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Same-Sex Marriage, The Great Equalizer*

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Abstract

When limited to heterosexual marriage, agents of different genders are not guaranteed to harvest the same payoff even conditional on having the same type, and even if all other factors, such as search costs or the distribution of partner types, are the same across genders. If same-sex marriage is legalized and there is a positive mass of agents who find marriage with both sexes acceptable, then only symmetric equilibria survive in symmetric environments.

Keywords: marriage markets, matching, gender equality, same-sex marriage.

JEL: C78, D1

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1 Introduction

Modern marriage is a civil contract that brings both spouses economic and contractual benefits. Inheritance issues, tax benefits, immigration status, adoption opportunities, etc., frequently depend upon marital status. Meanwhile, in many modern countries sex, cohabitation, and parenthood do not require marriage. Why then are many people, married and not, so strongly opinionated against same-sex marriage? Some of this resistance might be cultural or emotional, but we find an economic rationale for such an opposition. In this paper, we show that if same-sex marriage is prohibited, then asymmetric equilibria can arise in the marriage market, specifically that otherwise similar agents of different genders obtain different payoffs. Moreover, one of the genders can be systematically oppressed, meaning that all agents of one gender obtain a lower payoff than otherwise identical agents of the opposite gender. However, as we show in the paper, if same-sex marriage is allowed, then every marriage market equilibrium (in an otherwise gender-symmetric environment) is a symmetric one, meaning that agents' payoffs are gender-independent. We show that an arbitrarily tiny proportion of bisexual individuals is sufficient to guarantee gender-neutral market outcomes in the presence of same-sex marriage. This may be a reason for the advantaged gender to oppose such marriages.

Our model is based on the framework by Atakan (2006a), in which each agent has fixed per period search costs and the surplus of marriage is split according to the Nash bargaining solution. We show that once genders are formally introduced to this framework and only heterosexual marriage is allowed, then for each equilibrium in Atakan (2006a) there is a continuum of asymmetric equilibria. This gender inequality is maintained by limiting the set of marital partners to the opposite gender. Suppose that one of the genders expects a higher equilibrium payoff, which acts as a disagreement outcome in each current or potential match. This means that each agent of such gender is more demanding, so representatives of the dominated gender, being forced to marry representatives of the dominating gender, expect forthcoming matches to be equally demanding, and therefore accommodate such higher demands from current suitors, which leads to an asymmetric equilibrium. After illustrating the possibility of asymmetric outcomes, we allow for same-sex marriage. It turns out, that as long as there are some bisexual people, i.e. those who are able to accept marriage with both genders, only symmetric equilibrium outcomes are possible.

Remarkably, this result does not depend on the size of the bisexual cohort. The key mechanism is that allowing same-sex marriage improves a disagreement point for bisexuals of an oppressed gender, which lowers equilibrium payoffs for all agents of the advantaged gender, and thereby benefits all, even the heterosexual, agents of the dominated gender. This process unravels until all the gender-driven asymmetries disappear. However, asymmetries which are not related to institutional restrictions on marriage but arise due to differences in genders[1] may still remain. We show, however, that even such strong asymmetries might disappear in a purely bisexual society.

Our paper makes contributions to several strands of literature in the social sciences. The theoretical literature on marriage starts with Becker (1973) who showed that under the supermodularity of the marriage production function, marriage market equilibria feature positive assortative matching—"better" husbands get "better" wives. This result was subsequently extended by Atakan (2006a), who considered fixed search frictions, and by Shimer and Smith (2000), who considered time-dependent search frictions (which requires the log-supermodularity of the production function to obtain positive assortative matching) and by Smith (2006), who modelled a non-transferable utility (in which case a "class" equilibrium can arise: space of types gets broken into classes by ability, and higher class members of one gender marry higher class members of another gender). We contribute to this literature by establishing the existence of equilibrium in a matching model with fixed search costs and exogenous constraints on matching opportunities (Theorem 1).

The main focus of our paper is on gender asymmetries rather than the properties of the distribution of matches. Gender asymmetries in marriage outcomes were studied by Burdett and Coles (1997), where they arise due to the differences in equilibrium productivity type distributions, and in Bhaskar and Hopkins (2016) where such productivity type

[1] A biased gender ratio, as described in Abramitzky et al. (2011), is the most obvious asymmetry that can drive outcome asymmetries; see Burdett and Coles (1997) for more theoretical examples.

differences can arise due to the difference of returns to investments in productivity type. The nature of asymmetry in our paper is quite different, since asymmetric outcomes exist in a purely symmetric environment due to the distribution of bargaining power in equilibrium (see Proposition 2). Moreover, we show that asymmetries arising to the distribution of productivity types disappear in a bisexual environment with same-sex marriage.

A vast literature on intra-household allocation (see, e.g. Browning et al. (1994) and Browning and Chiappori (1998)) studies how the distribution of bargaining power affects intra-house consumption decisions. Although this literature mainly connects bargaining power to traits (age, income, etc.), some of the differences are explained by mere gender. Wright and Rogers (2011) provide an overview of the dynamics of gender inequality in labor distribution in US families that shows that the difference between genders were significant, but are diminishing with time, seemingly connected with better workplace opportunities for females. Not all inequality comes from the current status of either partner: Tichenor (1999) shows that even if a wife earns more than her husband, she does not necessarily enjoys more power in the family. This is perfectly consistent with our model: we can demonstrate an equilibrium where the wife of a better type collects a smaller lifetime payoff. Black et al. (2007) provide some statistics on same- and opposite-sex families in the US; notably, in same-sex couples, both partners are more likely to work. Oreffice (2011) documents that traits can affect the distribution of bargaining power differently in homosexual and heterosexual marriages.

Attitudes to same-sex marriage are significantly different between the two sexes. Olson et al. (2006) documents that females have substantially more positive attitudes about it than males. Lewis and Gossett (2008) also find females to be less opposed to same-sex marriage. Baunach (2012) notes a significant liberalization of public attitudes to same-sex marriage during the period from 1988 till 2010. She found that in all periods of study females had a significantly more positive attitude to same-sex marriage than males. She claims that "changing same-sex marriage attitudes are not due to demographic changes ... [R]ather, the liberalization in same-sex marriage attitudes ... is due primarily to a general societal change in attitudes", which can be interpreted as a change in equilibrium beliefs in our formal model.

The structure of the paper is as follows. The formal model is presented in section 2. In section 3 we define the equilibrium and prove its existence. Our key results on the impact of same-sex marriage restrictions on gender inequality are presented in section 4. In section 5 we discuss the role of our assumptions and possible extensions of our model.

2 Model

There are infinitely many agents in the model. Each agent is characterized by a two-dimensional type (i, x) with $i \in T, x \in [0,1]$. The set of types which can form a partnership is restricted with respect to $i \in T$: for $i, j \in T$, let $a_{ij} = 1$ if marriage is possible and $a_{ij} = 0$ if it is not[2], either due to sexual orientation or for legal reasons.[3] For example, if there are females and males in the population and if either same-sex marriage is prohibited or if all agents are heterosexual, then we have $a_{FF} = a_{MM} = 0$ and $a_{MF} = a_{FM} = 1$. We assume that at least some of $a_{ij} = 1$ for every i and impose $a_{ij} = a_{ji}$. Each type i appears in the population with the probability $q_i \in [0,1]$, $\sum_{i \in T} q_i = 1$. Let x be the "productivity" component of the type, which directly affects the pay-offs of the participants of the marriage market.

In every period agents meet a potential partner and bear costs $c > 0$. Agents can decide whether to accept or reject the match. If $a_{ij} = 1$ and both agents (i, x) and (j, y) agree to marry then they harvest joint production $f(x, y)$, which is defined by the production function $f : [0,1]^2 \to \mathbb{R}_{++}$, and then quit the market. Note that we assume that the output in the marriage is solely defined by the productivity component of the agents' types and is not related to the gender component.

[2]Our results can be directly extended to the case when $a_{ij} \in [0,1]$. In this case a_{ij} can be interpreted as the probability that a match between types i and j is possible. An alternative interpretation of $a_{i,j} \in (0,1)$ is that some kinds of marriage are legal but *repugnant* in the sense of Roth (2018): part of the surplus of such a marriage is dissipated.

[3]We interpret T as a set of restrictions derived from sexual orientation and legal constraints, but our model extends to other restrictions on possible matches, arising due to race, class, caste, etc.

Assumption 1. $f(\cdot,\cdot)$ *is positive, symmetric, increasing in both arguments and Lipschitz continuous of modulus* K.

This assumption implies that (i) higher productivity types are more attractive partners in marriage, and (ii) roles in marriage for both partners are equal. Lipschitz continuity is a technical assumption which is used in the proof of the existence of the equilibrium.

The productivity of an unmatched agent of type $i \in T$ is distributed according to the cumulative distribution function $G_i : [0,1] \to [0,1]$. We assume that $G_i(\cdot)$ has a continuous bounded density on $[0,1]$. We assume that when a married agent (i,x) leaves the market she is replaced with an agent of the same type, and therefore the distribution of types is stationary.

When agents (i,x) and (j,y) decide to marry, they produce $f(x,y)$ and divide it according to Nash's (1950) bargaining solution. Let $v_i(x)$ and $v_j(y)$ be the expected continuation values of rejecting the match and continuing searching, to be defined later. These values serve as a disagreement point in the Nash bargaining problem. Then, if the match is accepted by both players their payoffs are $\{v_i(x) + s_{ij}(x,y), v_j(y) + s_{ij}(x,y)\}$, where

$$s_{ij}(x,y) \equiv \frac{f(x,y) - v_i(x) - v_j(y)}{2}$$

is the *surplus*.

3 Definition and Existence of Equilibrium

Let $A_{ij}(x) \subseteq [0,1]$ be a set of productivity types with $j \in T$ acceptable by agent (i,x). Our setting imposes the following restriction

$$a_{ij} = 0 \Rightarrow A_{ij} = \emptyset. \tag{1}$$

The payoff function from a match between players (i,x) and (j,y) is specified by

$$\pi[x, y, A_{ij}(x), A_{ji}(y)] = \begin{cases} -c & \text{if } x \notin A_{ji}(y) \text{ or } y \notin A_{ij}(x) \\ -c + v_i(x) + s_{ij}(x, y) & \text{if } x \in A_{ji}(y) \text{ and } y \in A_{ij}(x) \\ 0 & \text{if matched in previous rounds} \end{cases} \tag{2}$$

Now we are ready to define the value function of player (i, x).

$$v_i(x) = \max_{\hat{A}_{ij}} \left\{ \sum_{j \in T} q_j \mathbb{E}_{j,y_t} \sum_{t=0}^{\infty} \pi[x, y_t, \hat{A}_{ij}, A_{ji}(y_t)] \right\} \tag{3}$$

with (1) satisfied for $\hat{A}_{ij}$. Notation $\mathbb{E}_{j,y_t}$ means that the expectation is taken with respect to $y_t \sim G_j$.

Definition 1. Search equilibrium *is a function* $v : T \times [0, 1] \to \mathbb{R}$ *and a strategy* $A_{ij}(x)$ *for each* $i \in T$, $x \in [0, 1]$ *such that*

1. $A_{ij}(x)$ *solves problem* (3) *given that all other types* $(j, y) \in T \times [0, 1]$ *are playing the strategy* $A_{jk}(y)$, $k \in T$ *and the payoff function* (2) *is defined according to* $v_i(x)$*;*

2. $v_i(x)$ *satisfies* (3) *given that all players* $(j, y) \in T \times [0, 1]$ *are playing* $A_{jk}(y)$, $k \in T$, *and payoff function* (2) *is defined according to* (3)*;*

3. *matching sets* $A_{ij}(y)$ *satisfy restriction* (1).

The following theorem establishes the existence of the equilibrium.

Theorem 1. *Under Assumption 1, the search equilibrium exists.*

The proof, which is similar to that by Atakan (2006b), is presented in Appendix B.

Denote by $M_{ij}(x)$ the matching sets of type (i, x), i.e. types (j, y) which both accept (i, x) and are accepted by (i, x). Suppose that agent (i, x) meets agent (j, y) and that $a_{ij} = 1$. Then, the value function can be represented as

$$v_{ij}(x,y) = \max\left\{ s_{ij}(x,y) + v_i(x), -c + \sum_{l \in T} q_l \mathbb{E}_{l,z} v_{i,l}(x,z) \right\} =$$

$$\max\left\{ s_{ij}(x,y) + v_i(x), v_i(x) \right\}$$

Thus, the match is accepted whenever surplus $s_{ij}(x, y)$ is non-negative.[4] As the same logic applies to player (j, y) we conclude that $M_{ij}(x) = \{(j, y) : s_{ij}(x, y) \geq 0, a_{ij} = 1\}$. If $a_{ij} = 0$ then $M_{ij}(x) = \emptyset$. The following Proposition proves that the **constant surplus condition** holds in equilibrium.

Proposition 1. *For all* $(i, x) \in T \times [0, 1]$

$$\sum_{j \in T} q_j \int_{M_{ij}(x)} s_{ij}(x,y) dG_j(y) = c. \quad (4)$$

4 Gender and Asymmetries

Once we have established the existence of equilibrium in our generalized model we can proceed with the analysis of the impact of marriage restrictions on gender inequality.

First, consider the model by Atakan (2006a). Since this model does not have any gender differences, we can treat it as an essentially one-gender model with $a_{11} = 1$. Since the division of the surplus cannot be conditioned on sex in such a setting, the equilibrium is necessarily symmetric. It satisfies the constant surplus condition (4). Let $\tilde{v}_c(x)$ be the value function associated with such equilibrium when the search cost is c.

Now we explore how the possibility of having different types of players affects the existence of asymmetric equilibria.

Definition 2. *An equilibrium is* ***asymmetric*** *if for some* $i, j \in T$ *and* $x \in [0, 1]$*:* $v_i(x) \neq v_j(x)$.

[4] We ignore superficial equilibria in which nobody marries nobody because everyone expects to be rejected.

We start our analysis with the world with two genders and no sexual orientation asymmetries $T = \{F, M\}$. A few questions may be asked here. Are there asymmetric equilibria in a model with two genders? If yes, what drives such asymmetries? Is it necessary to have two different productivity type distributions $G_F \neq G_M$ in order to have asymmetric equilibria? How does sexual orientation impact the existence of asymmetric outcomes? Namely, what is the difference between an environment when every agent is straight or same-sex marriage is forbidden ($a_{FM} = a_{MF} = 1$, $a_{FF} = a_{MM} = 0$) and an environment when all players are bisexual and same-sex marriage is allowed ($a_{FM} = a_{MF} = a_{FF} = a_{MM} = 1$)?

We start by considering an environment when same-sex marriage is forbidden, but otherwise the model is symmetric, i.e. $q_F = q_M = 1/2$ and $G_F(x) = G_M(x) = G(x)$. If everybody was bisexual and same-sex marriage was allowed, we would have the unisex equilibrium of Atakan (2006a) described above, with value function $\tilde{v}_c(x)$. Now, take the value function $\tilde{v}_{2c}(x)$ associated with the unisex equilibrium, but with double the search costs, to reflect that the chance of meeting the opposite gender agent is twice smaller. Let $\tilde{M}(x)$ be the matching set associated with such an equilibrium. Define

$$v_F(x) = \tilde{v}_{2c} - \Delta, \quad v_M(x) = \tilde{v}_{2c} + \Delta. \tag{5}$$

for some $\Delta > 0$. We claim that such value functions v_F, v_M together with matching sets $M_{FM}(x) = M_{MF}(x) = \tilde{M}(x)$, $M_{FF} = M_{MM} = \emptyset$ constitute an asymmetric equilibrium in a search economy when same-sex marriage is prohibited. First note that if the value functions are defined by (5) then the surplus remains the same as in the unisex economy:

$$s_{FM}(x, y) = s_{MF}(x, y) = \frac{f(x, y) - v_F(x) - v_M(y)}{2} = \frac{f(x, y) - \tilde{v}_{2c}(x) - \tilde{v}_{2c}(y)}{2} \equiv \tilde{s}(x, y)$$

and thus the matching sets are exactly the same as $\tilde{M}(x)$. Moreover, the optimal stopping problem is consistent with the value functions, i.e. if the agent is expected to get $\tilde{v}_{2c} \pm \Delta$ in the next round, this is also her current value function:

$$v_F(x) = -c + \frac{1}{2}[\tilde{v}_{2c}(x) - \Delta] + \frac{1}{2}\mathbb{E}\max\{\tilde{s}(x,y) + \tilde{v}_{2c} - \Delta, \tilde{v}_{2c} - \Delta\}$$
$$= \frac{1}{2}[\tilde{v}_{2c}(x) - 2\Delta] + \frac{1}{2}\left[-2c + \tilde{v}_{2c}(x) + \mathbb{E}\max\{\tilde{s}(x,y), 0\}\right] = \tilde{v}_{2c}(x) - \Delta$$

is defined recursively by $\tilde{v}_{2c}(x) = -2c + \tilde{v}_{2c}(x) + \mathbb{E}\max\{\tilde{s}(x,y), 0\}$. The same logic applies to v_M. This brings us to the following conclusion.

Proposition 2. *Suppose that $a_{FM} = a_{MF} = 1$ and $a_{FF} = a_{MM} = 0$, $q_F = q_M = 1/2$ and $G_F(x) = G_M(x)$ for all $x \in [0,1]$. Then there exists a continuum of asymmetric equilibria with $v_i(x) < v_j(x)$ for all values of $x \in [0,1]$.*

Note that our result does not simply say that there is unequal treatment of agents in equilibrium, meaning that the same productivity types get different payoffs depending on their gender (for example high productivity F's and low productivity M's are treated better than their opposite gender counterparts). We show that the difference in payoffs can be persistent across all productivity types, meaning that there can be a **systematic discrimination** against one of the genders. That is, all the F's can get lower payoffs than the M's of the same productivity type. These differences are not driven by asymmetries in the environment, which is symmetric, but are purely a result of coordination on a specific equilibrium outcome. This is in contrast to Burdett and Coles (1997) and others[5], where differences between gender payoffs are driven solely by differences in some gender characteristics, e.g. distributions of productivity types.

Next we consider an environment in which there are no hurdles for same-sex marriage: $a_{FM} = a_{MF} = a_{FF} = a_{MM} = 1$. Moreover, we allow for all sorts of asymmetries in gender distribution: $q_F \neq q_M$ and $G_F(x) \neq G_M(x)$. As the following Proposition establishes, even in such strikingly asymmetric environment all equilibria are necessarily symmetric.

[5]This is also the case in Bergstrom and Bagnoli (1993), Siow (1998), Chiappori and Oreffice (2008), Coles and Francesconi (2011), and Bhaskar and Hopkins (2016).

Proposition 3. *Suppose that $a_{FM} = a_{MF} = a_{FF} = a_{MM} = 1$. Then, in any equilibrium $v_F(x) = v_M(x)$ for all $x \in [0, 1]$.*

Proposition 2 highlights the fact that in the model with transferable utility gender differences in payoffs can arise purely because of exogenous gender restrictions on possible matches, while Proposition 3 shows that the absence of such restrictions leads to the equal treatment of genders even in asymmetric environments, e.g. such as in papers listed in Footnote 5. However, it relies on two important conditions: (i) that same-sex marriage is allowed and (ii) that all agents are willing to accept a same sex partner. Condition (i) is a policy issue and, as we have illustrated, the absence of institutional restrictions on same-sex marriage is generally good for gender equality. Condition (ii) relates to human nature and it is unreasonable to assume that it holds in real societies, since some of their members would find it impossible to marry a person of the same gender, regardless of his or her productivity characteristics. We intend to show that even having a tiny proportion of agents who are willing to accept partners of both genders is sufficient to guarantee gender equality in environments which are gender-symmetric. This key result is the main focus of the rest of this section.

Suppose that agents now differ both in their gender and their sexuality. We will distinguish heterosexual agents who can only match with the opposite gender and bisexual agents who can match with both genders.[6] Let the set of types be $T = \{FB, FH, MB, MH\}$. We make the following assumption on possible matches:

$$a_{iH,ij} = a_{ij,iH} = 0, \quad i \in \{F, M\},\ j \in \{B, H\}$$

and all other a's are equal to 1. That is, heterosexual people can only marry the opposite gender. Moreover, we impose the condition that the environment is symmetric with respect to genders.

[6] For the sake of brevity, we omit purely homosexual agents: their presence would not break the feedback loop from heterosexual to homosexual marriages via bisexuals that we will exploit. If anything, they would make our result easier to obtain by applying additional pressure towards making genders more equal.

Assumption 2. *$q_{FB} = q_{MB} = q$ and $q_{FH} = q_{MH} = 1 - q$ and $G_{FB}(x) = G_{MB}(x) = G_B(x)$ and $G_{FH}(x) = G_{MH}(x) = G_H(x)$ for all $x \in [0, 1]$.*

Without this assumption the difference in agents' payoffs can be driven purely by the composition of the available pool of matching candidates. If, say, there were only a few F's they would benefit at the expense of the MH's. The same logic applies to the differences between heterosexual and bisexual people: even in a symmetric environment bisexual people meet potentially suitable candidates more often than straight ones do and as a result obtain higher payoffs in equilibrium. To address this issue we redefine the notion of symmetry in the following way.

Definition 3. *An equilibrium is* **gender-symmetric** *if $v_{Fi}(x) = v_{Mi}(x)$ for all $x \in [0, 1]$ and $i = H, B$.*

Now we can proceed with the main result of our paper.

Proposition 4. *Suppose that Assumptions 1 and 2 hold, $q > 0$ and $f(\cdot, \cdot)$ is supermodular. Then all equilibria are gender-symmetric.*

Our previous analysis suggested that the presence of bisexual types should reduce gender inequality if same-sex marriage is possible. What is surprising is that gender inequality completely disappears for all values of $q > 0$, regardless of how small q is. The mechanism behind this result is as follows. Start with some asymmetric equilibrium in a situation when same-sex marriage was outlawed, and allow for same-sex marriage. Then the bisexual members of a dominated gender, say, the F's, will start matching with each other. This will drive up their disagreement point thereby making the M's less picky. This in turn will increase the payoffs for all the F's, bisexual or not. This process continues until all the gender-driven differences in payoffs are wiped out.

The supermodularity of the production function is usually assumed to obtain positive assortative matching, see Shimer and Smith (2000) and Atakan (2006a). In this paper we do not study the properties of matching distribution[7] and the supermodularity assumption

[7] Jepsen and Jepsen (2002) find evidence of positive assortative matching in both heterosexual and same-sex marriages in the US Census data.

in our model guarantees that the bisexuals of disadvantaged gender match with themselves in equilibrium. It is a sufficient condition for such matches, but it can be shown that it is not a necessary one.

5 Discussion

In this paper, we show that same-sex marriage might lead to a more egalitarian society: while people of differing abilities will still get different payoffs from marriage, people of the *same* ability and of the *same* attitude to same-sex marriage would get the same payoff.

Our model relies on the assumption that the willingness to participate in same-sex marriage is deterministic. This assumption can be easily replaced with the assumption that agents can participate in homosexual marriage with a certain probability. Such a replacement would be equivalent to assuming that a_{ij} are no longer drawn from a binary domain, but are real numbers between zero and one. The matching equilibrium still exists and the results of Proposition 4 hold as long as the probability of matching with the opposite gender is positive.

Our model can also encompass taste shocks or "love". It can be modelled by replacing $f(x,y)$ with $f(x,y)+\varepsilon$, where ε is a random shock. As long as the distribution of ε does not depend on gender, our main result in Proposition 4 holds.

We do not include purely homosexual types in our model, because they do not have any impact on gender equality, which is the main focus of our paper. However, it is obvious that if such agents existed in our model, they could only benefit from the introduction of same-sex marriage, since in the past they simply could not participate in the market. The impact of such agents on the surpluses earned by specific productivity types is unclear and would depend on the distribution of productivity types among homosexuals. However, as long as homosexuals of both genders are of equal measure and have the same distribution of traits, any matching market equilibrium is symmetric, provided that there is a positive mass of bisexuals.

We assume that the search costs are constant and type-independent. However, if the equilibrium exists, then the result of Proposition 4 holds for type-dependent search costs, $c(x)$, because the proof is based on the constant-surplus conditions (6)-(9) written out for one type alone, the most advantaged type. The same holds for the proportion of bisexual population among those whom this type can meet, $q(x)$. Having gender-dependent search costs, however, destroys the gender symmetry. If the search costs were value-dependent, for instance, featuring time discounting, the symmetric equilibrium could be guaranteed even without same-sex marriage.

We assumed that leaving agents are replaced with clones, which results in stationary productivity type distributions and marriage strategies. If this assumption is relaxed, then the equilibrium distribution of traits might be different for each of the two genders, leading to asymmetry in the payoffs. If all agents are bisexual, the equilibrium remains gender symmetric, but a small fraction of the bisexuals is no longer sufficient for guaranteeing gender symmetry.

With this paper, we show that same sex marriage helps to achieve equality between genders if the populations were originally symmetric. Institutional aspects borne by gender inequality, such as unequal access to education, healthcare or privacy, might produce different ability distributions in the two genders, even if ex-ante distributions were identical, reinforcing the gender inequality. Our propositions suggest that some inequality can be tolerated when the bisexual population is large enough; the proof of Proposition 3 is robust to differences across genders with respect to ability distribution or gender imbalances. We leave these issues for future research.

Appendix A: Proofs

Proof of Proposition 1. Note, that

$$v_i(x) = \sum_{j\in T} q_j \int_0^1 \max\{-c + v_i(x), -c + v_i(x) + s_{ij}(x,y)a_{ij}\} dG_j(y) =$$

$$-c + v_i(x) + \sum_{j\in T} q_j \int_{M_{ij}(x)} s_{ij}(x,y) dG_j(y)$$

Cancelling $v_i(x)$ and rearranging terms gives

$$\sum_{j\in T} q_j \int_{M_{ij}(x)} s_{ij}(x,y) dG_j(y) = c.$$

Proof of Proposition 3. Suppose that for some x we have $v_M(x) > v_F(x)$. From Proposition 1 it follows that

$$q_M \int_0^1 [f(x,y) - v_M(x) - v_M(y)]^+ dG_M(y) + q_F \int_0^1 [f(x,y) - v_M(x) - v_F(y)]^+ dG_F(y) = 2c$$

$$q_M \int_0^1 [f(x,y) - v_F(x) - v_M(y)]^+ dG_M(y) + q_F \int_0^1 [f(x,y) - v_F(x) - v_F(y)]^+ dG_F(y) = 2c$$

where $[z]^+ = \max\{z, 0\}$. Subtracting one equation from another yields

$$q_M \int_0^1 \left\{[f(x,y) - v_M(x) - v_M(y)]^+ - [f(x,y) - v_F(x) - v_M(y)]^+\right\} dG_M(y) +$$

$$q_F \int_0^1 \left\{[f(x,y) - v_M(x) - v_F(y)]^+ - [f(x,y) - v_F(x) - v_F(y)]^+\right\} dG_F(y) = 0$$

However, due to $v_M(x) > v_F(x)$ both summands are negative, so we arrive at a contradiction.

Proof of Proposition 4.

Define $z(x,y) \equiv f(x,y) - v_{FH}(x) - v_{FH}(y)$ and define

$$\Delta(x) \equiv v_{MH}(x) - v_{FH}(x)$$
$$\Delta_M(x) \equiv v_{MB}(x) - v_{MH}(x)$$
$$\Delta_F(x) \equiv v_{FB}(x) - v_{FH}(x)$$

i.e., $\Delta(x)$ is a premium for being male for type x conditional on being heterosexual, $\Delta_M(x)$ ($\Delta_F(x)$) is a premium for being bisexual conditional on being a male (female) of type x. Moreover, to define matching sets

$$MBMB(x) = \{y : z(x,y) - \Delta(x) - \Delta_M(x) - \Delta(y) - \Delta_M(y) > 0\}$$
$$MBFB(x) = \{y : z(x,y) - \Delta(x) - \Delta_M(x) - \Delta_F(y) > 0\}$$
$$MBFH(x) = \{y : z(x,y) - \Delta(x) - \Delta_M(x) > 0\}$$
$$MHFB(x) = \{y : z(x,y) - \Delta(x) - \Delta_F(y) > 0\}$$
$$MHFH(x) = \{y : z(x,y) - \Delta(x) > 0\}$$
$$FBMB(x) = \{y : z(x,y) - \Delta_F(x) - \Delta(y) - \Delta_M(y) > 0\}$$
$$FBMH(x) = \{y : z(x,y) - \Delta_F(x) - \Delta(y) > 0\}$$
$$FBFB(x) = \{y : z(x,y) - \Delta_F(x) - \Delta_F(y) > 0\}$$
$$FHMB(x) = \{y : z(x,y) - \Delta(y) - \Delta_M(y) > 0\}$$
$$FHMH(x) = \{y : z(x,y) - \Delta(y) > 0\}$$

Choose the type with the largest gender difference:

$$x_0 \in \arg\max_y \left(\max\{|v_{MH}(y) - v_{FH}(y)|, |v_{MB}(y) - v_{FB}(y)|\}\right)$$

Without loss of generality we assume that this type is male. We write out the optimal stopping conditions for type x_0 of various gender and sexual orientation combinations using our notation:

$$q\int_{MBMB(x_0)}[z(x_0,y)-\Delta(x_0)-\Delta_M(x_0)-\Delta(y)-\Delta_M(y)]dG_B(y)+ \\ q\int_{MBFB(x_0)}[z(x_0,y)-\Delta(x_0)-\Delta_M(x_0)-\Delta_F(y)]dG_B(y)+ \\ (1-q)\int_{MBFH(x_0)}[z(x_0,y)-\Delta(x_0)-\Delta_M(x_0)]dG_H(y)=2c \tag{6}$$

$$q\int_{MHFB(x_0)}[z(x_0,y)-\Delta(x_0)-\Delta_F(y)]dG_B(y)+ \\ (1-q)\int_{MHFH(x_0)}[z(x_0,y)-\Delta(x_0)]dG_H(y)=2c \tag{7}$$

$$q\int_{FBMB(x_0)}[z(x_0,y)-\Delta_F(x_0)-\Delta(y)-\Delta_M(y)]dG_B(y)+ \\ q\int_{FBFB(x_0)}[z(x_0,y)-\Delta_F(x_0)-\Delta_F(y)]dG_B(y)+ \\ (1-q)\int_{FBMH(x_0)}[z(x_0,y)-\Delta_F(x_0)-\Delta(y)]dG_H(y)=2c \tag{8}$$

$$q\int_{FHMB(x_0)}[z(x_0,y)-\Delta(y)-\Delta_M(y)]dG_B(y)+ \\ (1-q)\int_{FHMH(x_0)}[z(x_0,y)-\Delta(y)]dG_H(y)=2c \tag{9}$$

Now, suppose that $v_{MH}(x_0)-v_{FH}(x_0)\geq v_{MB}(x_0)-v_{FB}(x_0)$, i.e. the gender gap is maximal among straight people. Then, $\Delta(x_0)\geq\max\{\Delta(y),\Delta(y)+\Delta_M(y)-\Delta_F(y)\}$. Then, we get that

$$\int_{MHFB(x_0)}[z(x_0,y)-\Delta(x_0)-\Delta_F(y)]dG_B(y)\leq\int_{FHMB(x_0)}[z(x_0,y)-\Delta(y)-\Delta_M(y)]dG_B(y)$$

$$\int_{MHFH(x_0)}[z(x_0,y)-\Delta(x_0)]dG_H(y)\leq\int_{FHMH(x_0)}[z(x_0,y)-\Delta(y)]dG_H(y)$$

From (7) and (9) we get that both these expressions must hold as equalities, which implies that $\Delta(x_0)=\Delta(y)$ for all $y\in FHMH(x_0)=MHFH(x_0)$ and $\Delta(x_0)=\Delta(y)+\Delta_M(y)-\Delta_F(y)$ for all $y\in FHMB(x_0)=MHFB(x_0)$. Since $MHFB(x_0)\subset MHFH(x_0)$ we obtain that $\Delta_M(y) = \Delta_F(y)$ for all $y \in MHFB(x_0)$. Thus, we conclude that all the types suffer the same amount of discrimination regardless of their sexual orientation. This is equivalent to having the largest gender gap among bisexual people – the case we deal with next.

Finally, suppose that $v_{MH}(x_0) - v_{FH}(x_0) \leq v_{MB}(x_0) - v_{FB}(x_0)$, i.e. the gender gap is maximal among bisexual people. Then, $\Delta(x_0) + \Delta_M(x_0) - \Delta_F(x_0) \geq \max\{\Delta(y), \Delta(y) + \Delta_M(y) - \Delta_F(y)\}$. This implies that

$$\int_{MBMB(x_0)} [z(x_0, y) - \Delta(x_0) - \Delta_M(x_0) - \Delta(y) - \Delta_M(y)]dG_B(y) \leq \int_{FBFB(x_0)} [z(x_0, y) - \Delta_F(x_0) - \Delta_F(y)]dG_B(y)$$

$$\int_{MBFB(x_0)} [z(x_0, y) - \Delta(x_0) - \Delta_M(x_0) - \Delta_F(y)]dG_B(y) \leq \int_{FBMB(x_0)} [z(x_0, y) - \Delta_F(x_0) - \Delta(y) - \Delta_M(y)]dG_B(y)$$

$$\int_{MBFH(x_0)} [z(x_0, y) - \Delta(x_0) - \Delta_M(x_0)]dG_H(y) \leq \int_{FBMH(x_0)} [z(x_0, y) - \Delta_F(x_0) - \Delta(y)]dG_H(y)$$

Again, all these expressions must be equalities due to (6) and (8).

First we show that $FBFB(x_0)$ is non-empty using the supermodularity of $F(\cdot, \cdot)$. Suppose that $FBFB(x_0) = \emptyset$. Then, (i) $MBMB(x_0) = \emptyset$ and (ii) $v_{FB}(x_0) = v_{FH}(x_0)$. $FHMH(x_0) \neq \emptyset$. This implies that if $FBFB(x_0) = \emptyset$

$$f(x_0, x_0) - v_{FH}(x_0) - v_{FH}(x_0) - \Delta(x_0) < f(x_0, x_0) - v_{FH}(x_0) - v_{FH}(x_0) < 0.$$

Because both sexual orientations suffer equally from discrimination, we know that for all $y \in FHMH(x_0) = MHFH(x_0)$ the level of discrimination is constant: $\Delta(y) = \Delta(x_0)$ (see the case above). Now let

$$\overline{y} = \arg\max_{y \geq x_0}[f(x_0, y) - v_{FH}(x_0) - v_{FH}(y) - \Delta(y)]$$

$$\underline{y} = \arg\max_{y \leq x_0}[f(x_0, y) - v_{FH}(x_0) - v_{FH}(y) - \Delta(y)]$$

That is, $\overline{y}$ ($\underline{y}$) is the best possible match that is larger (smaller) than x_0. Now, the surplus is $s_{FHMH}(x, y) = \frac{1}{2}[f(x, y) - v_{FH}(x) - v_{FH}(y) - \Delta(y)]$. The supermodularity of $f(\cdot, \cdot)$ gives

$$s_{FHMH}(\overline{y}, y) - s_{FHMH}(\overline{y}, x_0) > s_{FHMH}(x_0, y) - s_{FHMH}(x_0, x_0)$$

Thus, since $s_{FHMH}(x_0, x_0) < 0$ we get that $s_{FHMH}(\overline{y}, y) > s_{FHMH}(\overline{y}, x_0) + s_{FHMH}(x_0, y)$ and because for all $y \in FHMH(x_0)$ we have $\Delta(y) = \Delta(x_0)$ we have that $s_{FHMH}(\overline{y}, y) > 2s_{FHMH}(x_0, y)$. Similarly, $s_{FHMH}(\underline{y}, y) > 2s_{FHMH}(x_0, y)$. Now, because the matching set is defined as a set where the surplus is positive we get that

$$\int_{FHMH(\overline{y})} s(\overline{y}, y) dG_H(y) \geq \int_{FHMH(x_0) \cap \{y \geq x_0\}} s(\overline{y}, y) dG_H(y) \geq 2 \int_{FHMH(x_0) \cap \{y \geq x_0\}} s(x_0, y) dG_H(y)$$

$$\int_{FHMH(\underline{y})} s(\underline{y}, y) dG_H(y) \geq \int_{FHMH(x_0) \cap \{y \leq x_0\}} s(\underline{y}, y) dG_H(y) \geq 2 \int_{FHMH(x_0) \cap \{y \leq x_0\}} s(x_0, y) dG_H(y)$$

Note, that since $FHMH(x_0) \neq 0$ (for otherwise the agent marries no one and gets the lifetime utility of negative infinity), then at least one ultimate inequality in either expressions is strict.

The same proof can be constructed for $FHMB(x_0)$.[8] Thus, we conclude that

$$
\begin{aligned}
c = (1-q)\int_{FHMH(x_0)} s(x_0,y)dG_H(y) + q\int_{FHMB(x_0)} s(x_0,y)dG_B(y) = \\
(1-q)\left[\int_{FHMH(x_0)\cap\{y\le x_0\}} s(x_0,y)dG_H(y) + \int_{FHMH(x_0)\cap\{y\ge x_0\}} s(x_0,y)dG_H(y)\right] + \\
q\left[\int_{FHMB(x_0)\cap\{y\le x_0\}} s(x_0,y)dG_B(y) + \int_{FHMB(x_0)\cap\{y\ge x_0\}} s(x_0,y)dG_B(y)\right] < \\
\frac{1}{2}(1-q)\int_{FHMH(\overline{y})} s(\overline{y},y)dG_H(y) + \frac{1}{2}q\int_{FHMB(\overline{y})} s(\overline{y},y)dG_B(y) + \\
\frac{1}{2}(1-q)\int_{FHMH(\underline{y})} s(\underline{y},y)dG_H(y) + \frac{1}{2}q\int_{FHMB(\underline{y})} s(\underline{y},y)dG_B(y) = c
\end{aligned}
$$

Thus, we arrive at a contradiction, and $FBFB(x_0)$ is non-empty.

Thus, for all $y \in MBMB(x_0) = FBFB(x_0)$ we obtain $\Delta(x_0) + \Delta_M(x_0) - \Delta_F(x_0) = -\Delta(y) - \Delta_M(y) + \Delta_F(y)$ and for all $y \in MBFB(x_0) = FBMB(x_0)$ we obtain $\Delta(x_0) + \Delta_M(x_0) - \Delta_F(x_0) = \Delta(y) + \Delta_M(y) - \Delta_F(y)$. Note also that any $y \in MBMB(x_0)$ such that $\Delta(y)+\Delta_M(y)-\Delta_F(y) > 0$ also is an element of $MBFB(x_0)$ and if $\Delta(y)+\Delta_M(y)-\Delta_F(y) > 0$ it must be an element of $FBMB(x_0)$. Thus, $MBMB(x_0) \cup MBFB(x_0)$ is non-empty and therefore for all y from this set it must hold that $\Delta(y) + \Delta_M(y) - \Delta_F(y) = 0$ and therefore $\Delta(x_0) + \Delta_M(x_0) - \Delta_F(x_0) = 0$.

Appendix B: Existence

The existence proof requires a sequence of Lemmas. Lemma 1 deals with the solution to the optimal stopping problem for an arbitrary choice of value functions. Lemma 2 establishes that the mapping of value functions defined in (10) is bounded, and thus we deal with a compact set of value functions. Lemma 3 establishes the continuity of this mapping. Then the existence result follows from Schauder's fixed point theorem.

Denote $\underline{f} = f(0,0)$, $\overline{f} = f(1,1)$. Let W be a set of functions $w : T \times [0,1] \rightarrow$

[8] Note that both $\Delta_M(y)$ and $\Delta_M(x_0)$ cancel on both sides of $s_{FHMB}(\overline{y},y) - s_{FHMB}(\overline{y},x_0) > s_{FHMB}(x_0,y) - s_{FHMB}(x_0,x_0)$.

$\left[-c + (\underline{f} - K)/2, (\overline{f} + K)/2\right]$. Pick up some $\mathbf{w} = \{w_i\}_{i \in T}$. Denote

$$\pi_{ij}^{\mathbf{w}}(x, y) = \frac{f(x, y) + w_i(x) - w_j(y)}{2} a_{ij}$$

That is, payoff either equals the Nash bargaining share of the surplus or zero, if the match is not admissible. Define

$$v_i^{\mathbf{w}}(x) = \max_{\hat{A}_{ij}} \left\{ \sum_{j \in T} q_j \mathbb{E}_{j, y_t} \sum_{t=0}^{\infty} \pi_{ij}^{\mathbf{w}}[x, y_t, \hat{A}_{ij}, A_{ji}(y_t)] \right\}, \quad \text{s.t. } (1) \tag{10}$$

where $\mathbb{E}_{j,y_t}$ means that y_t is distributed according to G_j.

Lemma 1. *For any given $\mathbf{w}$ the optimal stopping problem has a solution in stationary strategies and (j, y) is accepted by (i, x) if (i, j) satisfy (1) and $\pi_{ij}^{\mathbf{w}}(x, y) \geq v_i^{\mathbf{w}}(x)$.*

Proof. Existence of the optimal stopping rule is proved in Chapter 9 of Stokey et al. (1989). Suppose, that type (i, x) is matched with type (j, y) and now has to decide whether to accept the match. Denote $v_{ij}^{\mathbf{w}}(x, y)$ the value function of this decision. If the match is accepted, then the payoff is $\pi_{ij}^{\mathbf{w}}(x, y)$. If the match is rejected, then the game continues and the payoff is $\sum_{l \in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w}}(x, z)$. Thus,

$$v_{ij}^{\mathbf{w}}(x, y) = \max \left\{ \pi_{ij}^{\mathbf{w}}(x, y), -c + \sum_{l \in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w}}(x, z) \right\} = \max \left\{ \pi_{ij}^{\mathbf{w}}(x, y), v_i^{\mathbf{w}}(x) \right\}$$

which completes the proof. □

Lemma 2. *For all $(i, x) \in T \times [0, 1]$*

$$-c + \min \left\{ 0, \frac{\underline{f} - K}{2} \right\} \leq v_i^{\mathbf{w}}(x) \leq \frac{\overline{f} + K}{2}$$

and $v_i^{\mathbf{w}}(x)$ is Lipschitz-continuous of modulus K in x.

Proof. Due to the Lipschitz-continuity of f we have $\pi_{ij}^{\mathbf{w}}(x,y) \in \left[\min\{(\underline{f}-K)/2, 0\}, (\overline{f}+K)/2\right]$. When the matching set is empty $\pi_{ij}^{\mathbf{w}}(x,y) = 0$. As in the proof of Lemma 1, let $v_{ij}^{\mathbf{w}}(x,y)$ be the value obtained by type (i,x) when matched with type (j,y). We have that

$$v_{ij}^{\mathbf{w}}(x,y) \geq \pi_{ij}^{\mathbf{w}}(x,y) \geq \min\left\{0, \frac{\underline{f}-K}{2}\right\}.$$

Thus,

$$v_i^{\mathbf{w}}(x) = -c + \sum_{l \in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w}}(x,z) \geq -c + \min\left\{0, \frac{\underline{f}-K}{2}\right\}.$$

Similarly, we have that $v_{ij}^{\mathbf{w}}(x,y) \leq (\overline{f}+K)/2$ as the best possible match is accepted if feasible, and thus $v_i^{\mathbf{w}}(x) \leq (\overline{f}+K)/2$. Now, define $v_{ij}^{\mathbf{w},0}(x,y) = \pi_{ij}^{\mathbf{w}}(x,y)$ and

$$v_{ij}^{\mathbf{w},n}(x,y) = \max\left\{\pi_{ij}^{\mathbf{w}}(x,y), -c + \sum_{l \in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w},n-1}(x,z)\right\}$$

From Lipschitz-continuity of $v_{il}^{\mathbf{w},n-1}(x,y)$ and $\pi_{ij}^{\mathbf{w}}(x,y)$ follows Lipschitz-continuity of $v_{ij}^{\mathbf{w},n}(x,y)$ and therefore of $v_{ij}^{\mathbf{w}}(x,y) = \lim_{n\to\infty} v_{ij}^{\mathbf{w},n}(x,y)$. Thus, $v_i^{\mathbf{w}}(x) = -c + \sum_{l \in T} q_l \mathbb{E}_{l,y} v_{il}^{\mathbf{w},n-1}(x,y)$ is also Lipschitz-continuous of modulus K. □

Lemma 3. *Suppose* $\mathbf{w}_s \to \mathbf{w}$ *in supp norm, then* $\mathbf{v}^{\mathbf{w}_s} \to \mathbf{v}^{\mathbf{w}}$ *in supp norm, where* $\mathbf{v}^{\mathbf{w}_s} = \left\{v_j^{\mathbf{w}_s}\right\}_{j \in T}$.

Proof. Proof is by induction. Take $v_{ij}^{\mathbf{w_s},0}(x,y) = \pi_{ij}^{\mathbf{w_s}}(x,y)$ and $v_{ij}^{\mathbf{w},0}(x,y) = \pi_{ij}^{\mathbf{w}}(x,y)$. Then

$$\min_{t,z}\left[\pi_{ij}^{\mathbf{w_s}}(t,z) - \pi_{ij}^{\mathbf{w}}(t,z)\right] \leq v_{ij}^{\mathbf{w_s},0}(x,y) - v_{ij}^{\mathbf{w},0}(x,y) \leq \max_{t,z}\left[\pi_{ij}^{\mathbf{w_s}}(t,z) - \pi_{ij}^{\mathbf{w}}(t,z)\right]$$

from which follows that

$$\min_{t,z}\min_{l \in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z) - \pi_{il}^{\mathbf{w}}(t,z)\right] \leq v_{ij}^{\mathbf{w_s},0}(x,y) - v_{ij}^{\mathbf{w},0}(x,y) \leq \max_{t,z}\max_{l \in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z) - \pi_{il}^{\mathbf{w}}(t,z)\right]$$

Now, suppose that

$$\min_{t,z}\min_{l\in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z)-\pi_{il}^{\mathbf{w}}(t,z)\right]\leq v_{ij}^{\mathbf{w_s},n-1}(x,y)-v_{ij}^{\mathbf{w},n-1}(x,y)\leq \max_{t,z}\max_{l\in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z)-\pi_{il}^{\mathbf{w}}(t,z)\right]$$

for all (x,y). Recall that $v_{ij}^{\mathbf{w},n}(x,y)=\max\left\{\pi_{ij}^{\mathbf{w}}(x,y),-c+\sum_{l\in T}q_l\mathbb{E}_{l,z}v_{il}^{\mathbf{w},n-1}(x,z)\right\}$ for all $\mathbf{w}$. Now consider four cases.

1. Suppose that $\pi_{ij}^{\mathbf{w}}(x,y)\geq\sum_{l\in T}q_l\mathbb{E}_{l,z}v_{il}^{\mathbf{w},n-1}(x,z)$ and $\pi_{ij}^{\mathbf{w_s}}(x,y)\geq\sum_{l\in T}q_l\mathbb{E}_{l,z}v_{il}^{\mathbf{w_s},n-1}(x,z)$. In this case $v_{ij}^{\mathbf{w},n}(x,y)-v_{ij}^{\mathbf{w_s},n}(x,y)=\pi_{ij}^{\mathbf{w}}(x,y)-\pi_{ij}^{\mathbf{w_s}}(x,y)$ and thus

$$\begin{aligned}\min_{t,z}\min_{l\in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z)-\pi_{il}^{\mathbf{w}}(t,z)\right]\leq\min_{t,z}\left[\pi_{ij}^{\mathbf{w_s}}(t,z)-\pi_{ij}^{\mathbf{w}}(t,z)\right]\leq\\ v_{ij}^{\mathbf{w_s},n}(x,y)-v_{ij}^{\mathbf{w},n}(x,y)\leq\\ \max_{t,z}\left[\pi_{ij}^{\mathbf{w_s}}(t,z)-\pi_{ij}^{\mathbf{w}}(t,z)\right]\leq\max_{t,z}\max_{l\in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z)-\pi_{il}^{\mathbf{w}}(t,z)\right]\end{aligned}$$

2. Suppose that $\pi_{ij}^{\mathbf{w}}(x,y)<\sum_{l\in T}q_l\mathbb{E}_{l,z}v_{il}^{\mathbf{w},n-1}(x,z)$ and $\pi_{ij}^{\mathbf{w_s}}(x,y)<\sum_{l\in T}q_l\mathbb{E}_{l,z}v_{il}^{\mathbf{w_s},n-1}(x,z)$. In this case we have

$$v_{ij}^{\mathbf{w_s},n}(x,y)-v_{ij}^{\mathbf{w},n}(x,y)=\sum_{l\in T}q_l\mathbb{E}_{l,z}[v_{il}^{\mathbf{w_s},n-1}(x,z)-v_{il}^{\mathbf{w},n-1}(x,z)]$$

For all (x,z) we have

$$\begin{aligned}\sum_{l\in T}q_l\mathbb{E}_{l,z}[v_{il}^{\mathbf{w_s},n-1}(x,z)-v_{il}^{\mathbf{w},n-1}(x,z)]\geq\\ \sum_{l\in T}q_l\min_{t,z}\min_{l\in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z)-\pi_{il}^{\mathbf{w}}(t,z)\right]\geq\\ \min_{t,z}\min_{l\in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z)-\pi_{il}^{\mathbf{w}}(t,z)\right]\end{aligned}$$

where the first inequality is due to our induction assumption. Similarly

$$\sum_{l\in T} q_l \mathbb{E}_{l,z}[v_{il}^{\mathbf{w_s},n-1}(x,z) - v_{il}^{\mathbf{w},n-1}(x,z)] \leq \sum_{l\in T} q_l \max_{t,z}\max_{l\in T}[\pi_{il}^{\mathbf{w_s}}(t,z) - \pi_{il}^{\mathbf{w}}(t,z)] \leq \max_{t,z}\max_{l\in T}[\pi_{il}^{\mathbf{w_s}}(t,z) - \pi_{il}^{\mathbf{w}}(t,z)]$$

3. Suppose that $\pi_{ij}^{\mathbf{w}}(x,y) \geq \sum_{l\in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w},n-1}(x,z)$ and $\pi_{ij}^{\mathbf{w_s}}(x,y) < \sum_{l\in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w_s},n-1}(x,z)$. In this case we get

$$v_{ij}^{\mathbf{w_s},n}(x,y) - v_{ij}^{\mathbf{w},n}(x,y) = \sum_{l\in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w_s},n-1}(x,z) - \pi_{ij}^{\mathbf{w}}(x,y)$$

Note that in this case

$$\sum_{l\in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w_s},n-1}(x,z) - \pi_{ij}^{\mathbf{w}}(x,y) \leq \pi_{ij}^{\mathbf{w_s}}(x,y) - \pi_{ij}^{\mathbf{w}}(x,y)$$

and due to case 1 we have

$$v_{ij}^{\mathbf{w_s},n}(x,y) - v_{ij}^{\mathbf{w},n}(x,y) \leq \max_{t,z}\max_{l\in T}[\pi_{il}^{\mathbf{w_s}}(t,z) - \pi_{il}^{\mathbf{w}}(t,z)]$$

Also, because

$$\sum_{l\in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w_s},n-1}(x,z) - \pi_{ij}^{\mathbf{w}}(x,y) \geq \sum_{l\in T} q_l \mathbb{E}_{l,z}[v_{il}^{\mathbf{w_s},n-1}(x,z) - v_{il}^{\mathbf{w},n-1}(x,z)]$$

from case 2 we obtain

$$v_{ij}^{\mathbf{w_s},n}(x,y) - v_{ij}^{\mathbf{w},n}(x,y) \geq \min_{t,z}\min_{l\in T}[\pi_{il}^{\mathbf{w_s}}(t,z) - \pi_{il}^{\mathbf{w}}(t,z)]$$

4. Suppose that $\pi_{ij}^{\mathbf{w}}(x,y) < \sum_{l\in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w},n-1}(x,z)$ and $\pi_{ij}^{\mathbf{w_s}}(x,y) \geq \sum_{l\in T} q_l \mathbb{E}_{l,z} v_{il}^{\mathbf{w_s},n-1}(x,z)$.

This case in analogous to case 3.

We conclude that

$$\min_{t,z}\min_{l\in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z)-\pi_{il}^{\mathbf{w}}(t,z)\right]\leq\min_{t,z}\left[\pi_{ij}^{\mathbf{w_s}}(t,z)-\pi_{ij}^{\mathbf{w}}(t,z)\right]\leq$$
$$v_{ij}^{\mathbf{w_s},n}(x,y)-v_{ij}^{\mathbf{w},n}(x,y)\leq$$
$$\max_{t,z}\left[\pi_{ij}^{\mathbf{w_s}}(t,z)-\pi_{ij}^{\mathbf{w}}(t,z)\right]\leq\max_{t,z}\max_{l\in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z)-\pi_{il}^{\mathbf{w}}(t,z)\right]$$

and by taking limit with respect to n obtain

$$\min_{t,z}\min_{l\in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z)-\pi_{il}^{\mathbf{w}}(t,z)\right]\leq\min_{t,z}\left[\pi_{ij}^{\mathbf{w_s}}(t,z)-\pi_{ij}^{\mathbf{w}}(t,z)\right]\leq$$
$$v_{ij}^{\mathbf{w_s}}(x,y)-v_{ij}^{\mathbf{w}}(x,y)\leq$$
$$\max_{t,z}\left[\pi_{ij}^{\mathbf{w_s}}(t,z)-\pi_{ij}^{\mathbf{w}}(t,z)\right]\leq\max_{t,z}\max_{l\in T}\left[\pi_{il}^{\mathbf{w_s}}(t,z)-\pi_{il}^{\mathbf{w}}(t,z)\right]\quad(11)$$

Now, as regards $\mathbf{w}_s\to\mathbf{w}$ we have $\pi_{ij}^{\mathbf{w_s}}(t,z)\to\pi_{ij}^{\mathbf{w}}(t,z)$ for all i,j,t,z we conclude that both sides of (11) approach zero as $\mathbf{w}_s\to\mathbf{w}$ and therefore $v_{ij}^{\mathbf{w_s}}(x,y)\to v_{ij}^{\mathbf{w}}(x,y)$ which implies that $\mathbf{v}^{\mathbf{w}_s}\to\mathbf{v}^{\mathbf{w}}$. □

Finally, since $\mathbf{v}^{\mathbf{w}}$ is a continuous mapping of W onto itself and W is a compact subset of Banach space (due to Lemma 3) we obtain existence by the application of Schauder's theorem.

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