	0.6381
	s=5	0.8083	0.7436	0.8561	0.7822	0.8526	0.7534
	s=22	0.8287	0.9820	0.8562	0.9228	0.8913	0.9296
Correlation	s=1	0.9738	1.8924	0.9315	0.9412	0.9484	0.9838
	s=5	0.9855	1.2293	0.9610	0.9657	0.9775	0.9845
	s=22	1.0030	1.0868	1.0049	0.9877	0.9917	0.9893

Note: this table reports the ratio of the losses for the DCC-HEAVY models relative to the losses of DCC-GARCH models.

Table 5A: Step 2 correlation estimation results (3 stocks portfolio)

Stocks	DCC-GARCH		DCC-HEAVY		DCCX-GARCH		
AA&	0.029	0.950	0.680	0.180	0.039	0.758	0.107
1 2	(2.15)	(38.58)	(8.95)	(4.26)	(2.09)	(13.07)	(2.47)
AA&	0.017	0.962	0.803	0.197	0.016	0.799	0.181
3 4	(0.98)	(16.61)	(16.58)	(3.89)	(1.47)	(15.67)	(3.32)
AA&	0.012	0.980	0.354	0.365	0.012	0.751	0.155
5 7	(4.00)	(152.63)	(1.25)	(2.78)	(0.71)	(2.96)	(0.93)
AA&	0.012	0.979	0.390	0.333	0.018	0.405	0.319
8 9	(3.00)	(109.94)	(3.11)	(4.74)	(1.55)	(2.97)	(4.15)
AA&	0.019	0.958	0.814	0.135	0.007	0.821	0.124
10 1	(4.26)	(108.51)	(10.45)	(2.34)	(0.69)	(9.32)	(1.75)

Table 5B: Ratio of the losses (DCC-HEAVY vs DCC-GARCH)

		AA&1 2		AA&3 4		AA&5 7		AA&8 9		AA&10 1	
		RMSE	QLIK	RMSE	QLIK	RMSE	QLIK	RMSE	QLIK	RMSE	QLIK
Covariance	s=1	0.6216	0.7648	0.8120	0.7394	0.9541	0.7010	0.7832	0.6751	0.6192	0.6184
	s=5	0.6714	0.7890	0.7994	0.7843	0.9712	0.7525	0.8038	0.7852	0.6710	0.7112
	s=22	0.7062	1.2113	0.7583	0.8522	0.9939	0.9228	0.7924	1.0036	0.7043	0.9923
Variance	s=1	0.6925	0.6173	1.0535	0.7774	0.7327	0.6880	0.8069	0.6417	0.7714	0.6243
	s=5	0.7564	0.7078	0.9260	0.8174	0.7937	0.7539	0.8513	0.7684	0.7927	0.7130
	s=22	0.7787	1.0297	0.9509	0.8615	0.8522	0.9193	0.8841	0.9791	0.8195	0.9526
Correlation	s=1	1.0236	2.2874	0.9091	0.9022	0.7872	0.9717	1.0227	1.1877	0.9273	0.8861
	s=5	1.0202	1.3956	0.9337	0.9202	0.7631	0.9604	1.0214	1.1052	0.9570	0.9303
	s=22	1.0291	1.1306	0.9585	0.9357	0.7384	0.9453	1.0092	1.0336	0.9904	0.9832

Note: this table reports the ratio of the losses for the DCC-HEAVY models relative to the losses of DCC-GARCH models.

Table 6A: Step 2 correlation estimation results (2 stocks portfolio)

Stocks	DCC-GARCH		Step2:DCC-HEAVY		Step2:DCCX-GARCH		
AA&	0.018	0.959	0.820	0.151	0.001	0.819	0.151
1	(2.25)	(69.08)	(7.50)	(1.73)	(0.09)	(7.26)	(1.70)
AA&	0.014	0.982	0.742	0.136	0.012	0.983	-0.005
2	(2.17)	(100.12)	(7.09)	(2.35)	(2.28)	(107.03)	(-1.07)
AA&	0.022	0.962	0.818	0.182	0.019	0.827	0.151
2	(2.63)	(52.24)	(18.58)	(4.13)	(1.48)	(14.73)	(2.35)
AA&	0.015	0.977	0.876	0.118	0.009	0.875	0.109
4	(2.85)	(148.86)	(32.30)	(3.57)	(0.58)	(28.88)	(2.78)
AA&	0.011	0.977	0.165	0.406	0.023	0.207	0.383
5	(3.25)	(151.49)	(0.89)	(4.06)	(1.20)	(0.98)	(3.34)
AA&	0.011	0.986	0.697	0.222	-0.007	0.721	0.276
7	(1.50)	(82.86)	(1.79)	(0.85)	(.)	(.)	(.)
AA&	0.009	0.987	0.307	0.289	0.046	0.832	0.041
8	(1.41)	(89.01)	(1.53)	(3.03)	(2.88)	(5.52)	(0.56)
AA&	0.023	0.967	0.606	0.326	0.004	0.617	0.315
9	(3.32)	(89.55)	(4.72)	(3.32)	(0.20)	(3.67)	(2.31)
AA&	0.023	0.949	0.861	0.131	0.004	0.874	0.113
10	(3.40)	(74.77)	(21.03)	(2.89)	(0.14)	(7.38)	(0.76)

Table 6B: Ratio of the losses (DCC-HEAVY vs DCC-GARCH)

		AA&1		AA&2		AA&3		AA&4		AA&5	
		RMSE	QLIK	RMSE	QLIK	RMSE	QLIK	RMSE	QLIK	RMSE	QLIK
Covariance	s=1	0.6270	0.5930	0.6804	0.6445	0.7356	0.7195	0.7559	0.7773	0.6773	0.6892
	s=5	0.6780	0.6960	0.7154	0.7033	0.7427	0.7687	0.7611	0.7969	0.6936	0.7254
	s=22	0.7080	1.0640	0.7043	0.9182	0.7001	0.8197	0.7283	0.8740	0.6675	0.9095
Variance	s=1	0.9780	1.0180	1.0941	1.3556	0.8857	0.8490	0.9447	0.8973	1.0508	1.2829
	s=5	0.9890	1.0280	1.0852	1.2902	0.9254	0.8989	0.9809	0.9711	1.0052	1.0400
	s=22	1.0040	1.0230	1.0343	1.1454	0.9566	0.9354	1.0152	1.0169	0.9971	0.9990
Correlation	s=1	0.7030	0.6230	0.6853	0.6588	0.9333	0.7440	0.9959	0.7776	0.6865	0.6514
	s=5	0.7610	0.7150	0.7492	0.7136	0.8575	0.7925	0.9070	0.8007	0.7111	0.7211
	s=22	0.7730	1.0380	0.7643	0.9123	0.8585	0.8255	0.9369	0.8729	0.7267	0.9024

Note: this table reports the ratio of the losses for the DCC-HEAVY models relative to the losses of DCC-GARCH models.

Table 6C: Ratio of the losses (DCC-HEAVY vs DCC-GARCH)

		AA&7		AA&8		AA&9		AA&10	
		RMSE	QLIK		RMSE	QLIK			
Covariance	s=1	0.7789	0.7271	0.7000	0.6923	0.7934	0.6793	0.6877	0.6881
	s=5	0.7961	0.7688	0.7289	0.7648	0.8092	0.7821	0.7104	0.7293
	s=22	0.7357	0.8898	0.6993	0.9075	0.7897	0.9863	0.6725	0.7955
Variance	s=1	0.7856	0.7361	0.6996	0.6396	0.8598	0.6784	0.8036	0.6692
	s=5	0.8451	0.7769	0.7574	0.7429	0.8951	0.7768	0.8038	0.7213
	s=22	0.7871	0.8826	0.7700	0.9050	0.9253	0.9698	0.8256	0.7966
Correlation	s=1	1.0905	1.2574	1.1711	1.7134	0.9972	1.1036	0.9084	0.8453
	s=5	1.0852	1.2328	1.1373	1.4899	1.0004	1.0185	0.9481	0.9087
	s=22	1.0738	1.1934	1.0733	1.2417	0.9731	0.9323	0.9951	0.9796

Note: this table reports the ratio of the losses for the DCC-HEAVY models relative to the losses of DCC-GARCH models.