Cardiff Economics Working Papers

Working Paper No. E2019/4

Trade Wars under Oligopoly: Who Wins and is Free Trade Sustainable?

David R Collie

January 2019

ISSN 1749-6010

Cardiff Business School
Cardiff University
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
business.cardiff.ac.uk

Trade Wars under Oligopoly:

Who Wins and is Free Trade Sustainable?*

David R Collie

Cardiff Business School, Cardiff University

January 2019

Abstract

The outcome of a trade war (with import tariffs and export subsidies) between two countries is analysed in a Cournot duopoly and in a Bertrand duopoly with differentiated products. The model allows for asymmetries between the countries in terms of competitiveness. When the two countries are similar, both countries will be worse off in a trade war than under free trade, but the country with the uncompetitive firm may win the trade war when the asymmetries are sufficiently great. Hence, in an infinitely -repeated game, cost asymmetries make it difficult to sustain free trade using infinite Nash reversion. However, it is shown that both countries minimaxing each other by setting prohibitive import tariffs and export taxes is also a Nash equilibrium in trade policies that results in each country obtaining autarky welfare. In an infinitely-repeated game, it is much easier to sustain free trade using infinite minimax reversion when there are cost asymmetries than with infinite Nash reversion. In fact, free trade can be sustained even if the punishment phase lasts for only a few rounds. Since there are two Nash equilibria of the trade policy game, free trade can also be sustained in a finitely-repeated game.

Keywords: Retaliation; Tariffs; Cournot Oligopoly; Bertrand Oligopoly.

JEL Classification: F11; F12; F13.

---

* A preliminary version was presented at the European Trade Study Group (ETSG) annual conference at the SGH Warsaw School of Economics in September 2018 and at the Microeconomics Group at Cardiff Business School in October 2018. I would like to thank participants for their comments.

Address for correspondence: Professor David R. Collie, Cardiff Business School, Cardiff University, Aberconway Building, Colum Drive, Cardiff, CF10 3EU, United Kingdom. E-mail: Collie@cardiff.ac.uk.

1. Introduction

Since the formation of the General Agreement on Tariffs and Trade (GATT), several rounds of multilateral trade negotiations have brought tariffs down to historically very low levels. Following the Uruguay Round, the formation of the World Trade Organisation (WTO) in 1995 further strengthened the rules-based GATT/WTO trading regime that has created an extremely benign environment for international trade and encouraged globalisation, although there has not been a successfully completed round of multilateral trade negotiations since the formation of the WTO. However, since the inauguration of Donald Trump as US President in January 2017, there has been an increase in trade conflict with the US imposing tariffs on imports of steel and aluminium, provoking a trade war with China over the US-China trade deficit and concerns about the protection of US intellectual property in China, and threatening a trade war with the EU over automobile tariffs. He has also threatened that the US would quit the World Trade Organisation (WTO) unless there is significant reform of the organisation, and somewhat paradoxically, he has also called for zero tariffs, zero barriers, and zero subsidies.[1]

The first formal analysis of trade wars using game theory was undertaken by Johnson (1953) who considered the case of two large countries each producing two goods under perfect competition.[2] Since the countries are large, they both have market power in the world market and can unilaterally use an import tariff or an export tax to improve their terms of trade, but such a policy is a beggar-my-neighbour policy as any improvement in the terms of trade of one country involves a worsening of the terms of trade of the other country. This implies that there

[1] See various articles from ***Bloomberg***, the ***Financial Times*** and the ***Wall Street Journal***: What you need to know about the Trump steel tariffs, U.S. to Apply Tariffs on Chinese Imports, Restrict Tech Deals, China Retaliates Against Trump Tariffs With Duties on American Meat and Fruit, Donald Trump threatens to pull US out of the WTO, Tusk calls for WTO reform, Making Sense of President Trump's Call for Zero Tariffs, and EU Offer for No Auto Tariffs Is 'Not Good Enough,' Trump Says.

[2] For extensive surveys of the subsequent literature on trade agreements see Staiger (1995), Kowalczyk and Riezman (2013), Maggi (2014), Grossman (2016), and Bagwell and Staiger (2016).

is a strategic interdependence between the two countries when setting tariffs that can best be modelled using game theory. Johnson (1953) modelled the outcome of a trade war between the two welfare-maximising countries as the Nash equilibrium (NE) in tariffs of a static game. He showed that although the most likely outcome was that both countries lose as a result of the trade war (in the sense that welfare of both countries was lower at the NE in tariffs than under free trade), it is possible with asymmetries that one country could win the trade war.[3] In an endowment model with Cobb-Douglas utility functions, Kennan and Riezman (1988) show that if one country is substantially larger than the other country then it will win the trade war, and conclude that this possibility is by no means remote.[4] Importantly, Dixit (1987) showed that as well as the interior NE considered by Johnson (1953) both countries minimaxing each other by setting prohibitive tariffs is also a NE of the static game that results in autarky for both countries.

As the interior NE in tariffs is inefficient in the Johnson (1953) model, there is scope for trade negotiations to increase the welfare of both countries. Hence, Mayer (1981) argues that trade negotiations would lead to a Pareto-efficient outcome with domestic relative prices equal in both countries and with both countries better off than in the interior NE in tariffs. This would involve one country taxing trade and the other country subsidising trade, which is equivalent to free trade but with a lump-sum transfer from one country to the other country. He points out that free trade (with both countries setting zero tariffs) is not necessarily attainable by such negotiations one country may be better off in the interior NE in tariffs than under free trade when there are asymmetries.[5] Mayer (1981) did not explain how any such trade

[3] Despite this result virtually every commentator on the US-China trade war has asserted that it is always the case that everyone loses in a trade war so the result must be considered counterintuitive.

[4] A similar conclusion that country will win a trade war if its relative size is sufficiently large is reached by Syropoulos (2002) in a more general neoclassical model that includes production.

[5] The analysis of trade negotiations has been extended to the case when governments maximise a welfare functions that incorporates political economy considerations by Bagwell and Staiger (1999). They show that the terms of trade externality is the only rationale for trade agreements, and use their model to shed light on the principles of the GATT/WTO such as reciprocity. See Bagwell and Staiger (2016) for a survey of this literature.

agreement could be enforced between two sovereign countries, but Dixit (1987) used the folk theorem to explain how a trade agreement could be sustained in an infinitely repeated version of the Johnson (1953) game. This is fundamental to the enforceability or sustainability of trade agreements since as Grossman (2016, p.387) notes a formal trade agreement 'can only achieve those outcomes that are sustainable in an infinitely-repeated game'. Park (2000) and Bond and Park (2002) have noted that asymmetries between countries make it more difficult to enforce a trade agreement in an infinitely-repeated game and suggested a number of solutions including the use of transfers between countries.

Recently, the analysis of trade agreements has started to consider the implications of imperfect competition (or the *new trade theory*) that arguably results in new sources of international externalities. Ossa (2011) considers the firm-delocation externality under monopolistic competition while Mrázová (2011) considers the profit-shifting externality under oligopoly.[6] Bagwell and Staiger (2012a, 2012b) argue that in both cases the models can be re-interpreted so that the international externality in both cases is really just the usual terms of trade effect. Ossa (2014, 2012) considers a multi-country quantitative model that combines the terms of trade, profit-shifting, and political economy effects. Ossa (2012, p.469) argues that the *new trade theory* approach 'allows for a view of trade negotiations in which producer interests play a prominent role'.

The contribution of this paper will be to analyse trade wars and trade agreements between two countries under Cournot duopoly and under Bertrand duopoly with product differentiation and cost asymmetries. Before considering trade policy, it is shown that both countries always gain from multilateral free trade despite the cost asymmetries. Countries can intervene in international trade using an import tariff to extract rent and an export subsidy or

[6] Mrázová (2011) considers a symmetric Cournot oligopoly where firms have the same costs. Collie (1993) considers a trade agreement to prohibit profit-shifting export subsidies in an infinitely-repeated version of Brander and Spencer (1985) model with cost asymmetries.

tax to shift profits as in Brander and Spencer (1984, 1985). In a trade war, modelled as the interior NE trade policies, the outcome in the symmetric case is that both countries lose, but the country with the uncompetitive firm may win the trade war with cost asymmetries. Under oligopoly, a multilateral free trade agreement (zero import tariffs and zero export taxes/subsidies) is not efficient as efficiency would require subsidies due to the oligopolistic distortion, but widespread subsidisation seems to be politically and economically implausible.[7] However, given that history suggests that trade negotiations are time-consuming and costly, an efficient trade agreement would most likely be unachievable, and a more plausible outcome might be a focal point as suggested by Schelling (1960) who argued that:

> 'If we then ask what it is that can bring their expectations into convergence and bring the negotiation to a close, we might propose that it is the intrinsic magnetism of particular outcomes, especially those that enjoy prominence, uniqueness, simplicity, precedent, or some rationale that makes them qualitatively differentiable from the continuum of possible alternatives.' Schelling (1960, p.70)

With trade negotiations, it seems to be immediately obvious that multilateral free trade (zero import tariffs and zero export taxes/subsidies) satisfies all the criteria for a focal point. Therefore, this paper will consider the sustainability (or enforceability) of multilateral free trade in an infinitely-repeated version of the trade policy game. Using the threat of infinite Nash reversion, free trade is sustainable provided the cost asymmetries are not too great. However, as Dixit (1987) showed with perfect competition, both countries minimaxing each other with prohibitive import taxes and prohibitive export taxes is a NE in the trade policy game under oligopoly that results in autarky welfare for both countries.[8] Therefore, free trade can also be sustained by the threat of infinite minimax reversion, and in this case asymmetries are much less problematic as the critical discount factor is always significantly less than one.[9]

---

[7] Collie (2000b) provides a rationale for the EU's prohibition of state aid (production subsidies) and Collie (2000a) for the WTO's prohibition of export subsidies in oligopoly models where subsidies are financed with distortionary taxation.

[8] Under oligopoly with segmented markets, a prohibitive import tariff and a prohibitive export tax are required.

[9] Throughout the paper, infinite Nash reversion will be used as shorthand for infinite reversion to the interior NE, and infinite minimax reversion will be used as shorthand for infinite reversion to the minimax (autarky) NE.

Both infinite minimax reversion and infinite Nash reversion may seem implausible as the punishment phase lasts forever following any deviation, and countries would be expected to renegotiate following any deviation from free trade. An alternative is for the punishment phase with minimax reversion to last for only a few rounds, and then it turns out that the critical discount factor gets quite close to that with infinite minimax reversion and is lower than that with infinite Nash reversion. Since there are two NE of the constituent trade policy game, free trade can also be sustained in a finitely-repeated game as in Benoit and Krishna (1985). All the results under Bertrand duopoly are qualitatively similar to those under Cournot duopoly despite the interior NE trade policies (and those when a country deviates from free trade) including an export tax under Bertrand duopoly and an export subsidy under Cournot duopoly. However, it can be shown that it less likely that a country will win a trade war under Bertrand duopoly than under Cournot duopoly, and it is always easier to sustain free trade under Bertrand duopoly than under Cournot duopoly when using minimax reversion. Also, product differentiation makes it easier to sustain free trade under Cournot duopoly, and under Bertrand duopoly except when the products are close substitutes.

The paper also uses a somewhat novel approach to solving the Cournot and Bertrand duopoly models. Outputs, prices, profits and welfare are expressed in terms of the competitiveness of the two firms, and differences in welfare turn out to be quadratic forms in competitiveness. The competitiveness of the two firms can be replaced by a single parameter, the market share of a firm under free trade, since the market shares of the two firms must sum to unity, and hence the key results depend upon only two parameters: the market share of a firm under free trade and the degree of product substitutability. With cost symmetry, key results depend only upon the degree of product substitutability.

2. Cournot Oligopoly with Differentiated Products

The world consists of two countries: the home country labelled as country one and the foreign country labelled as country two. The industry in question is a Cournot duopoly consisting of a home firm and a foreign firm producing differentiated products, and selling in the segmented markets of the two countries.[10] The home firm labelled as firm one has constant marginal cost c_1 and the foreign firm labelled as firm two has constant marginal cost c_2. The *i*th firm produces output x_{ii} for its domestic market, which sells at price p_{ii}, and it exports output x_{ij} to the *j*th country, which sells at price p_{ij}. The *i*th country imposes a specific import tariff t_i on its imports and a specific export tax e_i on its exports. If an import tariff is negative, $t_i < 0$, then it is an import subsidy and if an export tax is negative, $e_i > 0$, then it is an export subsidy. The trade policy vector is defined as $\boldsymbol{\tau} = \left(t_i, e_i, t_j, e_j\right)'$. There are assumed to be no real trade costs such as transport costs in the model. In both countries, preferences of the representative consumer are derived from a quadratic, quasi-linear utility function:

$$U_i = \alpha\left(x_{1i} + x_{2i}\right) - \frac{\beta}{2}\left(x_{1i}^2 + x_{2i}^2 + 2\phi x_{1i} x_{2i}\right) + z_i \qquad i = 1,2 \tag{1}$$

where z_i is consumption in the *i*th country of a numeraire good produced by a perfectly-competitive industry using a constant returns to scale technology.[11] It is assumed that the parameters satisfy the following conditions: $\alpha > c_1$, $\alpha > c_2$, $\beta > 0$ and $\phi \in [0,1]$. The parameter ϕ is a measure of the degree of product substitutability that is equal to one when products are perfect substitutes and equal to zero when products are independent. In each

[10] For a survey of the literature on international trade under oligopoly, see Leahy and Neary (2011).
[11] The utility function is the same as in Clarke and Collie (2003).

country, utility maximisation by the representative consumer yields the inverse demand functions facing the home firm and the foreign firm, respectively:

$$p_{1i} = \alpha - \beta\left(x_{1i} + \phi x_{2i}\right) \qquad p_{2i} = \alpha - \beta\left(\phi x_{1i} + x_{2i}\right) \qquad i = 1,2 \tag{2}$$

The profits of the *i*th firm from sales in its domestic market are: $\pi_{ii} = \left(p_{ii} - c_i\right)x_{ii}$, and its profits from exports are: $\pi_{ij} = \left(p_{ij} - c_i - t_j - e_i\right)x_{ij}$, so its total profits are $\Pi_i = \pi_{ii} + \pi_{ij}$. Since preferences are assumed to be quasi-linear, consumer surplus is a valid measure of consumer welfare. Hence, the welfare of each country is given by the sum of consumer surplus, the profits of its domestic firm from domestic production and from exports, and government revenue (import tariff revenue and export tax revenue). Welfare can be expressed analytically using the indirect utility function or explicitly using consumer surplus derived from the quadratic utility function:

$$\begin{aligned} W_i &= V_i\left(p_{ii}, p_{ji}\right) + \Pi_i + t_i x_{ji} + e_i x_{ij} \\ &= \frac{\beta}{2}\left(x_{ii}^2 + x_{ji}^2 + 2\phi x_{ii} x_{ji}\right) + \Pi_i + t_i x_{ji} + e_i x_{ij} \qquad i, j = 1,2 \quad j \neq i \end{aligned} \tag{3}$$

In the Cournot duopoly, each firm sets its outputs to maximise its profits given the outputs of its competitor with both firms taking the trade policies of the two countries as given. Since the markets are assumed to be segmented and marginal costs are constant, the Cournot equilibrium outputs can be derived independently in each market. It is straightforward to solve for the Cournot equilibrium outputs of the home and foreign firms in the two markets, which yields:

$$x_{ii} = \frac{A_i + \phi\left(t_i + e_j\right)}{\beta \Phi_A} \qquad x_{ji} = \frac{A_j - 2\left(t_i + e_j\right)}{\beta \Phi_A} \qquad i, j = 1,2 \quad j \neq i \tag{4}$$

where $\Phi_A = 4-\phi^2 > 0$ and $A_i \equiv 2(\alpha - c_i) - \phi(\alpha - c_j)$, $i=1,2$ and $j \neq i$. The parameters A_1 and A_2 are measures of the competitiveness of the home firm and the foreign firm, respectively. The competitiveness of both firms is positive if there is an interior solution, where both firms sell positive quantities under free trade, $\boldsymbol{\tau} = \mathbf{0}$. For future reference, note from (4) that the market share of the *i*th firm under multilateral free trade, which is the same in both markets, is: $\mu_{Ai} = A_i / (A_i + A_j)$, and obviously: $\mu_{A1} + \mu_{A2} = 1$. Also, note that the Cournot equilibrium outputs in the home (foreign) market only depend upon the import tariff of the home (foreign) country and the export tax of the foreign (home) country. Imports of the *i*th country will be zero, $x_{ji} = 0$, if the sum of its import tariff and the *j*th country's export tax is prohibitive, $t_i + e_j \geq \overline{t}_i \equiv A_j / 2$.

Substituting the outputs (4) into the inverse demand functions (2) yields the Cournot equilibrium prices that can be used together with the Cournot equilibrium outputs to derive the Cournot equilibrium total profits of the home and foreign firm, respectively:

$$\Pi_i = \pi_{ii} + \pi_{ij} = \frac{\left(A_i + \phi t_i + \phi e_j\right)^2 + \left(A_i - 2t_j - 2e_i\right)^2}{\beta \Phi_A^2} \qquad i,j = 1,2 \quad j \neq i \tag{5}$$

Substituting the Cournot equilibrium outputs (4) and profits (5) into (3) yields welfare of the two countries as functions of the trade policies of the two countries:

$$\begin{aligned} W_{Ai} = W_{Ai}^F + \frac{1}{2\beta\Phi_A^2}\Big[& -2\phi^2 A_i e_i - 2\left(\left(2-\phi^2\right)A_j - \phi A_i\right)e_j + 2\left(2A_j + \phi A_i\right)t_i - 8A_i t_j \\ & -4\left(2-\phi^2\right)e_i^2 + \Phi_A e_j^2 - 3\Phi_A t_i^2 + 8t_j^2 + 4\phi^2 e_i t_j - 2\Phi_A e_j t_i \Big] \end{aligned} \tag{6}$$

where W_{Ai}^F is welfare under multilateral free trade, $\boldsymbol{\tau} = \mathbf{0}$, which can be written as a quadratic form in A_1 and A_2, where $\mathbf{A}' = \begin{bmatrix} A_i & A_j \end{bmatrix}$:

$$W_{Ai}^{F} = \frac{5A_i^2 + A_j^2 + 2\phi A_i A_j}{2\beta\left(4-\phi^2\right)^2} = \frac{1}{\beta\Psi^F}\mathbf{A}'\begin{bmatrix}\psi_{ii}^F & \psi_{ij}^F \\ \psi_{ij}^F & \psi_{jj}^F\end{bmatrix}\mathbf{A} \qquad i,j=1,2 \quad j\neq i \tag{7}$$

where $\Psi^F = 2\Phi_A^2$, $\psi_{ii}^F = 5$, $\psi_{ij}^F = \phi$, and $\psi_{jj}^F = 1$.

If the *i*th country pursues a policy of unilateral free trade, $t_i = e_i = 0$, while the *j*th country has a prohibitive tariff, $t_j = \overline{t}_j$, and a zero export tax/subsidy, $e_j = 0$, then the *i*th country will not export and its profits from exports will be zero, $\pi_{ij} = 0$. Setting $\pi_{ij} = 0$ in (5) and substituting into (3) yields welfare under unilateral free trade:

$$W_{Ai}^{U} = \frac{3A_i^2 + A_j^2 + 2\phi A_i A_j}{2\beta\Phi_A^2} = \frac{1}{\beta\Psi^U}\mathbf{A}'\begin{bmatrix}\psi_{ii}^U & \psi_{ij}^U \\ \psi_{ij}^U & \psi_{jj}^U\end{bmatrix}\mathbf{A} \qquad i,j=1,2 \quad j\neq i \tag{8}$$

where $\Psi^U = 2\Phi_A^2$, $\psi_{ii}^U = 3$, $\psi_{ij}^U = \phi$, and $\psi_{jj}^U = 1$.

Before analysing trade policy, it is useful to analyse whether there are gains from multilateral free trade and unilateral free trade in this model with cost asymmetries. Under autarky, each firm would have a monopoly in its own market and would not export to the other country. Setting the import tariffs and export taxes to their prohibitive levels, $t_1 + e_2 = \overline{t}_1 = A_2/2$ and $t_2 + e_1 = \overline{t}_2 = A_1/2$, in (6) yields welfare under autarky:

$$W_{Ai}^{0} = \frac{1}{\beta\Psi^0}\mathbf{A}'\begin{bmatrix}\psi_{ii}^0 & \psi_{ij}^0 \\ \psi_{ij}^0 & \psi_{jj}^0\end{bmatrix}\mathbf{A} = \frac{3}{8\beta}\left(\alpha - c_i\right)^2 \qquad i,j=1,2 \quad j\neq i \tag{9}$$

where $\Psi^0 = 8\Phi_A^2$, $\psi_{ii}^0 = 12$, $\psi_{ij}^0 = 6\phi$, and $\psi_{jj}^0 = 3\phi^2$.

To show that there are gains from multilateral free trade, subtract welfare under autarky (9) from welfare under multilateral free trade (7), which yields:

$$\Delta W_{Ai}^{F0} = W_{Ai}^{F} - W_{Ai}^{0} = \frac{1}{\beta\Psi^F\Psi^0}\mathbf{A}'\begin{bmatrix}\psi_{ii}^{F0} & \psi_{ij}^{F0} \\ \psi_{ij}^{F0} & \psi_{jj}^{F0}\end{bmatrix}\mathbf{A} > 0 \tag{10}$$

where $\psi_{ii}^{F0} = \psi_{ii}^{F}\Psi^{0} - \Psi^{F}\psi_{ii}^{0} = 16\Phi_A^2 > 0$, $\psi_{ij}^{F0} = \psi_{ij}^{F}\Psi^{0} - \Psi^{F}\psi_{ij}^{0} = -4\phi\Phi_A^2 < 0$, and $\psi_{jj}^{F0} = \psi_{jj}^{F}\Psi^{0} - \Psi^{F}\psi_{jj}^{0} = 2\left(4-3\phi^2\right)\Phi_A^2 > 0$. This quadratic form is positive definite as the principal diagonal elements of the matrix are both positive and the determinant is positive, $\det = 16\left(8-7\phi^2\right)\Phi_A^4 > 0$. Hence, welfare under multilateral free trade is always higher than welfare under autarky so there are always gains from multilateral free trade for both countries despite the cost asymmetries.

To analyse whether there are gains from unilateral free trade, subtract welfare under autarky (9) from welfare under unilateral free trade (8), which yields:

$$\Delta W_{Ai}^{U0} = W_{Ai}^{U} - W_{Ai}^{0} = \frac{1}{\beta\Psi^{U}\Psi^{0}}\mathbf{A}'\begin{bmatrix}\psi_{ii}^{U0} & \psi_{ij}^{UO} \\ \psi_{ij}^{U0} & \psi_{jj}^{U0}\end{bmatrix}\mathbf{A} \qquad (11)$$

where $\psi_{ii}^{U0} = \psi_{ii}^{U}\Psi^{0} - \Psi^{U}\psi_{ii}^{0} = 0$, $\psi_{ij}^{U0} = \psi_{ij}^{U}\Psi^{0} - \Psi^{U}\psi_{ij}^{0} = -4\phi\Phi_A^2 < 0$, and $\psi_{jj}^{U0} = \psi_{jj}^{U}\Psi^{0} - \Psi^{U}\psi_{jj}^{0} = 2\left(4-3\phi^2\right)\Phi_A^2 > 0$ Since one of the principal diagonals is zero and hence the determinant is negative, the quadratic form is indefinite. Using the definition of the market share under free trade, $A_j = \left(1-\mu_{Ai}\right)A_i/\mu_{Ai}$, the *i*th country will gain from trade if $2\psi_{ij}^{U0}\mu_{Ai} + \psi_{jj}^{U0}\left(1-\mu_{Ai}\right) > 0$ or $\mu_{Ai} < \mu_A^{U0} \equiv \psi_{jj}^{U0}\Big/\left(\psi_{jj}^{U0} - 2\psi_{ij}^{U0}\right)$, and in terms of the degree of product substitutability $\mu_A^{U0} = \left(4-3\phi^2\right)\Big/\left(\left(2-\phi\right)\left(2+3\phi\right)\right)$ Therefore, there are gains from unilateral free trade for the home country if $\mu_{A1} < \mu_A^{U0}$ and for the foreign country if $\mu_{A1} > 1-\mu_A^{U0}$ since $\mu_{A2} = 1-\mu_{A1}$. These critical values μ_A^{U0} and $1-\mu_A^{U0}$ are plotted against ϕ in figure 1, and they intersect when $\phi = 2/3$. When products are not close substitutes, $\phi < 2/3$, there is a region where both countries gain from unilateral free trade, and when products are close substitutes, $\phi > 2/3$, there is a region where both countries lose from unilateral free trade. This leads to the following proposition:

Proposition 1: *Under Cournot duopoly, there are always gains from multilateral free trade for both countries, and the ith country gains from unilateral free trade if* $\mu_{Ai} < (>) \mu_{A}^{U0}$.

Under Cournot duopoly with homogeneous products, Markusen (1981) shows that when countries differ in size the large country may lose from multilateral free trade, and Brander (1981) and Brander and Krugman (1983) show that both countries may lose from multilateral free trade when (real) trade costs are close to the prohibitive level. Collie (1996) shows that a country may lose from unilateral free trade unless its firm has a sufficient cost disadvantage. In this paper, where there are no (real) trade costs, the countries are identical in size, and the products are differentiated, both countries gain from multilateral free trade despite the cost asymmetries. The fact that one or both countries may lose from unilateral free trade while both countries gain from multilateral free trade shows clearly that there is a need for multilateral trade agreements.

3. Trade Wars under Cournot Oligopoly

Although there are gains from multilateral free trade under oligopoly, a country can unilaterally use trade policy to increase its welfare.[12] The government in each country has two trade taxes: an import tariff that affects its domestic market and an export tax (or subsidy) that affects its export market. Since markets are segmented, there is no interaction between the two trade taxes of a country. Under Cournot oligopoly, an import tariff can be used to extract rent from foreign firms (improve the terms of trade) and to shift profits to domestic firms as in Dixit (1984) or Brander and Spencer (1984) while an export subsidy can be used to shift profits to domestic firms as in Dixit (1984) or Brander and Spencer (1985). If a country sets its import

[12] For a survey of the extensive literature on strategic trade policy (trade policy under oligopoly), see Brander (1995) as well as Leahy and Neary (2011).

tariff to maximise its welfare (3) taking the export tax set by the other country as given, then the first-order condition is:

$$\frac{\partial W_i}{\partial t_i} = x_{ji}\left(1-\frac{\partial p_{ji}}{\partial t_i}\right)+\left(p_{ii}-c_i\right)\frac{\partial x_{ii}}{\partial t_i}+t_i\frac{\partial x_{ji}}{\partial t_i}=0 \qquad i,j=1,2 \quad j\neq i \qquad (12)$$

The first term is the terms of trade effect, which is positive as the price of imports increases by less than the amount of the tariff; the second term is the profit-shifting effect, which is positive as price is above marginal cost; and the third term is the effect of import volume on tariff revenue, which is negative if the tariff is positive as the tariff reduces imports. The first-order condition can also be derived from the explicit expression for welfare (6) and solved for the optimum tariff as a function of the other country's export tax, which yields:

$$\frac{\partial W_{Ai}}{\partial t_i}=\frac{\left(2A_j+\phi A_i\right)-3\Phi_A t_i-\Phi_A e_j}{\beta\Phi_A^2}=0 \quad \Rightarrow \quad t_i^*\left(e_j\right)=\frac{2A_j+\phi A_i-\Phi_A e_j}{3\Phi_A}>0 \qquad (13)$$

The optimum import tariff is positive, if the other country's export tax is less than the prohibitive trade cost, $e_j<\overline{t}_i\equiv A_j/2$, and it is decreasing in the other country's export tax, $\partial t_i/\partial e_j<0$, but it is independent of the other country's import tariff. Using the definition of competitiveness, the optimum import tariff can be rewritten as $t_i^*=\left(\alpha-c_j-e_j\right)/3$, which shows that the lower is the cost of the firm in the other country then the larger will be the optimum import tariff.[13]

If a country sets its export tax to maximise its welfare (3) taking the import tariff of the other country as given, then the first-order condition is:

$$\frac{\partial W_i}{\partial e_i}=\left(p_{ij}-c_i-t_j\right)\frac{\partial x_{ij}}{\partial e_i}+x_{ij}\frac{\partial p_{ij}}{\partial e_i}=0 \qquad i,j=1,2 \quad j\neq i \qquad (14)$$

[13] Also, the tariff is increasing in the export subsidy of the other country as with countervailing duties that were analysed by Dixit (1984) and Collie (1991).

The first term is the profit-shifting effect, which is negative as price is above marginal cost and the export tax reduces exports; and the second term is the terms of trade effect, which is positive as the export tax increases the price of exports. The first-order condition can also be derived from the explicit expression for welfare (6) and solved for the optimum export tax as a function of the other country's import tariff, which yields:

$$\frac{\partial W_{Ai}}{\partial e_i} = \frac{-\phi^2 A_i - 4\left(2-\phi^2\right)e_i + 2\phi^2 t_j}{\beta \Phi_A^2} = 0 \qquad \Rightarrow \qquad e_i\left(t_j\right) = -\frac{\phi^2\left(A_i - 2t_j\right)}{4\left(2-\phi^2\right)} < 0 \tag{15}$$

The optimum export tax is negative, if the other country's import tariff is less than the prohibitive tariff, $t_j < \overline{t}_j \equiv A_i/2$, so it is an export subsidy as one would expect under Cournot oligopoly, and it is increasing in the other country's import tariff, $\partial e_i/\partial t_j > 0$.

When a country unilaterally deviates from free trade, then its optimum import tariff and export tax are given by setting $t_j = e_j = 0$ in (13) and (15) which yields $t_i^D = \left(2A_j + \phi A_i\right)/3\Phi_A > 0$ and $e_i^D = -\phi^2 A_i/4\left(2-\phi^2\right) < 0$, Then, substituting these trade policies into welfare (6), yields the welfare from unilaterally deviating from free trade:

$$W_{Ai}^D = \frac{1}{\beta \Psi^D}\mathbf{A}'\begin{bmatrix}\psi_{ii}^D & \psi_{ij}^D \\ \psi_{ij}^D & \psi_{jj}^D\end{bmatrix}\mathbf{A} \tag{16}$$

where $\Psi^D = 24\left(2-\phi^2\right)\Phi_A^3$, $\psi_{ii}^D = 480 - 352\phi^2 + 68\phi^4 - 3\phi^6$, $\psi_{ij}^D = 4\phi\left(28 - 20\phi^2 + 3\phi^4\right)$, and $\psi_{jj}^D = 4\left(32 - 22\phi^2 + 3\phi^4\right)$ are all positive.

Obviously, if the other country passively pursues a policy of free trade then welfare with the optimum trade policy is higher than under free trade, $W_i^D > W_i^F$, see (A1). However, the other country will lose and hence it is likely to retaliate with the result that there will be a trade war. The trade war with both countries using their trade policies can be represented in this model by the NE in trade policies. Assuming an interior solution where intra-industry trade occurs, the interior NE trade policies are obtained by solving (13) and (15) for both countries, which yields:

$$t_i^N = \frac{\left(16-4\phi^2-\phi^4\right)A_i + 4\phi\left(2-\phi^2\right)A_j}{2G} > 0 \qquad e_i^N = -\frac{\phi^2\left(\left(8-3\phi^2\right)A_i - 2\phi A_j\right)}{2G} < 0 \quad (17)$$

where $G = \left(12-5\phi^2\right)\Phi_A > 0$. At the NE in trade policies, the outputs of the two firms are obtained by substituting the NE trade policies (17) into (4), which yields:

$$x_{ii}^N = \frac{4\left(3-\phi^2\right)A_i + \phi\left(2-\phi^2\right)A_j}{\beta G} > 0 \qquad x_{ji}^N = \frac{\left(8-3\phi^2\right)A_j - 2\phi A_i}{\beta G} \geq 0 \quad (18)$$

Therefore, there will be an interior solution where domestic production and imports are both positive in the *i*th market if $\left(8-3\phi^2\right)A_j - 2\phi A_i > 0$. Hence, exports of the *j*th firm to the *i*th market will be positive if the market share of the *j*th firm under free trade, μ_{Aj}, is greater than $\mu_A^X \equiv 2\phi / \left(8+2\phi-3\phi^2\right) \in \left[0, 2/7\right]$. Thus, there will be an interior solution in both markets if $\mu_A^X < \mu_{A1} < 1-\mu_A^X$, and intra-industry trade will occur in the oligopolistic industry.[14] This rules out the domestic firm from having such a large cost advantage that the Nash-equilibrium import tariff is prohibitive and imports are equal to zero. If there is an interior solution, then the NE trade policies are a positive import tariff and a negative export tax (export subsidy) in both countries, and it can be shown that $t_i^N + e_j^N > 0$ so the domestic firm is always protected. Total government revenue from trade taxes will be positive unless the competitiveness of the domestic firm is sufficiently high and then the export subsidy payments will exceed the import tariff revenue.

[14] Otherwise, there will be trade in only one direction from the country with the low-cost firm to the country with the high-cost firm.

Welfare in the interior NE in trade policies is obtained by substituting the NE trade policies (17) into welfare (6), which yields the quadratic form:

$$W_{Ai}^{N} = \frac{1}{\beta \Psi^{N}} \mathbf{A}' \begin{bmatrix} \psi_{ii}^{N} & \psi_{ij}^{N} \\ \psi_{ij}^{N} & \psi_{jj}^{N} \end{bmatrix} \mathbf{A} \tag{19}$$

where $\Psi^{N} = 2G^{2}$, $\psi_{ii}^{N} = 560 - 508\phi^{2} + 138\phi^{4} - 9\phi^{6}$, $\psi_{ij}^{N} = \phi\left(2-\phi^{2}\right)\left(68-27\phi^{2}\right)$, $\psi_{jj}^{N} = 192 - 76\phi^{2} - 31\phi^{4} + 12\phi^{6}$ are all positive.

The welfare effect of the trade war on a country is the difference between its welfare in the NE in trade policies and its welfare under free trade: $\Delta W_{Ai}^{NF} = W_{Ai}^{N} - W_{Ai}^{F}$, which is obtained by subtracting (7) from (19):

$$\Delta W_{Ai}^{NF} = \frac{1}{\beta \Psi^{N} \Psi^{F}} \mathbf{A}' \begin{bmatrix} \psi_{ii}^{NF} & \psi_{ij}^{NF} \\ \psi_{ij}^{NF} & \psi_{jj}^{NF} \end{bmatrix} \mathbf{A} \tag{20}$$

$$\begin{aligned} \psi_{ii}^{NF} &= \psi_{ii}^{N}\Psi^{F} - \Psi^{N}\psi_{ii}^{F} = -2\left(160 - 92\phi^{2} - 13\phi^{4} + 9\phi^{6}\right)\Phi_{A}^{2} < 0 \\ \psi_{ij}^{NF} &= \psi_{ij}^{N}\Psi^{F} - \Psi^{N}\psi_{ij}^{F} = -4\phi\left(4 + \phi^{2} - \phi^{4}\right)\Phi_{A}^{2} < 0 \\ \psi_{jj}^{NF} &= \psi_{jj}^{N}\Psi^{F} - \Psi^{N}\psi_{jj}^{F} = 8\left(3 - \phi^{2}\right)\left(4 + 5\phi^{2} - 3\phi^{4}\right)\Phi_{A}^{2} > 0 \end{aligned}$$

In the symmetric case, when $A_{1} = A_{2} = A$, as one would expect both countries are worse off in the NE in trade policies than under free trade:

$$\Delta W_{A}^{NF} = \frac{\left(\psi_{ii}^{NF} + 2\psi_{ij}^{NF} + \psi_{jj}^{NF}\right) A^{2}}{\beta \Psi^{N} \Psi^{F}} < 0 \tag{21}$$

This is negative since $\psi_{ii}^{NF} + 2\psi_{ij}^{NF} + \psi_{jj}^{NF} = -2\left(1+\phi\right)\left(2-\phi^{2}\right)^{2}\left(28 + 4\phi - 13\phi^{2} - 3\phi^{3}\right)\Phi_{A}^{2} < 0$. However, it is possible that one country could be better off in the NE in trade policies than under free trade. To see when a country may win a trade war, differentiate the welfare effect of the trade war (20) with respect to the competitiveness of the country's firm, which yields:

$$\frac{\partial \Delta W_{Ai}^{NF}}{\partial A_i} = 2\frac{\psi_{ii}^{NF} A_i + \psi_{ij}^{NF} A_j}{\beta \Psi^N \Psi^F} < 0 \tag{22}$$

Since this derivative is negative, see (20), the lower the competitiveness of the country's firm then the larger will be the welfare effect of the trade war so a country may win the trade war if its firm is sufficiently uncompetitive relative to its competitor. Since the market share under free trade of the *i*th firm is $\mu_{Ai} = A_i \big/ \left(A_i + A_j\right)$, the competitiveness of the other firm can be written as $A_j = \left(1-\mu_{Ai}\right) A_i \big/ \mu_{Ai}$ and substituting this into the quadratic form (20) yields:

$$\Delta W_{Ai}^{NF} = \frac{A_i^2}{\beta \Psi^N \Psi^F \mu_{Ai}^2}\left[\psi_{ii}^{NF} \mu_{Ai}^2 + 2\psi_{ij}^{NF}\left(1-\mu_{Ai}\right)\mu_{Ai} + \psi_{jj}^{NF}\left(1-\mu_{Ai}\right)^2\right] \tag{23}$$

The sign of the welfare effect depends upon the term in square brackets, which is a quadratic in μ_{Ai}, and hence a country will win a trade war if the market share under free trade of its firm is less than the critical value obtained by setting the quadratic equal to zero and solving for μ_A^{NF}:

$$\mu_{Ai} < \mu_A^{NF} \equiv \frac{\psi_{jj}^{NF}}{\psi_{jj}^{NF} - \psi_{ij}^{NF} + \sqrt{\left(\psi_{ij}^{NF}\right)^2 - \psi_{ii}^{NF}\psi_{jj}^{NF}}} \tag{24}$$

The home country will win a trade war if $\mu_{A1} < \mu_A^{NF}$, and, since $\mu_{A1} + \mu_{A2} = 1$, the foreign country will win a trade war if $\mu_{A1} > 1 - \mu_A^{NF}$. The critical market shares, μ_A^{NF} and $1-\mu_A^{NF}$, are plotted against ϕ in figure 2 together with μ_A^X and $1-\mu_A^X$, which show where exports of the home firm and the foreign firm will be equal to zero. In the region below μ_A^{NF}, the home country wins and the foreign country loses the trade war, whereas in the region above $1-\mu_A^{NF}$, the foreign country wins and the home country loses the trade war. Note that a sufficient condition for a country to win a trade war is that its exports in the NE in trade policies are equal to zero! In the region between μ_A^{NF} and $1-\mu_A^{NF}$, which includes the symmetric case, both countries are worse off than under free trade, and this region becomes narrower as the degree of product substitutability, ϕ, increases.

The change in world welfare as a result of the trade war is the total welfare of the two countries in the interior NE in trade policies minus the total welfare of the two countries under free trade, $\Delta\Omega_A^{NF} = \Delta W_{A1}^{NF} + \Delta W_{A2}^{NF}$, and evaluating using (20) yields the quadratic form:

$$\Delta\Omega_A^{NF} = \frac{1}{\beta\Psi^N\Psi^F}\mathbf{A}'\begin{bmatrix}\psi_{ii}^{NF}+\psi_{jj}^{NF} & 2\psi_{ij}^{NF}\\ 2\psi_{ij}^{NF} & \psi_{ii}^{NF}+\psi_{jj}^{NF}\end{bmatrix}\mathbf{A} < 0 \tag{25}$$

Since $\psi_{ii}^{NF}+\psi_{jj}^{NF} = -2\left(4-3\phi^2\right)\left(28-13\phi^2+\phi^4\right)\Phi_A^2 < 0$ and $\psi_{ij}^{NF} < 0$ from (20), all elements of the matrix are negative so the quadratic form is negative for $\mathbf{A} > 0$. Therefore, a trade war will always reduce world welfare even if one country wins. These results lead to the following proposition:

Proposition 2: *Under Cournot duopoly, the ith country will win (and the other country will lose) the trade war if* $\mu_{Ai} < \mu_A^N$, *and if* $\mu_A^N < \mu_{Ai} < 1-\mu_A^N$ *then both countries lose the trade war. World welfare is always lower in a trade war than under free trade.*

Counterintuitively, if the foreign firm has a sufficiently large cost advantage and consequently a large market share under free trade, then the home country can win the trade war.[15] If the foreign firm has lower costs than the home firm, then the foreign firm will face a higher import tariff than the home firm in the NE in trade policies (13), and will receive a higher export subsidy than the home firm. Hence, the home government can extract a larger amount of rent from the foreign firm with its import tariff than the foreign government can

[15] In contrast, Collie (1993) shows that the country with the most competitive firm may win an export subsidy war in the Brander and Spencer (1985) model.

extract from the home firm with its import tariff. Obviously, the import tariffs reduce consumer surplus, but if the cost advantage of the foreign firm is sufficiently large then the gain in rent from the import tariff will outweigh the loss of consumer surplus and the lower profits from exports of the home firm. If neither firm has a significant cost advantage, then the outcome of any trade war is going to be that both countries lose. In this case, the trade policy game is like the classic prisoners' dilemma with both countries worse off in the interior NE in trade policies (trade war) than under free trade.

The interior NE in trade policies is not unique as both countries minimaxing each other is also a NE in trade policies where the outcome is autarky.[16] If each country sets a prohibitive import tariff, $t_i = \overline{t}_i = A_j/2$, and a prohibitive export tax, $e_i = \overline{t}_j = A_i/2$, then the other country setting a prohibitive import tariff and a prohibitive export tax is a best reply.[17] Since Proposition 1 showed that there are always gains from multilateral free trade, both countries will lose a trade war where each country sets a prohibitive import tariff and export tax, and the outcome is autarky with welfare, W_{Ai}^0, given by (9). The difference in welfare between the interior NE in trade policies and the minimax NE in trade policies is $\Delta W_{Ai}^{N0} = W_{Ai}^N - W_{Ai}^0$, which is obtained by subtracting (9) from (19):

$$\Delta W_{Ai}^{N0} = \frac{1}{\beta \Psi^N \Psi^0} \mathbf{A}' \begin{bmatrix} \psi_{ii}^{N0} & \psi_{ij}^{N0} \\ \psi_{ij}^{N0} & \psi_{jj}^{N0} \end{bmatrix} \mathbf{A} > 0 \qquad (26)$$

[16] In an insightful survey on strategic aspects of trade policy, Dixit (1987) points out that autarky, as well as the usual *interior* NE considered by Johnson (1953), is a NE in trade policies under perfect competition, but does not consider this possibility when discussing trade policy under oligopoly later in the survey.

[17] Under oligopoly, with segmented markets, an import tariff and an export tax are both required for the two countries *minimaxing* each other to be a NE in trade policies that results in the autarky outcome. Trade could occur if a country subsidised exports or imports, but this would decrease its welfare given the import tariff or export tax of the other country, so it will not occur in the *minimax* NE in trade policies.

$$\psi_{ii}^{N0} = \psi_{ii}^{N}\Psi^{0} - \Psi^{N}\psi_{ii}^{0} = 8\left(128 - 148\phi^{2} + 63\phi^{4} - 9\phi^{6}\right)\Phi_{A}^{2} > 0$$
$$\psi_{ij}^{N0} = \psi_{ij}^{N}\Psi^{0} - \Psi^{N}\psi_{ij}^{0} = -4\phi\left(8 - 3\phi^{2}\right)\left(20 - 7\phi^{2}\right)\Phi_{A}^{2} < 0$$
$$\psi_{jj}^{N0} = \psi_{jj}^{N}\Psi^{0} - \Psi^{N}\psi_{jj}^{0} = 2\left(768 - 736\phi^{2} + 236\phi^{4} - 27\phi^{6}\right)\Phi_{A}^{2} > 0$$

Since the principal diagonal elements of the matrix are both positive and the determinant is positive, $\det = 48\left(2-\phi^{2}\right)\left(16-9\phi^{2}\right)^{2}\Phi_{A}^{7} > 0$, the quadratic form is positive definite. This leads to the following proposition:

Proposition 3: *Under Cournot duopoly, welfare of both countries is higher in the interior NE in trade polices than in the minimax NE in trade policies.*

Since the interior NE in trade policies is Pareto superior to autarky, one would argue that it is the most likely outcome of the static (one-shot) trade policy game. Trade policy is often described and modelled as a classic prisoners' dilemma but, strictly speaking, this is not correct in this model for a couple of reasons. Firstly, it is possible for one country to win the trade war in the case of the interior NE in trade policies unlike the classic prisoners' dilemma where both players lose. Secondly, there are multiple NE in trade policies whereas there is a unique NE in dominant strategies in the classic prisoners' dilemma. The implications of these observations for the sustainability of co-operation (free trade) when the game is repeated will be considered in the next section.

4. Sustaining Free Trade under Cournot Oligopoly

To avoid the perceived prisoners' dilemma in trade policy, the GATT and the WTO were established to facilitate multilateral trade negotiations. Since countries are sovereign and there is no supra-national authority, any international trade agreement has to be self-enforcing so the ultimate enforcement mechanism available in the WTO is for members to retaliate if a WTO member does not honour its commitments. Hence, one might argue that the WTO sustains free trade (or almost free trade with very low tariffs and hardly any export subsidies) by the threat of retaliation (withdrawal of concessions) if a country deviates from free trade. This can be modelled as an infinitely-repeated game where co-operation (free trade) is sustained by the threat of retaliation if a country deviates from free trade. The Folk Theorem implies that co-operation can be sustained as a subgame perfect equilibrium if the discount factor is sufficiently large. The sustainability of free trade as a subgame-perfect NE will be analysed where the countries use Nash-reversion trigger strategies as in Friedman (1971) and where the countries minimax each other as in Fudenberg and Maskin (1986).

Now consider an infinitely-repeated version of the constituent game considered in the previous sections where both countries have the same discount factor $\delta \in (0,1)$.[18] With infinite Nash-reversion trigger strategies, the strategy of each country is to play free trade until the other country deviates by using its optimum trade policies, and thereafter for both countries to play their interior NE trade policies. Assuming that the other country uses infinite Nash-reversion trigger strategies, a country will play free trade if the discounted present value of welfare under free trade exceeds the welfare from unilaterally deviating from free trade for one round followed by the welfare in the interior NE forever afterwards: $W_{Ai}^{F}/(1-\delta) > W_{Ai}^{D} + \delta W_{Ai}^{N}/(1-\delta)$. Setting both sides equal and solving for the critical discount factor of each country required to sustain free trade yields:[19]

$$\delta_{Ai}^{N\infty} = \frac{W_{Ai}^{D} - W_{Ai}^{F}}{W_{Ai}^{D} - W_{Ai}^{N}} = \frac{\Delta W_{Ai}^{DF}}{\Delta W_{Ai}^{DN}} \qquad i = 1,2 \qquad (27)$$

Free trade is sustainable as a subgame-perfect NE if the discount factor is greater than the critical discount factor for both countries, $\delta \geq Max\left\{\delta_{A1}^{N\infty}, \delta_{A2}^{N\infty}\right\}$. Then, substituting in the

[18] It is assumed that the firms always produce their static Cournot-Nash outputs and do not collude on outputs. This is a reasonable assumption given that collusion is generally illegal in most countries whereas governments can negotiate about co-operation on trade policy at the WTO or elsewhere such as the G7 or G20.

[19] The denominator may be negative, see (A2), but this is only the case if welfare in the *interior* NE is higher than welfare under free trade in which case free is not sustainable as a subgame perfect NE.

welfare differences from (A1) and (A2) yields the critical discount factor in terms of the parameters of the model A_i, A_j and ϕ. Since the market share of the *i*th firm under free trade is $\mu_{Ai} = A_i / (A_i + A_j)$, the competitiveness of the *j*th firm can be expressed as $A_j = (1-\mu_{Ai}) A_i / \mu_{Ai}$, and if this is substituted into the critical discount factors then they will be functions of just two parameters: the market share of the home firm under free trade, μ_{Ai}, and the degree of product substitutability, ϕ, since all the *Psi* depend only upon ϕ:

$$\delta_{Ai}^{N\infty}(\mu_{Ai},\phi) = \left(\frac{\Psi^N}{\Psi^F}\right) \frac{\psi_{ii}^{DF}\mu_{Ai}^2 + 2\psi_{ij}^{DF}(1-\mu_{Ai})\mu_{Ai} + \psi_{jj}^{DF}(1-\mu_{Ai})^2}{\psi_{ii}^{DN}\mu_{Ai}^2 + 2\psi_{ij}^{DN}(1-\mu_{Ai})\mu_{Ai} + \psi_{jj}^{DN}(1-\mu_{Ai})^2} \tag{28}$$

The critical discount factors of the two countries are plotted against μ_{A1} in figure 3 for different values of $\phi = \{0, 1/2, 1\}$, but only for the parameter values where there is an interior solution. Since free trade is only sustainable if $\delta \geq Max\{\delta_{A1}^{N\infty}, \delta_{A2}^{N\infty}\}$, it will be sustainable in the region *I* above both of the curves in figure 3. Clearly, it is easier to sustain free trade when the firms are symmetric, $\mu_{A1} = 1/2$, and cost asymmetries make it harder to sustain free trade. If a country can win a trade war, $W_{Ai}^N > W_{Ai}^F$, then (27) implies that its critical discount factor will be greater than one and free trade will not be sustainable. Therefore, free trade is only sustainable in the region where both countries are worse off in the trade war than under free trade.

Since autarky is also a NE of the constituent game, free trade can also be sustained by the threat of infinite reversion to the minimax NE. Proposition 3 showed that welfare in the minimax NE is always lower than welfare in the *interior* NE, Hence, the punishment is more severe, and it should be easier to sustain free trade using the threat of the minimax NE rather than the *interior* NE. Assuming that the other country uses infinite minimax-reversion trigger strategies, a country will play free trade if the discounted present value of welfare under free trade exceeds the welfare from unilaterally deviating from free trade for one round followed by the welfare in the minimax NE forever afterwards: $W_{Ai}^{F}/(1-\delta) > W_{Ai}^{D} + \delta W_{Ai}^{0}/(1-\delta)$. Setting both sides equal and solving for the critical discount factor of each country required to sustain free trade yields:

$$\delta_{Ai}^{M\infty} = \frac{W_{Ai}^{D} - W_{Ai}^{F}}{W_{Ai}^{D} - W_{Ai}^{0}} = \frac{\Delta W_{Ai}^{DF}}{\Delta W_{Ai}^{D0}} \qquad i = 1,2 \tag{29}$$

Substituting in the welfare changes from (A1) and (A3), and then using $A_j = (1-\mu_{Ai})A_i/\mu_{Ai}$, yields the critical discount factor of each country required to sustain free trade using infinite minimax reversion:

$$\delta_{Ai}^{M\infty}(\mu_{Ai},\phi) = \left(\frac{\Psi^0}{\Psi^F}\right)\frac{\psi_{ii}^{DF}\mu_{Ai}^2 + 2\psi_{ij}^{DF}(1-\mu_{Ai})\mu_{Ai} + \psi_{jj}^{DF}(1-\mu_{Ai})^2}{\psi_{ii}^{D0}\mu_{Ai}^2 + 2\psi_{ij}^{D0}(1-\mu_{Ai})\mu_{Ai} + \psi_{jj}^{D0}(1-\mu_{Ai})^2} \tag{30}$$

The critical discount factors of the two countries are plotted against μ_{A1} in figure 4 for different values of $\phi = \{0, 1/2, 1\}$, but only for the parameter values where there is an interior solution. Since free trade is only sustainable if $\delta \geq Max\{\delta_{A1}^{M\infty}, \delta_{A2}^{M\infty}\}$, it will be sustainable in the region *I* above both of the curves in figure 4. Since both countries are always worse off under autarky than under free trade, $W_{Ai}^{0} < W_{Ai}^{F}$, (29) implies that the critical discount factor of both firms will always be less than one. In fact, as can be seen in figure 4, the highest value of the critical discount factor is $\delta_{Ai}^{M\infty} = 4/5$.

Comparing (27) and (29), it is clear that $\delta_{Ai}^{N\infty} > \delta_{Ai}^{M\infty}$ if $W_{Ai}^{N} > W_{Ai}^{0}$, but this was proved in Proposition 3, and therefore it follows that $Max\{\delta_{A1}^{N\infty}, \delta_{A2}^{N\infty}\} > Max\{\delta_{A1}^{M\infty}, \delta_{A2}^{M\infty}\}$. Hence, it is always easier to sustain free trade with infinite minimax reversion than with infinite Nash reversion. This leads to the following proposition:

Proposition 4: *Under Cournot duopoly, it is easier to sustain free trade in an infinitely-repeated game using infinite minimax reversion than using infinite Nash reversion.*

To compare the critical discount factors $\delta_{Ai}^{N\infty}$ and $\delta_{Ai}^{M\infty}$, figure 5 shows these two discount factors for both countries for the case when $\phi = 1/2$. Free trade is sustainable in the region labelled *I* with infinite Nash reversion, whereas it is sustainable in the regions labelled *I*, *II*, *III*, and *IV* with infinite minimax reversion. Cost asymmetries have much less effect on the sustainability of free trade with infinite minimax reversion than with infinite Nash reversion.

For the rest of this section, it will be assumed that the firms are symmetric so $\mu_{A1} = \mu_{A2} = 1/2$, and substituting this into (28) and (30), yields the critical discount factors with infinite Nash reversion and infinite minimax reversion, which are the same for both of the countries and depend only upon ϕ:

$$\delta_A^{N\infty}(\phi) = \left(\frac{\Psi^N}{\Psi^F}\right)\frac{\psi_{ii}^{DF} + 2\psi_{ij}^{DF} + \psi_{jj}^{DF}}{\psi_{ii}^{DN} + 2\psi_{ij}^{DN} + \psi_{jj}^{DN}} \qquad \delta_A^{M\infty}(\phi) = \left(\frac{\Psi^0}{\Psi^F}\right)\frac{\psi_{ii}^{DF} + 2\psi_{ij}^{DF} + \psi_{jj}^{DF}}{\psi_{ii}^{D0} + 2\psi_{ij}^{D0} + \psi_{jj}^{D0}} \quad (31)$$

The critical discount factor $\delta_A^{N\infty}$ is increasing in the degree of product substitutability from $3/10$ for independent products, $\phi = 0$, up to $245/373 \approx 0 \cdot 657$ for homogeneous products, $\phi = 1$. The critical discount factor $\delta_A^{M\infty}$ is increasing in the degree of product substitutability from $1/10$ for independent products, $\phi = 0$, up to $1/2$ for homogeneous products, $\phi = 1$. These are plotted against ϕ in figure 6 where it can be seen that $\delta_A^{M\infty} < \delta_A^{N\infty}$ and that both are increasing in ϕ, which leads to the following proposition:

Proposition 5: *Under Cournot duopoly, the critical discount factors $\delta_A^{N\infty}$ and $\delta_A^{M\infty}$ are both increasing in the degree of product substitutability, ϕ.*

When free trade is sustained by infinite Nash reversion and infinite minimax reversion, the punishment phase continues forever after any deviation from free trade. This punishment is credible since playing the NE forever is a subgame perfect equilibrium, so it is not rational for an individual country to deviate from the punishment phase. However, if the two countries were to forgive and forget the deviation from free trade and return to co-operation then both countries would be better off than continuing in the punishment phase. If the countries know that renegotiation will occur after a deviation, then the punishment will not be credible. This possibility can be avoided by the use of (weakly) renegotiation-proof strategies so that the country that deviates is punished, but the cheated country is better off than under free trade in the punishment phase. Then, the cheated country will not agree to forgive and forget then return to co-operation since it is better off punishing the deviating country. The idea that equilibria of infinitely-repeated games should be (weakly) renegotiation proof implicitly assumes that the countries are able to instantaneously and costlessly renegotiate their agreement and then immediately return to free trade. However, the length of multilateral trade negotiations such as the Uruguay Round suggests that this is not very likely.[20] Given that renegotiation is likely to be a lengthy process, it is not really necessary that the cheated country is better off in the punishment phase than under free trade if the punishment phase lasts only a few rounds. Also, as has been shown, the more severe is the punishment then the easier it will be to sustain free trade. This suggest that sustaining free trade by the threat of the minimax reversion for a limited number of rounds followed by a return to free trade might be the best solution. When the punishment phase lasts for one round, then free trade is sustainable as a subgame-perfect NE if the welfare from free trade for two rounds exceeds the welfare from deviation for one round followed by the welfare from autarky for one round: $W_A^F + \delta W_A^F > W_A^D + \delta W_A^0$. Hence, the

[20] The Uruguay Round of multilateral trade negotiations under the auspices of the GATT led to the formation of the WTO, it started in 1986 and was completed successfully in 1994. There has not been a successful round of multilateral trade negotiations since the formation of the WTO in 1995.

critical discount factor is $\delta_A^{M1} \equiv \left(W_A^D - W_A^F\right) / \left(W_A^F - W_A^0\right) = \Delta W_{Ai}^{DF} / \Delta W_{Ai}^{F0}$, and using the welfare differences from (A1) and (10) yields:

$$\delta_A^{M1}(\phi) = \left(\frac{\Psi^0}{\Psi^D}\right) \frac{\psi_{ii}^{DF} + 2\psi_{ij}^{DF} + \psi_{jj}^{DF}}{\psi_{ii}^{F0} + 2\psi_{ij}^{F0} + \psi_{jj}^{F0}} \tag{32}$$

Similarly, if the punishment phase lasts for two rounds then free trade is sustainable if $\left(1+\delta+\delta^2\right)W_A^F > W_A^D + \delta\left(1+\delta\right)W_A^0$ and the critical discount factor δ_A^{M2} is given by the solution of $\delta\left(1+\delta\right) = \Delta W_{Ai}^{DF} / \Delta W_{Ai}^{F0}$. Also, if the punishment phase lasts for three rounds then free trade is sustainable if $\left(1+\delta+\delta^2+\delta^3\right)W_A^F > W_A^D + \delta\left(1+\delta+\delta^2\right)W_A^0$ and the critical discount factor δ_A^{M3} is given by the solution of $\delta\left(1+\delta+\delta^2\right) = \Delta W_{Ai}^{DF} / \Delta W_{Ai}^{F0}$. It is possible to solve both equations for explicit solutions and these are plotted against ϕ in figure 7 together with $\delta_A^{N\infty}$, $\delta_A^{M\infty}$, and δ_A^{M1}. All the critical discount factors are increasing in ϕ, so again product differentiation makes it easier to sustain free trade. As the number of rounds of the punishment phase increases then the critical discount factor decreases, it is lower than $\delta_A^{N\infty}$ when the punishment phase lasts for two rounds, $\delta_A^{M2} < \delta_A^{N\infty}$, and when the punishment phase lasts for three rounds, the critical discount factor is very close to $\delta_A^{M\infty}$. Generally, if the punishment phase lasts for z rounds then it can be shown that $\delta_A^{Mz} < 1$ if $z > \delta_A^{M\infty} / \left(1-\delta_A^{M\infty}\right)$.[21] These results leads to the following proposition:

Proposition 6: *Under Cournot duopoly, with minimax reversion for z rounds the critical discount factors δ_A^{Mz} is increasing in ϕ. Also, $\delta_A^{M1} \le 1$, and $\delta_A^{M2} < \delta_A^{N\infty}$.*

---

[21] See (A4) in appendix A. With cost asymmetries, it was shown that the highest value for $\delta_{Ai}^{M\infty} = 4/5$, then the punishment phase would need to be more than four rounds, $z > 4$.

In a finitely-repeated game with T rounds, since there are two NE of the constituent game, it is possible to sustain free trade for a number of rounds as in Benoit and Krishna (1985). The countries co-operate by playing free trade for the first $T-t$ rounds then play the interior NE trade policies for the final t rounds of the game, where $T-t \geq 1$ and $t \geq 1$. If a country deviates from free trade, then both countries play the minimax NE trade policies for the remaining rounds of the game. Free trade is sustainable for $T-1$ rounds if there is no incentive for either country to deviate in the penultimate round of the game, which will be the case if $W_A^F + \delta W_A^N > W_A^D + \delta W_A^0$.[22] Hence, the critical discount factor required to sustain free trade in the finitely-repeated game is $\delta_A^{T-1}(\phi) = \left(W_A^D - W_A^F\right) \big/ \left(W_A^N - W_A^0\right) = \Delta W_A^{DF} / \Delta W_A^{N0}$, and substituting in (A1) and (26) yields:

$$\delta_A^{T-1}(\phi) = \left(\frac{\Psi^D \Psi^F}{\Psi^N \Psi^0}\right) \frac{\psi_{ii}^{DF} + 2\psi_{ij}^{DF} + \psi_{jj}^{DF}}{\psi_{ii}^{N0} + 2\psi_{ij}^{N0} + \psi_{jj}^{N0}} \tag{33}$$

Similarly, free trade is sustainable for $T-2$ rounds if there is no incentive for either country to deviate in round $T-2$ of the game, which will be the case if $W_A^F + \delta(1+\delta) W_A^N > W_A^D + \delta(1+\delta) W_A^0$, and the critical discount factor $\delta_A^{T-2}(\phi)$ is given by the solution of $\delta(1+\delta) = \Delta W_A^{DF} / \Delta W_A^{N0}$. Also, free trade is sustainable for $T-3$ rounds if there is no incentive for either country to deviate in round $T-3$ of the game, which will be the case if $W_A^F + \delta\left(1+\delta+\delta^2\right) W_A^N > W_A^D + \delta\left(1+\delta+\delta^2\right) W_A^0$, and the critical discount factor $\delta_A^{T-3}(\phi)$ is given by the solution of $\delta\left(1+\delta+\delta^2\right) = \Delta W_A^{DF} / \Delta W_A^{N0}$. It is possible to solve both equations for explicit solutions for δ_A^{T-2} and δ_A^{T-3}, which are plotted against ϕ in figure 8 together with δ_A^{T-1} and, for comparison, $\delta_A^{N\infty}$, and $\delta_A^{M\infty}$. All the critical discount factors are increasing in the

[22] If there is no incentive to deviate in the penultimate round, then there is no incentive to deviate in earlier rounds as the punishment phase will be longer.

degree of product substitutability, so again product differentiation makes it easier to sustain free trade. As t increases the critical discount factor decreases and when $t \geq 3$ then free trade is always sustainable for a discount factor sufficiently close to one.

5. Bertrand Oligopoly with Differentiated Products

A common concern about the literature on trade policy under oligopoly is that the results are not robust, and that Cournot oligopoly and Bertrand oligopoly yield quite different results. To address this concern and to allow a comparison, now consider the situation where the two firms in Section 3 compete in prices rather than quantities so that there is a Bertrand duopoly rather than a Cournot duopoly. International trade under Bertrand duopoly with differentiated products has been analysed by Clarke and Collie (2003), and this section will use the same approach. Inverting the inverse demand functions (2), assuming that $\phi \in [0,1)$ so the case of perfect substitutes is ruled out as usual, yields the demand functions facing the home firm and the foreign firm in the two markets:

$$x_{1i} = \frac{\alpha(1-\phi) - p_{1i} + \phi p_{2i}}{\beta(1-\phi^2)} \qquad x_{2i} = \frac{\alpha(1-\phi) + \phi p_{1i} - p_{2i}}{\beta(1-\phi^2)} \qquad i = 1,2 \tag{34}$$

In the Bertrand duopoly, each firm sets its price to maximise its profits given the price set by its competitor, and with both firms taking the trade policies of the two countries as given. Since the markets are assumed to be segmented and marginal costs are constant, the Bertrand equilibrium prices and sales can be derived independently in each market. It is straightforward to solve for the Bertrand equilibrium prices of the home firm and the foreign firm in the two markets as functions of trade policies:

$$p_{ii} = c_i + \frac{B_i + \phi(t_i + e_j)}{(4-\phi^2)} \qquad p_{ji} = c_j + t_i + e_j + \frac{B_j - (2-\phi^2)(t_i + e_j)}{(4-\phi^2)} \tag{35}$$

where $B_i = \left(2-\phi^2\right)\left(\alpha - c_i\right) - \phi\left(\alpha - c_j\right)$, $i = 1,2$ and $j \neq i$, are measures of the competitiveness of the firms under Bertrand duopoly. Note that the definition of competitiveness under Bertrand duopoly is not the same as under Cournot duopoly. Substituting these prices into the demand functions (34) yields the sales of the home firm and the foreign firm in the two markets:

$$x_{ii} = \frac{B_i + \phi\left(t_i + e_j\right)}{\beta \Phi_B} \qquad x_{ji} = \frac{B_j - \left(2-\phi^2\right)\left(t_i + e_j\right)}{\beta \Phi_B} \qquad i, j = 1,2 \;\; j \neq i \tag{36}$$

where $\Phi_B = \left(1-\phi^2\right)\left(4-\phi^2\right) > 0$ if $\phi \in [0,1)$. If there is an interior solution where both firms sell positive quantities in both markets under free trade, $\boldsymbol{\tau} = \mathbf{0}$, then $B_i > 0$ for $i = 1,2$. Hence, the market share of the *i*th firm under free trade is $\mu_{Bi} = B_i \big/ \left(B_i + B_j\right)$. Imports of the *i*th country will be zero if $t_i + e_j \geq \tilde{t}_i \equiv B_j \big/ \left(2-\phi^2\right)$; however, the *i*th firm will only be able to set the monopoly price if $t_i + e_i \geq \overline{t}_i \equiv A_j/2 = \left(\left(4-3\phi^2\right)B_j + \phi^3 B_i\right)\beta \big/ 2\Phi$; and when $t_i + e_j \in \left[\tilde{t}_i, \overline{t}_i\right)$ there are no imports but the threat of competition reduces the price of the *i*th firm below its monopoly price.[23] From (35) and (36), the total profits (profits in the domestic market plus profits from exports) of the firms are:

$$\Pi_i = \pi_{ii} + \pi_{ij} = \frac{\left(B_i + \phi\left(t_i + e_j\right)\right)^2 + \left(B_i - \left(2-\phi^2\right)\left(t_j + e_i\right)\right)^2}{\beta\left(4-\phi^2\right)\Phi_B} \tag{37}$$

Substituting the sales (36) and the profits (37) into (3) yields welfare of the two countries as functions of their trade policies:

[23] For a more detailed explanation, see Clarke and Collie (2003), who present the best-reply functions of the firms allowing for this possibility. However, the analysis in this paper will only consider the interior solution where both firms have positive sales.

$$W_{Bi} = W_{Bi}^F + \frac{1}{2\beta\left(4-\phi^2\right)\Phi_B}\Big[2\phi^2 B_i e_i - 2\left(2B_j - \phi B_i\right)e_j + 2\left(\left(2-\phi^2\right)B_j + \phi B_i\right)t_i$$
$$-4\left(2B_i - \phi^2 B_j\right)t_j - 4\left(2-\phi^2\right)e_i^2 + \left(4-\phi^2\right)e_j^2 - \left(3-2\phi^2\right)\left(4-\phi^2\right)t_i^2 \qquad (38)$$
$$+2\left(2-\phi^2\right)^2 t_j^2 - 2\phi^2\left(2-\phi^2\right)e_i t_j - 2\Phi_B e_j t_i\Big]$$

where W_{Bi}^F is welfare under multilateral free trade, $\boldsymbol{\tau} = \mathbf{0}$, which can be written as a quadratic form in B_i and B_j, where $\mathbf{B}' = \begin{bmatrix} B_i & B_j \end{bmatrix}$:

$$W_{Bi}^F = \frac{1}{\beta\Theta^F}\mathbf{B}'\begin{bmatrix} \theta_{ii}^F & \theta_{ij}^F \\ \theta_{ij}^F & \theta_{jj}^F \end{bmatrix}\mathbf{B} \qquad (39)$$

where $\Theta^F = 2\Phi_B^2 > 0$, $\theta_{ii}^F = 5 - 4\phi^2 > 0$, $\theta_{ij}^F = \phi > 0$, and $\theta_{jj}^F = 1$.

Under autarky, each firm has a monopoly in its own market so autarky welfare is still given by (9) but, for future welfare comparisons, it is more helpful to rewrite it as a quadratic form in terms of $\mathbf{B}$ rather than $\mathbf{A}$, by noting that $A_i = \left(\left(4-3\phi^2\right)B_i + \phi^3 B_j\right)\big/\Phi_B$:

$$W_{Bi}^0 = \frac{1}{\beta\Theta^0}\mathbf{B}'\begin{bmatrix} \theta_{ii}^0 & \theta_{ij}^0 \\ \theta_{ij}^0 & \theta_{jj}^0 \end{bmatrix}\mathbf{B} = \frac{3}{8\beta}\left(\alpha - c_i\right)^2 \qquad i,j = 1,2 \quad j \neq i \qquad (40)$$

where $\Theta^0 = 8\Phi_B^2 > 0$, $\theta_{ii}^0 = 3\left(2-\phi^2\right)^2 > 0$, $\theta_{ij}^0 = 3\phi\left(2-\phi^2\right) > 0$, and $\theta_{jj}^0 = 3\phi^2 > 0$.

To show that there are gains from multilateral free trade, subtract welfare under autarky (40) from welfare under multilateral free trade (39), which yields the quadratic form in $\mathbf{B}$:

$$\Delta W_{Bi}^{F0} = W_{Bi}^F - W_{Bi}^0 = \frac{1}{\beta\Theta^F\Theta^0}\mathbf{B}'\begin{bmatrix} \theta_{ii}^{F0} & \theta_{ij}^{F0} \\ \theta_{ij}^{F0} & \theta_{jj}^{F0} \end{bmatrix}\mathbf{B} > 0 \qquad (41)$$

$$\theta_{ii}^{F0} = \Theta^0\theta_{ii}^F - \Theta^F\theta_{ii}^0 = 2\left(8 - 4\phi^2 - 3\phi^4\right)\Phi_B^2 > 0$$
$$\theta_{ij}^{F0} = \Theta^0\theta_{ij}^F - \Theta^F\theta_{ij}^0 = -2\phi\left(2 - 3\phi^2\right)\Phi_B^2 < 0$$
$$\theta_{jj}^{F0} = \Theta^0\theta_{jj}^F - \Theta^F\theta_{jj}^0 = 2\left(4 - 3\phi^2\right)\Phi_B^2 > 0$$

The matrix is positive definite as the principal diagonal elements are both positive and the determinant is positive, $\det = 16\left(1-\phi^2\right)\left(8-3\phi^2\right)\Phi_B^4 > 0$. Hence, welfare under multilateral free trade is always higher than welfare under autarky so there are always gains from trade despite the cost asymmetries. It can also be shown that there are always gains from unilateral free trade, and this leads to the following proposition:

Proposition 7: *Under Bertrand duopoly, there are always gains from multilateral free trade for both countries, and there are always gains from unilateral free trade.*

This follows from the result in Clarke and Collie (2003) that there are always gains from unilateral free trade under Bertrand duopoly, and hence that there are always gains from multilateral free trade.

6. Trade Wars under Bertrand Oligopoly

Under Bertrand oligopoly, an import tariff can be used to extract rent from foreign firms and to shift profits to the domestic firm as in Cheng (1988), and an export tax can be used to shift export profits to the domestic country as in Eaton and Grossman (1986). When the *i*th country unilaterally deviates from multilateral free trade, then its optimum import tariff and export tax are given by maximising its welfare (38) with $t_j = e_j = 0$, which yields:

$$t_i^D = \frac{\phi B_i + \left(2-\phi^2\right)B_j}{\left(3-2\phi^2\right)\left(4-\phi^2\right)} > 0, \qquad e_i^D = \frac{\phi^2 B_i}{4\left(2-\phi^2\right)} > 0 \tag{42}$$

The country sets a positive import tariff and a positive export tax when it unilaterally deviates from free trade, whereas the optimum policy under Cournot duopoly was a positive import tariff and a negative export tax (an export subsidy). Substituting these trade policies, $\boldsymbol{\tau}_i^D = \left(t_i^D, e_i^D, 0, 0\right)$, into welfare (38) yields the welfare of the *i*th country when it unilaterally deviates from free trade:

$$W_{Bi}^{D} = \frac{1}{\beta \Theta^{D}} \mathbf{B}' \begin{bmatrix} \theta_{ii}^{D} & \theta_{ij}^{D} \\ \theta_{ij}^{D} & \theta_{jj}^{D} \end{bmatrix} \mathbf{B} \qquad i, j = 1, 2, j \neq i \tag{43}$$

where $\Theta^{D} = 8\left(2-\phi^{2}\right)\left(3-2\phi^{2}\right)\left(4-\phi^{2}\right)\Phi_{B}^{2}$, $\theta_{ii}^{D} = 480 - 1056\phi^{2} + 844\phi^{4} - 299\phi^{6} + 45\phi^{8} - 2\phi^{10}$, $\theta_{ij}^{D} = 4\left(2-\phi^{2}\right)\left(14-14\phi^{2}+3\phi^{4}\right)$, and $\theta_{jj}^{D} = 4\left(32-54\phi^{2}+33\phi^{4}-9\phi^{6}+\phi^{8}\right)$ are all positive. Obviously, the *i*th country will gain from unilaterally deviating from multilateral free trade while the other country pursues a policy of free trade, $\Delta W_{Bi}^{DF} = W_{Bi}^{D} - W_{Bi}^{F} > 0$ from (B1) in the appendix, and the other country will lose. Consequently, the other country is likely to retaliate with the result that there will be a trade war, which can be represented in this model by the NE in trade policies. At the interior NE in trade policies, each country sets its trade policy to maximise its welfare (38) given the trade policy of the other country. Assuming an interior solution where intra-industry trade occurs, the interior NE trade policies are:

$$\begin{aligned} t_{i}^{N} &= \frac{4\phi\left(2-\phi^{2}\right)B_{i} + \left(16-20\phi^{2}+9\phi^{4}-\phi^{6}\right)B_{j}}{\left(2-\phi^{2}\right)H} > 0 \\ e_{i}^{N} &= \phi^{2}\frac{\left(8-7\phi^{2}+\phi^{4}\right)B_{i} - \phi\left(2-\phi^{2}\right)B_{j}}{\left(2-\phi^{2}\right)H} > 0 \end{aligned} \tag{44}$$

where $H = \left(4-\phi^{2}\right)\left(12-9\phi^{2}+\phi^{4}\right) > 0$. The interior NE import tariff is clearly positive since $16-20\phi^{2}+9\phi^{4}-\phi^{6} > 0$, and the NE export tax is positive if the exports of the country are positive. This can be checked by substituting the NE trade policies into (34), which yields the exports of the *i*th country:

$$x_{ij}^{N} = \frac{\left(8-7\phi^{2}+\phi^{4}\right)B_{i} - \phi\left(2-\phi^{2}\right)B_{j}}{\beta\left(1-\phi^{2}\right)\left(2-\phi^{2}\right)H} > 0 \qquad i, j = 1, 2\ j \neq i \tag{45}$$

The numerator is the same as the numerator of the interior NE export tax so if there is intra-industry trade in the NE in trade policies then both NE export taxes will be positive. From

(45), the exports of the ith country will be positive if $\mu_{Bi} > \mu_B^X = \phi\left(2-\phi^2\right)/\left(2+\phi\right)\left(4-\phi-3\phi^2+\phi^3\right)$, where $\mu_B^X \in \left[0, 1/3\right]$. The welfare of the countries in the interior NE in trade policies are obtained by substituting the NE trade policies (44) into (38), which yields:

$$W_{Bi}^N = \frac{1}{\beta\Theta^N}\mathbf{B}'\begin{bmatrix}\theta_{ii}^N & \theta_{ij}^N \\ \theta_{ij}^N & \theta_{jj}^N\end{bmatrix}\mathbf{B} \qquad i,j=1,2,\, j \neq i \tag{46}$$

$$\begin{aligned}
\Theta^N &= 2\left(1-\phi^2\right)\left(2-\phi^2\right)H^2 > 0 \\
\theta_{ii}^N &= 1120-2984\phi^2+3080\phi^4-1542\phi^6+385\phi^8-45\phi^{10}+2\phi^{12} > 0 \\
\theta_{ij}^N &= 2\phi\left(2-\phi^2\right)\left(68-95\phi^2+39\phi^4-4\phi^6\right) > 0 \\
\theta_{jj}^N &= \left(12-9\phi^2+\phi^4\right)\left(32-58\phi^2+45\phi^4-17\phi^6+2\phi^8\right) > 0
\end{aligned}$$

The welfare effect of a trade war on a country is the difference between its welfare in the interior NE in trade policies and its welfare under free trade, $\Delta W_{Bi}^{NF} = W_{Bi}^N - W_{Bi}^F$, which is obtained by subtracting (39) from (46):

$$\Delta W_{Bi}^{NF} = \frac{1}{\beta\Theta^N\Theta^F}\mathbf{B}'\begin{bmatrix}\theta_{ii}^{NF} & \theta_{ij}^{NF} \\ \theta_{ij}^{NF} & \theta_{jj}^{NF}\end{bmatrix}\mathbf{B} \tag{47}$$

where $\theta_{ii}^{NF} = \Theta^F\theta_{ii}^N - \Theta^N\theta_{ii}^F < 0$, $\theta_{ij}^{NF} = \Theta^F\theta_{ij}^N - \Theta^N\theta_{ij}^F$, $\theta_{jj}^{NF} = \Theta^F\theta_{jj}^N - \Theta^N\theta_{jj}^F$, and the quadratic form is indefinite, but it can be shown that $\theta_{ii}^{NF} + 2\theta_{ij}^{NF} + \theta_{jj}^{NF} < 0$.

In the symmetric case, when $B_i = B_j = B$, as expected and as in the case of Cournot duopoly, both countries are worse off in the interior NE in trade policies than under free trade:

$$\Delta W_B^{NF} = \frac{\left(\theta_{ii}^{NF} + 2\theta_{ij}^{NF} + \theta_{jj}^{NF}\right)B^2}{\beta\Theta^N\Theta^F} < 0 \tag{48}$$

In the asymmetric case, since the market share under free trade of the *i*th firm is $\mu_{Bi} = B_i / (B_i + B_j)$, the competitiveness of the other firm can be written as $B_j = (1 - \mu_{Bi}) B_i / \mu_{Bi}$ and substituting this into the quadratic form yields:

$$\Delta W_{Bi}^{NF} = \frac{B_i^2}{\beta \Theta^N \Theta^F \mu_{Bi}^2} \left[\theta_{ii}^{NF} \mu_{Bi}^2 + 2\theta_{ij}^{NF} (1 - \mu_{Bi}) \mu_{Bi} + \theta_{jj}^{NF} (1 - \mu_{Bi})^2 \right] \tag{49}$$

When the exports of the *i*th firm are zero, $\mu_{Bi} = \mu_B^X$, it can be shown that $\Delta W_{Bi}^{NF} > (<) 0$ if $\phi < (>) \phi_B^{NF} \approx 0 \cdot 774$, and when the exports of the *j*th firm are zero, $\mu_{Bi} = 1 - \mu_B^X$, it can be shown that $\Delta W_{Bi}^{NF} < 0$. Therefore, if $\phi < \phi_B^{NF}$ then the *i*th country can win a trade war, and solving the quadratic in square brackets in (49), the *i*th country will win a trade war if its market share under free trade is less than the critical value:

$$\mu_{Bi} < \mu_B^{NF} \equiv \frac{\theta_{jj}^{NF}}{\theta_{jj}^{NF} - \theta_{ij}^{NF} + \sqrt{\left(\theta_{ij}^{NF}\right)^2 - \theta_{ii}^{NF} \theta_{jj}^{NF}}} \tag{50}$$

The home country will win the trade war if $\mu_{B1} < \mu_B^{NF}$ and, since $\mu_{B1} + \mu_{B2} = 1$, the foreign country will win the trade war if $\mu_{B1} > 1 - \mu_B^{NF}$. Plotting μ_B^{NF} and $1 - \mu_B^{NF}$ against ϕ in figure 9 together with μ_B^X and $1 - \mu_B^X$, which show where exports of the home firm and the foreign firm are zero. In the region between μ_B^{NF} and $1 - \mu_B^{NF}$, both countries lose the trade war, while in the region below μ_B^{NF}, the home country wins and the foreign country loses the trade war, and in the region above $1 - \mu_B^{NF}$, the foreign country wins and the home country loses the trade war. When the products are close substitutes, $\phi > \phi_B^{NF}$, it is always the case that both countries will lose the trade war.

The change in world welfare as a result of the trade war is the total welfare of the two countries in the interior NE in trade policies minus the total welfare of the two countries under multilateral free trade, $\Delta\Omega_B^{NF} = \Delta W_{B1}^{NF} + \Delta W_{B2}^{NF}$, and evaluating using (47) yields the quadratic form:

$$\Delta\Omega_B^{NF} = \frac{1}{\beta\Theta^N\Theta^F}\mathbf{B}'\begin{bmatrix}\theta_{ii}^{NF}+\theta_{jj}^{NF} & 2\theta_{ij}^{NF} \\ 2\theta_{ij}^{NF} & \theta_{ii}^{NF}+\theta_{jj}^{NF}\end{bmatrix}\mathbf{B} \tag{51}$$

where it can be shown that $\theta_{ii}^{NF}+\theta_{jj}^{NF}<0$, and that the determinant of the matrix is positive, $\det = \left(\theta_{ii}^{NF}+\theta_{jj}^{NF}\right)^2 - 4\left(\theta_{ij}^{NF}\right)^2 > 0$. Hence, the quadratic form is negative definite so there is a reduction in world welfare as a result of the trade war as one would expect and as happens under Cournot duopoly. These results lead to the following proposition:

Proposition 8: *Under Bertrand duopoly, if $\mu_{Bi} < \mu_B^{NF}$ then the ith country wins the trade war (and the other country loses), and if $\mu_B^N < \mu_{B1} < 1-\mu_B^N$ then both countries lose. World welfare is always lower in a trade war than under multilateral free trade.*

As was the case under Cournot duopoly, it is possible that the country with the uncompetitive firm may win the trade war. To compare the outcome of a trade war under Bertrand duopoly with that under Cournot duopoly, figures 2 and 9 can be combined to produce figure 10. Here, it can be seen that both countries lose a trade war under Cournot duopoly in region *I*, and under Bertrand duopoly in regions *I*, *II* and *III*. The home country wins the trade war under Cournot duopoly in regions *II* and *IV*, and under Bertrand duopoly in region *IV*. The foreign country wins the trade war under Cournot duopoly in regions *III* and *V*, and under Bertrand duopoly in region *V*. Therefore, it is less likely that either country will win a trade war and more likely that both countries will lose a trade war under Bertrand duopoly than under Cournot duopoly.[24]

[24] One has to be careful when interpreting this comparison as the same exogenous parameters (demand parameters and marginal costs) will result in different market shares under Cournot duopoly than under Bertrand duopoly.

As in the case of Cournot duopoly, the interior NE is not unique as both countries minimaxing each other is also a NE in trade policies where the outcome is autarky. If a country sets a prohibitive import tariff, $t_i = \overline{t}_i$, and a prohibitive export tax, $e_i = \overline{t}_j$, then the other country setting a prohibitive import tariff and a prohibitive export tax is a best-reply. Since Proposition 7 showed that there were always gains from multilateral free trade, both countries will lose a trade war where the outcome is the minimax NE in trade policies. The difference in welfare between the interior NE in trade policies and the minimax NE in trade policies is $\Delta W_{Bi}^{N0} = W_{Bi}^{N} - W_{Bi}^{0}$, which is obtained by subtracting (40) from (46):

$$\Delta W_{Bi}^{N0} = \frac{1}{\beta \Theta^N \Theta^0} \mathbf{B}' \begin{bmatrix} \theta_{ii}^{N0} & \theta_{ij}^{N0} \\ \theta_{ij}^{N0} & \theta_{jj}^{N0} \end{bmatrix} \mathbf{B} \tag{52}$$

where $\theta_{ii}^{N0} = \Theta^0 \theta_{ii}^N - \Theta^N \theta_{ii}^0 > 0$, $\theta_{ij}^{N0} = \Theta^0 \theta_{ij}^N - \Theta^N \theta_{ij}^0$, $\theta_{jj}^{N0} = \Theta^0 \theta_{jj}^N - \Theta^N \theta_{jj}^0 > 0$, and the determinant of the matrix is positive, $\theta_{ii}^{N0}\theta_{jj}^{N0} - \left(\theta_{ij}^{N0}\right)^2 > 0$. Hence, the quadratic form is positive definite, which leads to the following proposition:

Proposition 9: *Under Bertrand duopoly, welfare of both countries is higher in the interior NE in trade polices than in the minimax (autarky) NE in trade policies.*

As in the case of Cournot duopoly, the interior NE in trade policies is Pareto superior to the minimax NE in trade policies. Therefore, reversion to the minimax NE in trade policies will be a more severe punishment than reversion to the interior NE in trade policies when looking at the sustainability of free trade.

7. Sustaining Free Trade under Bertrand Oligopoly

Free trade can be sustained as a subgame perfect NE in an infinitely-repeated game using infinite Nash reversion trigger strategies if the discount factor is greater than the critical value for both countries. Analogously to (27) the critical discount factor for sustaining free trade using infinite Nash reversion is $\delta_{Bi}^{N\infty} = \left(W_{Bi}^{D} - W_{Bi}^{F}\right) / \left(W_{Bi}^{D} - W_{Bi}^{N}\right) = \Delta W_{Bi}^{DF} / \Delta W_{Bi}^{DN}$, then substituting in welfare differences from (B1), (B2), and $B_j = \left(1 - \mu_{Bi}\right) B_i / \mu_{Bi}$ yields:

$$\delta_{Bi}^{N\infty}\left(\mu_{Bi}, \phi\right) = \left(\frac{\Theta^{N}}{\Theta^{F}}\right) \frac{\theta_{ii}^{DF}\mu_{Bi}^{2} + 2\theta_{ij}^{DF}\left(1-\mu_{Bi}\right)\mu_{Bi} + \theta_{jj}^{DF}\left(1-\mu_{Bi}\right)^{2}}{\theta_{ii}^{DN}\mu_{Bi}^{2} + 2\theta_{ij}^{DN}\left(1-\mu_{Bi}\right)\mu_{Bi} + \theta_{jj}^{DN}\left(1-\mu_{Bi}\right)^{2}} \tag{53}$$

Since all the *Theta* only depend upon the degree of product substitutability, the critical discount factor is a function of only two variables: the market share of the firm, μ_{Bi}, and the degree of product substitutability, ϕ. The critical discount factors of the two countries are plotted against μ_{A1} in figure 11 for different values of $\phi = \{1/10, 9/10\}$ for the parameter values where there is an interior solution. Since free trade is sustainable if $\delta > Max\left\{\delta_{B1}^{N\infty}, \delta_{B2}^{N\infty}\right\}$, it will be sustainable in the region labelled *I*. Clearly, as in the case of Cournot duopoly, it is easier to sustain free trade when firms are symmetric, $\mu_{B1} = 1/2$, and cost asymmetries make it harder to sustain free trade.

Since autarky is also a NE of the constituent game, free trade can be sustained as a subgame perfect NE in an infinitely-repeated game using infinite minimax reversion trigger strategies if the discount factor is greater than the critical discount factor for both countries. Analogously to (29) the critical discount factor for sustaining free trade using infinite minimax reversion is $\delta_{Bi}^{M\infty} = \left(W_{Bi}^{D} - W_{Bi}^{F}\right) / \left(W_{Bi}^{D} - W_{Bi}^{0}\right) = \Delta W_{Bi}^{DF} / \Delta W_{Bi}^{D0}$, and substituting in welfare differences from (B1), (B3), and $B_j = \left(1 - \mu_{Bi}\right) B_i / \mu_{Bi}$ yields:

$$\delta_{Bi}^{M\infty}\left(\mu_{Bi}, \phi\right) = \left(\frac{\Theta^{0}}{\Theta^{F}}\right) \frac{\theta_{ii}^{DF}\mu_{Bi}^{2} + 2\theta_{ij}^{DF}\left(1-\mu_{Bi}\right)\mu_{Bi} + \theta_{jj}^{DF}\left(1-\mu_{Bi}\right)^{2}}{\theta_{ii}^{D0}\mu_{Bi}^{2} + 2\theta_{ij}^{D0}\left(1-\mu_{Bi}\right)\mu_{Bi} + \theta_{jj}^{D0}\left(1-\mu_{Bi}\right)^{2}} \tag{54}$$

The critical discount factors of the two countries are plotted against μ_{B1} in figure 12 for different values of $\phi = \{1/10, 9/10\}$ for the parameter values where there is an interior solution. Since free trade is sustainable if $\delta > Max\left\{\delta_{B1}^{M\infty}, \delta_{B2}^{M\infty}\right\}$, it will be sustainable in the region labelled *I*. Clearly, as in the case of Cournot duopoly, it is easier to sustain free trade when firms are symmetric, $\mu_{B1} = 1/2$, and that cost asymmetries make it harder to sustain free trade although the effect of cost asymmetries is not that strong when products are close substitutes, $\phi = 9/10$.

As in the case of Cournot duopoly, it is clear that $\delta_{Bi}^{N\infty} > \delta_{Bi}^{M\infty}$ if $W_{Bi}^{N} > W_{Bi}^{0}$, but this was proved in Proposition 9, and therefore it follows that $Max\left\{\delta_{B1}^{N\infty}, \delta_{B2}^{N\infty}\right\} > Max\left\{\delta_{B1}^{M\infty}, \delta_{B2}^{M\infty}\right\}$. Hence, it is always easier to sustain free trade with the threat of reversion to the minimax NE in trade policies than to the interior NE in trade policies. This leads to the following proposition:

Proposition 10: *Under Bertrand duopoly, it is easier to sustain free trade in an infinitely-repeated game using infinite minimax reversion than using infinite Nash reversion.*

To compare the critical discount factors $\delta_{Bi}^{N\infty}$ and $\delta_{Bi}^{M\infty}$ figure 13 shows these two discount factors for both countries plotted against μ_{B1} when $\phi = 1/2$. Free trade is sustainable in the region labelled *I* using the threat of reversion to the interior NE whereas it is sustainable in the regions labelled *I*, *II*, *III* and *IV* using the threat of reversion to the minimax NE in trade policies. Cost asymmetries have less effect on the sustainability of free trade using the threat of reversion to the minimax NE than using the threat of reversion to the interior NE in trade policies. Comparing figure 13 for Bertrand duopoly with figure 5 for Cournot duopoly shows that the results are qualitatively similar.

For the rest of this section, it will be assumed that the firms are symmetric so $\mu_{B1} = \mu_{B2} = 1/2$, and substituting this into (53) and (54) yields the critical discount factors for infinite Nash reversion and infinite minimax reversion, which are the same for both countries:

$$\delta_B^{N\infty}(\phi)=\left(\frac{\Theta^N}{\Theta^F}\right)\frac{\theta_{ii}^{DF}+2\theta_{ij}^{DF}+\theta_{jj}^{DF}}{\theta_{ii}^{DN}+2\theta_{ij}^{DN}+\theta_{jj}^{DN}} \qquad \delta_B^{M\infty}(\phi)=\left(\frac{\Theta^0}{\Theta^F}\right)\frac{\theta_{ii}^{DF}+2\theta_{ij}^{DF}+\theta_{jj}^{DF}}{\theta_{ii}^{D0}+2\theta_{ij}^{D0}+\theta_{jj}^{D0}} \tag{55}$$

The two critical discount factors are plotted in figure 14 against ϕ where it can be seen that $\delta_B^{N\infty}$ is increasing in ϕ, whereas $\delta_B^{M\infty}$ is increasing in ϕ up to $\phi_B^M \approx 0\cdot 738$ then decreasing in ϕ and it goes to zero as $\phi \to 1$. This leads to the following proposition:

Proposition 11: *Under Bertrand duopoly, the critical discount factors* $\delta_B^{N\infty}$ *is increasing in* ϕ *while* $\delta_B^{M\infty}$ *is increasing (decreasing) in* ϕ *if* $\phi<(>)\phi_B^M \approx 0\cdot 738$.

As it may be considered implausible that countries would stay in the minimax NE forever after any deviation from free trade, it is worthwhile considering again the case when the length of the punishment phase is limited to a number of rounds. Free trade can be sustained as a subgame perfect NE in the infinitely-repeated game using the threat of reversion to the minimax NE for one round followed by a return to free trade. The critical discount factor required to sustain free trade is $\delta_B^{M1}=\left(W_B^D-W_B^F\right)/\left(W_B^F-W_B^0\right)=\Delta W_B^{DF}/\Delta W_B^{F0}$, and then substituting in the welfare differences from (B1) and (41) yields:

$$\delta_B^{M1}(\phi)=\left(\frac{\Theta^0}{\Theta^D}\right)\frac{\theta_{ii}^{DF}+2\theta_{ij}^{DF}+\theta_{jj}^{DF}}{\theta_{ii}^{F0}+2\theta_{ij}^{F0}+\theta_{jj}^{F0}} \tag{56}$$

This critical discount factor is plotted against ϕ in figure 15 together with $\delta_B^{N\infty}$ and $\delta_B^{M\infty}$. It can be seen δ_B^{M1} is increasing in ϕ up to ϕ_B^M then decreasing in ϕ and goes to zero as $\phi \to 1$. Also, δ_B^{M1} is very close to $\delta_B^{M\infty}$ and significantly lower than $\delta_B^{N\infty}$. If the length of the punishment phase was increased from one round to two or three rounds, then the critical discount factor would get even closer to $\delta_B^{M\infty}$. These results lead to the following proposition:

Proposition 12: *Under Bertrand duopoly, with minimax reversion for z rounds the critical discount factors* δ_A^{Mz} *increasing (decreasing) in* ϕ *if* $\phi < (>) \phi_B^M \approx 0 \cdot 738$*, and* $\delta_B^{M1} < \delta_B^{N\infty}$.

In a finitely-repeated game with T rounds, as in Benoit and Krishna (1985), co-operation can be sustained since there are two NE of the constituent game. The countries co-operate by playing free trade for first $T-t$ rounds then play the interior NE trade policies for the final t rounds, and if a country deviates then all countries play the minimax (autarky) NE trade policies. The critical discount factor required to sustain free trade for $T-1$ rounds of the finitely-repeated game is $\delta_B^{T-1} = \left(W_B^D - W_B^F\right) / \left(W_B^N - W_B^0\right) = \Delta W_B^{DF} / \Delta W_B^{N0}$, and then substituting in the welfare differences from (B1) and (52) yields:

$$\delta_B^{T-1}(\phi) = \left(\frac{\Theta^D \Theta^F}{\Theta^N \Theta^0}\right) \frac{\theta_{ii}^{DF} + 2\theta_{ij}^{DF} + \theta_{jj}^{DF}}{\theta_{ii}^{N0} + 2\theta_{ij}^{N0} + \theta_{jj}^{N0}} \tag{57}$$

Similarly, free trade is sustainable for $T-2$ rounds if the discount factor is greater than the critical discount factor δ_B^{T-2}, which is given by the solution of $\delta(1+\delta)\left(W_B^N - W_B^0\right) = W_B^D - W_B^F$. Figure 16 shows the critical discount factors δ_B^{T-1} and δ_B^{T-2} plotted against ϕ together with $\delta_B^{N\infty}$ and $\delta_B^{M\infty}$ for comparison. Both critical discount factors δ_B^{T-1} and δ_B^{T-2} are increasing in ϕ up to $\phi_B^T \approx 0 \cdot 662$ then decreasing in ϕ and both go to zero as $\phi \to 1$. Also, they are both lower than $\delta_B^{N\infty}$ and get fairly close to $\delta_B^{M\infty}$.

Finally, the sustainability of free trade under Cournot oligopoly can be compared with the sustainability of free trade under Bertrand oligopoly. The critical discount factors for infinite Nash reversion $\delta_A^{N\infty}$ and $\delta_B^{N\infty}$ are plotted against ϕ in Figure 17, where it can be seen that $\delta_B^{N\infty} < \delta_A^{N\infty}$ for $\phi < \phi_{AB}^N \approx 0 \cdot 905$ and $\delta_A^{N\infty} < \delta_B^{N\infty}$ for $\phi > \phi_{AB}^N$. Hence, using the threat of infinite Nash reversion, it is easier to sustain free trade under Bertrand duopoly than under

Cournot duopoly except when products are very close substitutes. The critical discount factors for infinite minimax reversion $\delta_A^{M\infty}$ and $\delta_B^{M\infty}$ are plotted against ϕ in Figure 18, where it can be seen that $\delta_B^{M\infty} < \delta_A^{M\infty}$ for $\phi \in (0,1)$. Free trade is sustainable in region *A* under Cournot duopoly and in regions *A* and *B* under Bertrand duopoly. Hence, using the threat of infinite minimax reversion, it is always easier to sustain free trade under Bertrand duopoly than under Cournot duopoly. The critical discount factors for minimax reversion for one round δ_A^{M1} and δ_B^{M1} are plotted against ϕ in Figure 19, where it can be seen that $\delta_B^{M1} < \delta_A^{M1}$ for $\phi \in (0,1)$. Hence, using the threat of minimax reversion for one round, it is always easier to sustain free trade under Bertrand duopoly than under Cournot duopoly. In fact, it is straightforward to extend the result to minimax reversion for *z* rounds and show that $\delta_B^{Mz} < \delta_A^{Mz}$. The critical discount factors for the finitely-repeated game δ_A^{T-1} and δ_B^{T-1} are plotted against ϕ in Figure 20, where it can be seen that $\delta_B^{T-1} < \delta_A^{T-1}$ for $\phi \in (0,1)$. Hence, in the finitely-repeated game using the threat of minimax reversion, it is always easier to sustain free trade for $T-1$ rounds under Bertrand duopoly than under Cournot duopoly. These results lead to the final proposition:

Proposition 13: *When using the threat of minimax reversion to sustain free trade, the critical discount factors are lower under Bertrand duopoly than under Cournot duopoly,* $\delta_B^{M\infty} < \delta_A^{M\infty}$, $\delta_B^{M1} < \delta_A^{M1}$, $\delta_B^{Mz} < \delta_A^{Mz}$, $\delta_B^{T-1} < \delta_A^{T-1}$, *and* $\delta_B^{T-t} < \delta_B^{T-t}$ *for* $\phi \in (0,1)$.

When using the threat of minimax reversion to sustain free trade, the incentive to deviate from free trade is lower under Bertrand duopoly than under Cournot duopoly as the price cost margin is lower, and the punishment for deviating from free trade is larger under Bertrand duopoly than under Cournot duopoly as the gains from trade are larger. Therefore, it is easier to sustain free trade under Bertrand duopoly than under Cournot duopoly when using the threat of reversion to the minimax NE.

8. Conclusions

This paper has analysed trade wars and trade agreements between two countries under Cournot and Bertrand duopoly with differentiated products and cost asymmetries, where each country used an import tariff and an export tax/subsidy. In a trade war, modelled as the interior NE in trade policies, the outcome in the symmetric case was that both countries lose, but the country with the uncompetitive firm may win the trade war with cost asymmetries. Then, since free trade is clearly a focal point, the sustainability of free trade (zero import tariffs, zero export taxes/subsidies, and zero transfers) was considered in an infinitely-repeated game. Using infinite Nash reversion, free trade was sustainable provided the cost asymmetries were not too great. It was shown that free trade was also sustainable using infinite minimax reversion (that results in autarky welfare for both countries), and in this case asymmetries were much less problematic as the critical discount factor was always significantly less than one. However, both infinite Nash reversion and infinite minimax reversion seemed implausible as the punishment phase lasts forever following any deviation from free trade. An alternative was for the punishment phase with minimax reversion to last for only a few rounds, and then it turned out that the critical discount factor gets quite close to that with infinite minimax reversion and is lower than that with infinite Nash reversion. All the results under Bertrand duopoly were qualitatively similar to those under Bertrand duopoly despite the NE trade policies (and those when a country deviates from trade) including an export tax under Bertrand duopoly and an export subsidy under Cournot duopoly. However, it was shown that it is always easier to sustain free trade under Bertrand duopoly than under Cournot duopoly when using minimax reversion. Also, product differentiation makes it easier to sustain free trade under Cournot duopoly, and under Bertrand duopoly except when the products are close substitutes. Possible extensions to the analysis would be to consider the cases of many countries, many firms, differences in market size, and trade blocs.

Appendix A: Welfare Comparisons under Cournot Duopoly

Some additional welfare comparisons are required for the derivation of the critical discount factors required to sustain free trade under Cournot duopoly. Firstly, the welfare gain from deviation compared to free trade, which is obtained by subtracting (7) from (16):

$$\Delta W_{Ai}^{DF} = W_{Ai}^{D} - W_{Ai}^{F} = \frac{1}{\beta \Psi^{D} \Psi^{F}} \mathbf{A}' \begin{bmatrix} \psi_{ii}^{DF} & \psi_{ij}^{DF} \\ \psi_{ij}^{DF} & \psi_{jj}^{DF} \end{bmatrix} \mathbf{A} \tag{A1}$$

$$\begin{aligned}
\psi_{ii}^{DF} &= \psi_{ii}^{D}\Psi^{F} - \Psi^{D}\psi_{ii}^{F} = 2\phi^2\left(8+8\phi^2-3\phi^4\right)\Phi_A^2 > 0 \\
\psi_{ij}^{DF} &= \psi_{ij}^{D}\Psi^{F} - \Psi^{D}\psi_{ij}^{F} = 16\phi\left(2-\phi^2\right)\Phi_A^2 > 0 \\
\psi_{jj}^{DF} &= \psi_{jj}^{D}\Psi^{F} - \Psi^{D}\psi_{jj}^{F} = 32\left(2-\phi^2\right)\Phi_A^2 > 0
\end{aligned}$$

The quadratic form is positive for $\mathbf{A} > \mathbf{0}$, since all the elements of the matrix are positive. Secondly, the welfare gain from deviation compared to the *interior* NE in trade policies, which is obtained by subtracting (19) from (16):

$$\Delta W_{Ai}^{DN} = W_{Ai}^{D} - W_{Ai}^{N} = \frac{1}{\beta \Psi^{D} \Psi^{N}} \mathbf{A}' \begin{bmatrix} \psi_{ii}^{DN} & \psi_{ij}^{DN} \\ \psi_{ij}^{DN} & \psi_{jj}^{DN} \end{bmatrix} \mathbf{A} \tag{A2}$$

$$\begin{aligned}
\psi_{ii}^{DN} &= \psi_{ii}^{D}\Psi^{N} - \Psi^{D}\psi_{ii}^{N} = 2\left(15360-19200\phi^2+7488\phi^4-496\phi^6-244\phi^8+33\phi^{10}\right)\Phi_A^2 > 0 \\
\psi_{ij}^{DN} &= \psi_{ij}^{D}\Psi^{N} - \Psi^{D}\psi_{ij}^{N} = 16\phi\left(2-\phi^2\right)\left(8-3\phi^2\right)\left(24-6\phi^2-\phi^4\right)\Phi_A^2 > 0 \\
\psi_{jj}^{DN} &= \psi_{jj}^{D}\Psi^{N} - \Psi^{D}\psi_{jj}^{N} = -32\phi^2\left(2-\phi^2\right)\left(216-226\phi^2+78\phi^4-9\phi^6\right)\Phi_A^2 < 0
\end{aligned}$$

Since one principal diagonal element of the matrix is positive and the other is negative, the quadratic form is indeterminate. It can be shown that ΔW_{Ai}^{DN} is positive (negative) if $\mu_{Ai} > (<) \mu_A^{DN}$ where $\mu_A^{DN} > \mu_A^{X}$ for $\phi < \phi_A^{DN} \approx 0 \cdot 973$ and $\mu_A^{DN} < \mu_A^{NF}$, and therefore, $\Delta W_{Ai}^{DN} < 0$ if $W_{Ai}^{N} > W_{Ai}^{D} > W_{Ai}^{F}$, in which case free trade would never be sustainable using infinite Nash reversion. Thirdly, the welfare gain from deviation compared to the minimax (autarky) NE in trade policies, which is obtained by subtracting (9) from (16):

$$\Delta W_{Ai}^{D0} = W_{Ai}^{D} - W_{Ai}^{0} = \frac{1}{\beta \Psi^{D} \Psi^{0}} \mathbf{A}' \begin{bmatrix} \psi_{ii}^{D0} & \psi_{ij}^{D0} \\ \psi_{ij}^{D0} & \psi_{jj}^{D0} \end{bmatrix} \mathbf{A} \tag{A3}$$

$$\begin{aligned}
\psi_{ii}^{D0} &= \psi_{ii}^{D}\Psi^{0} - \Psi^{D}\psi_{ii}^{0} = 8\left(8-3\phi^2\right)\left(24-8\phi^2+\phi^4\right)\Phi_A^2 > 0 \\
\psi_{ij}^{D0} &= \psi_{ij}^{D}\Psi^{0} - \Psi^{D}\psi_{ij}^{0} = -16\phi\left(2-\phi^2\right)\left(8-3\phi^2\right)\Phi_A^2 < 0 \\
\psi_{jj}^{D0} &= \psi_{ij}^{D}\Psi^{0} - \Psi^{D}\psi_{ij}^{0} = 8\left(2-\phi^2\right)\left(8-3\phi^2\right)^2\Phi_A^2 > 0
\end{aligned}$$

The two principal diagonal elements of the matrix are positive, and the determinant of the matrix is positive, $\det = 192\left(2-\phi^2\right)\left(8-3\phi^2\right)^2 \Phi_A^7 > 0$, so the quadratic form is positive definite.

Note that the critical discount factors with infinite minimax reversion and minimax reversion for *z* rounds can be defined as:

$$\frac{\delta_A^{M\infty}}{1-\delta_A^{M\infty}} \equiv \frac{\Delta W_A^{DF}}{\Delta W_A^{F0}} \qquad \delta_A^{Mz} + \left(\delta_A^{Mz}\right)^2 \ldots + \left(\delta_A^{Mz}\right)^z \equiv \frac{\Delta W_A^{DF}}{\Delta W_A^{F0}} \tag{A4}$$

Since the right-hand side of both identities is equal and the left-hand side of the second equation is equal to less than z when $\delta_A^{Mz} < 1$, therefore $\delta_A^{Mz} < 1$ if $z > \delta_A^{M\infty} \big/ \left(1-\delta_A^{M\infty}\right)$.

Appendix B: Welfare Comparisons under Bertrand Duopoly

Some additional welfare comparisons are required for the derivation of the critical discount factors required to sustain free trade under Bertrand duopoly. Firstly, the welfare gain from deviation compared to free trade, which is obtained by subtracting (39) from (43):

$$\Delta W_{Bi}^{DF} = W_{Bi}^{D} - W_{Bi}^{F} = \frac{1}{\beta \Theta^D \Theta^F} \mathbf{B}' \begin{bmatrix} \theta_{ii}^{DF} & \theta_{ij}^{DF} \\ \theta_{ij}^{DF} & \theta_{jj}^{DF} \end{bmatrix} \mathbf{B} \tag{B1}$$

$$\begin{aligned}
\theta_{ii}^{DF} &= \theta_{ii}^{D}\Theta^F - \Theta^D \theta_{ii}^{F} = 2\phi^2\left(1-\phi^2\right)\left(8+8\phi^2-11\phi^4+2\phi^6\right)\Phi_B^2 > 0 \\
\theta_{ij}^{DF} &= \theta_{ij}^{D}\Theta^F - \Theta^D \theta_{ij}^{F} = 8\phi\left(1-\phi^2\right)\left(2-\phi^2\right)^2 \Phi_B^2 > 0 \\
\theta_{jj}^{DF} &= \theta_{jj}^{D}\Theta^F - \Theta^D \theta_{jj}^{F} = 8\left(1-\phi^2\right)\left(2-\phi^2\right)^3 \Phi_B^2 > 0
\end{aligned}$$

The quadratic form is positive for $\mathbf{B} > \mathbf{0}$, since all the elements of the matrix are positive. Secondly, the welfare gain from deviation compared to the *interior* NE in trade policies, which is obtained by subtracting (46) from (43):

$$\Delta W_{Bi}^{DN} = W_{Bi}^{D} - W_{Bi}^{N} = \frac{1}{\beta \Theta^D \Theta^N} \mathbf{B}' \begin{bmatrix} \theta_{ii}^{DN} & \theta_{ij}^{DN} \\ \theta_{ij}^{DN} & \theta_{jj}^{DN} \end{bmatrix} \mathbf{B} \tag{B2}$$

$$\theta_{ii}^{DN} = \theta_{ii}^{D}\Theta^N - \Theta^D \theta_{ii}^{N} > 0 \qquad \theta_{ij}^{DN} = \theta_{ii}^{D}\Theta^N - \Theta^D \theta_{ii}^{N} > 0 \qquad \theta_{jj}^{DN} = \theta_{jj}^{D}\Theta^N - \Theta^D \theta_{jj}^{N} > 0$$

The quadratic form is positive for $\mathbf{B} > \mathbf{0}$, since all the elements of the matrix are positive. Thirdly, the welfare gain from deviation compared to the *minimax* NE in trade policies, which is obtained by subtracting (40) from (43):

$$\Delta W_{Bi}^{D0} = W_{Bi}^{D} - W_{Bi}^{0} = \frac{1}{\beta \Theta^D \Theta^0} \mathbf{B}' \begin{bmatrix} \theta_{ii}^{D0} & \theta_{ij}^{D0} \\ \theta_{ij}^{D0} & \theta_{jj}^{D0} \end{bmatrix} \mathbf{B} \tag{B3}$$

$$\begin{aligned}
\theta_{ii}^{D0} &= \theta_{ii}^{D}\Theta^0 - \Theta^D \theta_{ii}^{0} = 8\left(192-360\phi^2+184\phi^4+7\phi^6-24\phi^8+4\phi^{10}\right)\Phi_B^2 > 0 \\
\theta_{ij}^{D0} &= \theta_{ij}^{D}\Theta^0 - \Theta^D \theta_{ij}^{0} = -8\phi\left(2-\phi^2\right)\left(16-46\phi^2+33\phi^4-6\phi^6\right)\Phi_B^2 \\
\theta_{jj}^{D0} &= \theta_{jj}^{D}\Theta^0 - \Theta^D \theta_{jj}^{0} = 8\left(2-\phi^2\right)\left(8-5\phi^2\right)\left(8-9\phi^2+2\phi^4\right)\Phi_B^2 > 0
\end{aligned}$$

The two principal diagonal elements of the matrix are positive and the determinant of the matrix of the matrix is positive, $\det = 64\left(4-\phi^2\right)^2\left(2-\phi^2\right)\left(3-2\phi^2\right)\left(64-80\phi^2+21\phi^4+2\phi^6\right)\Phi_B^2 > 0$, so the quadratic form is positive definite.

Appendix C: Discount Factors under Bertrand Duopoly

The critical discount factor with infinite minimax reversion and with minimax reversion for one round can be defined as:

$$\frac{\delta_B^{M\infty}}{1-\delta_B^{M\infty}} \equiv \frac{\Delta W_B^{DF}}{\Delta W_B^{F0}} \qquad \delta_B^{M1} \equiv \frac{\Delta W_B^{DF}}{\Delta W_B^{F0}} \tag{C1}$$

Differentiating these identities with respect to the degree of product substitutability yields:

$$\frac{1}{\left(1-\delta_B^{M\infty}\right)^2}\frac{\delta_B^{M\infty}}{\partial\phi} = \frac{\partial}{\partial\phi}\left(\frac{\Delta W_B^{DF}}{\Delta W_B^{F0}}\right) \qquad \frac{\partial\delta_B^{M1}}{\partial\phi} = \frac{\partial}{\partial\phi}\left(\frac{\Delta W_B^{DF}}{\Delta W_B^{F0}}\right) \tag{C2}$$

Hence, the signs of both derivatives will be the same and both will have a turning point at $\phi_B^M \approx 0\cdot 738$, and the same argument applies to δ_B^{Mz}. A similar argument can be used to show that δ_B^{T-1}, δ_B^{T-2}, and δ_B^{T-t} all have a turning point at $\phi_B^T \approx 0\cdot 662$. Also, using the same argument as in (A4), the critical discount factor with minimax revision for *z* rounds $\delta_B^{Mz} < 1$ if $z > \delta_B^{M\infty} \Big/ \left(1-\delta_B^{M\infty}\right)$.

References

Bagwell, Kyle, and Robert W. Staiger. 1999. "An Economic Theory of GATT." *The American Economic Review* 89 (1):215-248. doi: https://doi.org/10.1257/aer.89.1.215.

Bagwell, Kyle, and Robert W. Staiger. 2012a. "The economics of trade agreements in the linear Cournot delocation model." *Journal of International Economics* 88 (1):32-46. doi: https://dx.doi.org/10.1016/j.jinteco.2012.01.013.

Bagwell, Kyle, and Robert W. Staiger. 2012b. "Profit Shifting and Trade Agreements in Imperfectly Competitive Markets." *International Economic Review* 53 (4):1067-1104. doi: https://doi.org/10.1111/j.1468-2354.2012.00712.x.

Bagwell, Kyle, and Robert W. Staiger. 2016. "Chapter 8 - The Design of Trade Agreements." In *Handbook of Commercial Policy*, edited by Kyle Bagwell and Robert W. Staiger, 435-529. North-Holland. doi: https://doi.org/10.1016/bs.hescop.2016.04.005.

Benoit, Jean-Pierre, and Vijay Krishna. 1985. "Finitely Repeated Games." *Econometrica* 53 (4):905-922. doi: https://doi.org/10.2307/1912660.

Bond, Eric W., and Jee-Hyeong Park. 2002. "Gradualism in Trade Agreements with Asymmetric Countries." *The Review of Economic Studies* 69 (2):379-406. doi: https://doi.org/10.1111/1467-937X.00210.

Brander, James A. 1981. "Intra-Industry Trade in Identical Commodities." *Journal of International Economics* 11 (1):1-14. doi: https://dx.doi.org/10.1016/0022-1996(81)90041-6.

Brander, James A. 1995. "Chapter 27 Strategic trade policy." In *Handbook of International Economics*, 1395-1455. Elsevier. doi: https://doi.org/10.1016/s1573-4404(05)80007-3.

Brander, James A., and Paul R. Krugman. 1983. "A Reciprocal Dumping Model of International-Trade." *Journal of International Economics* 15 (3-4):313-321. doi: https://dx.doi.org/10.1016/S0022-1996(83)80008-7.

Brander, James A., and Barbara J. Spencer. 1984. "Trade warfare: Tariffs and cartels." *Journal of International Economics* 16 (3–4):227-242. doi: https://dx.doi.org/10.1016/S0022-1996(84)80002-1.

Brander, James A., and Barbara J. Spencer. 1985. "Export subsidies and international market share rivalry." *Journal of International Economics* 18 (1–2):83-100. doi: https://dx.doi.org/10.1016/0022-1996(85)90006-6.

Cheng, Leonard K. 1988. "Assisting Domestic Industries Under International Oligopoly: The Relevance of the Nature of Competition to Optimal Policies." *The American Economic Review* 78 (4):746-758.

Clarke, Roger, and David R. Collie. 2003. "Product differentiation and the gains from trade under Bertrand duopoly." *Canadian Journal of Economics* 36 (3):658-673. doi: https://doi.org/10.1111/1540-5982.t01-2-00007.

Collie, David. 1991. "Export Subsidies and Countervailing Tariffs." *Journal of International Economics* 31 (3-4):309-324. doi: https://doi.org/10.1016/0022-1996(91)90041-4.

Collie, David R. 1993. "Profit-Shifting Export Subsidies and the Sustainability of Free Trade." *Scottish Journal of Political Economy* 40 (4):408-419. doi: https://doi.org/10.1111/j.1467-9485.1993.tb00663.x.

Collie, David R. 1996. "Gains and losses from unilateral free trade under oligopoly." *Recherches Economiques de Louvain / Louvain Economic Review* 62 (2):191-202. doi: https://doi.org/10.1017/S0770451800055706.

Collie, David R. 2000a. "A Rationale for the WTO Prohibition of Export Subsidies: Strategic Export Subsidies and World Welfare." *Open Economies Review* 11 (3):229-245. doi: https://doi.org/10.1023/A:1008322823299.

Collie, David R. 2000b. "State aid in the European Union: The prohibition of subsidies in an integrated market." *International Journal of Industrial Organization* 18 (6):867-884. doi: https://dx.doi.org/10.1016/S0167-7187(98)00051-4.
Dixit, Avinash. 1984. "International Trade Policy for Oligopolistic Industries." *The Economic Journal* 94:1-16. doi: https://doi.org/10.2307/2232651.
Dixit, Avinash. 1987. "Strategic aspects of trade policy." In *Advances in Economic Theory: Fifth World Congress*, edited by Truman Fassett Bewley, 329-362. Cambridge: Cambridge University Press. doi: https://doi.org/10.1017/CCOL0521340446.009.
Eaton, Jonathan, and Gene M. Grossman. 1986. "Optimal Trade and Industrial Policy Under Oligopoly." *The Quarterly Journal of Economics* 101 (2):383-406. doi: https://doi.org/10.2307/1891121.
Friedman, James W. 1971. "A Non-cooperative Equilibrium for Supergames." *The Review of Economic Studies* 38 (1):1-12. doi: https://dx.doi.org/10.2307/2296617.
Fudenberg, Drew, and Eric Maskin. 1986. "The Folk Theorem in Repeated Games with Discounting or with Incomplete Information." *Econometrica* 54 (3):533-554. doi: https://doi.org/10.2307/1911307.
Grossman, Gene M. 2016. "Chapter 7 - The Purpose of Trade Agreements." In *Handbook of Commercial Policy*, edited by Kyle Bagwell and Robert W. Staiger, 379-434. North-Holland. doi: https://doi.org/10.1016/bs.hescop.2016.04.016.
Johnson, Harry G. 1953. "Optimum Tariffs and Retaliation." *The Review of Economic Studies* 21 (2):142-153. doi: https://doi.org/10.2307/2296006.
Kennan, John, and Raymond Riezman. 1988. "Do Big Countries Win Tariff Wars?" *International Economic Review* 29 (1):81-85. doi: https://doi.org/10.2307/2526808.
Kowalczyk, Carsten, and Raymond Riezman. 2013. "Trade Agreements." In *Palgrave Handbook of International Trade*, edited by Daniel Bernhofen, Rod Falvey, David Greenaway and Udo Kreickemeier, 367-388. London: Palgrave Macmillan UK. doi: https://doi.org/10.1007/978-0-230-30531-1_13.
Leahy, Dermot, and J. Peter Neary. 2011. "Oligopoly and Trade." In *Palgrave Handbook of International Trade*, edited by Daniel M. Bernhofen, Rodney Falvey, David Greenaway and Udo Kreickemeier. Basingstoke: Palgrave Macmillan. doi: https://doi.org/10.1007/978-0-230-30531-1_7.
Maggi, Giovanni. 2014. "Chapter 6 - International Trade Agreements." In *Handbook of International Economics*, edited by Gita Gopinath, Elhanan Helpman and Kenneth Rogoff, 317-390. Elsevier. doi: https://doi.org/10.1016/B978-0-444-54314-1.00006-9.
Markusen, James R. 1981. "Trade and the Gains from Trade with Imperfect Competition." *Journal of International Economics* 11 (4):531-551. doi: https://doi.org/10.1016/0022-1996(81)90033-7.
Mayer, Wolfgang. 1981. "Considerations on Negotiated Tariff Adjustments." *Oxford Economic Papers* 33 (1):135-153. doi: https://doi.org/10.1093/oxfordjournals.oep.a041494.
Mrázová, Monika. 2011. Trade Agreements when Profits Matter. London School of Economics.
Ossa, Ralph. 2011. "A “New Trade” Theory of GATT/WTO Negotiations." *Journal of Political Economy* 119 (1):122-152. doi: https://doi.org/10.1086/659371.
Ossa, Ralph. 2012. "Profits in the “New Trade” Approach to Trade Negotiations." *American Economic Review* 102 (3):466-469. doi: https://doi.org/10.1257/aer.102.3.466.
Ossa, Ralph. 2014. "Trade Wars and Trade Talks with Data†." *American Economic Review* 104 (12):4104-4146. doi: https://doi.org/10.1257/aer.104.12.4104.

Park, Jee-Hyeong. 2000. "International trade agreements between countries of asymmetric size." *Journal of International Economics* 50 (2):473-495. doi: https://doi.org/10.1016/S0022-1996(99)00006-9.

Schelling, Thomas C. 1960. *The Strategy of Conflict*. Cambridge, MA: Harvard University Press.

Staiger, Robert W. 1995. "Chapter 29 International rules and institutions for trade policy." In *Handbook of International Economics*, 1495-1551. Elsevier. doi: https://doi.org/10.1016/S1573-4404(05)80009-7.

Syropoulos, Constantinos. 2002. "Optimum Tariffs and Retaliation Revisited: How Country Size Matters." *The Review of Economic Studies* 69 (3):707-727. doi: https://doi.org/10.1111/1467-937X.t01-1-00028.

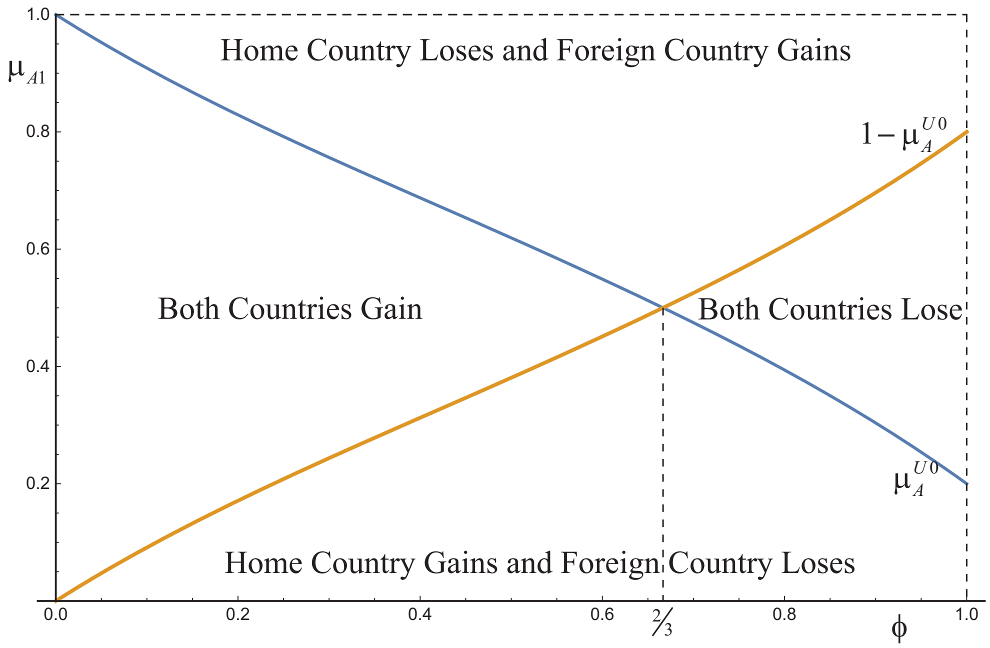

Figure 1: Gains and Losses from Unilateral Free Trade under Cournot Duopoly

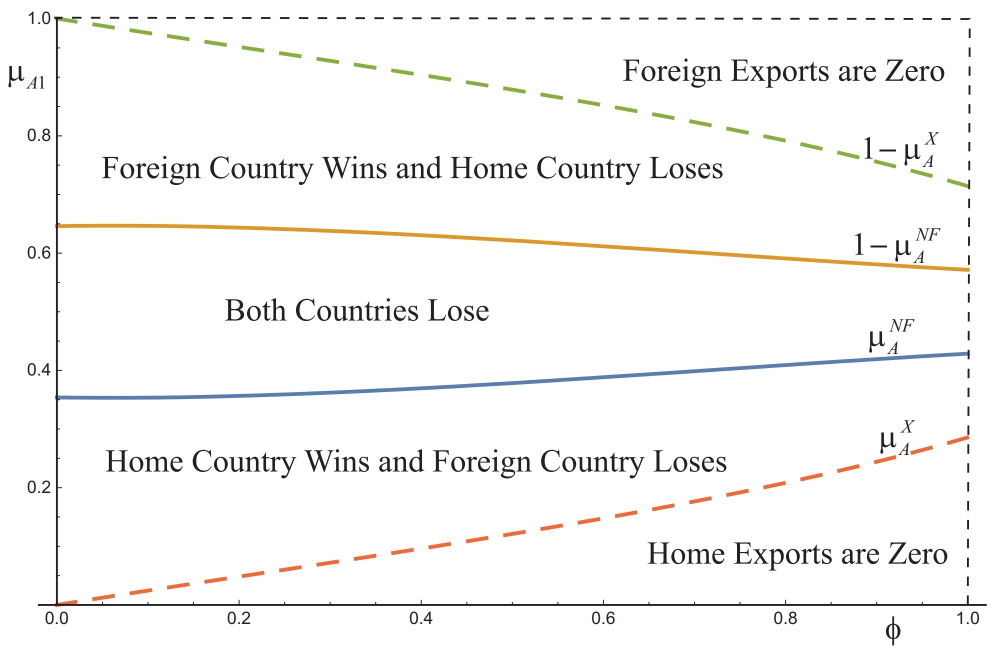

Figure 2: NE in Trade Policies under Cournot Duopoly:
Winners and Losers in Trade War

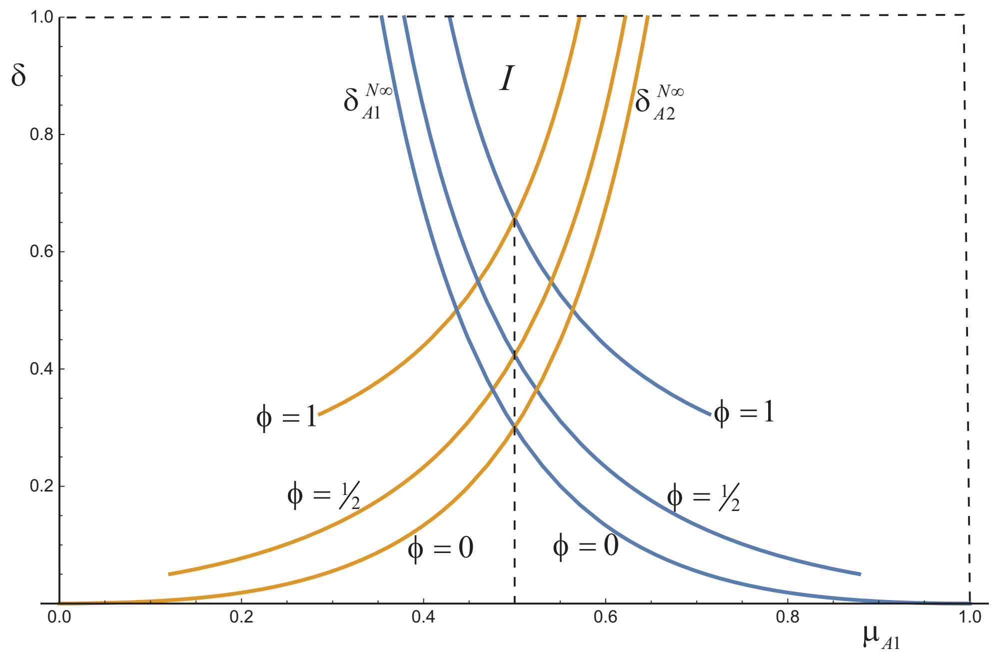

Figure 3: Critical Discount Factors using Nash-Reversion Trigger Strategies under Cournot Duopoly

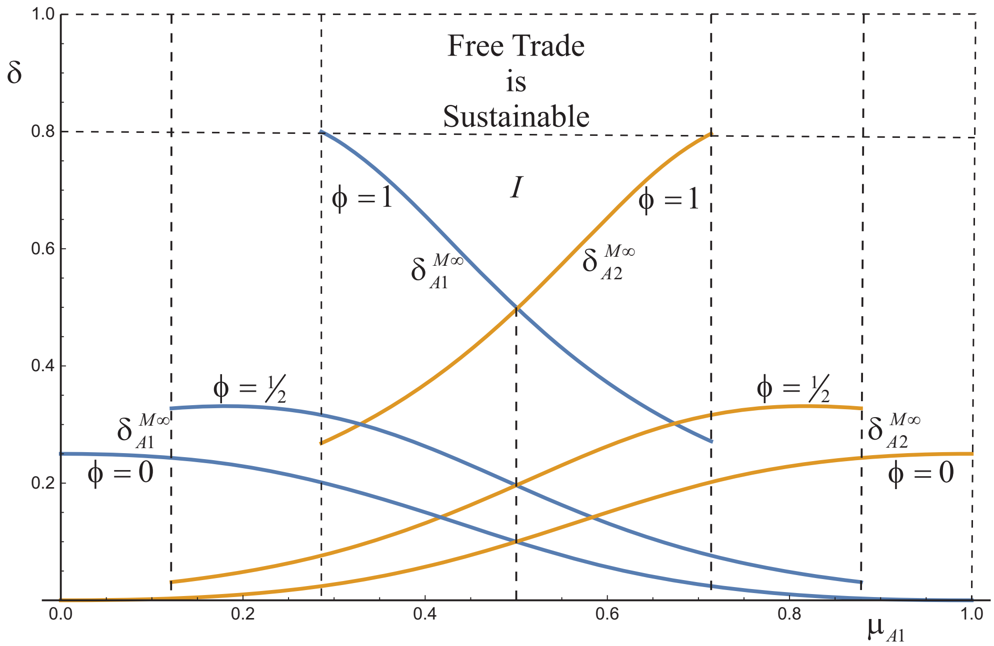

Figure 4: Critical Discount Factors using Minimax-Reversion Trigger Strategies under Cournot Duopoly

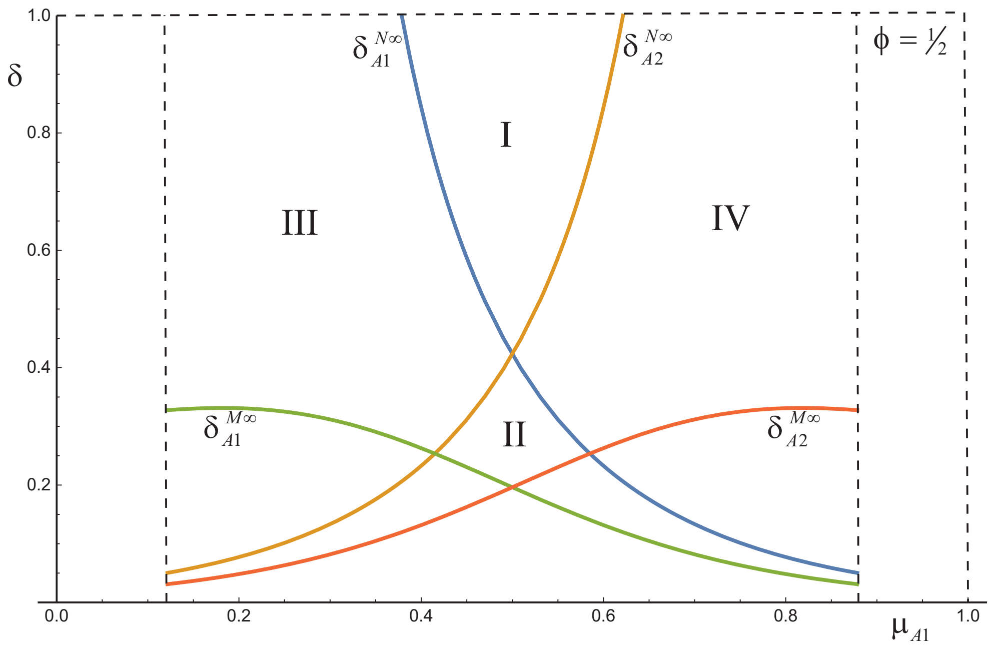

Figure 5: Comparison of Critical Discount Factors under Cournot Duopoly

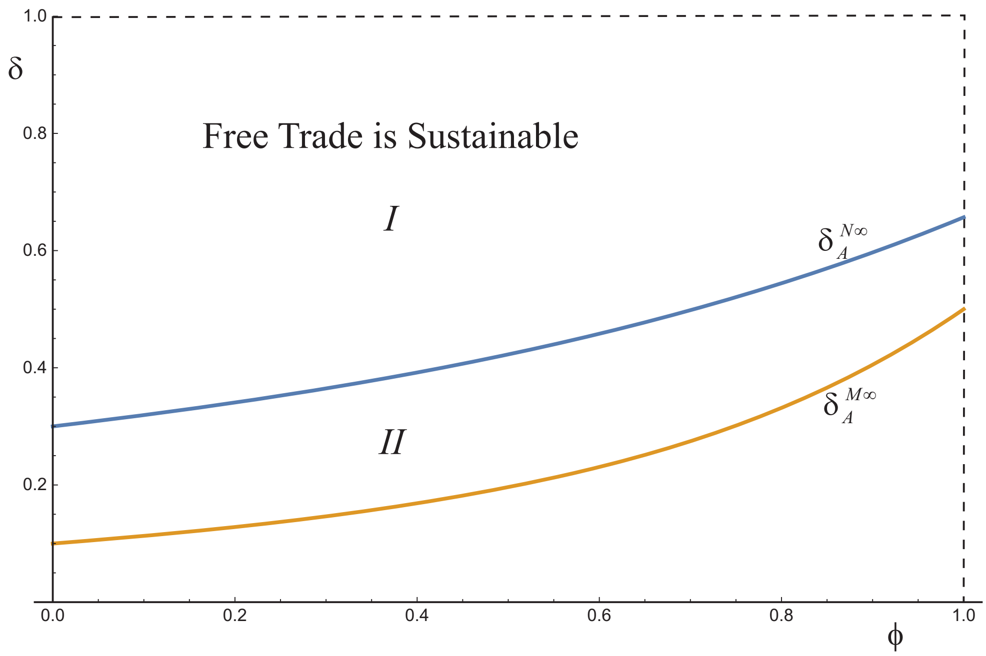

Figure 6: Critical Discount Factors and Product Differentiation under Cournot Duopoly

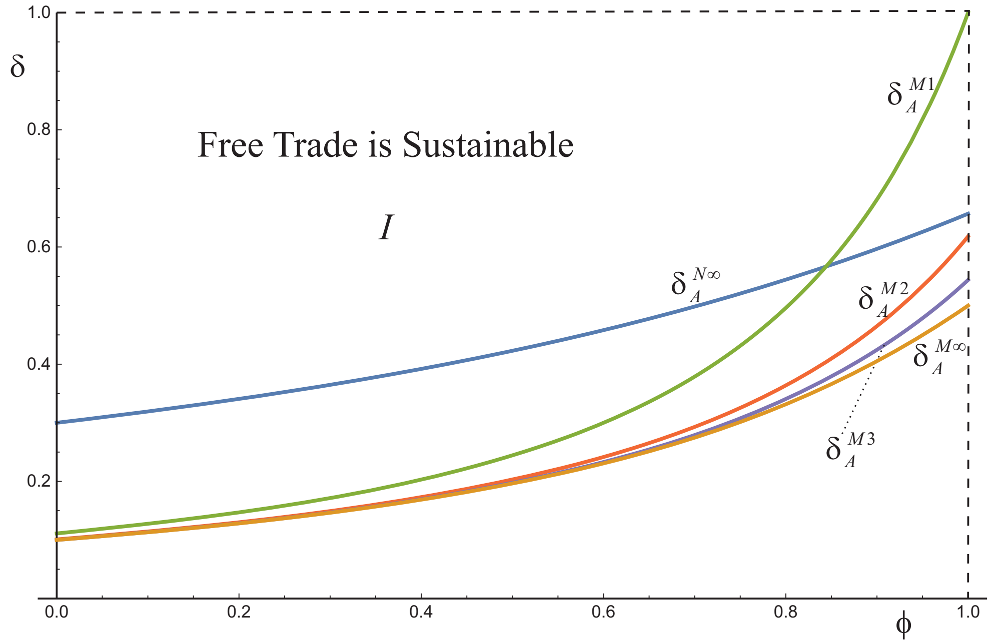

Figure 7: Minimax Reversion for a Limited Number of Rounds under Cournot Duopoly

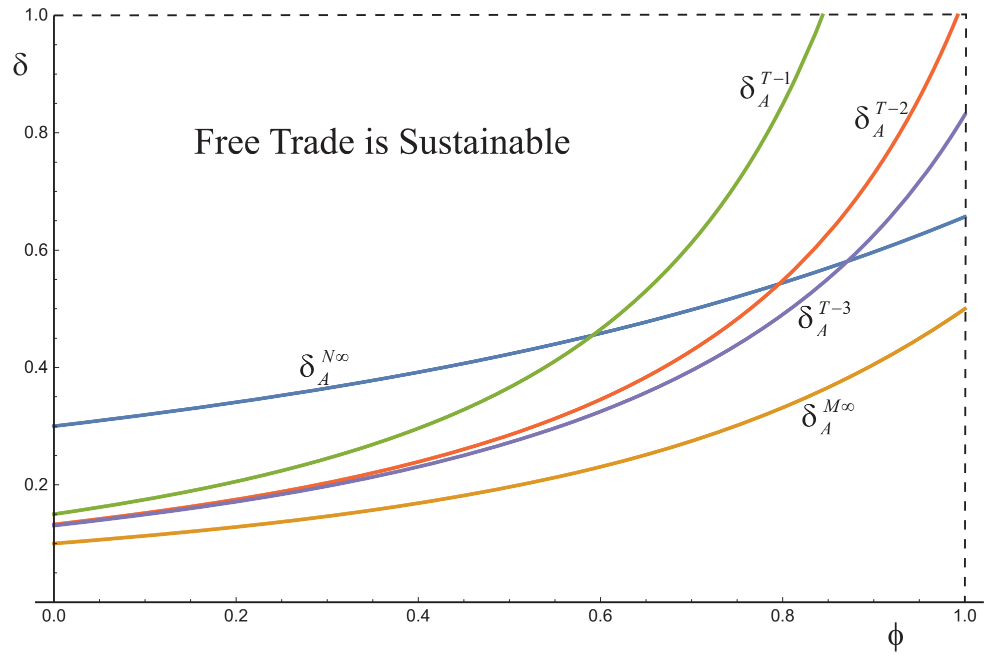

Figure 8: Critical Discount Factors in Finitely-Repeated Game under Cournot Duopoly

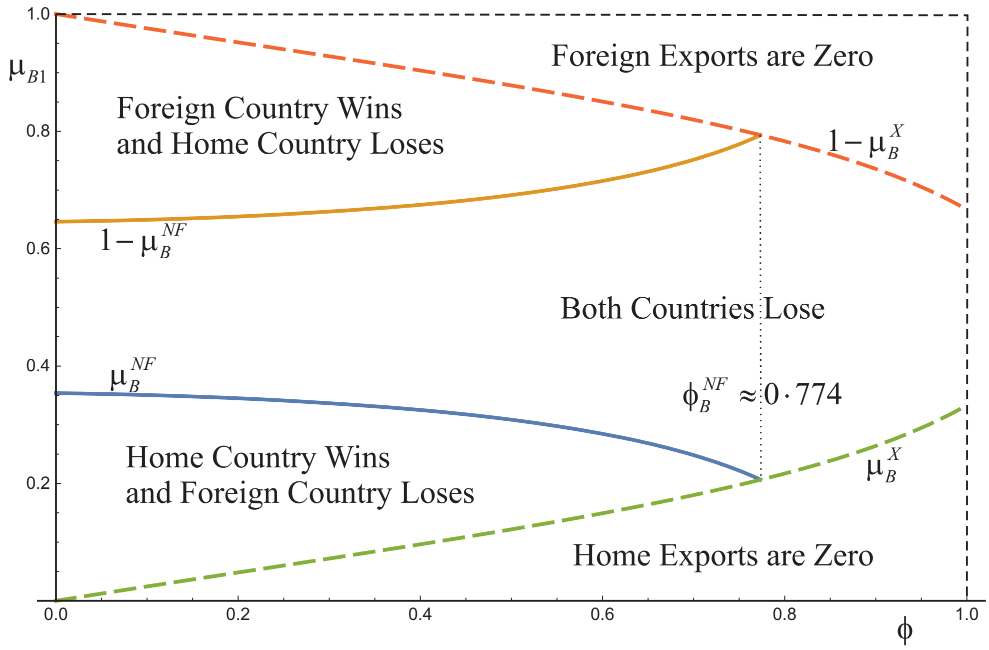

Figure 9: NE in Trade Policies under Bertrand Duopoly: Winners and Losers in Trade War

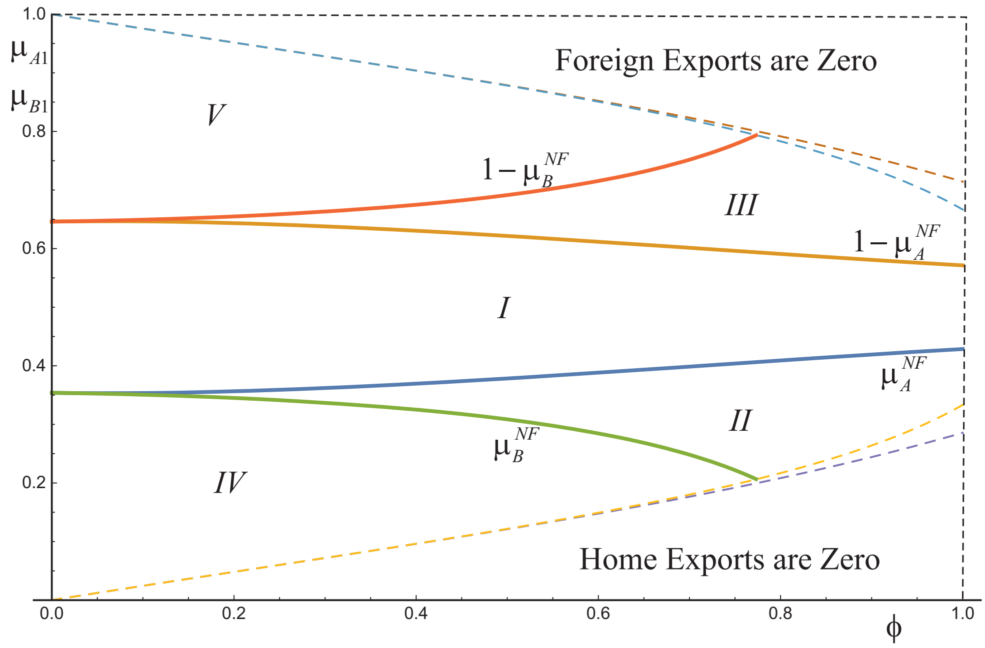

Figure 10: Winners and Losers in a Trade War under Cournot Duopoly and under Bertrand Duopoly

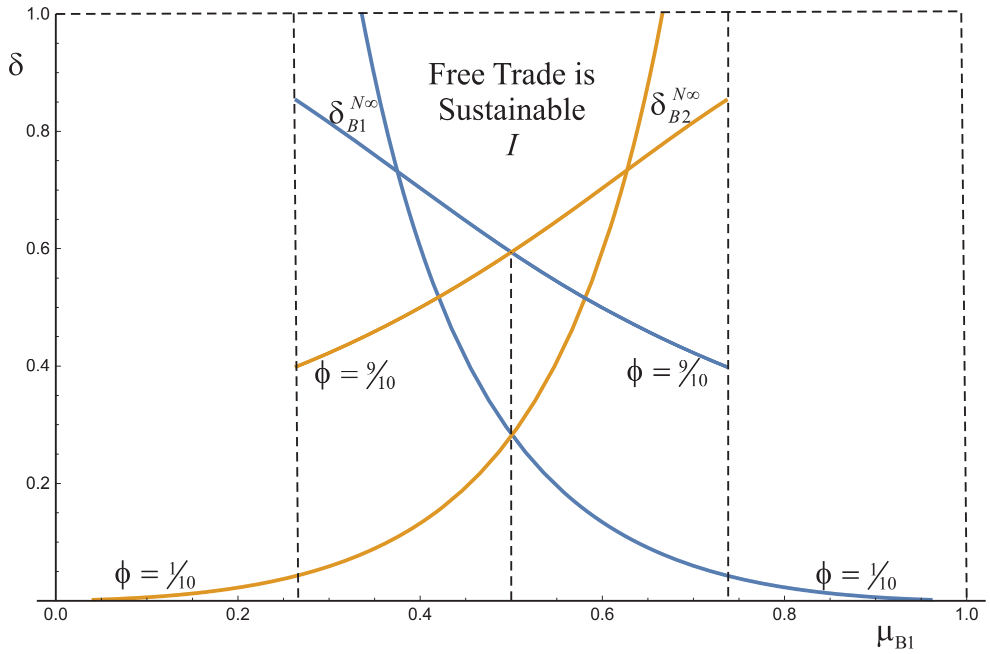

Figure 11: Critical Discount Factors using Nash-Reversion Trigger Strategies under Bertrand Duopoly

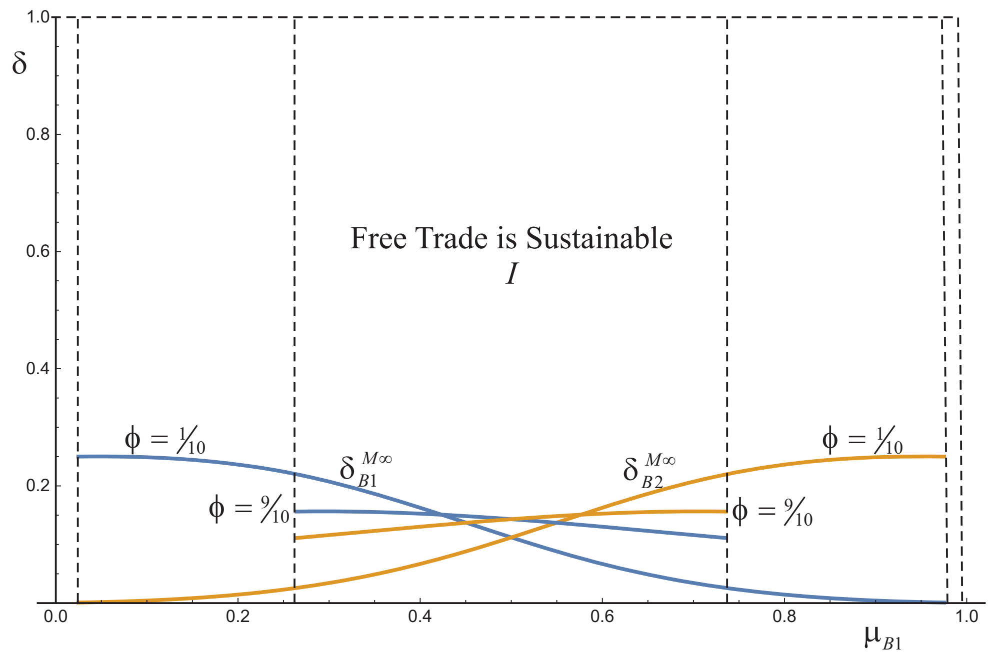

Figure 12: Critical Discount Factors using Minimax-Reversion Trigger Strategies under Bertrand Duopoly

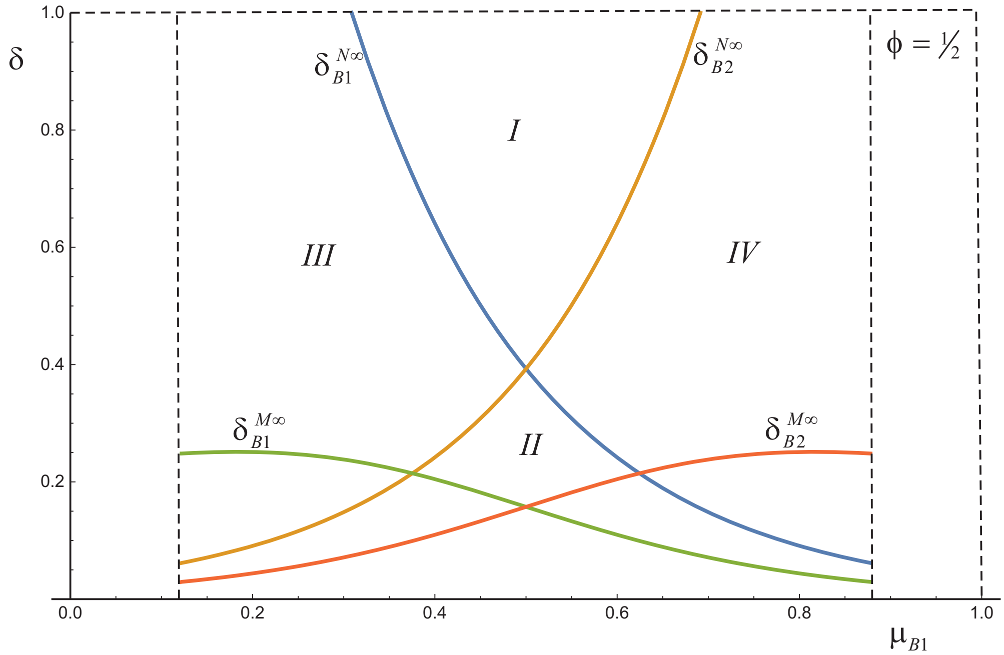

Figure 13: Comparison of Critical Discount Factors under Bertrand Duopoly

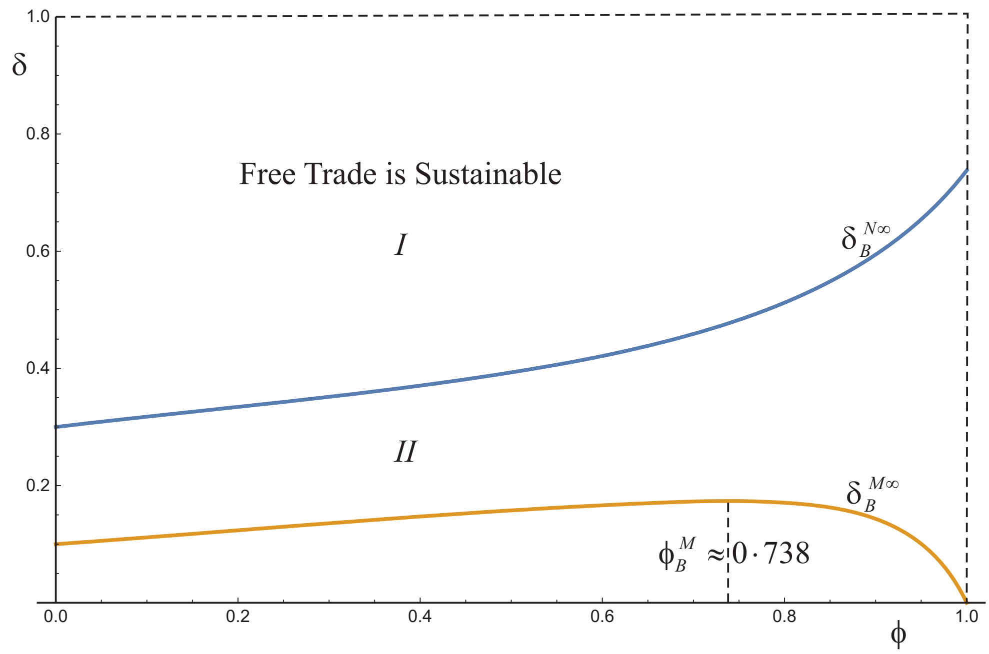

Figure 14: Critical Discount Factors and Product Differentiation under Bertrand Duopoly

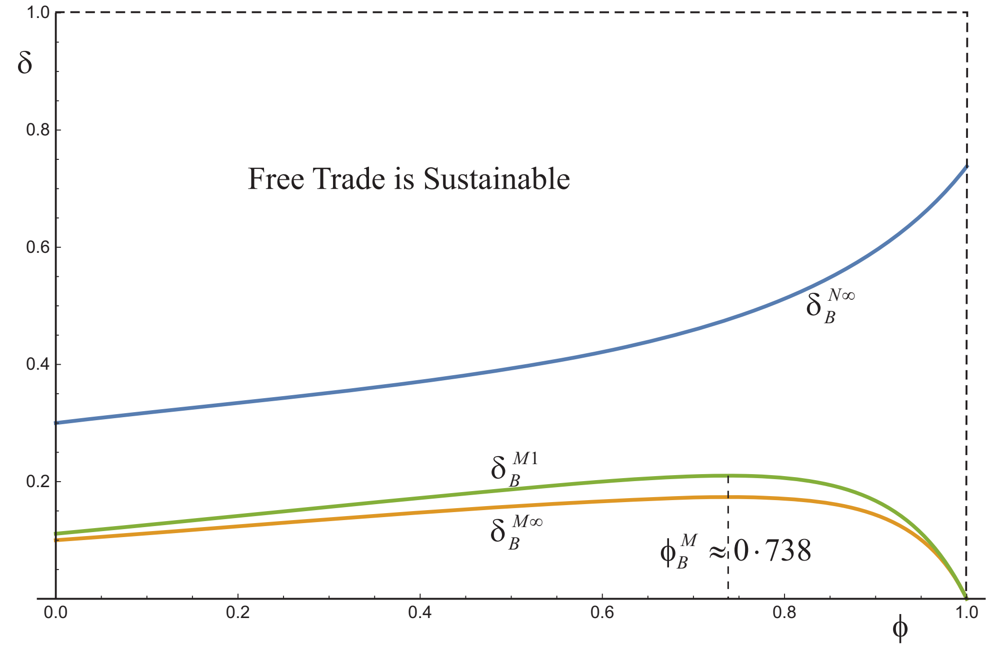

Figure 15: Minimax Reversion for One Round under Bertrand Duopoly

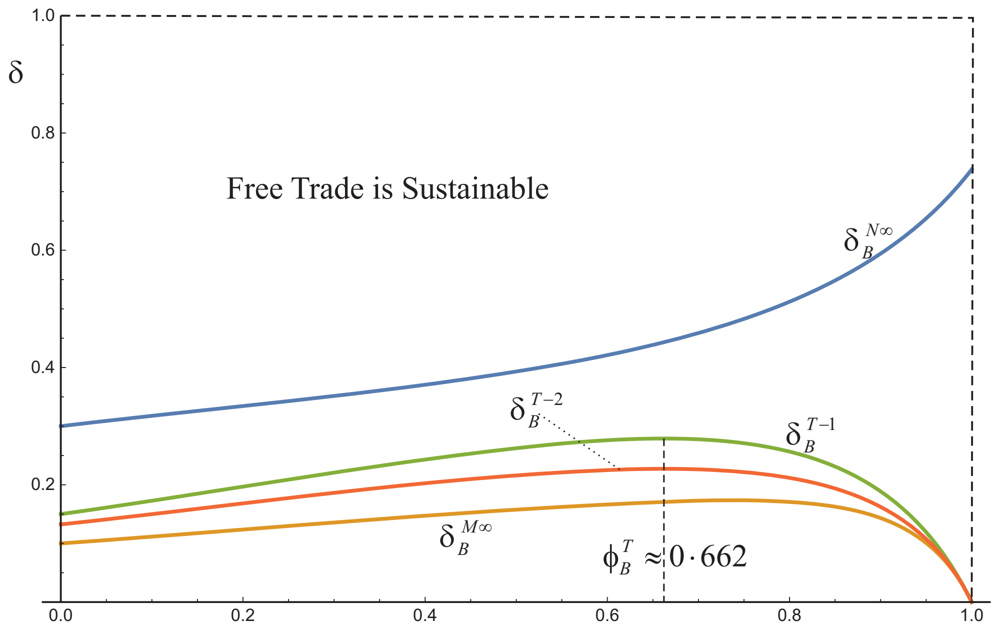

Figure 16: Critical Discount Factors in Finitely-Repeated Game under Bertrand Duopoly

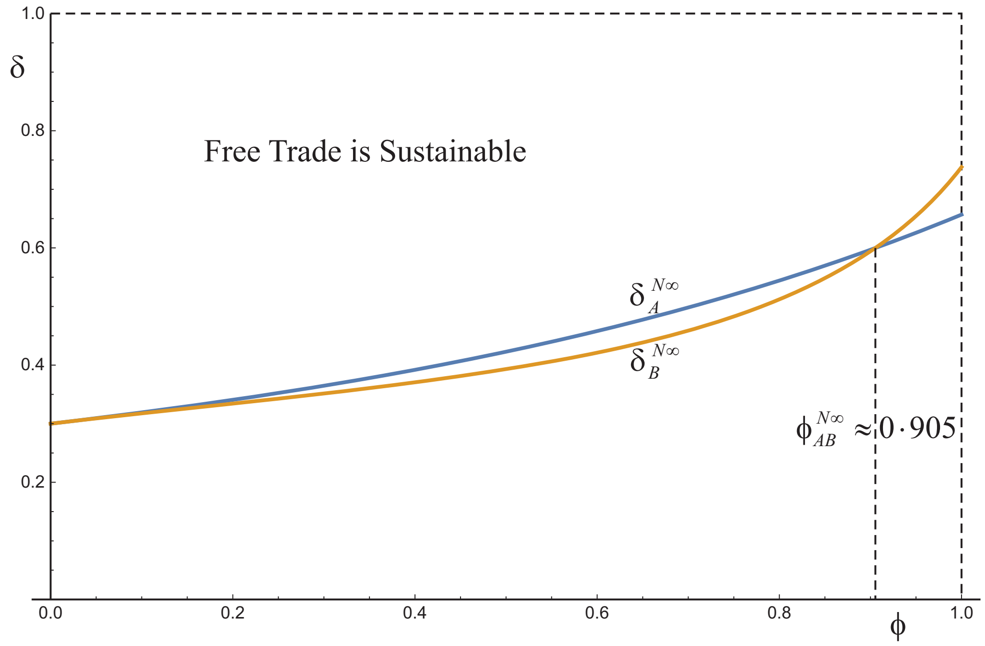

Figure 17: Nash Reversion under Cournot Duopoly and Bertrand Duopoly

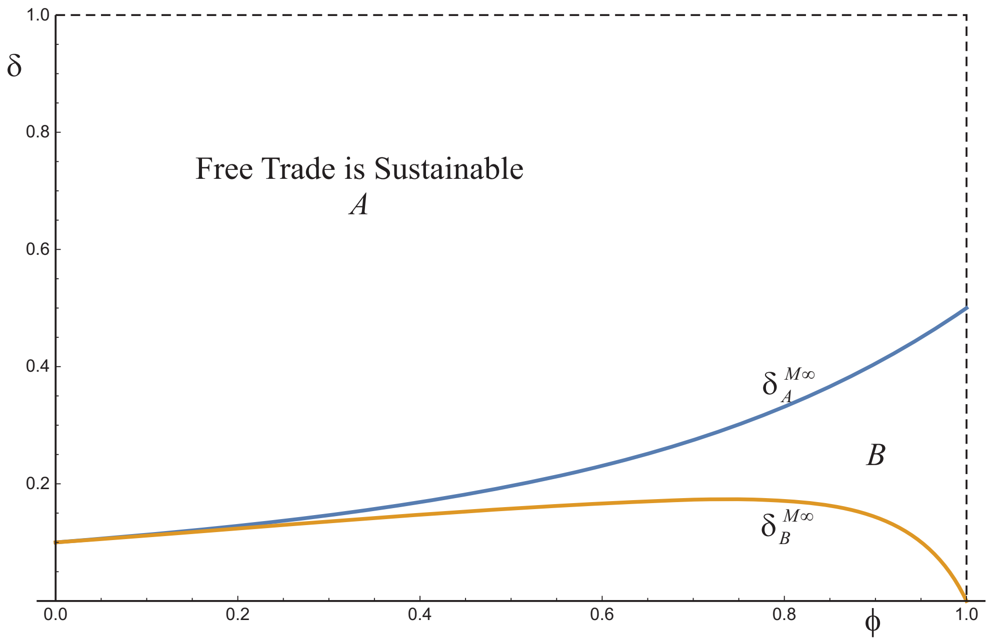

Figure 18: Minimax Reversion under Cournot Duopoly and Bertrand Duopoly

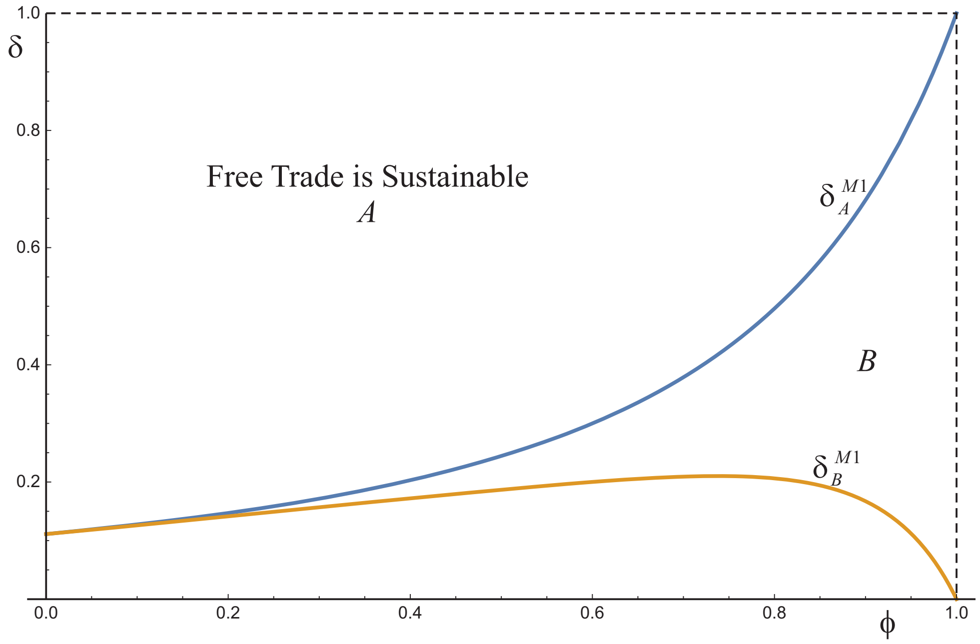

Figure 19: Minimax Reversion for One Round under Cournot Duopoly and Bertrand Duopoly

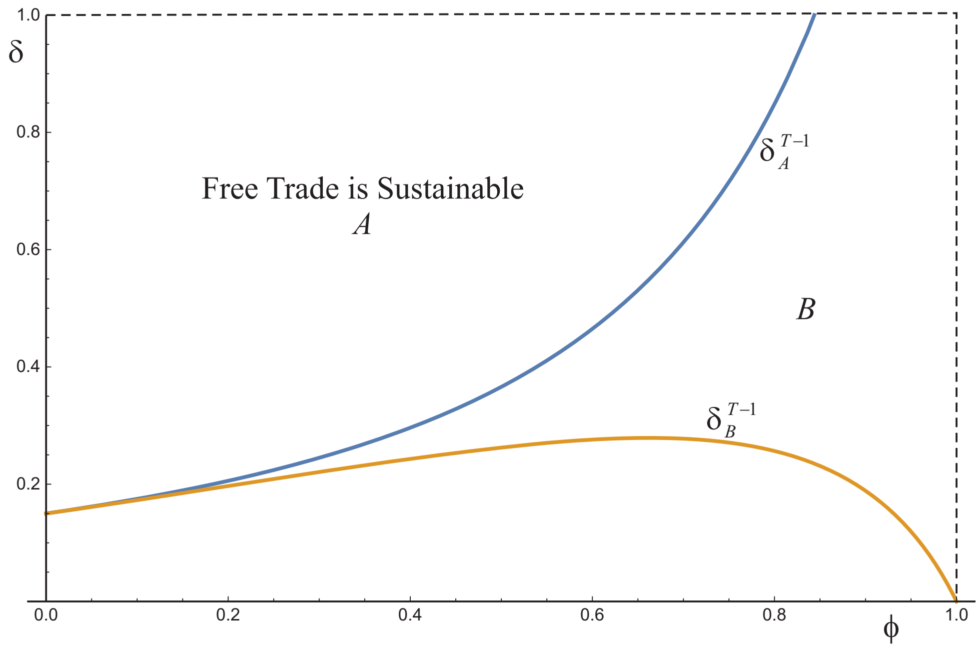

Figure 20: Finitely-Repeated Game under Cournot Duopoly and Bertrand Duopoly