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Abstract

Why do contests exist in settings where negotiation provides a costless alternative? I assess a new explanation: parties may be overconfident about their ability or optimistic about their chances of winning. For both parties in a contest, this hubris: (i) reduces the incentive to exit the contest; (ii) reduces effort; and (iii) increases expected payoffs. Whilst hubris leads to the contest being preferred to costless negotiation, the welfare loss is nonmonotonic in either behavioural bias. Keywords: Contests; Optimism bias; Overconfidence bias; Negotiation. JEL: C71, D74, D91.

1 Introduction

From political campaigning to litigation, contests are a widely-observed source of inefficiency. Resources are employed in an attempt to win a prize – power, compensation – that could otherwise be productive. Why contests arise in settings where negotiation provides a costless alternative, is an open question.

Several explanations already exist. The prize may be indivisible, as in a patent race, precluding a settlement (Loury 1979). There may be information problems, as in gang wars, that prevent agreement (Bester and Wärneryd 2006), or cause contests to generate valuable signals (Long 2015). The immediate cost of a contest may be outweighed by long-run benefits (Garfinkel and Skaperdas 2000).

Overconfidence is often cited as a driver of an important form of contest: war (Johnson 2004; Hardie et al. 2011). In this context, it refers to two, related, behavioural biases.

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Overconfidence bias causes individuals to overestimate their ability relative to others’ [Hirshleifer and Luo 2001]. Optimism bias causes them to overestimate the probability of good outcomes [Brunnermeier and Parker 2005].

I assess conditions under which these biases cause negotiations failures that trigger contests, and derive their welfare implications. Proofs are presented in online appendices. Two players, with different biases, must determine the division of a common-value prize. They first engage in costless negotiation. If agreement is reached, the prize is divided and the game ends. Disagreement triggers a generalised Tullock contest [Tullock 1980]. Players simultaneously exert effort to increase their probability of winning the entire prize. Overconfidence and optimism both cause the players to overestimate their chances of winning, so I refer to them collectively as hubris.

Several behavioural contests have previously been analysed, including those incorporating utility from winning, prosocial preferences, status-seeking and bounded rationality (see Sheremeta 2015 for a review). Closest to the current work is one on cumulative loss aversion [Baharad and Nitzan 2008]. Both cumulative loss aversion and hubris distort players’ perception of their probability of winning, although in different ways. I also differ by allowing for ex ante negotiation and considering heterogeneous biases.

2 Model

Two risk-neutral players, $i$ and $j$, are required to split a divisible prize with common value $V$. The game proceeds in two stages, outlined in Figure 1. First, players negotiate. Without loss of generality, this takes the form of an ultimatum game. $i$ offers $V_j$ to $j$, keeping $V - V_j$ for herself.

If $j$ accepts $i$’s offer, the prize is shared and the game ends. If $j$ rejects, players enter a generalised Tullock contest. They simultaneously choose effort, costing $x_i, x_j \geq 0$. $V$ is then awarded by means of a lottery, where $i$’s probability of winning is:

$$p(x_i, x_j) = \begin{cases} \frac{x_i}{x_i + x_j}, & x_i > 0 \text{ or } x_j > 0 \\ \frac{1}{2}, & x_i = x_j = 0 \end{cases},$$

and $r > 0$.

When the game is played between unbiased objective players, it is straightforward to show that negotiation always succeeds in pure strategy subgame perfect equilibrium. The contest stage is never reached. That we often observe contests in settings where

\footnote{Available at https://sites.google.com/site/ianlongecon/research}
such negotiation is possible is puzzling. What could cause negotiation to fail, and the
contest to be triggered, in subgame perfect equilibrium?

I propose a new answer: both players suffer from (heterogeneous) behavioural biases.
Overconfidence bias causes $i$ to incorrectly multiply $x_i^r$ by some $\sigma_i \geq 1$ in $p(x_i, x_j)$,
following the asymmetric abilities contest success function of Tullock (1980). Optimism
bias causes her to raise $p(x_i, x_j)$ to the power $1 - \theta_i$, where $\theta_i \in [0, 1]$. Whilst the true
structure of the game is unchanged, her subjective probability of winning becomes:

$$
\mu(x_i, x_j|\sigma_i, \theta_i) = \begin{cases} 
\left(\frac{\sigma_i x_i^r}{\sigma_i x_i^r + x_j^r}\right)^{1 - \theta_i}, & \text{if } x_i > 0 \text{ or } x_j > 0 \\
\left(\frac{1}{2}\right)^{1 - \theta_i}, & \text{if } x_i = x_j = 0
\end{cases}.
$$

Player $j$ is similarly afflicted, and so players’ perceptions of $p(x_i, x_j)$ diverge.

The biases’ effects are illustrated in Figure 2. Dashed lines represent the (correct)
beliefs of objective players. Thinner solid lines represent larger biases. For any $(x_i/x_j)^r$, 
overconfidence increases $\mu(x_i, x_j|\sigma_i, \theta_i)$ by causing $i$ to overestimate the impact of her
effort on her chances of winning (Panel a). Optimism directly increases her subjective
probability (Panel b). Since both biases have qualitatively identical effects, I refer to
them collectively as hubris, and a player who suffers from either as hubristic.

Following earlier investigations of behavioural contests, I assume that each player
observes her opponent’s biases. This ensures that any negotiation failure arises solely
from the existence of hubris, rather than from information problems.

Although players are hubristic, they are otherwise rational. Their actions are chosen to maximise their expected payoff, even though those expectations are incorrect. The equilibrium concept employed is subgame perfection.

3 Equilibrium

The game is solved by backward induction, starting with the solution to the contest.

3.1 Contest

Player $i$’s expected payoff from the contest is:

$$
\pi_i(x_i, x_j; \sigma_i, \theta_i) = \mu(x_i, x_j; \sigma_i, \theta_i) V - x_i.
$$

(2)

She chooses effort to maximise (2), given $j$’s effort. Her best response, $\hat{x}_i(x_j)$, is implicitly defined by:

$$
\frac{\partial \mu}{\partial x_i} \bigg|_{x_j, \sigma_i, \theta_i} V - 1 = 0
$$

$$
\iff \frac{r(1 - \theta_i)\sigma_i^{1-\theta_i}\hat{x}_i(x_j)^{r(1-\theta_i)-1}x_j^r}{[\sigma_i\hat{x}_i(x_j)^r + x_j^{r-\theta_i}]} V - 1 = 0.
$$

(3)
For a given $x_j$, $\hat{x}_i$ is only affected by $i$’s own hubris. $i$ views $j$’s hubris as a mistake, and does not adjust her effort in response to changes in its magnitude. $j$’s best response is similarly defined.

**Proposition 1** If:

$$ r \leq \min \left\{ \frac{2 - \theta_i}{1 - \theta_i}, \frac{2 - \theta_j}{1 - \theta_j} \right\}, \tag{4} $$

then a unique pure strategy effort profile, $\{x^*_i, x^*_j\}$, solves the contest subgame.

Condition (4) ensures that neither player prefers exiting the contest to exerting effort according to (3), which is necessary and sufficient for the existence of a pure strategy effort profile that solves the contest. Suppose that $x_j = \tilde{x}_j$ is large enough that $\pi_i[\hat{x}_i(\tilde{x}_j), \tilde{x}_j; \sigma_i, \theta_i] < 0$. $i$ prefers $x_i = 0$, which yields $\pi_i = 0$, to $x_i = \hat{x}_i(\tilde{x}_j)$. $j$ now has a profitable deviation, as any $x_j > 0$ guarantees her the prize. By exerting effort arbitrarily close to zero, she receives $\pi_j = V > V - \hat{x}_j$. $\{0, \hat{x}_j\}$ cannot be a solution to the contest.

If $\theta_i = 0$ or $\theta_j = 0$ then (4) reduces to $r < 2$. This standard condition for existence of a pure strategy solution with objective players ensures that some rents are not dissipated. As players become optimistic, their beliefs diverge in their favour and (4) is relaxed. Increasingly convinced that they will win, each player is willing to endure higher effort by their opponent and remain in the contest.

The contest solution is illustrated in Figure 3 at the intersection of players’ best responses. $\hat{x}_i$, for example, initially involves positive effort, before discontinuously dropping to $x_i = 0$ when $x_j$ becomes large enough to cause $i$ to exit. (4) ensures that the discontinuities always occur at effort levels above that of the contest solution. When $i$’s hubris increases, she is willing to remain in the contest at levels of $x_j$ that would previously have caused her to exit.

### 3.2 Negotiation

At the negotiation stage, players’ disagreement utilities are their expected payoffs in the contest subgame: $\pi^*_i = \pi(x^*_i, x^*_j; \sigma_i, \theta_i)$ and $\pi^*_j = \pi(x^*_j, x^*_i; \sigma_j, \theta_j)$. $j$ will accept $i$’s offer if and only if $V_j \geq \pi^*_j$.

There are two possibilities. If $\pi^*_i + \pi^*_j \leq V$, $i$ offers $V_j = \pi^*_j$, which $j$ accepts. $i$ receives $V - \pi^*_j \geq \pi^*_i$. The unique pure strategy subgame perfect equilibrium involves successful negotiation.

If:

$$ \pi^*_i + \pi^*_j > V, \tag{5} $$

5
making an acceptable offer would leave \( i \) with \( V - V_j < \pi_i^* \). \( i \) prefers to trigger the contest. She offers some \( V_j < \pi_j^* \), which \( j \) rejects. In any pure strategy subgame perfect equilibrium, negotiations fail and the contest is triggered.

With objective players, \( \pi_i^* + \pi_j^* = V - x_i^* - x_j^* < V \), so negotiations succeed. With hubristic players, this is no longer necessarily the case:

**Proposition 2** If players are sufficiently hubristic, then negotiations fail in pure strategy subgame perfect equilibrium.

To illustrate the intuition, consider first the effect of an increase in \( i \)'s hubris on the effort profile that solves the contest (Figure 3). Although the direct impact on \( i \)'s best response is nonmonotonic, \( \tilde{x}_i \) can be shown to always decline at \( x_j^* \). Moreover, \( \{x_i^*, x_j^*\} \) is always found in a range in which \( x_i \) and \( x_j \) are strategic complements. Any decline in \( \tilde{x}_i \) causes a reduction in \( x_j \) which, in turn, induces further declines in \( x_i \). In the new contest solution, **both** players exert less effort.

Now consider the effect on players’ expected payoffs from the contest. For given effort levels, an increase in \( i \)'s hubris directly increases her expected payoff by further distorting her subjective probability of winning (Figure 2). If she then chooses to unilaterally reduce \( x_i \) below \( x_i^* \), her expected payoff must increase further. This fall in \( x_i \) increases \( j \)'s expected payoff, as it becomes more likely that she wins the contest. If \( j \) subsequently
chooses to reduce \( x_j \) below \( x_j^* \), both players’ payoffs must increase again. An increase in \( i \)'s hubris increases both \( \pi_i^* \) and \( \pi_j^* \).

In (5), \( \pi_i^* + \pi_j^* \) is thus increasing in either player’s hubris. Mildly hubristic players are close enough to objective that negotiation is still preferable. However, as hubris increases along either dimension, (5) is eventually satisfied and negotiations fail. The more overconfident a player is, the less optimistic they need to be to trigger the contest.

### 3.3 Welfare

Social welfare is given by:

\[
W = \begin{cases} 
V, & \pi_i^* + \pi_j^* \leq V \\
V - x_i^* - x_j^*, & \pi_i^* + \pi_j^* > V
\end{cases}
\]

If negotiations succeed, players share the whole prize. If they fail, one player wins the prize, whilst both expend effort.

**Proposition 3** Social welfare is nonmonotonic in hubris.

For low levels of hubris, the unique pure strategy subgame perfect equilibrium outcome is a negotiated solution. No welfare is lost. As hubris increases, negotiations eventually fail and the contest is triggered. Welfare discontinuously declines at this point. Further increases in hubris cause both players reduce their equilibrium contest effort (Figure 3). Welfare is subsequently increasing hubris.

### 4 Conclusion

From lobbying to litigation, parties often incur costs in an attempt to secure a prize. Yet, in many settings, negotiation could generate the same expected shares of the prize without the need to incur any costs. Why, then, are contests so prevalent? One possibility is that participants are hubristic. Each is sufficiently convinced that they would win any resulting contest that their inflated claims cannot both be satisfied during negotiations.

Whilst hubris would still likely cause negotiation failure under alternative contest success functions, the welfare implications may differ. The model could also be extended to more than two players. Multiple players would likely cause negotiation failure at lower levels of hubris, as a greater number of inflated claims would need to be met. The opponent’s biases are observable in the model, to highlight the effect of hubris.
Unobservable biases may cause more negotiation failure, due both to hubris and information problems. Introducing dynamics may eventually allow players to learn about, and correct, their biases, enabling successful negotiation.

References


