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Patrick Minford, Zhirong Ou and Zheyi Zhu

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Cardiff Business School
Cardiff University
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
business.cardiff.ac.uk

Can a small New Keynesian model of the world economy with risk-pooling match the facts?

Patrick Minford
Cardiff University and CEPR

Zhirong Ou*
Cardiff University

Zheyi Zhu
Cardiff University

Abstract

We ask whether a model of the US and Europe trading with the rest of the world can match the facts of world behaviour in a powerful indirect inference test. One version has uncovered interest parity (UIP), the other risk-pooling. Both pass the test but the most probable is risk-pooling. This is consistent with risk-pooling failing a number of single equation tests, as has been found in past work; we show that these tests will typically reject risk-pooling when it in fact prevails. World economic behaviour under risk-pooling shows much stronger spillovers than under UIP with opposite monetary responses to the exchange rate. We argue that the risk-pooling model therefore demands more attention from policy-makers.

Keywords: Open economy, UIP, risk-pooling, test, Indirect Inference

JEL Classification: C12, E12, F41

1 Introduction

In this paper we have a twofold empirical aim: to discover whether a three-country New Keynesian model of the world economy can match world data behaviour and as part of that endeavour whether a risk-pooling variant of that model can also do so. We do this using a testing and estimation approach, indirect inference, that has been found in recent work (see Le, et al., 2016, and Meenagh, et al., 2018 for comprehensive surveys of this work) to heavily dominate other non-Bayesian methods in the small samples we typically have to deal with in open economy macroeconomics. When as here there are fundamental questions of what modelling assumptions are appropriate, and these are the very things we wish to test, Bayesian methods, which rely on generally agreed priors, cannot be used. To anticipate our results, we find that they overturn much of the conventionally believed previous empirical findings on the issues here. It is therefore important for readers to be thoroughly aware of the power of the indirect inference methods we use, even though they are not yet widely familiar among open economy macroeconomists.

A number of efforts have been made to create a DSGE New Keynesian model of the world economy with several countries, usually the US, Europe, and the rest of the world. These models however so far have not been shown to be able to match data behaviour according to the powerful indirect inference test- for example, Chari, et al. (2002), Kollman (2016) and Le, et al. (2010). These models have been estimated in various ways but the general consensus has been that while some moments can be matched a general matching of such a model to the facts of the world economy is not possible. Yet much success has been reported in matching single economy models to these economies' data behaviour; for example, DSGE models of the US (e.g. Le, Meenagh and Minford, 2016) and China (e.g. Le et al., 2014) separately successfully match those economies' facts.

*Corresponding author. Address: B14, Aberconway building, Colum Drive, Cardiff, UK, CF10 3EU. Tel: +44 (0)29 2087 5190. Email: ouz@cardiff.ac.uk

On the particular issue of consumer risk-pooling across borders it is generally agreed according to a variety of direct empirical tests that there is no evidence of it or even of a weaker version of it in the form of uncovered interest parity, UIP. Examples are for UIP Delcoure, et al. (2003) and Isard (2006), and for consumption risk-pooling Obstfeld (1989), Backus and Smith (1993), Canova and Ravn (1996), Crucini (1999), Hess and Shin (2000), Razzak (2013). The empirical testing in this work has been via predictive tests on the exchange rate based on the single equation relationship for UIP or regression of the single equation for risk-pooling; cointegration tests are also used. However there are considerable difficulties with these approaches, which we deal with carefully below. In this paper we embed these relationships in a full DSGE model and test the model as a whole. Non-rejection implies success of the model in all its parts.

In this paper we attempt to find a simple New Keynesian model of the world economy where in essence we take the three-equation set-up of Gali, et al. (1999) and extend it to embrace three economies, the US, Europe and the rest of the world (but this last included only for its trade and not in a complete model since our focus here is on the behaviour of the major developed countries when linked together). We then use our powerful indirect inference test of its ability to match the data behaviour in the two economies. It turns out that because the cross-equation restrictions on a three-country model are dense, the empirical tests we are using may have been set at too demanding a level which has misled some previous researchers, including some of us ourselves, into premature dismissal of these models.

We focus in particular on the capital movement relationships in these models: UIP and consumer risk-pooling. With highly sophisticated financial markets capable of providing insurance it has seemed a puzzle that the evidence noted above does not favour either UIP or risk-pooling. However one of the problems in assessing this evidence has been that all the variables in these hypotheses are endogenous, creating difficult econometric issues.

Given the financial crisis and the upheavals it has caused in monetary and regulatory policy, we have had to approach monetary and related policy issues in a way that would not complicate our simple set-up by creating non-linear regime switches to the zero bound and the accompanying adoption of Quantitative Easing (QE, aggressive open market operations) and stringent bank regulation. These are important issues, tackled in recent work for the US by Le, et al. (2016, 2018) in the context of the closed continental US economy. Instead of focusing on these issues by such means, we assume that the relevant interest rate in these models is the corporate bond rate (AAA rated corporate bond yield for US, and equally weighted average of France and Germany Corp bond yield rate for EA). This rate did not hit the zero bound, unlike the rate on government bonds. By implication of this choice of reference interest rate, we think of monetary policy as influencing it by various policy means, including bank regulation, QE and direct changes in central bank lending/deposit rates for banks (which of course have gone negative at times). Thus our Taylor Rule relates to this commercial credit rate according to this interpretation.

In the rest of this paper we first describe the model, in section 2, in both its standard version with uncovered interest parity and non-contingent bonds and also its risk-pooling version with fully contingent bonds. In section 3 we go on to test the two versions, after estimation, against the data behaviour of the two countries: we carefully discuss the way our testing method works and the power of the test we use. In section 4 we compare and contrast the two versions, in their responses to shocks. Section 5 concludes.

2 A simple open economy framework

2.1 The standard model with non-contingent bonds

We model a world economy comprising two countries (US and EA) and the rest of the world. The US and EA share the same model structure, while the rest of the world is included to pick up trade happening indirectly between the US and EA economies. To save space, we present in the following the basic model structure from the point of view of US that we refer to as the 'home' country (denoted with subscript H). Unless necessary, we omit to present the EA economy, which is 'foreign' (denoted with subscript F) to US and have variables denoted with asterisk. In Appendix A we provide the full listing of the log-linearised model.

2.1.1 Households

The representative household's preference is given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \epsilon_t \left(\frac{1}{1-\sigma} C_t^{1-\sigma} - \frac{1}{1+\varphi} {N_t}^{1+\varphi} \right) \tag{1}$$

where β is the discount factor, ϵ_t is the time preference shock, σ is the inverse of consumption elasticity, φ is the Inverse of labour elasticity, and C_t is the aggregate consumption index defined as:

$$C_t = \left[(1-\alpha)^{\frac{1}{\eta}} {C_{H,t}}^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $C_{H,t} = \left[\int_0^1 C_t(h)^{\frac{\gamma-1}{\gamma}} dh \right]^{\frac{\gamma-1}{\gamma}}$ is the CES index of goods produced in home country, and $C_{F,t} = \left[\int_0^1 C_t(f)^{\frac{\gamma^*-1}{\gamma^*}} df \right]^{\frac{\gamma^*-1}{\gamma^*}}$ is that of goods produced in foreign country. $\eta > 0$ is the degree of substitution between domestic and foreign goods (Armington, 1969). α is the degree of openness (and we assume $\alpha^* = \alpha$). $\gamma, \gamma^* > 0$ are the price elasticities of differentiated goods.

We assume complete financial market in both the home and foreign economies. Households who invest both in domestic and foreign bonds, B_t and B_t^*, have budget constraint:

$$P_t C_t + E_t \left(\frac{B_{t+1}}{1+R_t} \right) + E_t \left(\frac{S_t B_{t+1}^*}{1+R_t^*} \right) \leq B_t + S_t B_t^* + W_t N_t + TR_t \tag{3}$$

where $P_t = \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$ is the consumer price index (See Gali and Monacelli, 2005), S_t is the nominal exchange rate defined as units of home currency per unit of foreign currency (\$/¢), R_t and R_t^* are the home and foreign nominal interest rates, W_t is the nominal wage, and TR_t is the lump-sum transfer.

The optimisation problem of households is to maximise (1), subject to (3), by choosing C_t, N_t, B_{t+1} and B_{t+1}^*. The optimal conditions with respect to C_t and B_{t+1} lead to the Euler equation:

$$\beta \frac{E_t \epsilon_{t+1}}{\epsilon_t} \left(\frac{E_t C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{E_t P_{t+1}} (1+R_t) = 1 \tag{4}$$

which can be log-linearised to be:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} \left(R_t - E_t \pi_{t+1} - \bar{r} + E_t \ln \epsilon_{t+1} - \ln \epsilon_t \right) \tag{5}$$

where $c_t = \ln C_t$, $E_t \pi_{t+1} = E_t(\ln P_{t+1}) - \ln P_t = E_t p_{t+1} - p_t$ is the expected CPI inflation, and $\bar{r} = \beta^{-1} - 1$ is the steady-state real interest rate.

The optimal conditions with respect to B_{t+1} and B_{t+1}^* imply:

$$\frac{1+R_t}{1+R_t^*} = \frac{E_t S_{t+1}}{S_t} \tag{6}$$

which can be log-linearised to find the uncovered interest parity (UIP):

$$R_t - R_t^* = E_t \left(s_{t+1} - s_t \right) \tag{7}$$

where $s_t = \ln S_t$. Let real exchange rate be $Q_t = \frac{S_t P_t^*}{P_t}$ and therefore $q_t = s_t + p_t^* - p_t$ in log-linearised form. The UIP condition can be rewritten in real terms as:

$$E_t q_{t+1} - q_t = (R_t - E_t \pi_{t+1}) - (R_t^* - E_t \pi_{t+1}^*) \tag{8}$$

2.2 Firms

Following Calvo (1983), we let a fraction $(1-\theta)$ of firms re-optimise prices $P_t(h)$ in each period, while the rest θ keep theirs. Firms resetting prices maximise:

$$E_0 \sum_{k=0}^{\infty} \theta^k M_{t,t+k} \left[P_t(h) Y_{t+k}(h) - P_{H,t+k} MC_{t+k} Y_{t+k}(h) \right] \tag{9}$$

by choosing $P_t(h)$, subject to demand:

$$Y_{t+k}(h) = \left(\frac{P_t(h)}{P_{H,t+k}} \right)^{-\gamma} Y_{t+k} \tag{10}$$

where MC_{t+k} is the real marginal cost at $t+k$. The first order condition implies the optimal reset price $\tilde{P}_t(h)$, which can be log-linearised and combined with the price index of domestic goods ($P_{H,t} = \left[\int_0^1 P_t(h)^{1-\gamma} dh \right]^{\frac{1}{1-\gamma}}$) linearised around a zero-inflation steady-state as usual, to find the new Keynesian Phillips curve for domestic inflation:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \lambda \widehat{mc}_t \tag{11}$$

where $\lambda = \frac{(1-\beta\theta)(1-\theta)}{\theta}$, and '^' denotes the percentage deviation of a variable from the steady-state level.

In open economy, the general price level reflects also imported products. Since the general price index in log-linearised form is:

$$p_t = (1-\alpha) p_{H,t} + \alpha p_{F,t} \tag{12}$$

the CPI inflation can be shown as:

$$\pi_t = (1-\alpha)\pi_{H,t} + \alpha(p_{F,t} - p_{F,t-1}) \tag{13}$$

which can be further simplified, using the real exchange rate equation ($q_t = s_t + p_t^* - p_t$), to:

$$\pi_t = \pi_{H,t} + \frac{\alpha}{1-2\alpha}(q_t - q_{t-1}) \tag{14}$$

Substituting (14) into (11), it yields the open-economy New Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \lambda \widehat{mc}_t - \frac{\alpha}{1-2\alpha} \left[\beta E_t (q_{t+1} - q_t) - (q_t - q_{t-1}) \right] \tag{15}$$

To find the real marginal cost in (15), let the production function of the whole economy be:

$$Y_t = A_t N_t \tag{16}$$

where A_t is productivity and N_t is labour input. The unit cost of output is W_t/Y_t where W_t is the nominal wage rate. The real marginal cost per unit of output is therefore:

$$\frac{\partial (W_t/P_{H,t})/Y_t}{\partial N_t} = \frac{W_t}{P_{H,t} A_t} \tag{17}$$

which can be log-linearised to:

$$mc_t = w_t - p_{H,t} - a_t \tag{18}$$

where $w_t = \ln W_t$ and $a_t = \ln A_t$.

2.3 The IS-PC-Taylor rule model

The above model can be condensed to the well-known IS-Phillips curves model where for our 'world model' variant here we also assume the following trade equations (All expressed in log; again, seeing US as the home economy):

Home import from foreign country:

$$im^{US}_{EA,t} = \mu y_t - \psi q_t \tag{19}$$

Home import from the rest of the world[1]:

$$im^{US}_{W,t} = \nu y_t \tag{20}$$

Trade balance of the world economy:

$$\Xi im^{US}_{W,t} + (1-\Xi) im^{EA}_{W,t} = \digamma ex^{US}_{W,t} + (1-\digamma) ex^{EA}_{W,t} \tag{21}$$

where Ξ and $\digamma$ are the steady-state import/export ratios, and the LHS of the equation can be seen as the output of the rest of the world:

$$y^{RoW}_t = \Xi im^{US}_{W,t} + (1-\Xi) im^{EA}_{W,t}$$

The World's relative demand for US and EA products is set by:

$$ex^{US}_{W,t} = ex^{EA}_{W,t} + \psi^{RoW} q_t \tag{22}$$

Assume that the home economy clears at $Y_t = C_t + NX_t$ where NX_t is the net export. The national income identity can be log-linearised and combined the Euler equation and the trade equations to find the IS curve of the home economy:

$$y_t = E_t y_{t+1} - c\frac{1}{\sigma}\Theta(R_t - E_t\pi_{t+1} - \bar{r}) - xz_1\Theta E_t \Delta y^*_{t+1} - xz_2\Theta E_t \Delta y^{RoW}_{t+1} - xz_4 \Theta E_t \Delta q_{t+1} + \varepsilon^{IS}_t \tag{23}$$

where c and x are the steady-state consumption and export ratios, Θ, z_1, z_2 and z_4 are combinations of structural parameters, and ε^{IS}_t is the equation error which can be interpreted as the demand shock (See details of derivation in Appendix B).

The Phillips curve can be rewritten to reflect the relationship between CPI inflation and the 'output gap' by combining (15) and (18), using the national income identity, to be:

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa_a (y_t - y^p_t) - \frac{\alpha}{1-2\alpha}\left[\beta E_t(q_{t+1} - q_t) - (q_t - q_{t-1})\right] + \varepsilon^{PP}_t \tag{24}$$

where $\kappa_a = \lambda(\sigma\frac{1}{c}\Theta^{-1} + \varphi)$, and ε^{PP}_t is the supply shock. In particular, we assume that 'potential output' y^p_t follows a random walk process with drift (in the log form) as:

$$y^p_t - y^p_{t-1} = \Gamma^{y^p} + \delta(y^p_{t-1} - y^p_{t-2}) + \varepsilon^{yp}_t \tag{25}$$

to reflect permanent impact of the productivity shock (ε^{yp}_t). Γ^{y^p} in (25) is the deterministic trend of the potential output, and $\delta < 0$ ensures that the process is trend-stationary.

[1] We assume import from RoW is only affected domestic income for simplicity.

The model can be closed by setting a rule for monetary policy which we let it follow a Taylor rule:

$$R_t = \rho R_{t-1} + (1-\rho)[\phi_\pi \pi_t + \phi_y (y_t - y_t^p)] + \phi_q (q_t - q_t^{ss}) + \varepsilon_t^R \tag{26}$$

where ρ measures the inertia of policy, ϕ_π and ϕ_y are the responses to inflation and output, and ε_t^R is the policy error. Here we allow for international monetary cooperation such that monetary policy also responds to fluctuations of the real exchange rate. On this occasion, home interest rate rises if home currency depreciates; the responsiveness is measured by ϕ_q.

Thus, equations (19) - (26), together with the UIP condition (8) and the 'foreign' equations omitted for the EA, constitute a simple 'world' model that we list in full in Appendix A and treat as the benchmark model.

2.4 The risk-pooling and UIP variants of the model

As was noted by Chari et al. (2002), given that this model has bond UIP via non-contingent nominal bonds, it produces real UIP which generates expected risk-pooling from an initial position. That is to say that from wherever the real exchange rate is today, it is expected that future consumption in the two countries will move together adjusting for movement in the real exchange rate. This comes about because the real interest rate differential is equal to the expected change in the real exchange rate (due to UIP) and also to the expected change in the consumption differential (adjusted for the risk-aversion parameter) due to the two Euler equations. Therefore expected consumption will move together in the two countries apart from the effect of the changing real exchange rate.

This risk-pooling is 'dynamic' because it is disturbed by shocks to consumption preferences (there is no shock to UIP in the model because second moments are all constant). Viewed over time from some initial date, risk-pooling is close to being delivered (exactly if utility is logarithmic in consumption). But there is no insurance against preference shocks. If however consumers have access to contingent nominal bonds, full risk-pooling occurs, insuring against all shocks, so that the real exchange rate is deterministically related to the ratio of foreign to home consumption.

This can be shown formally as follows — following Chari, et al. (2002):

a) full risk-pooling via state-contingent nominal bonds:

let the price at time t=0 (when the state was x_0) of a home nominal state-contingent bond paying 1 pound (home currency) in state x_t be:

$$n(x_t, x_0) = \beta f(x_t, x_0) \frac{U_c(x_t, x_0)}{P(x_t, x_0)} / \frac{U_c(x_0)}{P(x_0)} \tag{27}$$

where β is time-preference and $f(x_t, x_0)$ is the probability of x_t occurring given x_0 has occurred. Now note that foreign consumers can also buy this bond freely via the foreign exchange market (where S is home currency per foreign currency as above) and its value as set by them will be:

$$n(x_t, x_0) = \beta f(x_t, x_0) \frac{U_c^*(x_t, x_0) S(x_t, x_0)}{P^*(x_t, x_0)} / \frac{U_c^*(x_0) S(x_0)}{P^*(x_0)} \tag{28}$$

Here they are equating the expected marginal utility of acquiring this pound bond with foreign currency, with the marginal utility of a unit of foreign currency at time 0. Plainly the price paid by the foreign consumer must be equal by arbitrage to the price paid by the home consumer. Equating these two equations yields:

$$\frac{U_c(x_t, x_0)}{P(x_t, x_0)} / \frac{U_c(x_0)}{P(x_0)} = \frac{U_c^*(x_t, x_0) S(x_t, x_0)}{P^*(x_t, x_0).} / \frac{U_c^*(x_0) S(x_0)}{P^*(x_0)} \tag{29}$$

Now we note that the terms for the period t=0 are the same for all x_t so that for all t from t=0 onwards:

$$\frac{U_c(x_t, x_0)}{U_c^*(x_t, x_0)} = \kappa \frac{P(x_t, x_0) S(x_t, x_0)}{P^*(x_t, x_0).} \tag{30}$$

where $\kappa = \frac{U_c(x_0)}{P(x_0)} / \frac{U_c^*(x_0)S(x_0)}{P^*(x_0)}$ is a constant.

Let us parameterise as above $U = C_t^{(1-\sigma)}\epsilon/(1-\sigma)$ and let $q_t = -\ln P + \ln P^* + s_t$ be the real exchange rate (where a rise is a US, Home, depreciation) as in our notation elsewhere; ϵ is the shock to time-preference. Then this yields the risk-pooling condition:

$$\sigma(\ln C_t - \ln C_t^*) = q_t - v_t \tag{31}$$

ignoring the constant: v is the difference between the logs of the two countries' time-preference errors (These errors will also form part of the two IS shocks).

To see that this implies the UIP relationship, use the Euler equations for consumption (e.g. for home consumers $\ln C_t = -\frac{1}{\sigma}\left(\frac{R_t - E_t\pi_{t+1}}{1-B^{-1}} - \ln \epsilon_t\right)$ where R_t is the nominal interest rate and B^{-1} is the forward operator keeping the date of expectations constant). Substituting for consumption into the risk-pooling equation gives us $E_t q_{t+1} - q_t = (R_t - E_t\pi_{t+1}) - (R_t^* - E_t\pi_{t+1}^*)$.

b) when there are only non-contingent bonds then arbitrage forces UIP. When this is substituted back into the Euler equations it yields:

$$\sigma(1-B^{-1})(\ln C_t - \ln C_t^*) = (1-B^{-1})(q_t - v_t) \tag{32}$$

Hence now the risk-pooling condition occurs in expected form from where it currently is. But any shocks may disturb it in the future.

Thus with full risk-pooling under state-contingent bonds relative consumption is exactly correlated with the real exchange rate and time-preference shocks. But under non-contingent bonds it is subject to all shocks: it is only expected to be correlated exactly from where it currently is.

We continue with the model under both these variants: our 'default' variant contains UIP, and we consider the risk-pooling variant as an explicit alternative where the UIP equation (8) is replaced with the risk-pooling equation (31).

3 Single equation tests of UIP and risk-pooling

3.1 Single-equation tests of UIP

As noted in the introduction, there is a large empirical literature testing UIP and risk-pooling by single equation methods. Begin with the UIP equation: $(R_t - E_t\pi_{t+1}) - (R_t^* - E_t\pi_{t+1}^*) - E_t q_{t+1} + q_t = 0$.

Here the usual single equation test is a predictive test, to see whether the actual future real exchange rate obeys the rational expectation prediction of the equation. Thus $q_{t+1} = E_t q_{t+1} + e_{t+1}$; and so:

$$q_{t+1} = (R_t - E_t\pi_{t+1}) - (R_t^* - E_t\pi_{t+1}^*) + q_t + e_{t+1}$$

becomes a predictive equation. Many authors have found it predicts poorly and have concluded that UIP does not hold.

We can bootstrap our model here to replicate the property of this test under the null hypothesis of UIP. We do it for the real exchange rate. We compare the error a researcher would find from the actual sample data for these variables and what would be found from data simulated by the whole model including UIP when bootstrapped. The latter implies the true standard deviation of e_{t+1}; the standard deviation in the actual data sample that has been typically used in this test is much smaller. The actual exchange rate will thus seem to exceed its two-standard-error limits very frequently.

We find that the standard error of the forecast is understated (the t-value is overstated) by 42%. Therefore the 'results' that the real exchange rate is 'badly forecast' by the forward rate are in most cases likely to be insignificant. Table 1 shows our findings.

Table 1: Standard deviation of forecast errors for real exchange rate

Full model with bootstrapped simulations	0.039
Single equation with actual data	0.028

This Table is constructed as follows. The forecast test is based on a single sample of one-period-ahead errors, taken from the actual data itself. These errors have a standard deviation on 0.028 approximately. The t-values for the forecast test will be based on these errors. However, the true forecast errors are derived from the structural model and its shocks: it is these that cause the forecast to go wrong. We can compute the standard deviation of these true forecast errors by bootstrapping the shocks in the structural model many times to discover what the sample forecast errors could be. As the Table shows the standard deviation of these forecast errors, the true ones according to the model, is approximately 0.039. This is 42% higher than the standard error in the data sample; hence the t-values of the forecasting tests used are overstated by 42% also. Intuitively, the reason for this is that a single sample will understate the true potential variation in the exchange rate because that sample contains just one drawing of the structural shocks and their impact on the structural model. This sample will tend to be from the centre of the distribution, with few extreme values from the structural shocks and some cancellations of the effects of different shocks.

Another possible test of UIP would be a cointegration test between q_{t+1} and $R_t - R_t^* + q_t$. This amounts to a test of whether e_{t+1} is stationary. We examined 1000 bootstraps derived either from the sample data or from the full structural model, and in both cases we find virtually all are stationary. This is not surprising since q is generally found to be I(1) so that $q_{t+1} - q_t$ will be stationary, while the interest differential is stationary. So the error is the sum of two stationary processes. Thus a cointegration test of UIP will generally pass it.

Thus in these single equation tests, though its forecasts will generally be found to be cointegrated with the actual data, UIP may well seem to forecast significantly badly. .

3.2 Single-equation tests of risk-pooling

If we turn now to the single equation tests of the risk-pooling equation, we see that they either regress a time series- of the consumption differential against the real exchange rate- or the cross-section, or both in a panel regression. They include a random i.i.d. error.

Because both the consumption ratio and the real exchange rate are non-stationary, one may also carry out a cointegration test, to see whether the two series vary together as the risk-pooling hypothesis states: this test tests whether the error from an OLS regression is stationary or not, using an ADF test.

The problem however is that the risk-pooling equation includes the relative shock to consumers' time-preference as derived above. This is an exogenous variable, not an i.i.d. shock. It could be recovered from the two countries' Euler equations; but this is not usually done and if done would need to respect the rational expectations restrictions on expected future consumption in the Euler equation coming from the whole model solution. It is plainly an important time-series shock; and in cross-sections between countries at different income levels it would typically differ both across country and over time. Simply leaving it out creates omitted variable bias for the estimated equation- a serious and possibly fatal specification error.

We checked what this might do to a time-series regression based on the sample data used in our study by bootstrapping the risk-pooling model to obtain the relevant variables in this regression[2]. We found that the downward bias introduced was 70%. Table 2 shows our findings.

[2]For the risk-pooling equation, we bootstrap 1000 draws of the US consumption, EA consumption, real exchange rate, respectively, simulated by the structural model by assumption is the 'true' model. We then estimate the single-equation regression $\ln C_{t,s} - \ln C_{t,s}^* = \beta_{0,s} + \beta_{1,s} q_{t,s} + \varepsilon_{t,s}$ where $s = 1, 2, ...1000$ for each bootstrap.

Table 2: 'True' vs OLS estimate of parameter of risk-pooling equation

	Estimated coefficient	S.E
'True' parameter value ($\frac{1}{\sigma}$)	0.6269	-
Mean OLS estimate of risk-pooling equation ($\hat{\beta}_1$)	0.1852	0.24

Table 3: Number of stationary residual series according to the ADF test

lag = 1	lag = 2	lag = 3	lag = 4	lag = 5	lag = 6	lag = 7	lag = 8
With drift							
251	173	145	126	117	114	108	88
With drift and trend							
182	119	99	69	71	68	70	76

The Table shows the true coefficient (0.6269) according to the risk-pooling model, which is found when the risk-pooling error process is included. When a simple regression is performed with this error process omitted, the coefficient estimated on 1000 bootstrap samples from the risk-pooling model averages 0.1852, with a standard error of 0.24. It follows that such regressions will on average find an insignificant relationship.

When we turn to cointegration tests on the errors resulting from these OLS regressions we find that they will generally be non-stationary, as table 3 shows. Depending on the exact form of the test, 25% or less of the errors (out of the 1000 bootstraps) are found be stationary, and so to imply cointegration. The vast majority imply lack of cointegration. Yet the risk-pooling model from which these errors come implies cointegration on the true equation. The general finding of lack of cointegration comes from the omitted variable, the risk-pooling error process. Effectively it is this omitted shock that ensures cointegration.

What we have found therefore is that if the risk-pooling model is correct the regressions performed on sample data generated from it will find an insignificant relationship between the real exchange rate and relative consumption and also a lack of cointegration because the key error in this relationship is omitted – an important mis-specification.

4 The method of Indirect Inference

We now turn to an indirect inference test of full models with either UIP or risk-pooling embedded in them. The idea is that any major fault in these models should lead to their rejection with high likelihood; this power of our test is something we establish below with Monte Carlo simulations. Plainly either UIP or risk-pooling are key relationships in the model that if wrong should produce rejection: both have strong implications for the behaviour of all the model's variables.

Indirect Inference is a relatively unfamiliar method of estimation and testing. We use it here because we need a method that will powerfully reject a mis-specified model in the small sample that we have (around 168 quarterly observations). The two main alternatives today are Bayesian estimation with strong priors or Maximum Likelihood (equivalent to Bayesian estimation with flat priors).

The former is an appropriate method when much is already known about the issue at hand, so that priors can be set out that command general assent; often the case in the physical sciences and indeed in some parts of the social sciences. However, this condition does not apply here: the macroeconomics of the world economy is not much explored and remains controversial.

Maximum Likelihood estimation is based on minimising the model's now-casting prediction errors and its associated test is based on the likelihood implied by these errors. The two main difficulties of this method are first that it exhibits high estimation bias in small samples and second that the power of the test in small samples is also rather limited and in particular its power to reject a mis-specified model is close to zero, because such a model can be fitted closely to the data, so creating small errors. Le et al. (2016) carried out a Monte Carlo comparison of this method with Indirect Inference, treating the widely used Smets and Wouters (2007) model of the US as the true model, and concluded that, while indeed ML methods suffered from these problems, by contrast Indirect Inference offered very low bias and potentially large power. The method involves first describing the data behaviour in the sample by an 'auxiliary model', for which we use a VAR; and then simulating the DSGE model by bootstrapping its innovations to create many parallel samples (or histories) from each of which implied auxiliary model coefficients are estimated, generating a distribution of these coefficients according to the DSGE model. We then ask whether the VAR coefficients found in the actual data sample (actual history) came from this distribution with a high enough probability to pass the Wald test (where we put the test threshold at 5%). Notice that when we bootstrap these shocks we do so by time vector, that is to say we draw all the innovations for one period together when we randomly select shocks. This preserves any simultaneous correlation between them which may well be important because a single event source can trigger shocks all over the economy — as in the recent financial crisis.

4.1 The auxiliary model

The state-space representation of log-linearized DSGE model in general has a restricted VARMA representation for the endogenous variables or a finite order VAR model. However, if the observed data are non-stationary, following Meenagh, et al. (2012) and Le, et al. (2015a), an unrestricted version of VECM can be used as an auxiliary model when errors are stationary. The VECM model is an approximation of the reduced form of DSGE model and can be represented as a cointegrated VAR with exogenous variables (VARX) model.

Suppose the structural model can be written in log linearized form as:

$$A(L)y_t = B(L)E_t y_{t+1} + C(L)x_t + D(L)e_t \tag{33}$$

where y_t is a vector of endogenous variables with dimension $p \times 1$ and x_t is a vector of exogenous variables with dimension $q \times 1$. We assume x_t are non-stationary and follows a unit root process:

$$\Delta x_t = a(L)\Delta x_{t-1} + d + c(L)\epsilon_t \tag{34}$$

The disturbances e_t and ϵ_t are both vectors of i.i.d. error processes with zero means. L denotes the lag operator and $A(L), (B(L), a(L), c(L)$ are polynomial functions having roots lying outside the unit circle.

The general solution of y_t is given by:

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)\epsilon_t \tag{35}$$

where f is a vector of constants and polynomial functions in lag operator. Since y_t and x_t are both non-stationary, the solution has p cointegrating relationships such that:

$$\begin{aligned} y_t &= [I - G(L)]^{-1}[H(L)x_t + f] \\ &= \Pi x_t + g \end{aligned} \tag{36}$$

where Π is a $p \times p$ matrix with a rank $0 \leq r < p$, with r being the number of linearly independent cointegrating vectors. In long run, the solution to the model is given by:

$$\overline{y_t} = \Pi \overline{x_t} + g \tag{37}$$

$$\overline{x_t} = [1 - a(1)]^{-1}[dt + c(1)\xi_t] \tag{38}$$

$$\xi_t = \sum_{i=0}^{t-1} \epsilon_{t-s} \tag{39}$$

where $\overline{y_t}$ and $\overline{x_t}$ are the long run solution to y_t and x_t respectively. The generic solution of $\overline{x_t}$ can be decomposed into a deterministic trend $\overline{x_t}^d = [1-a(1)]^{-1} dt$ and a stochastic trend $\overline{x_t}^s = [1-a(1)]^{-1} c(1)\xi_t$.

The solution of y_t in equation (35) can be re-written as in the Cointegrated VECM with a mixed moving average process ω_t:

$$\begin{aligned} \Delta y_t &= -[I-G(L)](y_{t-1}-\Pi x_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t \\ \omega_t &= M(L)e_t + N(L)\epsilon_t \end{aligned} \tag{40}$$

The VECM can be approximated by:

$$\Delta y_t = -K(y_{t-1}-\Pi x_{t-1}) + R(L)\Delta y_{t-1} + S(L)\Delta x_t + g + \zeta_t \tag{41}$$

where ζ_t is an i.i.d. process with zero mean. Since $g = \overline{y}_{t-1} - \Pi \overline{x}_{t-1}$, the VECM can also be written as:

$$\Delta y_t = -K\left[(y_{t-1}-\overline{y}_{t-1}) - \Pi(x_{t-1}-\overline{x}_{t-1})\right] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t \tag{42}$$

Either of equation (41) or (42) can serve as the auxiliary model. In particular, (42) distinguishes between the effect of the trend component of x_t and the temporary deviation of x_t from trend; it can be rewritten to be a VARX(1) in level:

$$y_t = [I-K]\, y_{t-1} + \Pi K \overline{x}_{t-1} + \eta_t + v_t \tag{43}$$

where $\overline{x}_{t-1}$ contains the stochastic trends in the exogenous variables, η_t is included to pick up the deterministic trends in y_t, and v_t is a vector of the error terms.

For doing the Wald test, we calculate the Wald statistic where we account for the VAR coefficients of the lagged endogenous variables $(I-K)$ and the variances of the VAR errors $Var(v_t)$ that we take as descriptors of the data. We are not interested in matching the time trends and the coefficients of the exogenous variables (the two potential outputs on this occasion), and we assume that the model coefficients yielding these balanced growth paths and effects of trend productivity on the steady state are chosen accurately.

4.2 Choosing the variables to be matched by Indirect Inference

A central question to be addressed in testing a model by Indirect Inference is choosing the power of the test. In practice this is equivalent to choosing which variables to put in the auxiliary model – which here we put in the form of a VAR. Other forms of auxiliary model could be used instead- such as moments or IRFs- with similar results, as discussed in Minford, Wickens and Xu (2017). Le, et al. (2016) show that, as the number of variables and the order of the VAR rise, the power of the test increases up to the point where the full reduced form VAR of the model is reached. For example in Smets and Wouters (2007), the full reduced from is a VAR(4) in seven variables, implying some 200 VAR coefficients in all. Plainly each of these carries additional information about the implications of the model for the data.

Policy-maker using a macro model (or any other user) would like to find a model that passes the test, in order to make progress in assessing the effects of policy and also the accuracy of the assessment. A model that does not pass the test cannot be of any use in this respect. On the other hand the test needs also to have considerable power in order to discriminate between good and bad models and to ensure that the model chosen is reasonably accurate. Thus the most powerful II test will reject any model that is as little as 1% inaccurate; effectively only admitting a model that 'is the real world'. The least powerful may admit models of considerable inaccuracy.

To assess how many variables should be included and what order of VAR requires us to examine the power of various combinations on the type of model we are investigating. This can be done by Monte Carlo simulation. After some experimentation with different variables and VAR orders we found that just two key variables in a VAR(1) – the two outputs from each country – provide substantial power in testing this two-country model. We include these two variables and also the variances of their residuals in our auxiliary model.

Table 4: Rejection rates of falsified model at the 5 percent-UIP mode

Percent falsify	Structural coeffs	AR coeffs	Structural and AR coeffs
1%	11.2	9.8	28.0
2%	20.3	10.4	46.8
3%	48.9	11.1	81.0
5%	86.4	13.5	100.0
7%	100.0	19.2	100.0
10%	100.0	51.7	100.0
15%	100.0	86.3	100.0
20%	100.0	91.6	100.0

4.2.1 The power of test for a two-variable VARX(1) – some Monte Carlo experiments

In this section we examine the relative power of the II test on alternative false models. We do so by first estimating both the UIP and risk-pooling models which we treat as the 'true' models, and then using them to generate 1000 sets of simulated data by bootstrapping the structural shocks identified over the sample period. These simulated data are then fitted to a VARX(1) for a distribution of the VAR parameters to be found. This also gives us a distribution of the Wald statistics of the true models which we know at the 95 percentile (i.e. at the 5% level of significance) 5% of the true model simulations will be rejected. The corresponding Wald statistic is the critical value of the Wald test at the 5% significance level. To evaluate the power of the test against the two models, in the next step we falsify each of them by biasing their parameters by a percentage and generate false simulations with the biased models. We then find the distribution of the Wald statistics just as before, but in this case we calculate the rejection rate by using the 5% critical values found with the true models. We try different degrees of falseness, and we allow for mis-specification from just the structural parameters, or just the autoregressive parameters of shocks, or both.

The Monte Carlo experiment results for each of the models are reported in tables 4 and 5, respectively. It can be seen that the power of the test with just the two outputs is very high. When all parameters of the UIP model are falsified by only 5%, the model is rejected 100% of the time; with only 3% falsification it is rejected 81% of the time. The test power for the risk-pooling model is similar.If we add just one variable to the two outputs, the real exchange rate, the power rises sharply for both models, as can be seen from table 6.

In recent work (Meenagh et al., 2018) on Indirect Inference in small samples, it has been found that the test power tends to rise with the number of variables in the auxiliary VAR as we find here. However, the test power is rather insensitive to which variables are included in the auxiliary VAR; thus here we would expect similar test power with any other two variables, such as the two consumptions, or the two interest rates. The test power is also fairly insensitive to whether one uses a VAR for the two variables or a set of moments or a set of impulse response functions (IRFs), provided the number of each in the auxiliary model is similar. Thus a two-variable VAR(1) implies four VAR coefficients plus the two VAR error variances, six 'descriptors' in all. Around the same number of moments or IRFs should be selected for similar power.

4.3 Data and Calibration

We now confront the model described above to the quarterly US and euro area data between 1970Q1 and 2011Q4 which we plot in figure 1. Certain parameters are fixed throughout; others are calibrated to begin with and then re-estimated by Indirect Inference.

Of the fixed parameters, we set the discount factor (β) for both economies to 0.99 to imply a steady-state annual real interest rate of 4%. The steady-state consumption-to-output ratio (c) is set to 0.66 for US and

Table 5: Rejection rates of falsified model at the 5 percent-RP model

Percent falsify	Structural coeffs	AR coeffs	Structural and AR coeffs
1%	10.4	7.8	13.7
2%	15.6	10.0	29.4
3%	33.7	12.2	69.6
5%	82.2	13.0	99.6
7%	99.9	18.9	100.0
10%	100.0	45.4	100.0
15%	100.0	75.2	100.0
20%	100.0	88.0	100.0

Table 6: Rejection rates of falsified model when added RXR for both models

	UIP Model	Risk Pooling Model
Percent falsify	Structural and AR coeffs	Structural and AR coeffs
1%	42.4	27.3
2%	55.3	46.8
3%	81.6	77.3
5%	100.0	100.0
7%	100.0	100.0
10%	100.0	100.0
15%	100.0	100.0
20%	100.0	100.0

0.55 for EA, while the export ratios (x) are 0.12 and 0.30, respectively. Other fixed parameters/steady-state ratios are also listed in tables 7 and 8.

For the parameters that are to be re-estimated later, we mainly follow Smets and Wouters (2007) in setting the starting values: the intertemporal elasticity of substitution σ and elasticity of labour supply φ are set to 1.38 and 1.83, respectively; the Calvo non-adjusting probability is 0.66, which suggests nominal prices are on average adjusted every three quarters. The persistence of nominal interest rate is set to 0.81, while the response to inflation is 2.04 and that to output gap is 0.12. In our specification we let nominal interest rate respond also to changes in real exchange rate and the response is 0.5. For the euro area, we follow Smets and Wouters (2003): thus σ and φ are 1.39 and 2.50, respectively; the Calvo parameter is 0.9; the Taylor rule coefficients are 0.96 (persistence), 1.69 (inflation response) and 0.12 (output response), and we let the real exchange rate response be the same as that of the US. These calibrated values are listed in table 9 in comparison to the estimated values.

Table 7: Fixed parameters in both models

Parameter	Definition	Value
β	Discount factor	0.99
μ	Income elasticity of US import from EA	1.00
ν	Income elasticity of US import from RoW	1.00
ψ	Exchange rate elasticity of US import from EA	0.80
μ^*	Income elasticity of EA import from US	1.00
ν^*	Income elasticity of EA import from RoW	1.00
ψ^*	Exchange rate elasticity of EA import from US	0.80
ψ^{RoW}	Exchange rate elasticity of RoW import from US relative to EA	0.80

Figure 1: Observed Data series in both models

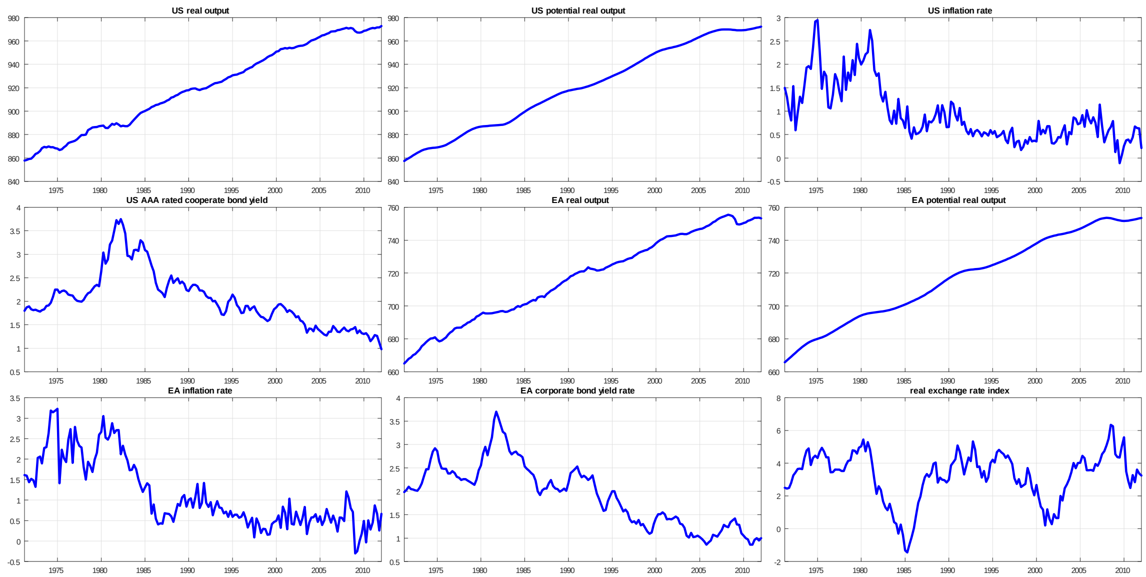


Table 8: Steady state ratio in both models

Parameter	Definition	Value
c	Steady-state US consumption to US output ratio	0.66
x	Steady-state US export to US output ratio	0.12
c^*	Steady-state EA consumption to EA output ratio	0.55
x^*	Steady-state EA export to EA output ratio	0.30
m_1	Steady-state US export to EA to US output ratio	0.02
m_2	Steady-state US export to RoW to US output ratio	0.10
n_1	Steady-state US import from EA to US output ratio	0.03
n_2	Steady-state US import from RoW to US output ratio	0.09
F	Steady-state US export to RoW to RoW output ratio	0.60
Ξ	Steady-state US import from RoW to RoW output ratio	0.40

Table 9: Estimated parameters

Parameter	Definition	Calibration				II Estimation			
		UIP		R-P		UIP		R-P	
		US	EA	US	EA	US	EA	US	EA
σ	Inverse of intertemporal cons. elasticity	1.380	1.390	1.380	1.390	3.550	2.051	1.595	1.540
φ	Inverse of labour elasticity	1.830	2.500	1.830	2.500	2.658	3.460	3.507	2.052
θ	Calvo-non-adjusting probability	0.660	0.960	0.660	0.960	0.653	0.669	0.832	0.877
λ	$\frac{(1-\beta\theta)(1-\theta)}{\theta}$	0.179	0.027	0.179	0.027	0.188	0.167	0.036	0.018
κ_α	$\lambda(\sigma\frac{1}{c}\Theta^{-1}+\varphi)$	0.665	0.008	0.665	0.008	1.628	1.234	0.221	0.092
α	Degree of openness	0.400	0.400	0.400	0.400	0.158	0.158	0.236	0.236
ρ	Monetary policy inertia	0.810	0.960	0.810	0.960	0.217	0.561	0.940	0.776
ϕ_π	Monetary policy response to inflation	2.040	1.690	2.040	1.690	2.618	3.944	2.385	2.017
ϕ_y	Monetary policy response to output	0.12	0.120	0.12	0.120	0.591	0.117	0.893	0.752
ϕ_q	Monetary policy response to RXR	0.500	0.500	0.500	0.500	0.143	0.128	0.949	0.876
	Wald percentile	100		100		92.6		80.4	
	T-stat (p-value)	6.60		12.17		1.413(0.074)		0.847(0.196)	

4.4 The models' performance

In table 9 we report the test results for the two models according to Indirect Inference. Not surprisingly, the calibrated models are severely rejected. However, after re-estimation, the UIP model can jointly match the behaviour of the two outputs with a t-statistic of 1.4 and Wald percentile of 92.6, thus a p-value of 0.074. This result is in line with the empirical finding of Le et al. (2013) that a large UIP-based world model of the US and the EA, essentially following the full Smets-Wouters specification in both continents, matched a VAR using the subset of the two outputs.

What is an entirely new finding is that a risk-pooling model will also jointly match the same behaviour. Furthermore, it does so with a considerably higher probability, with a t-statistic of 0.8 and a Wald percentile of 80.4, thus a p-value of 0.196- nearly three times that of UIP.

4.5 The processes of shocks

We can extract the structural shocks of both models from the unfiltered data and fit each of them to a time-series process to check the properties of them (For productivity of the two countries we simply use the potential output data that we extracted from the time series of outputs using the HP filter). We plot these processes in figures 2 and 3. For each of them, we test their stationarity using the ADF test. Table 10 shows the two productivity processes are I(1) processes, which supports our specification for them (as in equation 25). The two demand shocks and the two monetary policy shocks are trend stationary, while the two supply shocks are stationary.

We fit all the shock series to an AR(1) process (while the two productivities are kept to be ARIMA (1,1,0) processes as assumed). We estimate the persistence of all these processes using the Limited Information Maximum Likelihood method (McCallum, 1976; Wickens, 1982) and report the estimates in the same table.

Table 10: Stationarity and persistence of shock processes

Shocks	P-value		Stationarity	Persistence	
US	UIP	R-P		UIP	R-P
Demand	0.01	0.00	I(0) + trend	0.88	0.62
Productivity	0.52	0.52	I(1)	0.95	0.95
Supply	0.00	0.00	I(0)	0.45	0.63
Policy	0.01	0.04	I(0) + trend	0.83	0.76
EA					
Demand	0.00	0.05	I(0) + trend	0.96	0.81
Productivity	0.73	0.73	I(1)	0.95	0.95
Supply	0.00	0.00	I(0)	0.57	0.03
Policy	0.00	0.08	I(0) + trend	0.83	0.92
Risk pooling	–	0.09	I(0) + trend	–	0.94

Figure 2: Structural shocks implied by the UIP model: 1971-2011

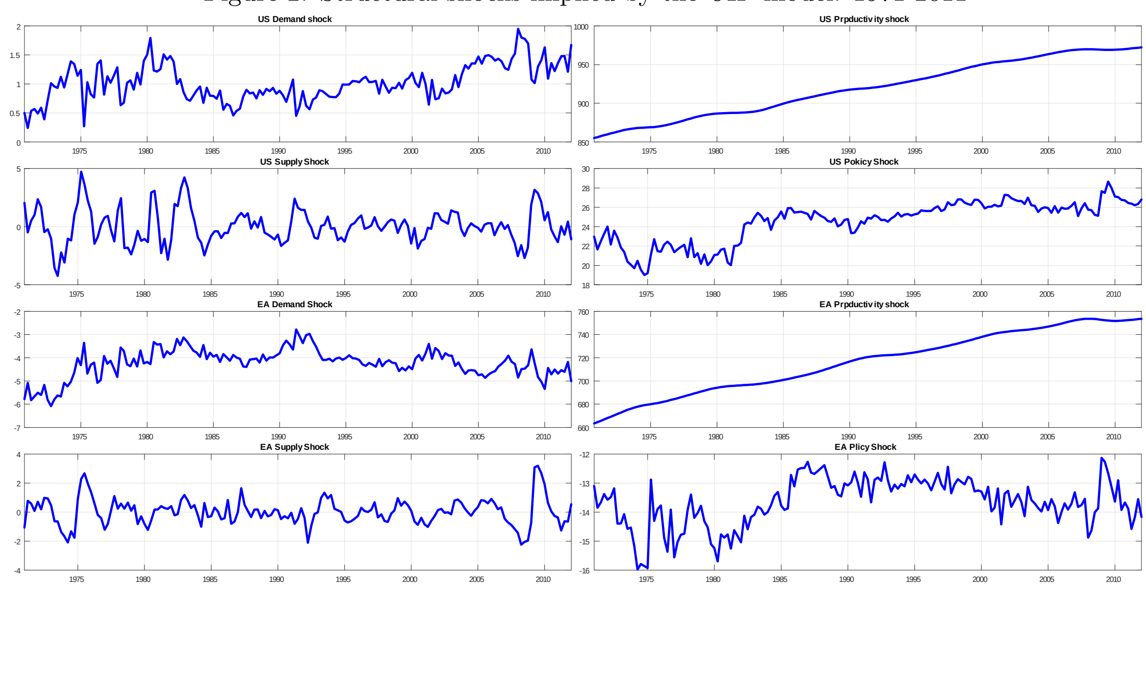
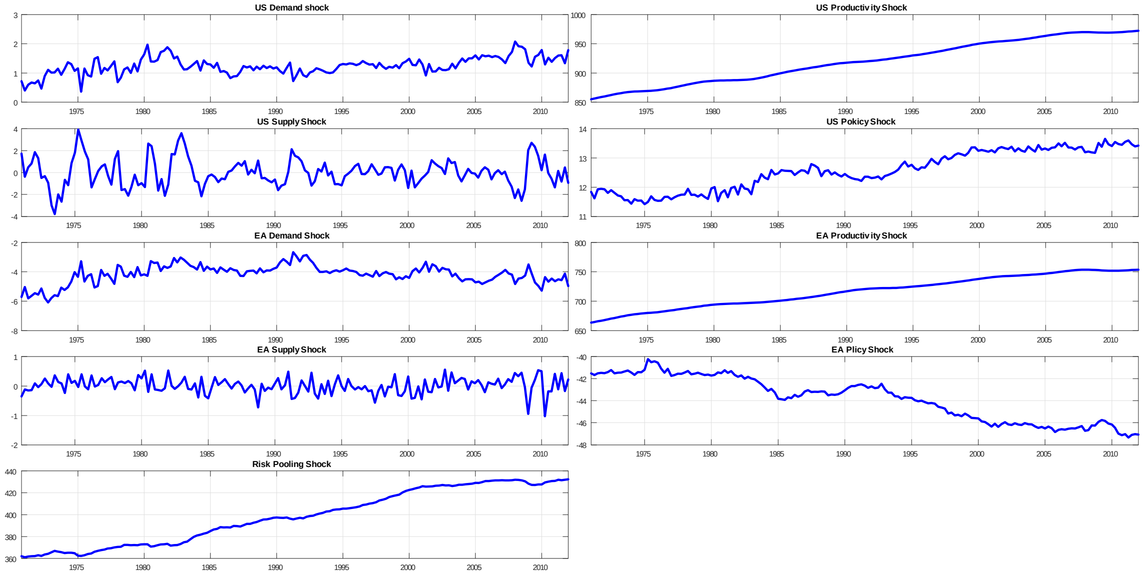


Figure 3: Structural shocks implied by the risk-pooling model: 1971-2011



5 The model's workings

5.1 Impulse response functions

5.1.1 The standard UIP model

What emerges from our results for the standard UIP model is that the two continents are essentially self-contained. Spillovers on real variables are small, as can be seen from the UIP variance decomposition over any length of period (as detailed below in table 11, and tables 13 and **??** in Appendix D). They are frustrated partly by movements in interest rates and in the real exchange rate. Both central banks respond to the real exchange rate with interest rate changes, while the real exchange rate in turn responds to real interest rate differentials. This pattern conforms to a standard model of the open economy under floating, where the floating exchange rate allows interest rate movements in each country to dampen its own shocks as well as any spillovers from shocks abroad. Central banks in effect control cross-continent integration by pursuing their own objectives and forcing the exchange rate to adjust. Thus home shocks dominate the home economy real variables; foreign shocks are largely neutralised.

This pattern can be seen in the IRFs for individual shocks, where in all cases spillovers to foreign output and consumption are small. Thus for example a US demand shock — see figure 4 — raises US output on impact by 1.4%, raises EA inflation and interest rates and so lowers EA output by 0.2%, while an EA demand shock — figure 5 — raises EA output by 1.5%, raises US real interest rates and so lowers US output by 0.4%.

5.1.2 The risk-pooling model

The risk-pooling model in effect opens up a direct channel of insurance between consumers in different continents, removing power from central banks to separate the economies. One can think of this risk-pooling mechanism as enabling foreign consumers to transfer resources directly to home consumers hit by a downturn; these resources are then spent by home consumers who thereby bid for foreign supplies, their own being short. This raises the relative price of foreign supplies, causing a real depreciation in the home exchange rate. What matters here for the real exchange rate reaction is the elasticity of foreign supply which

Figure 4: Impulse responses to a US Demand shock under UIP and Risk Pooling

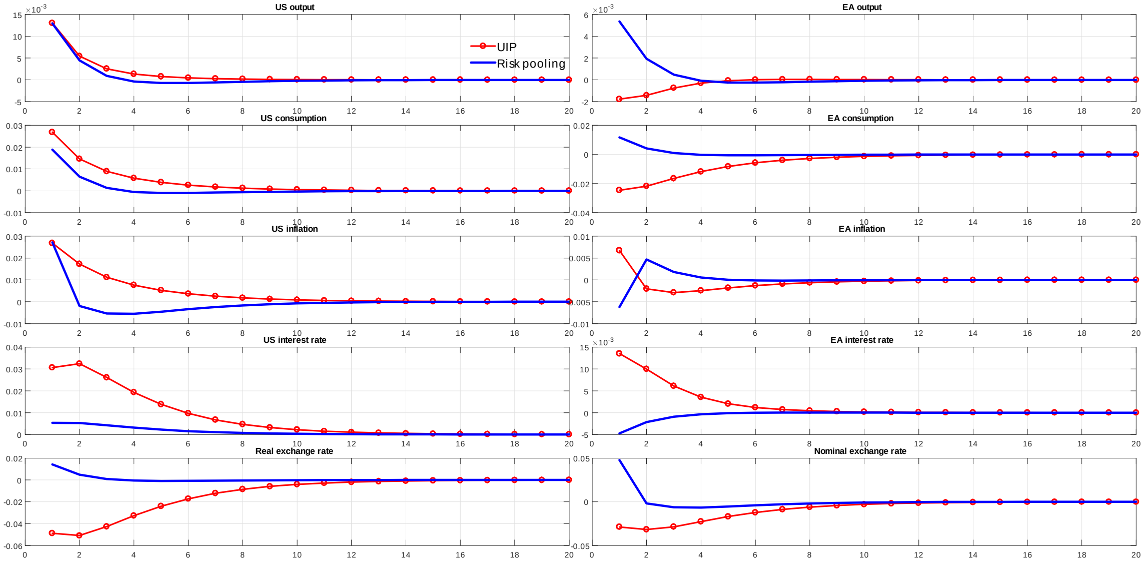
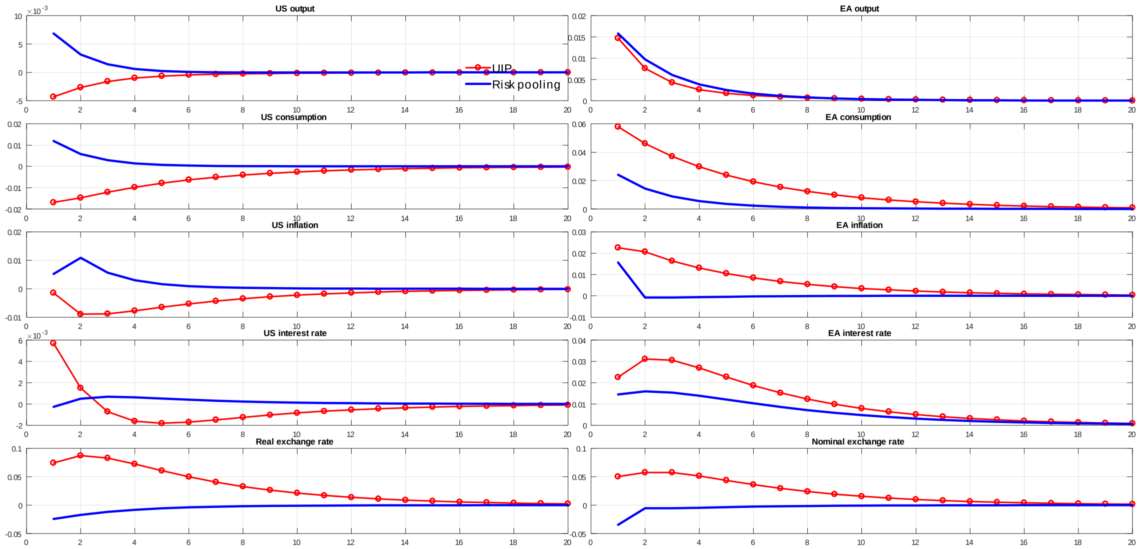


Figure 5: Impulse responses to a EA Demand shock under UIP and Risk Pooling



in this New Keynesian model is dictated by the Calvo stickiness parameter. The estimated stickiness of the US and EA are similar enough to imply that both EA and US output supply have a similar inelasticity.

The risk-pooling model therefore creates much greater integration of the two economies. Both US and EA shocks now spill over into the other continent. Take the demand shock below as an example. The US demand shock raises US output by 1.4% and EA output by 0.6%, while the US real exchange rate depreciates by nearly 2%. Central banks react to the home effects of the home shocks in a familiar way, in this case raising interest rates; but the foreign central banks, while reacting normally to the spillovers, react mainly to the sharp real exchange rate movements which push them in a direction opposite to the familiar one. Thus on the US demand shock EA interest rates fall in the attempt to dampen the real appreciation of the euro, while on the EA demand shock US interest rates fall to dampen the real appreciation of the dollar. In effect central banks are being forced to help the spillover process by dampening the real exchange rate reaction coming from supply inelasticity. Hence whereas central banks largely frustrate the ability of consumers to profit from spillovers under UIP, under risk-pooling consumers make free use of spillovers and force central banks to help the process along.

To save space we report the impulse responses of the other shocks in Appendix C.

5.2 Variance decomposition

If we now turn to the variance decomposition of the two models, we find that the real spillovers under risk-pooling are substantially larger than under UIP, as emerged from our impulse response functions. Taking the short-run (the two-year case) as an example, when there are demand shocks under UIP the variance share of the output spillover is 0.7% of that of the home output for the US demand shock and 7% for the EA demand shock (See table 11). The corresponding percentages under Risk-pooling are 6% and 35% (Table 12).

In Appendix D, we also report the decompositions for longer horizons (10 years and 40 years). As one would expect, the longer the time horizon the more the variances are dominated by productivity shocks. Indeed, we find too under Risk-pooling there are spillovers, but none to speak of under UIP.

5.3 Historical decomposition

When we compare the historical decomposition of the two models, we see that the UIP model is the lack of spillovers (all in yellow) into output of either continent from the other (Figures 6 and 8). We also see that interest rates in each continent are a key instrument by which these spillovers are frustrated since the other continent's shocks (in yellow) bulk large in each continent's monetary responses (Figures 7 and 9). When we turn to the case of the risk-pooling model, we see larger output spillovers (again in yellow) in both directions (Figures 10 and 12). As for interest rates again we see how in each continent interest rates respond to foreign shocks (Figures 11 and 13) — here because of their effects on the exchange rate, the response pattern is quite different.

Table 11: Variance Decompotion over 2 years (UIP)

Shocks	US				EA					
	y	c	π	r	y^*	c^*	π^*	r^*	RXR	NXR
US Demand	17.52	21.25	0.92	8.93	0.12	1.70	0.52	5.00	2.73	0.18
US Productivity	1.59	1.98	0.94	2.17	0.09	0.31	0.16	0.02	0.19	0.22
US Supply	1.14	0.78	16.18	14.05	0.39	0.41	0.53	2.78	1.53	3.58
US Policy	70.31	55.76	76.25	68.19	10.56	9.20	8.97	6.82	43.42	84.73
EA Demand	4.17	14.28	0.96	0.62	61.41	75.40	11.78	32.69	29.74	4.63
EA Productivity	0.11	0.22	0.20	0.27	0.59	0.66	0.17	0.33	0.08	0.07
EA Supply	1.00	1.48	0.79	1.86	3.58	0.96	38.47	44.74	8.50	0.40
EA Policy	4.13	4.24	3.79	3.90	23.25	11.34	39.30	7.62	13.82	6.18
Total	100	100	100	100	100	100	100	100	100	100

Table 12: Variance decompotion over 2 years (Risk Pooling)

Shocks	US				EA					
	y	c	π	r	y^*	c^*	π^*	r^*	RXR	NXR
US Demand	3.27	2.64	0.02	8.34	0.19	0.52	4.60	1.69	2.01	0.26
US Productivity	1.54	1.24	0.18	0.08	0.08	0.23	8.64	1.82	1.00	0.07
US Supply	7.62	6.16	3.76	8.58	0.45	1.25	1.21	2.34	4.38	0.10
US Policy	64.85	56.30	76.31	68.33	11.60	17.39	12.07	1.97	14.64	89.93
EA Demand	13.75	18.22	3.80	0.25	39.33	23.66	0.14	77.05	31.11	6.02
EA Productivity	0.00	0.04	0.93	1.24	1.02	0.41	11.30	0.96	2.42	0.15
EA Supply	0.41	0.56	0.06	0.03	1.20	0.73	13.21	7.22	0.96	0.14
EA Policy	8.08	12.06	11.16	8.96	34.74	19.44	24.17	3.81	35.84	7.00
Risk pooling	0.64	2.78	3.76	4.17	11.39	36.38	24.66	3.14	7.64	0.33
Total	100	100	100	100	100	100	100	100	100	100

Figure 6: Historical decomposition for US Output (UIP)

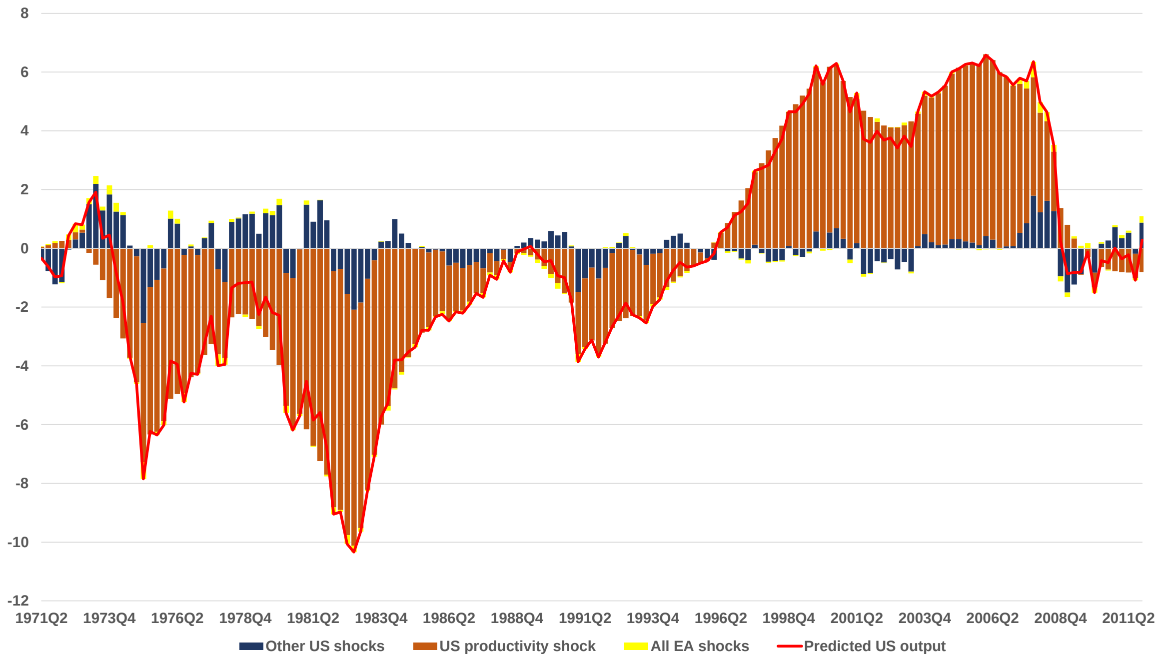


Figure 7: Historical decomposition for US Interest Rate (UIP)

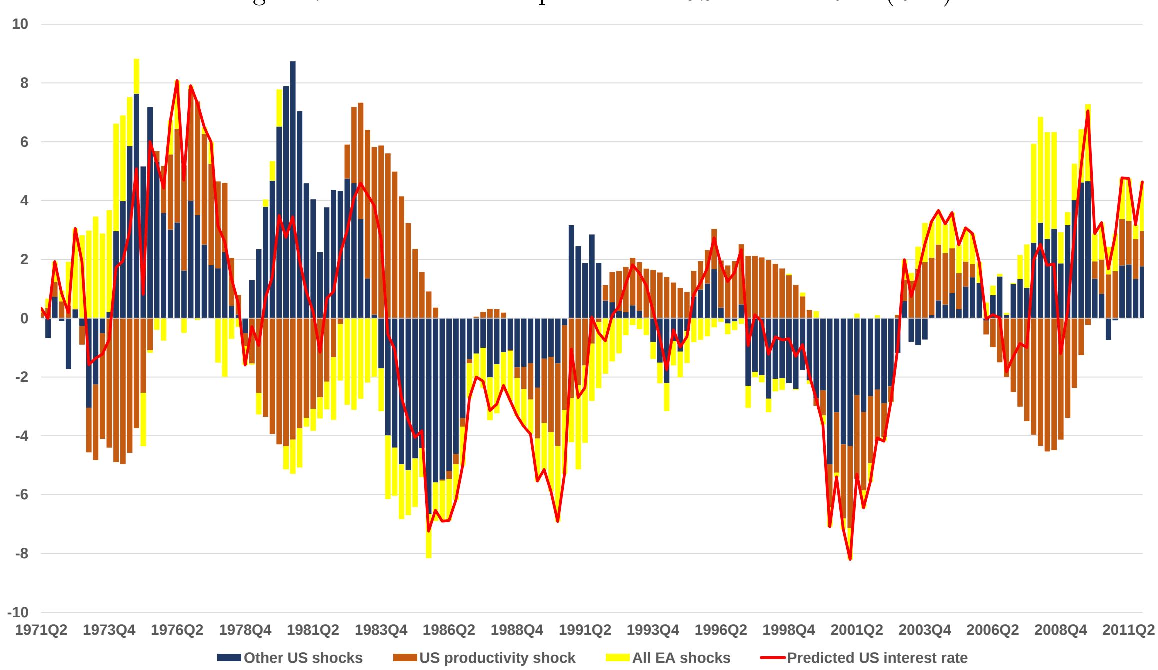


Figure 8: Historical decomposition for EA Output (UIP)

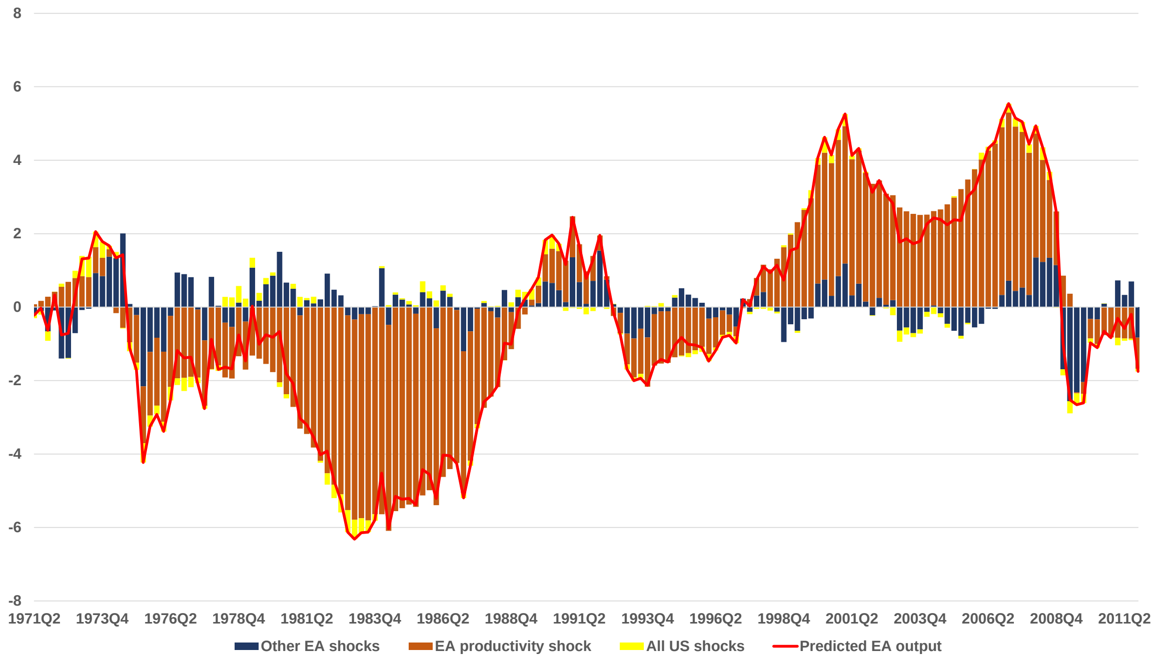


Figure 9: Historical decomposition for EA Interest Rate (UIP)

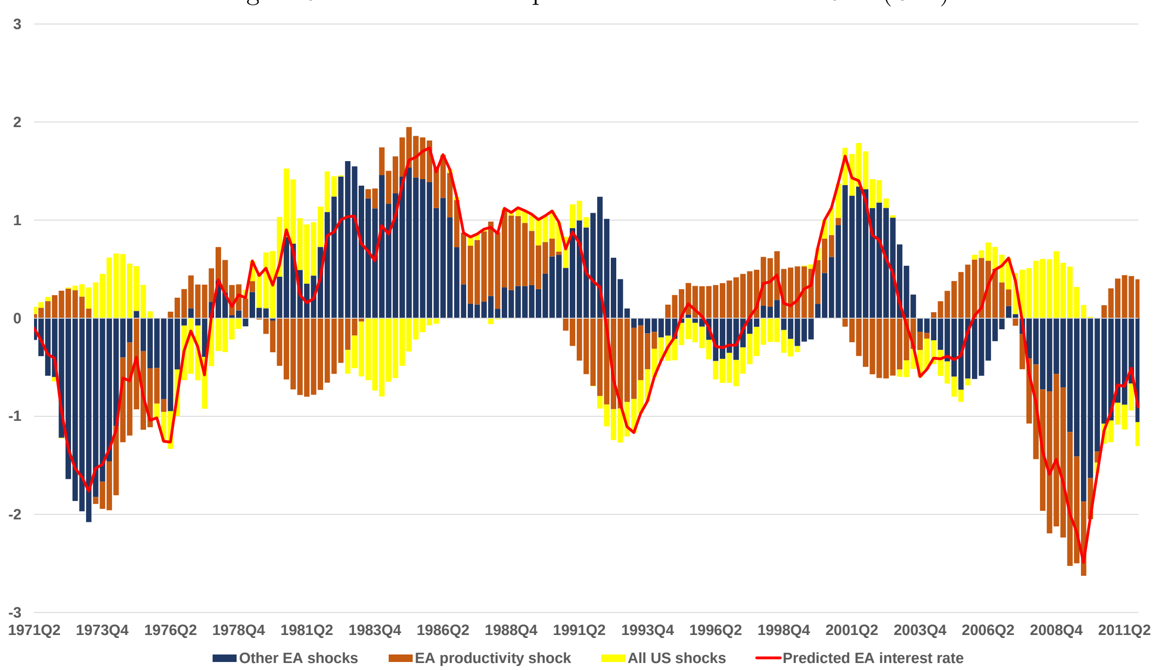


Figure 10: Historical decomposition for US output (Risk Pooling)

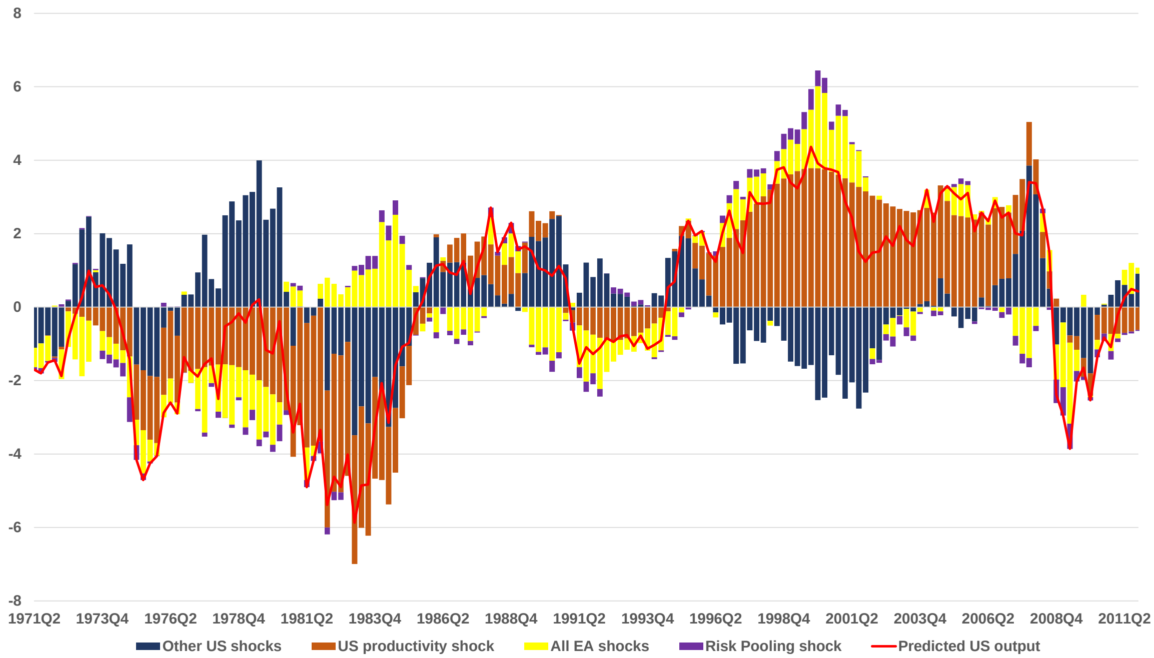


Figure 11: Historical decomposition for US Interest Rate (Risk Pooling)

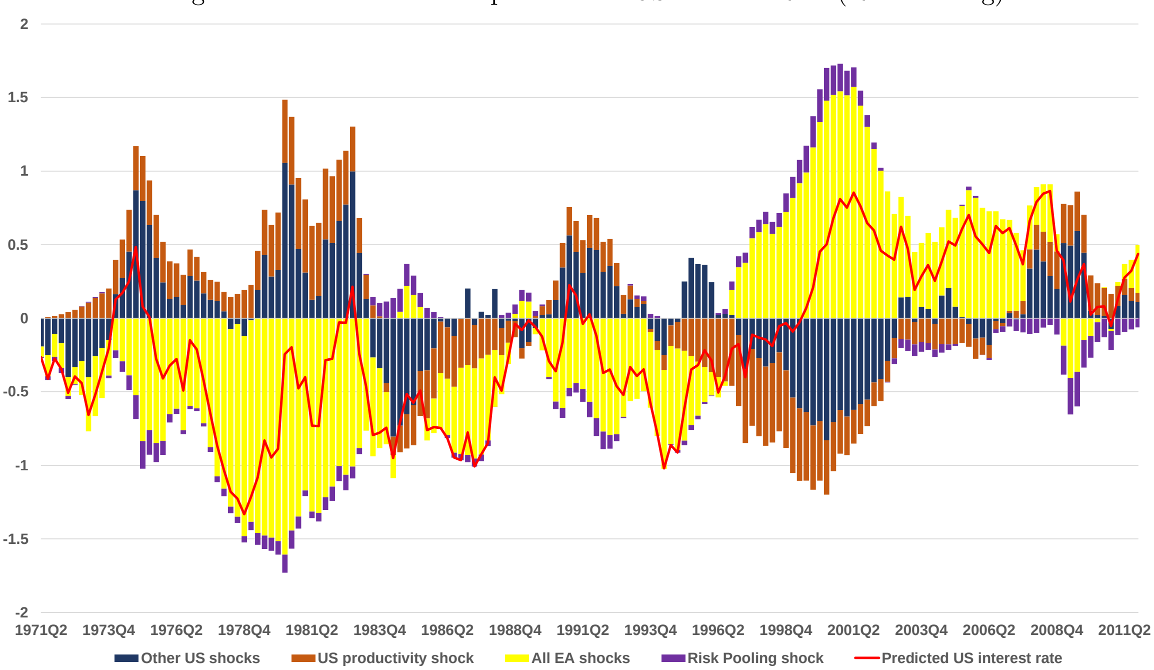


Figure 12: Shock decomposition for EA Output (Risk-pooling)

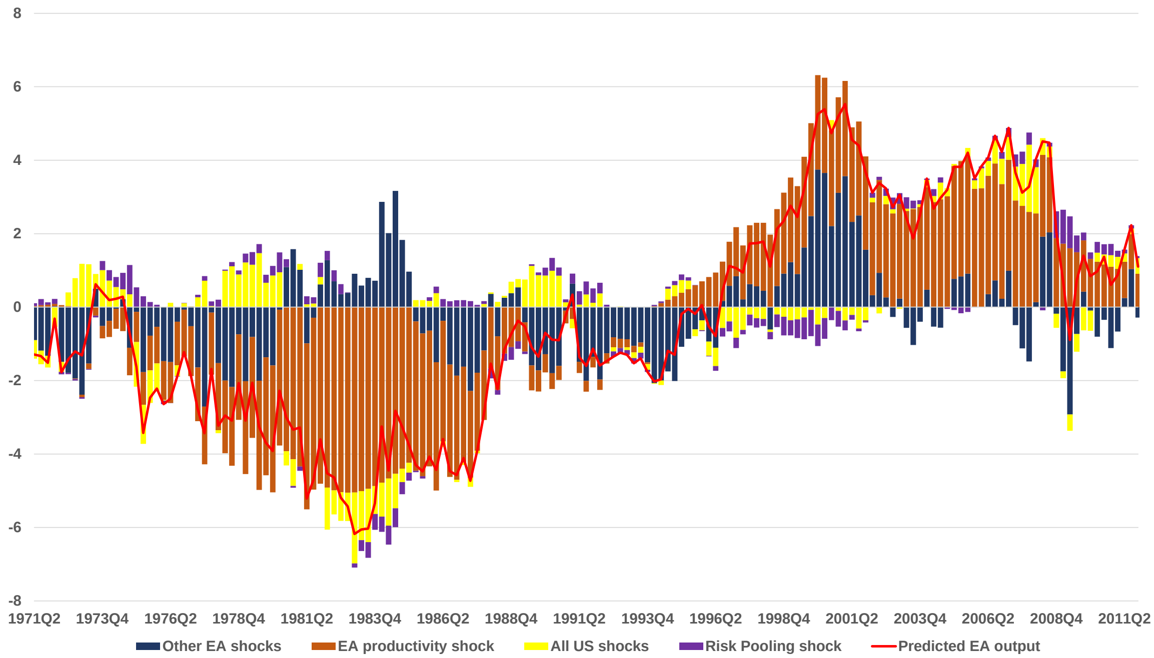
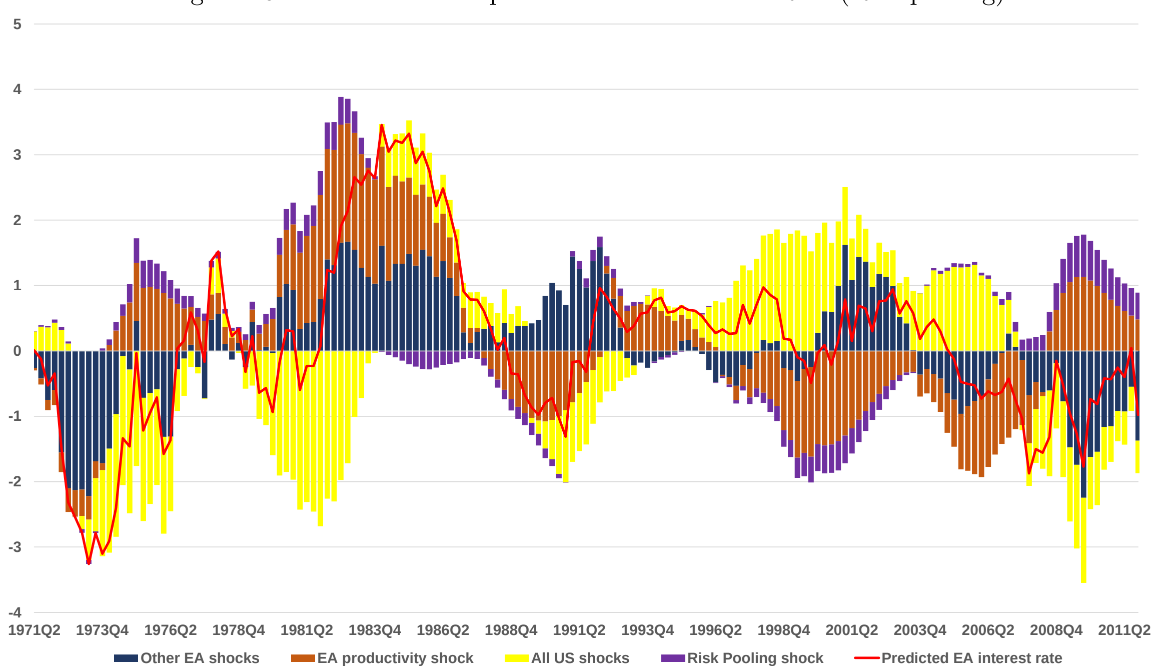


Figure 13: Historical decomposition for EA Interest Rate (Risk-pooling)



6 Conclusion

In this paper our first aim was to find a World model of two continents plus a Rest of World sector that could match selected data in a powerful Indirect Inference test. Our second aim was to discover whether there was risk-pooling in such a model. We did find such a model both in a UIP version in which there was 'dynamic' risk-pooling from non-contingent bonds tradeable across borders; and also in a version with risk-pooling provided by state-contingent bonds. Of these two versions the risk-pooling one was considerably more probable than the UIP version but both passed our tests.

In the UIP version of this model we found rather familiar features: each continent, US and EA, responds almost entirely to its own non-stationary productivity shocks, with stationary demand/supply shocks having limited spillover effects. Each economy is largely insulated from the other by the floating exchange rate, with monetary policy largely unresponsive to it. By contrast, in the risk-pooling version, behaviour turned out to be materially different. The two continents were in this closely integrated by private insurance markets achieved through contingent assets. Shocks in one continent cause consumers to acquire resources in the other where they spend them, driving up relative prices (the real exchange rate) to generate supply. These real exchange rate movements then force monetary policy to lean against them and so boost these spillovers.

Previous statistical tests of both UIP and risk-pooling have used single-equation methods, which we explain are likely to reject the model spuriously; and we confirm this from Monte Carlo experiments. This is to our knowledge the first time that a powerful statistical test has been performed on a full world model embodying these hypotheses with their distinctive effects on the behaviour of all variables. The fact that the full risk-pooling hypothesis has passed this test with a high p-value suggests that it deserves serious attention from policy-makers looking for a relevant model with which to discuss international monetary and other business cycle policy.

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Appendix

A Listing of model

- US

IS curve:

$$y_t = E_t y_{t+1} - c\frac{1}{\sigma}\Theta(R_t - E_t\pi_{t+1} - \bar{r}) - xz_1\Theta E_t\Delta y^*_{t+1} - xz_2\Theta E_t\Delta y^{RoW}_{t+1} - xz_4\Theta E_t\Delta q_{t+1} + \varepsilon^{IS}_t \tag{A.1}$$

Phillips curve:

$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa_a(y_t - y^p_t) - \frac{\alpha}{1-2\alpha}\left[\beta E_t(q_{t+1} - q_t) - (q_t - q_{t-1})\right] + \varepsilon^{PP}_t \tag{A.2}$$

Taylor rule:

$$R_t = \rho R_{t-1} + (1-\rho)[\phi_\pi \pi_t + \phi_y(y_t - y^p_t)] + \phi_q(q_t - q^{ss}_t) + \varepsilon^R_t \tag{A.3}$$

Productivity:

$$y^p_t - y^p_{t-1} = \Gamma^{y^p} + \delta(y^p_{t-1} - y^p_{t-2}) + \varepsilon^{yp}_t \tag{A.4}$$

US import from EA:

$$im^{US}_{EA,t} = \mu y_t - \psi q_t \tag{A.5}$$

US import from RoW:

$$im^{US}_{W,t} = \nu y_t \tag{A.6}$$

- EA

IS curve:

$$y^*_t = E_t y^*_{t+1} - c^*\frac{1}{\sigma^*}\Theta^*(R^*_t - E_t\pi^*_{t+1} - \bar{r}^*) - x^*z_3\Theta^* E_t\Delta y_{t+1} + x^*z_2\Theta^* E_t\Delta y^{RoW}_{t+1} + x^*z_4\Theta^* E_t\Delta q_{t+1} + \varepsilon^{IS^*}_t \tag{A.7}$$

Phillips curve:

$$\pi^*_t = \beta^* E_t\pi^*_{t+1} + \kappa^*_\alpha(y^*_t - y^{p*}_t) + \alpha^*(\beta^* E_t\Delta q_{t+1} - \Delta q_t) + \varepsilon^{PP^*}_t \tag{A.8}$$

Taylor rule:

$$R^*_t = \rho^* R^*_{t-1} + (1-\rho^*)[\phi^*_\pi \pi^*_t + \phi^*_y(y^*_t - y^{p*}_t)] - \phi^*_q(q_t - q^{ss}_t) + \varepsilon^{R^*}_t \tag{A.9}$$

Productivity:

$$y^{p*}_t - y^{p*}_{t-1} = \Gamma^{y^p*} + \delta^*(y^{p*}_{t-1} - y^{p*}_{t-2}) + \varepsilon^{yp*}_t \tag{A.10}$$

EA import from US:

$$im^{EA}_{US,t} = \mu^* y^*_t + \psi^* q_t \tag{A.11}$$

EA import from RoW:

$$im^{EA}_{W,t} = \nu^* y^*_t \tag{A.12}$$

- Rest of the world

World trade balance:

$$\Xi im^{US}_{W,t} + (1-\Xi) im^{EA}_{W,t} = \digamma ex^{US}_{W,t} + (1-\digamma) ex^{EA}_{W,t} \tag{A.13}$$

World output:

$$y^{RoW}_t = \Xi im^{US}_{W,t} + (1-\Xi) im^{EA}_{W,t} \tag{A.14}$$

World's relative demand for US and EA products:

$$ex^{US}_{W,t} = ex^{EA}_{W,t} + \psi^{RoW} q_t \tag{A.15}$$

- Real exchange rate determination
  - UIP variant:

$$E_t q_{t+1} - q_t = (R_t - E_t \pi_{t+1}) - (R^*_t - E_t \pi^*_{t+1}) \tag{A.16}$$

  - Risk-pooling variant:

$$\sigma(c_t - c^*_t) = q_t - v_t \tag{A.17}$$

- Real exchange rate in the steady state[3]:

$$q^{ss}_t = \frac{n_1 \mu + n_2 \nu - m_2 \Xi \nu}{n_1 \psi + m_1 \psi' + m_2 (1-\digamma) \psi''} y^p_t - \frac{m_1 \mu' + m_2 (1-\Xi) \nu'}{n_1 \psi + m_1 \psi' + m_2 (1-\digamma) \psi''} y^{p*}_t \tag{A.18}$$

- All shocks in the model are assumed to follow an AR(1) process.

[3] This is found by imposing the long-run restriction of trade balance (thus, $nx_t = 0$) on the US net export equation and solving for the real exchange rate.

B Derivation of the IS curve (US example)

Given the Euler equation $c_t = E_t c_{t+1} - \frac{1}{\sigma}(R_t - E_t\pi_{t+1} - \bar{r} + E_t \ln \epsilon_{t+1} - \ln \epsilon_t)$, and the market clearing condition in its log-linearised form $y_t = cc_t + xnx_t$ (where net export nx_t is defined as $nx_t = \ln X_t - \ln M_t = \frac{m_1}{x} im^{EA}_{US,t} + \frac{m_2}{x} ex^{US}_{W,t} - (\frac{n_1}{x} im^{US}_{EA,t} + \frac{n_2}{x} im^{US}_{W,t})$), we substitute the latter into the former to replace c_t, as the following:

a. Solve for c_t using the market clearing condition:

$$
\begin{aligned}
c_t &= \frac{1}{c}\{y_t - xnx_t\} \\
&= \frac{1}{c}\left\{y_t - x\left[\frac{m_1}{x}(\mu^* y_t^* + \psi^* q_t) + \frac{m_2}{x}(y_t^{RoW} + (1-F)\psi^{RoW} q_t) - \frac{n_1}{x}(\mu y_t - \psi q_t) - \frac{n_2}{x}(\nu y_t)\right]\right\} \\
&= \frac{1}{c}\left\{\left[1 + (\frac{n_1}{x}\mu + \frac{n_2}{x}v)\right] y_t - x(\frac{m_1}{x}\mu^*) y_t^* - x(\frac{m_2}{x}) y_t^{RoW} - x\left[\frac{m_1}{x}\psi^* + (1-F)\frac{m_2}{x}\psi^{RoW} + \frac{n_1}{x}\psi\right] q_t\right\} \\
&= \frac{1}{c}\left(\Theta^{-1} y_t - xz_1 y_t^* - xz_2 y_t^{RoW} - xz_4 q_t\right)
\end{aligned}
$$

where $z_1 = \frac{m_1}{x}\mu^*, z_2 = \frac{m_2}{x}, z_3 = \frac{n_1}{x}\mu + \frac{n_2}{x}v, z_4 = \frac{m_1}{x}\psi^* + (1-F)\frac{m_2}{x}\psi^{RoW} + \frac{n_1}{x}\psi, \Theta^{-1} = 1 + xz_3$.

b. Substitute the solution for c_t into the Euler equation to find:

$$
\begin{aligned}
& \frac{1}{c}\left(\Theta^{-1} y_t - xz_1 y_t^* - xz_2 y_t^{RoW} - xz_4 q_t\right) \\
= \;& E_t\left[\frac{1}{c}\left(\Theta^{-1} y_{t+1} - xz_1 y_{t+1}^* - xz_2 y_{t+1}^{RoW} - xz_4 q_{t+1}\right) - \frac{1}{\sigma}(R_t - E_t\pi_{t+1} - \bar{r} + E_t \ln \epsilon_{t+1} - \ln \epsilon_t)\right]
\end{aligned}
$$

and then rearrange to find the IS equation allowing for demand disturbance:

$$
y_t = E_t y_{t+1} - c\frac{1}{\sigma}\Theta(R_t - E_t\pi_{t+1} - \bar{r}) - xz_1\Theta E_t \Delta y_{t+1}^* - xz_2\Theta E_t \Delta y_{t+1}^{RoW} - xz_4\Theta E_t \Delta q_{t+1} + \varepsilon_t^{IS}
$$

where $\varepsilon_t^{IS} = c\frac{1}{\sigma}\Theta(E_t \ln \epsilon_{t+1} - \ln \epsilon_t)$.

C Other impulse response functions

- Productivity shock

Figure 14: Impulse responses to a US Productivity shock under UIP and Risk Pooling

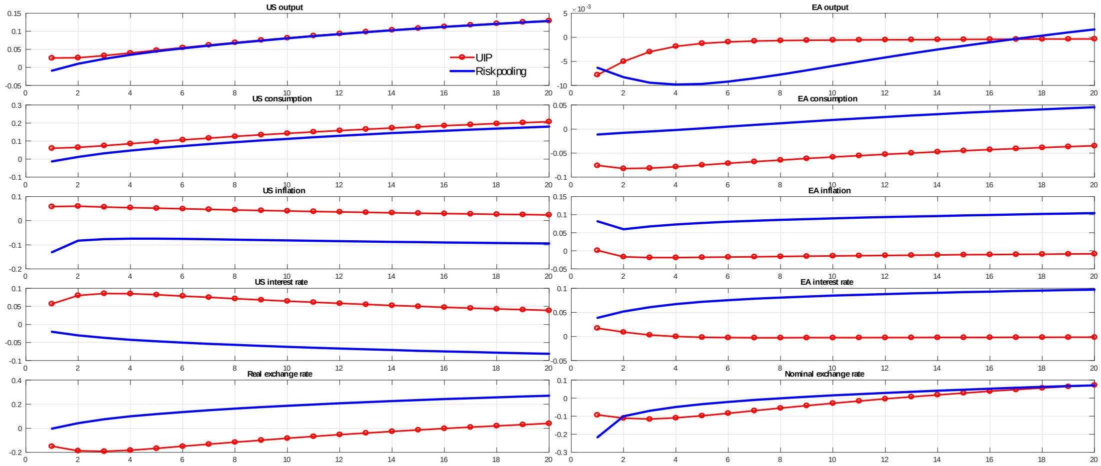
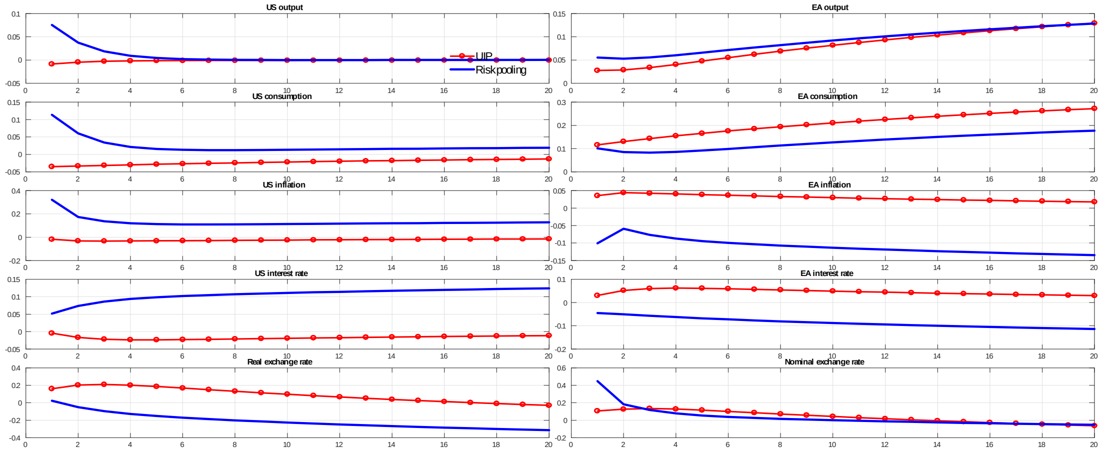


Figure 15: Impulse responses to a EA Productivity shock under UIP and Risk Pooling



- Monetary policy shock

Figure 16: Impulse responses to a US policy shock under UIP and Risk Pooling

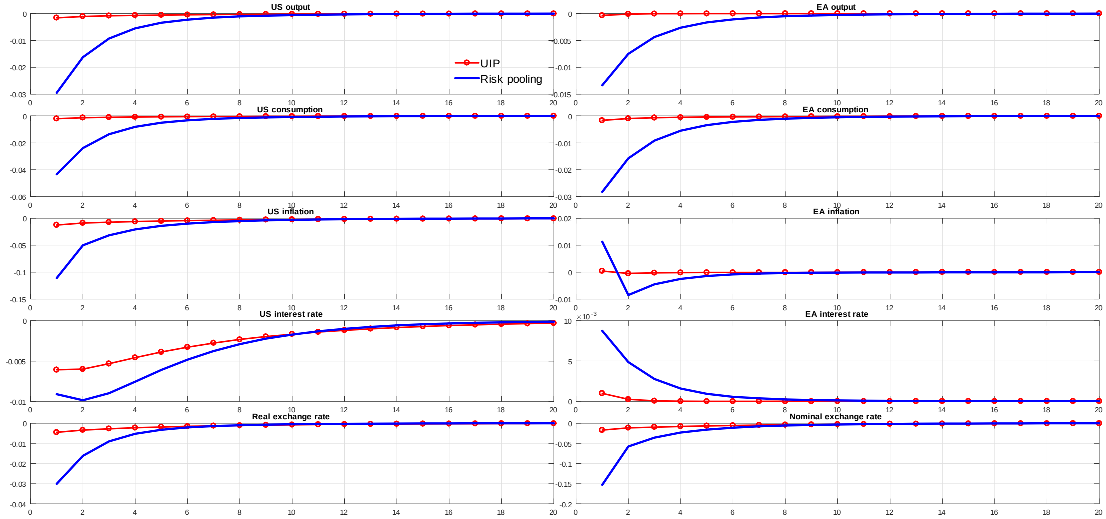
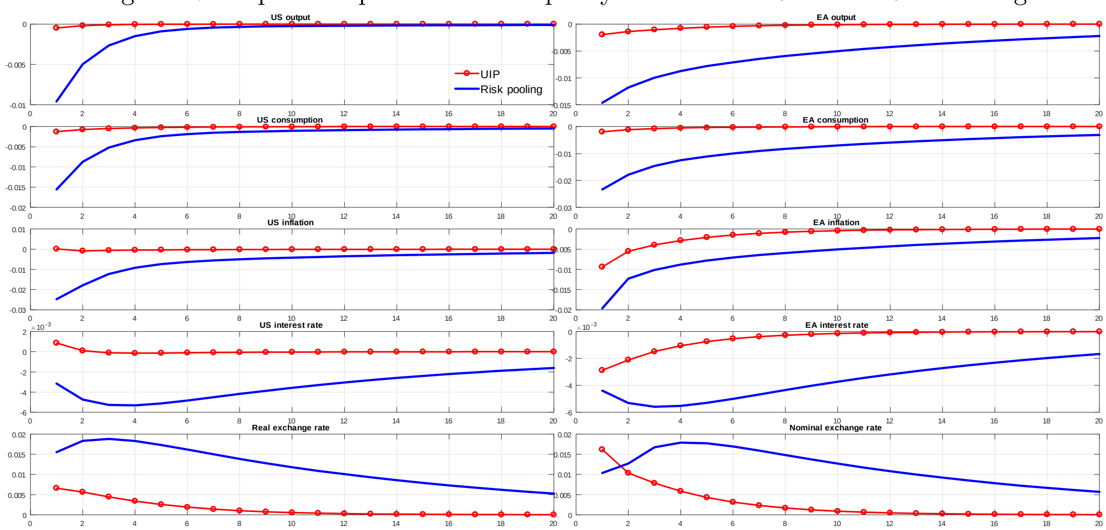


Figure 17: Impulse responses to a EA policy shock under UIP and Risk Pooling



- Supply Shock

Figure 18: Impulse responses to a US supply shock under UIP and Risk Pooling

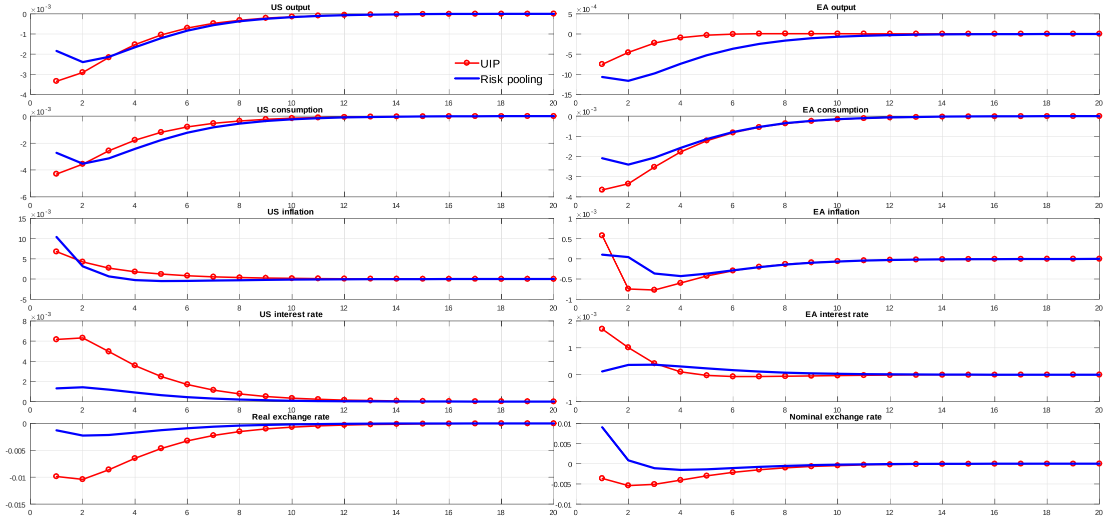
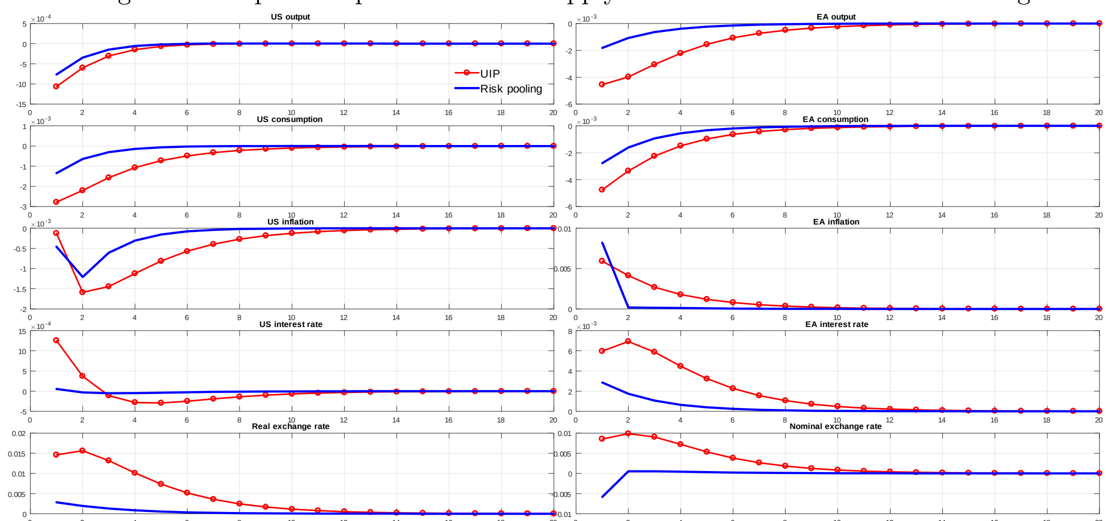
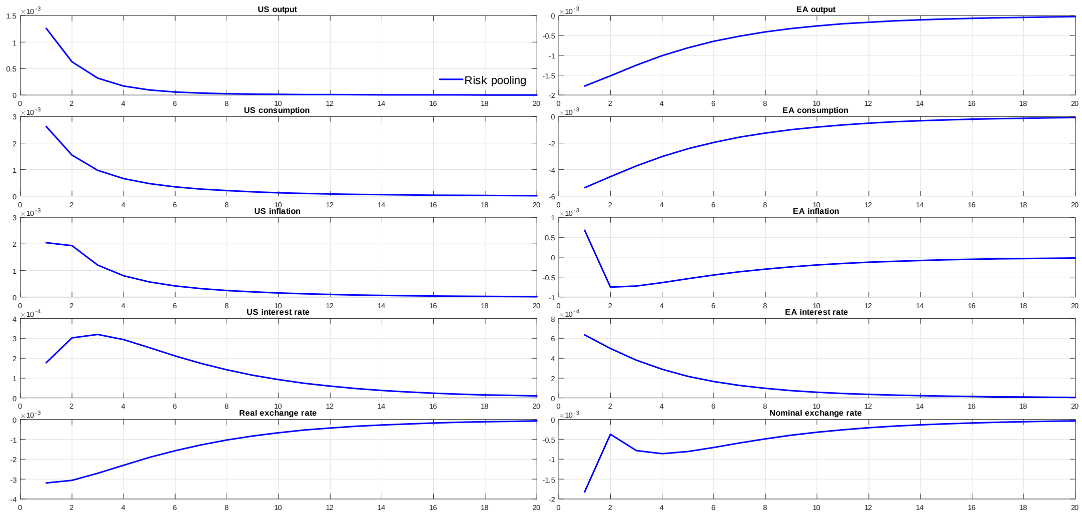


Figure 19: Impulse responses to a EA supply shock under UIP and Risk Pooling



- Risk pooling Shock

Figure 20: Impulse responses to a risk pooling shock under Risk Pooling model



D Variance decomposition over longer horizons

Table 13: Model with UIP - 10 years

Shocks	US				EA					
	y	c	π	r	y^*	c^*	π^*	r^*	RXR	NXR
US Demand	13.43	15.58	1.30	10.70	0.18	1.85	0.57	7.18	2.84	0.24
US Productivity	42.38	42.9	7.10	14.34	0.42	2.94	1.15	0.19	7.05	8.24
US Supply	0.46	0.29	11.76	8.72	0.20	0.47	0.30	2.09	0.82	2.35
US Policy	35.78	25.03	68.97	56.50	6.38	12.19	7.52	4.72	32.00	70.57
EA Demand	2.76	11.84	1.25	0.61	50.89	45.11	11.89	42.67	27.28	4.98
EA Productivity	0.08	0.11	0.28	0.33	5.01	4.84	0.32	0.51	2.04	1.51
EA Supply	0.34	0.46	0.49	0.98	1.54	1.09	18.48	28.42	3.86	0.22
EA Policy	4.78	3.70	8.85	7.82	35.44	31.50	59.76	14.23	24.12	11.89
Total	100	100	100	100	100	100	100	100	100	100

Table 14: Model with Risk Pooling - 10 years

Shocks	US				EA					
	y	c	π	r	y^*	c^*	π^*	r^*	RXR	NXR
US Demand	2.15	1.90	1.59	1.61	0.18	0.40	0.48	0.04	1.14	0.39
US Productivity	59.35	51.57	26.57	41.22	3.22	9.54	40.33	22.49	35.65	2.41
US Supply	2.64	2.34	3.98	3.72	0.24	0.50	0.36	0.12	1.31	0.08
US Policy	24.82	23.64	20.07	3.68	6.69	7.79	0.72	0.11	4.70	77.87
EA Demand	4.34	6.40	7.084	0.20	18.75	8.82	0.12	53.99	8.74	4.36
EA Productivity	0.01	0.27	16.03	27.66	8.09	2.57	25.37	13.19	10.74	0.54
EA Supply	0.12	0.18	0.02	0.01	0.53	0.25	0.00	0.04	0.24	0.09
EA Policy	5.91	10.06	12.24	8.48	42.77	18.37	18.24	5.23	27.52	13.69
Risk pooling	0.66	3.64	12.42	13.42	19.53	51.75	14.38	4.79	9.96	0.58
Total	100	100	100	100	100	100	100	100	100	100

Table 15: Model with UIP - 40 years

Shocks	US				EA					
	y	c	π	r	y^*	c^*	π^*	r^*	RXR	NXR
US Demand	1.78	2.25	1.03	7.87	0.04	0.89	0.42	5.99	0.63	0.08
US Productivity	91.64	91.10	14.94	27.93	0.42	3.65	2.25	0.42	53.10	45.28
US Supply	0.07	0.06	11.92	8.18	0.10	0.14	0.28	2.22	0.23	1.05
US Policy	5.18	4.26	59.88	45.42	2.63	3.38	6.06	4.29	7.73	27.03
EA Demand	0.32	1.16	0.89	0.40	17.10	29.83	7.79	31.56	5.35	1.55
EA Productivity	0.05	0.10	1.03	1.12	60.30	49.28	1.10	1.98	24.37	19.15
EA Supply	0.07	0.11	0.61	1.13	0.92	0.35	21.43	37.24	1.34	0.12
EA Policy	0.87	0.96	9.70	7.94	18.49	12.49	66.67	16.31	7.35	5.74
Total	100	100	100	100	100	100	100	100	100	100

Table 16: Model with Risk Pooling - 40 years

Shocks	US				EA					
	y	c	π	r	y^*	c^*	π^*	r^*	RXR	NXR
US Demand	0.18	0.18	1.69	1.51	0.04	0.09	0.02	0.46	0.11	0.17
US Productivity	95.47	92.89	30.37	40.28	14.65	44.49	33.13	40.30	61.90	31.16
US Supply	0.28	0.27	1.92	1.32	0.06	0.15	0.06	0.13	0.15	0.04
US Policy	2.67	2.84	19.92	6.14	1.88	2.37	0.09	0.43	0.56	43.55
EA Demand	0.39	0.64	3.91	0.06	4.42	2.25	24.71	0.04	0.88	2.04
EA Productivity	0.05	0.88	31.28	43.38	54.42	18.94	34.24	42.06	30.10	12.36
EA Supply	0.02	0.03	0.03	0.00	0.18	0.09	0.07	0.00	0.04	0.06
EA Policy	0.84	1.60	8.72	4.94	15.93	7.39	3.68	8.74	4.38	10.14
Risk pooling	0.11	0.67	2.16	2.36	8.42	24.23	3.99	8.13	1.88	0.47
Total	100	100	100	100	100	100	100	100	100	100