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Testing a model of UK growth – a causal role for R&D subsidies

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We show that a DSGE model in which subsidies to private sector R&D stimulate economic growth, following the predictions of semi-endogenous growth theory, can account for the joint behaviour of UK output and total factor productivity for 1981-2010. R&D subsidies are measured as government-funded R&D performed by the private sector as a proportion of total private sector R&D. We estimate and test the performance of the model using Indirect Inference, and also investigate the robustness of the results using a Monte Carlo exercise. Our findings indicate that sharp cuts in R&D subsidies tend to have highly persistent growth effects in the UK.

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1. INTRODUCTION

Since Schultz (1953) and Griliches (1958) an influential literature has linked R&D activity to economic growth, and the R&D growth channel is now taken as given by many. For instance, Warda (2005) states simply that "Innovation is the engine of growth in a knowledge economy, and Research and Development (R&D) is the key ingredient of the innovation process," going on to say that "Government has a major supporting role in this area by providing a favourable business environment, including appropriate and competitive incentive programs for R&D." (p.2) However, while plainly innovation must cause productivity growth by definition, the causal link between subsidies, corporate R&D and successful innovation remains less empirically certain at the macroeconomic level. This paper therefore investigates a structural model which embeds that key growth hypothesis. The research question is whether direct government R&D subsidies have incentivised the private sector to conduct R&D, and so enhanced innovation and productivity growth in the UK over the sample (1981-2010). The model is tested and estimated using Indirect Inference methods (Le et al., 2012, 2016).

The power of the Indirect Inference test has been shown to be strong in a variety of model contexts, using numerical methods - see for instance Le et al. (2016). We conduct a further Monte Carlo study for the particular model used here which reinforces that conclusion. The exercise allows us to construct uncertainty bounds around our estimates and around the quantitative conclusions derived using the model. For this reason, the present paper is a useful complement to existing empirical work on the macroeconomic impact of direct R&D subsidies. By estimating the DSGE model and testing it in this way, the conclusions cannot be said to rely on an untested calibration.

Further value of the indirect estimation and testing approach taken here lies in the ability to specify a particular causal mechanism for growth in the DSGE model; hence there is no question surrounding the exogeneity of policy in the model. This approach therefore bypasses difficulties associated with potential regressor endogeneity which are so hard to address conclusively in macro-level regressions, while retaining the idea that hypotheses can be tested by classical econometric methods (an idea that receives less attention in the DSGE literature). To check the model's identification, we apply the numerical identification test proposed in Le et al. (2017).

Another advantage is that we can look at a single country, the UK, without imposing homogeneity assumptions across a sample of countries which may actually differ in the relationship between R&D subsidies and growth. As a backdrop for the analysis we take an open economy model which has been shown elsewhere to account well for the UK macroeconomy's behaviour (Meenagh et al., 2010), and add an unambiguous role for R&D subsidies which affect innovation incentives at the microfoundation level. The UK is a highly open economy and we judge that openness to be an important feature in an empirical analysis such as this. In the model, temporary R&D policy shocks generate medium-term growth episodes; like Comin and Gertler (2006), we investigate both short and medium-term business cycle dynamics.

Finally, we note continuing controversy over the importance of direct R&D subsidies to the private sector. R&D policy programmes represent a considerable outlay of public money, but whether they actually generate growth is debated. R&D expenditures and patent numbers are convenient measurables often used as proxies for innovation outputs in empirical studies, but how far they capture innovation is questionable (Danguy et al. 2010). These proxies may be more closely correlated with non-innovative activities. Firms may patent as a signal to capital markets or to earn through licensing revenues, for instance. Increased R&D expenditures resulting from subsidies may also be channeled straight into researcher wage increases, since researcher supply is relatively inelastic (Goolsbee, 1998). Direct support for business R&D is a key plank of the UK government's industrial strategy, so the policy relevance of this question continues to be high (HM Government, 2017).

We find robust evidence in this paper of a positive impact of shocks to direct R&D subsidies on the path of Total Factor Productivity (TFP) and output. The estimated structural model is used to simulate the impact of a one-off, one percentage point shock to direct subsidies which dies out gradually; this generates a long-lasting growth episode in TFP. The episode translates into an increase in the average annual growth rate of output of 0.2 percentage points per annum for nearly two decades.

A review of some existing literature on R&D-driven growth is given in Section 2, focusing on the macroeconomic literature. Section 3 outlines the DSGE model including the growth process. Empirical work follows in Section 4, including an outline of the methodology and data, estimation results and a variance decomposition for the estimated DSGE model. We also report the results of our Monte Carlo exercise on the power of the testing method applied here, as well as simulation results for a controlled temporary R&D policy reform using the estimated model. Section 5 concludes.

2. LITERATURE

In the New Endogenous Growth theory, spillovers overcome diminishing returns to accumulable factors in the aggregate production function, generating sustained economic growth. They also undermine private incentives to innovate since the innovator cannot appropriate the full return from his investment (e.g. Aghion and Howitt, 1992; Romer, 1990). Supposing that a downward incentive effect dominates, the broad flavour of policy recommendations coming out of these models is that research activities should be subsidised directly - or indirectly through fiscal incentives - in order to bring private returns into line with the social rate, and that protection of intellectual property rights should be increased, enabling the innovator to appropriate more of the returns to his investment despite the non-rivalry of knowledge outputs. The underlying structure of the environment can also play a role, depending on the particular model; competition policy and the reduction of barriers to entry and other market frictions may increase the innovation rate (see discussion in Aghion et al. 2013).

Pure endogenous growth models in the style of Romer (1990) predict large long-run growth responses to changes in the scale of the economy's R&D sector; but while R&D activity (in terms of labour inputs and investment) increased dramatically in the last century, long-run growth rates were largely stable. Since Jones (1995), a second generation of 'semi-endogenous' R&D-driven growth models has emerged which imply a weaker scale effect, allowing R&D and policies incentivising it to have important transitional effects on growth but not to determine the long-run. The choice of semi- versus fully endogenous growth mechanism can imply significantly different optimal R&D tax and subsidy policies; see Sener (2008) for discussion. We discuss semi-endogenous growth, given our own empirical focus on the transitional growth effects of R&D policy.

A number of existing DSGE models explore the macroeconomic impacts of R&D policies by simulation, embedding a semi-endogenous R&D-driven growth mechanism and making additional modelling choices which offer various insights. For instance, policymakers may increase innovation through the R&D channel by subsidising human capital accumulation, exploiting complementarities between the two activities that arise through the use of highly skilled workers as an input to the R&D process. This complementarity is modelled in Papageorgiou and Perez-Sebastian (2006) and explored in Varga and 't Veld (2011) among others. Cozzi et al. (2017) take a Schumpeterian approach in which the technology frontier evolves semi-endogenously, combining creative destruction with price stickiness.

McMorrow and Roeger (2009) examine the impact of R&D policy on growth in a global DSGE model calibrated to the EU and to the US. They add the semi-endogenous growth mechanism in Jones (1995) to the European Commission's QUEST III model (Ratto et al. 2009), finding that subsidies to R&D make only a modest contribution to productivity growth. The supply of high-skilled workers is constrained so the subsidy impact is largely absorbed by increases in researcher wages (cf. Goolsbee, 1998). Of course the overall impact is constrained by the semi-endogenous growth assumption. In the short run there is reallocation of high-skilled labour from the production sectors to the research sector, which dampens output directly following the reform (this is the case in the model we propose as well).

A key issue in such models is calibration of the R&D externality parameter.[1] This is generally either set based on the panel econometric literature or set indirectly by other para-

[1] Where international spillovers are included there is both a domestic R&D externality parameter and an international externality to be calibrated.

meter choices, themselves calibrated to results from econometric studies (e.g. Papageorgiou and Perez-Sebastian, 2006). McMorrow and Roeger (2009) calibrate externalities to panel regression estimates from Botazzi and Peri (2007) and Coe and Helpman (1995). Bye et al. (2011) use a CGE model of a small open economy calibrated to Norway to simulate innovation policy reforms; while they calibrate the growth process from econometric results, they note that estimates are scarce for their purposes and rely heavily on sensitivity tests. Of course the simulated policy impacts produced from calibrated DSGE models depend strongly on calibration choices. The difficulties of interpretation posed by macro-level regressions of growth or productivity on policy variables are well known - causality is hard to establish and the scarcity of strong, exogenous instruments for potentially endogenous regressors leaves such regressions prone to bias.[2] We therefore opt not to calibrate the growth process in our model from this literature, given that the magnitude of the parameter on R&D policy is pivotal for our conclusions.

Other notable papers in this literature are Comin and Gertler (2006) and Comin et al. (2009), who provide New Keynesian models of the US. They use a modified expanding varieties mechanism for technological change but add a role for technology absorption, where absorption is costly. Though some parameters are estimated using Bayesian methods, the technological parameters are still calibrated from econometric studies: Comin and Gertler (2006) readily acknowledge the difficulty of calibrating these parameters and that estimates are "crude" (p.541).

The only exception to this calibration strategy that we know of at the time of writing is Cozzi et al. (2017), who estimate structural parameters for a New Keynesian creative destruction model of the US using Bayesian methods, including the parameters featuring in the growth process. The structure of their model differs from ours, but we note their relatively high estimate of the intertemporal knowledge spillover parameter. This implies that shocks affecting R&D intensity will have long-lasting macroeconomic effects. They also find a high persistence for exogenous R&D policy shocks, consistent with our own results below. We prefer a frequentist estimation strategy here since our reading of the empirical literature does not suggest an appropriate prior for parameters governing the R&D subsidy impact in our UK model. The approach taken here also allows us to evaluate the model's performance together

[2] Macro-regression studies are often defended on the grounds that "they help us update our priors about the impact of certain types of policies" (Rodrik, 2012, p. 141) and that "even simple or partial correlations can restrict the range of possible causal statements that can be made" (Wacziarg, 2002, p. 909), but when models are not identified it is not clear that this is defensible.

with the estimated parameter, using the Indirect Inference test. Formal econometric evaluation of DSGE models is now receiving increasing attention in the literature; see Giacomini (2013).

Before presenting the UK model in the next section, we highlight some of our modelling choices in the context of the DSGE literature discussed here - one notable difference being that we abstract from knowledge spillovers in the growth process. Growth occurs in this model due to the representative agent's decision to spend time 'innovating'; the resulting innovation is excludably donated to the firm, of which the agent is sole shareholder.[3] The assumption simplifies the model considerably while allowing the important testable policy implication to emerge, that R&D subsidies stimulate productivity growth. The broader DSGE literature accomodates increasing theoretical complexity which is insightful; our aim is to strip back this complexity for the time being and see whether we find robust empirical evidence for a simple DSGE model in which R&D subsidies cause TFP behaviour. This is a nontrivial question, since there is a strong possibility that the causation works in the opposite direction, or that the effect is simply negligible and that an exogenous growth model is more appropriate. If support is found for the simple mechanism we propose here, we can proceed to model the microfoundations with more complexity.

3. MODEL

We adapt the open economy Real Business Cycle model in Meenagh et al. (2010), adding an endogenous growth process based on Meenagh et al. (2007). It is a two-country model with a single industry; one broad type of consumption good is traded internationally, but home goods sector production is differentiated from the foreign product. Consumers demand both home goods and imported goods. The home country is calibrated to the UK economy and the foreign country represents the rest of the world; its size therefore allows us to treat foreign prices and consumption demand as exogenous. International markets are cleared by the real exchange rate.

The model is a standard UK workhorse in terms of expected macroeconomic and open economy reactions. It is used as a testing vehicle to examine whether the productivity path is systematically affected by shocks to R&D subsidies in the UK - a relationship derived below from the model's microfoundations. This model has the added advantage for the UK

[3] While the firm makes zero profits, the agent obtains the full benefit of productivity increases through resulting real wage increases.

of capturing real exchange rate movements while abstracting from monetary policy, which underwent several regime changes in the UK during this period. Since the calibrated UK model has performed well in similar tests (Meenagh et al., 2010), the introduction of the R&D policy variable should test whether this policy hypothesis alone has caused the rejection.

3.1. Consumer Problem

The consumer chooses consumption (C_t) and leisure (x_t) to maximise lifetime utility, U:

$$U = \max E_0[\sum_{t=0}^{\infty} \beta^t u(C_t, x_t)] \tag{1}$$

$u(.)$ takes the form:

$$u(C_t,x_t) = \theta_0 \frac{1}{(1-\rho_1)} \gamma_t C_t^{(1-\rho_1)} + (1-\theta_0) \frac{1}{(1-\rho_2)} \xi_t x_t^{(1-\rho_2)} \tag{2}$$

$\rho_1, \rho_2 > 0$ are coefficients of relative risk aversion; γ_t and ξ_t are preference shocks, and $0 < \theta_0 < 1$ is consumption preference.

The agent divides time among three activities: leisure, labour N_t supplied to the firm for the real wage w_t, and an activity z_t that is unpaid at t but known to have important future returns. The time endowment is:

$$N_t + x_t + z_t = 1 \tag{3}$$

Here the consumer chooses leisure, consumption, domestic and foreign bonds (b, b^f) and bonds issued by the firm to finance its capital investment ($\tilde{b}$), and new shares (S^p) purchased at price q, subject to the real terms budget constraint.[4]

$$\begin{gathered} C_t + b_{t+1} + Q_t b_{t+1}^f + q_t S_t^p + \tilde{b}_{t+1} = w_t N_t - T_t + b_t(1+r_{t-1}) + \\ Q_t b_t^f (1+r_{t-1}^f) + (q_t + d_t) S_{t-1}^p + (1+\hat{r}_{t-1}) \tilde{b}_t \end{gathered} \tag{4}$$

The taxbill T_t is defined further below. The only taxed choice variable in the model is z_t; all other taxes are treated as lump sum to rule out wealth effects. Since the choice of z_t is left aside until Section 3.4 on endogenous growth, the taxbill is not relevant at this stage of the problem. $Q_t = \frac{P_t^f}{P_t}.\hat{E}_t$ gives relative consumer prices. The nominal exchange rate $\hat{E}_t$ is

[4] Price P_t of the consumption bundle is numeraire

assumed fixed; Q_t is then the relative import price.[5] Higher Q_t implies a real depreciation of domestic goods on world markets and hence an increase in competitiveness; this can be thought of as a real exchange rate depreciation.

The consumer's first order conditions yield the Euler equation (5), the intratemporal condition (6),[6] real uncovered interest parity (7), and the share price formula (8). First order conditions on $\tilde{b}_{t+1}$ and b_{t+1} combine for $\hat{r}_t = r_t$. Indeed, returns on all assets (S_t^p, b_{t+1}, $\tilde{b}_{t+1}$ and b^f_{t+1}) are equated.

$$\frac{1}{(1+r_t)}\gamma_t C_t^{-\rho_1} = \beta E_t[\gamma_{t+1} C_{t+1}^{-\rho_1}] \tag{5}$$

$$\frac{U_x}{U_c}|_{U=0} = \frac{(1-\theta_0)\xi_t x_t^{-\rho_2}}{\theta_0 \gamma_t C_t^{-\rho 1}} = w_t \tag{6}$$

$$(1+r_t) = E_t \frac{Q_{t+1}}{Q_t}(1+r_t^f) \tag{7}$$

$$q_t = \frac{q_{t+1}+d_{t+1}}{(1+r_t)} = \sum_{i=1}^{\infty} \frac{d_{t+i}}{\prod\limits_{j=0}^{i-1}(1+r_{t+j})} \tag{8}$$

Equation 8 rests on the further assumption that q_t does not grow faster than the interest rate, $\lim_{i\to\infty} \frac{q_{t+i}}{\prod\limits_{j=0}^{i-1}(1+r_{t+j})} = 0$.

The domestic country has a perfectly competitive final goods sector, producing a version of the final good differentiated from the product of the (symmetric) foreign industry. The model features a multi-level utility structure (cf. Feenstra et al. 2014). The level of C_t chosen above must satisfy the expenditure constraint,

$$C_t = p_t^d C_t^d + Q_t C_t^f \tag{9}$$

$p_t^d \equiv \frac{P_t^d}{P_t}$. C_t^d and C_t^f are chosen to maximise $\tilde{C}_t$ via the following utility function (equation 10), subject to the constraint that $\tilde{C}_t \leqslant C_t$.

$$\tilde{C}_t = [\omega(C_t^d)^{-\rho} + (1-\omega)\varsigma_t(C_t^f)^{-\rho}]^{-\frac{1}{\rho}} \tag{10}$$

At a maximum the constraint binds; $0 < \omega < 1$ denotes domestic preference bias. Import

[5] b^f_{t+1} is a real bond - it costs what a unit of the foreign consumption basket (C_t^*) would cost, i.e. P_t^* (the foreign CPI). In domestic currency, this is $P_t^* \hat{E}_t$. Assuming $P_t^* \simeq P_t^f$ (i.e. exported goods from the home country have little impact on the larger foreign country) the unit cost of b^f_{t+1} is Q_t.

[6] Later we show that the return on labour time, w_t, is equal at the margin to the return on z_t.

demand is subject to a shock, ς_t. The elasticity of substitution between domestic and foreign varieties is constant at $\sigma = \frac{1}{1+\rho}$. First order conditions imply the relative demands for the imported and domestic goods:

$$\frac{C_t^f}{C_t} = \left(\frac{(1-\omega)\varsigma_t}{Q_t}\right)^{\sigma} \tag{11}$$

$$\frac{C_t^d}{C_t} = \left(\frac{\omega}{p_t^d}\right)^{\sigma} \tag{12}$$

Given equation 11 above, the symmetric equation for foreign demand for domestic goods (exports) relative to general foreign consumption is

$$(C_t^d)^* = C_t^* \left(\left(1-\omega^F\right)\varsigma_t^*\right)^{\sigma^F} (Q_t^*)^{-\sigma^F} \tag{13}$$

* signifies a foreign variable; ω^F and σ^F are foreign equivalents to ω and σ. Q_t^* is the foreign equivalent of Q_t, import prices relative to the CPI, and $\ln Q_t^* \simeq \ln p_t^d - \ln Q_t$.[7] An expression for p_t^d as a function of Q_t follows from the maximised equation 10:

$$1 = \omega^{\sigma}(p_t^d)^{\rho\sigma} + [(1-\omega)\varsigma_t]^{\sigma} Q_t^{\rho\sigma} \tag{14}$$

A first order Taylor expansion around a point where $p^d \simeq Q \simeq \varsigma \simeq 1$, with $\sigma = 1$, yields a loglinear approximation for this:

$$\ln p_t^d = \hat{k} - \frac{1-\omega}{\omega}\frac{1}{\rho}\ln\varsigma_t - \frac{1-\omega}{\omega}\ln Q_t \tag{15}$$

The export demand equation is then

$$\ln(C_t^d)^* = \check{c} + \ln C_t^* + \sigma^F \frac{1}{\omega}\ln Q_t + \varepsilon_{ex,t} \tag{16}$$

where $\check{c}$ collects constants and $\varepsilon_{ex,t} = \sigma^F[\ln\varsigma_t^* + \frac{1-\omega}{\omega}\frac{1}{\rho}\ln\varsigma_t]$.

Assuming no capital controls, the real balance of payments constraint is satisfied.

$$\Delta b_{t+1}^f = r_t^f b_t^f + \frac{p_t^d EX_t}{Q_t} - IM_t \tag{17}$$

[7] $Q_t^* = \frac{P_t^d}{P_t^*}$ - since $Q_t = \frac{P_t^f}{P_t}$ and P_t is numeraire, $Q_t = P_t^f$. If domestic export prices hardly influence the foreign CPI then $P_t^* \simeq P_t^f$.

3.2. Firm Problem

The representative firm produces the final good via a Cobb Douglas function with constant returns to scale, where A_t is total factor productivity:

$$Y_t = A_t K_t^{1-\alpha} N_t^{\alpha} \tag{18}$$

There are diminishing marginal returns to labour and capital. The firm also faces convex adjustment costs to capital. The firm undertakes investment, purchasing new capital via debt issue ($\tilde{b}_{t+1}$) at t; the cost $\hat{r}_t$ is payable at $t+1$. Bonds are issued one for one with capital units demanded: $\tilde{b}_{t+1} = K_t$.The cost of capital covers the return demanded by debt-holders, capital depreciation δ and adjustment costs, $\tilde{a}_t$.[8] The profit function is:

$$\pi_t = Y_t - \tilde{b}_{t+1}(\hat{r}_t + \delta + \kappa_t + \tilde{a}_t) - (\tilde{w}_t + \chi_t)N_t$$

$\tilde{w}_t$ is the real unit cost of labour; κ_t and χ_t are cost shocks capturing random movements in marginal tax rates. The consumer's first order conditions showed $\hat{r}_t = r_t$. Substituting for $\tilde{b}_{t+1} = K_t$ and $\hat{r}_t = r_t$, profits are:

$$\pi_t = Y_t - K_t(r_t + \delta + \kappa_t) - \frac{1}{2}\zeta(\Delta K_t)^2 - (\tilde{w}_t + \chi_t)N_t \tag{19}$$

Here adjustment costs are explicit, having substituted $\tilde{b}_{t+1}\tilde{a}_t = K_t\tilde{a}_t = \frac{1}{2}\zeta(\Delta K_t)^2$. Parameter ζ is constant.

The firm chooses K_t and N_t to maximise expected profits, taking r_t and $\tilde{w}_t$ as given. Assume free entry and a large number of firms operating under perfect competition. The optimality condition for K_t equates the marginal product of capital (net of adjustment costs and depreciation) to its price, plus cost shock – d is the firm's discount factor. Rearranged, this gives a non-linear difference equation in capital.

$$K_t = \frac{1}{1+d}K_{t-1} + \frac{d}{1+d}E_t K_{t+1} + \frac{(1-\alpha)}{\zeta(1+d)}\frac{Y_t}{K_t} - \frac{1}{\zeta(1+d)}(r_t + \delta) - \frac{1}{\zeta(1+d)}\kappa_t \tag{20}$$

[8] the adjustment cost attached to $\tilde{b}_{t+1}$ is: $\tilde{b}_{t+1}\tilde{a}_t = \tilde{b}_{t+1}.\frac{1}{2}\zeta\left(\tilde{b}_{t+1} + \frac{\tilde{b}_t^2}{\tilde{b}_{t+1}} - 2\tilde{b}_t\right) = \frac{1}{2}\zeta(\Delta\tilde{b}_{t+1})^2$

Given capital demand, the firm's investment, I_t, follows via the capital accumulation identity.

$$K_t = I_t + (1-\delta)K_{t-1} \tag{21}$$

The optimal labour choice gives the firm's labour demand condition:

$$N_t = \alpha . \frac{Y_t}{\tilde{w}_t + \chi_t} \tag{22}$$

Internationally differentiated goods introduce a wedge between the consumer real wage, w_t, and the real labour cost for the firm, $\tilde{w}_t$.[9] The wedge is

$$p_t^d = \frac{w_t}{\tilde{w}_t} \tag{23}$$

implying, via 15, the following relationship:

$$\ln w_t = \hat{k} + \ln \tilde{w}_t - \frac{1-\omega}{\omega} \ln Q_t - \frac{1-\omega}{\omega} \frac{1}{\rho} \ln \varsigma_t \tag{24}$$

3.3. Government

The government spends on the consumption good (G_t) subject to its budget constraint.

$$G_t + b_t(1 + r_{t-1}) = T_t + b_{t+1} \tag{25}$$

Spending is assumed to be non-productive (transfers). As well as raising tax revenues T_t the government issues one-period bonds. Each period, revenues cover spending and the current interest bill: $T_t = G_t + r_{t-1}b_t$ and so $b_t = b_{t+1}$. Therefore government debt is fixed in the model. Revenue T_t is made up as follows.

$$T_t = \Phi_t - s_t z_t \tag{26}$$

s_t is a proportional subsidy rate on time spent in activity z_t. Φ_t, a lumpsum tax capturing the revenue effects of all other tax instruments, responds to changes in $s_t z_t$ to keep tax revenue neutral in the government budget constraint. Government spending is modeled as an exogenous

[9] The firm's real cost of labour is the nominal wage W_t relative to domestic good price, P_t^d, while the real consumer wage is W_t relative to the general price P_t.

trend stationary AR(1) process.

$$\ln G_t = g_o + g_1 t + \rho_g \ln G_{t-1} + \eta_{g,t} \tag{27}$$

where $\mid \rho_g \mid < 1$ and $\eta_{g,t}$ is a white noise innovation.

3.4. Productivity Growth

Assume that productivity growth is a linear function of time spent in some innovation-enhancing activity z_t.

$$\frac{A_{t+1}}{A_t} = a_0 + a_1 z_t + u_t \tag{28}$$

where $a_1 > 0$. z_t is the systematic channel through which policy incentives, s_t, drive growth.[10] Here z_t is assumed to be time spent in R&D.

The model is similar to Lucas (1990) where growth depends on time spent accumulating human capital. In the short term the return to labour (for a given level of human capital) is foregone to raise the human capital stock. The endogenous growth process below is adapted from Meenagh et al. (2007) to a decentralised framework.

The consumer chooses z_t to maximise utility (eqn.s 1 and 2), subject to equations 3 ,4 and 26. We assume the consumer's shareholdings are equivalent to a single share:[11] $S_t^p = \bar{S} = 1$. The rational agent expects z_t to raise her consumption possibilities through her role as the firm's sole shareholder. She knows that, given equation 28, a marginal change in z_t permanently raises productivity from $t+1$. This higher productivity is fully excludable and donated to the atomistic firm she owns; higher productivity is anticipated to raise household income via firm profits paid out as dividends, d_t (everything leftover from revenue after labour and capital costs are paid). The choice is thought not to affect economy-wide aggregates; all prices are taken as parametric (note that the productivity increase is not expected to increase the consumer real wage here, though it does so in general equilibrium - cf. Boldrin and Levine, 2002 and 2008).[12]

Substituting into the first order condition for z_t using equation 28 and rearranging for $\frac{A_{t+1}}{A_t}$ yields (after some approximation)

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[10] All other factors that systematically affect growth are therefore in the error term.

[11] This allows the substitution in the budget constraint that $q_t S_t^p - (q_t + d_t) S_{t-1}^p = -d_t$.

[12] Given the time endowment $1 = N_t + x_t + z_t$, the agent has indifference relations between z_t and x_t, between x_t and N_t, and z_t and N_t. The intratemporal condition in 6 gives the margin between x_t and N_t; here we focus on the decision margin between z_t and N_t, so the margin between z_t and x_t is implied. Therefore the substitution $N_t = 1 - x_t - z_t$ can be made in the budget constraint.

$$\frac{A_{t+1}}{A_t} = a_1 \cdot \frac{\frac{\beta\rho_\gamma}{1-\beta\rho_\gamma} \cdot \frac{Y_t}{C_t}}{\frac{w_t}{C_t}(1 - s'_t)} \tag{29}$$

The full derivation is given in the Appendix. We focus on $\frac{s_t}{w_t} \equiv s'_t$, a unit free measure with the dimensions of a rate unlike s_t which, like the wage, is an amount payable per unit of time. A first order Taylor expansion of the righthand side of equation 29 around $s'_t = s'$ gives a linear relationship between $\frac{A_{t+1}}{A_t}$ and s'_t of the form

$$d \ln A_{t+1} = b_0 + b_1 s'_t + \varepsilon_{A,t} \tag{30}$$

where $b_1 = a_1 \cdot \frac{\frac{\beta\rho_\gamma}{1-\beta\rho_\gamma} \frac{Y}{C}}{\frac{w}{C}(1-s')^2}$.[13] Note that this relationship came out of the first order condition for z_t. The household chooses z_t taking all other sources of productivity growth as exogenous. Equation 30 drives the behaviour of the model in simulations.

There are notable aspects of the R&D growth channel that we abstract from here. We do not include distance to the global technological frontier; nor do we explicitly include spillovers in the micro-foundations. There is no suggestion that growth is in reality as simple as this model suggests. We look simply at whether the approximations made here are empirically justifiable.

Substituting into 29 using 28 reveals a relationship between z_t and s'_t. Define $\frac{\partial z_t}{\partial s'_t} \equiv c_1$, assumed constant. This parameter enters the simulation explicitly in the producer labour cost equation:

$$\ln \tilde{w}_t = const_4 + \rho_2 \ln N_t + \rho_1 \ln C_t + \left[\frac{1-\omega}{\omega}\right]^\sigma \ln Q_t - \rho_2 2 c_1 s'_t + e_{w,t} \tag{31}$$

where $e_{w,t} = -\ln \gamma_t + \ln \xi_t + \frac{1}{\rho}\left[\frac{1-\omega}{\omega}\right]^\sigma \ln \varsigma_t$ – so the unit labour cost shock is a combination of preference shocks to consumption and leisure and to import demand. This equation is derived from the intratemporal condition (equation 6) which governs labour supply choices (full derivation in Appendix). Since s'_t is an incentive to R&D, $c_1 > 0$ and hence $\frac{d \ln \tilde{w}_t}{ds'_t} > 0$ and equally $\frac{d \ln N_t}{ds'_t} < 0$, as equation 31 is simply the labour supply condition rearranged. The worker's response to a higher subsidy rate on z_t is to reduce time spent in ordinary employment.

[13] Other terms in the expansion are treated as part of the error term.

3.5. Closing the model

Goods market clearing in volume terms is:

$$Y_t = C_t + I_t + G_t + EX_t - IM_t \tag{32}$$

All asset markets also clear.

A transversality condition is also required to ensure a balanced growth equilibrium is reached for this open economy, to rule out growth financed by insolvent borrowing rather than growing fundamentals. The restriction on the balance of payments is that the long run change in net foreign assets (the capital account) is zero. At a notional date T when the real exchange rate is constant, the cost of servicing the current debt is met by an equivalent trade surplus.

$$r_T^f b_T^f = -\left(\frac{p_T^d.EX_T}{Q_T} - IM_T\right) \tag{33}$$

The numerical solution path is forced to be consistent with the constraints this condition places on the rational expectations. In practice it constrains household borrowing since government solvency is ensured already, and firms do not borrow from abroad. When solving the model, the balance of payments constraint is scaled by output so that the terminal condition imposes that the ratio of debt to gdp must be constant in the long run, $\Delta \hat{b}_{t+1}^f = 0$ as $t \to \infty$, where $\hat{b}_{t+1}^f = \frac{b_{t+1}^f}{Y_{t+1}}$. The model is loglinearised before solution and simulation; the full model listing is in Appendix B.

3.6. Exogenous variables

Stationary exogenous variables are are shocks to real interest rates (Euler equation), labour demand, real wages, capital demand, export demand and import demand. These are not directly observed but are implied as the difference between the data and the model predictions. Those differences $e_{i,t}$ are treated as trend stationary AR(1) processes:

$$e_{i,t} = a_i + b_i t + \rho_i e_{i,t-1} + \eta_{i,t} \tag{34}$$

$\eta_{i,t}$ is an i.i.d mean zero innovation term; i identifies the shock. We model foreign consumption demand, government consumption, foreign interest rates and policy variable s_t' similarly. AR(1) coefficients ρ_i are estimated. Where expectations enter, they are estimated using a robust instrumental variable technique (Wickens, 1982; McCallum,1976); they are the one step ahead predictions from an estimated VECM. Where $a_i \neq 0$ and $b_i \neq 0$, detrended residual $\hat{e}_i$ is used:

$$\hat{e}_{i,t} = \rho_i \hat{e}_{i,t-1} + \eta_{i,t} \tag{35}$$

$$\hat{e}_{i,t} = e_{i,t} - \hat{a}_i - \hat{b}_i t \tag{36}$$

The innovations $\eta_{i,t}$ are approximated by the fitted residuals from estimation of equation 35, $\hat{\eta}_{i,t}$. The Solow residual $\ln A_t$ is modelled as a unit root process with drift driven by a stationary AR(1) shock and by exogenous variable s'_t, following equation 30.

$$\ln A_t = d + \ln A_{t-1} + b_1 s_{t-1} + e_{A,t} \tag{37}$$

$$e_{A,t} = \rho_A e_{A,t-1} + \eta_{A,t} \tag{38}$$

Deterministic trends are removed from exogenous variables since they enter the model's balanced growth path. We focus here on how the economy deviates from steady state in response to shocks - in particular, stationary shocks to R&D subsidies. Such shocks will have a permanent shift effect on the path of TFP via its unit root. Due to their persistence they also generate transitional TFP growth episodes above long-run trend.

4. EMPIRICAL WORK

4.1. Indirect Inference Methods

The model in the preceding section is tested and estimated using the Indirect Inference method from Le et al. (2011). The approach is similar to traditional RBC moment-matching, but adds a formal test for the closeness of moments. Samples generated from the bootstrapped model and the observed data are described atheoretically by an auxiliary model, used as a basis for the comparison. The full methodology is given in Le et al. (2016). We describe it briefly here.

Given parameter set θ, J bootstrap simulations are generated from the DSGE model. Having added back the effects of deterministic trends removed from shocks, an auxiliary model is estimated for all J pseudo-samples. The estimated auxiliary model coefficient vectors a_j ($j = 1, ..., J$) yield the variance-covariance matrix Ω of the DSGE model's implied distribution for these coefficients. Hence the small-sample distribution for the Wald statistic $WS(\theta)$ is obtained:

$$WS(\theta) = (a_j - \overline{a_j(\theta)})' W(\theta)(a_j - \overline{a_j(\theta)}) \tag{39}$$

$\overline{a_j(\theta)}$ is the mean of the J estimated vectors and $W(\theta) = \Omega(\theta)^{-1}$ is the inverse of the estimated variance-covariance matrix. The test statistic, $WS^*(\theta)$, is $WS^*(\theta) = (\hat{\alpha} - \overline{a_j(\theta)})' W(\theta)(\hat{\alpha} - \overline{a_j(\theta)})$ – this depends on the distance between $\overline{a_j(\theta)}$ and $\hat{\alpha}$, where $\hat{\alpha}$ is the coefficient vector estimated from the UK data. Inference proceeds by comparing the percentile of the Wald distribution in which the test statistic falls with the chosen size of the test; for 5% significance, a percentile above 95% signifies rejection. We can present the same information as a Mahalanobis distance or as a p-value.

For estimation, a 'simulated annealing' algorithm performs the indirect inference Wald test for points inside a bounded parameter space. We search for a structural parameter set such that the restrictions the model imposes, including the causal relationship from R&D subsidies to TFP, do not lead it to be rejected as a data generating process. This is discussed further below.

4.2. Data

4.2.1. UK Macroeconomic Data

We use unfiltered data from 1981 to 2010. For problems inherent in data filtering see e.g. Hamilton (2016). Here we are interested in relatively long growth episodes in response to shocks propagated through non-stationary TFP; the risk of mistaking that response for a change in underlying trend and removing it is high with the HP filter (cf. Comin and Gertler, 2006). The auxiliary model is therefore a Vector Error Correction Model since the data is non-stationary; this is discussed further in section 4.3. Key UK macroeconomic data is plotted in Figure 1. Data sources are listed in Appendix C.

4.2.2. Data on R&D Subsidies

The hypothesis is that $b_1 > 0$ in equation 30, i.e. s_t' encourages the growth driver z_t, defined here as R&D. Since z_t itself is not included in model simulations, the choice of data for s_t identifies the growth channel. Data is available post-1981 for R&D. The policy variable used is the ratio of business-performed R&D expenditure (BERD) financed directly by government, to the total level of BERD (all sources of funding). This is referred to below as the

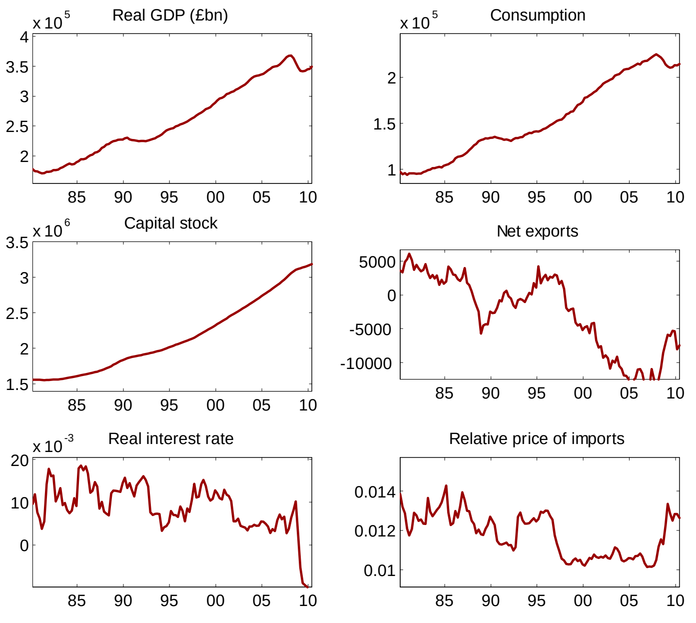

FIG. 1 Key quarterly UK data for 1980-2010.

subsidy rate. Aggregate data on BERD from 1981 is annual with missing values at 1982 and 1984. Missing values are interpolated as the arithmetic average of the two contiguous values (robustness checks on the interpolation choice show it has no impact on results). The ratio is interpolated from annual to a quarterly frequency using a constant average match interpolation. Figure 2 plots the series for constant average and quadratic average interpolation. The detrended subsidy variable is modeled as a persistent but stationary AR(1) process (see exogenous variables section above).

This R&D subsidy variable excludes fiscal incentives to R&D which have increased in the UK since 2000, so it is only a partial proxy for policy incentives to R&D. However, fiscal incentives as measured by the OECD B-Index may affect R&D and productivity growth differently to direct subsidies (e.g. Foreman-Peck, 2013), so it is not immediately clear that we should combine them into a single index. Likewise, no indicator of intellectual property rights spans a long enough timeframe for this investigation; and for the UK such an indicator would show little time series variation. We could resort to patent counts to proxy innovation policy, but a) these are an outcome and may not be a good proxy for policy and b) they respond in a way that may have nothing to do with productivity (see e.g. van Pottelsberghe, 2011, for related

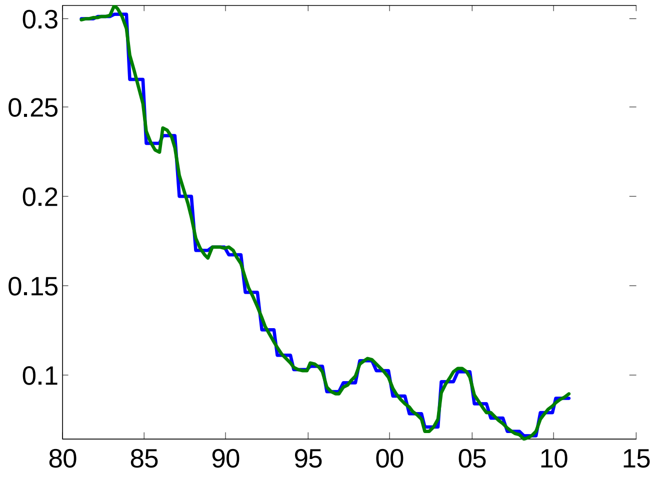

FIG. 2 Business R&D Subsidy Variable. Ratio of Government Funded BERD to Total BERD. Constant and average match interpolation. Source, OECD

literature). For these reasons, the subsidy variable employed here is preferred.

4.3. Auxiliary Model

The full solution to the structural model can be represented as a cointegrated VECM rearranged as a VARX(1) – see Appendix D. The general form is

$$y_t = [I - K]y_{t-1} + K\Pi x_{t-1} + n + \phi t + q_t \tag{40}$$

The error q_t contains suppressed lagged difference regressors, while t captures the deterministic trend in $\bar{x}_t$ (the balanced growth behaviour of the exogenous variables) affecting both the endogenous and exogenous variables. x_{t-1} contains unit root variables, present to control for the impact of past shocks on the long run path of both x and y. This VARX(1) approximation to the reduced form of the model is the unrestricted auxiliary model used to assess the closeness of model-simulated samples to the observed data.

The focus is on the transitional growth of output and TFP and whether our assumptions about the causal role of R&D policy are correct, so we use a 'directed' Wald (Le et al. 2011). Endogenous variables in the auxiliary VARX(1) are therefore output and TFP, while exogenous lagged variables are the subsidy variable and net foreign assets, b^f_{t-1}. The latter captures the stochastic trend in the model through its unit root.

The test is whether the model replicates the features not just of output and productivity taken singly, but the joint behaviour of those variables conditional on the behaviour of any non-stationary predetermined variables and of the policy variable. Although this VARX(1) is a severe approximation of the model's solution, the power of the test remains strong; the small sample properties of Indirect Inferences are discussed for a variety of models in Le et al. (2016). Since Monte Carlo studies can be model-dependent, we also investigate the power of the test in this particular context in Section 4.6 below.

The vector a_j used to construct the Wald distribution (eq. 39) includes OLS estimates of coefficients on the lagged endogenous and exogenous variables, as well as the variances of the fitted auxiliary model errors; the same coefficients make up vector $\hat{\alpha}$ estimated on the observed data. The VARX errors are also tested for stationarity. The trend term in the VARX(1) captures the deterministic trend in the data and simulations. Since the focus of the study is on the stochastic trend resulting from the shocks, the deterministic trend is not part of the Wald test on which the model's performance is evaluated.

4.4. Indirect Inference Testing and Estimation Results

We first test a baseline calibration of the model, using parameter values from Meenagh et al (2010). We then estimate a number of the structural parameters of the model using Indirect Inference and report those estimates as well as the Wald test statistic for the model with that set of parameters. The structural parameters we estimate are listed in Table 3. They are generally preference-related parameters, as well as the policy-growth parameter, for which no strong priors exist. Due to the attention paid in the literature to adjustment inertia in the response of R&D to policy determinants (Guellec and van Pottelsberghe, 2000; Westmore, 2013; Di Comite et al. 2015), we also test and estimate the model with a 4 quarter lag in the subsidy rate, whereas the baseline model assumes a 1 quarter lag. Some structural model coefficients are kept fixed throughout, at values taken from Meenagh et al. (2010); see Table 1. Long run ratios featuring in the loglinearised model for $\frac{M}{Y}$, $\frac{X}{Y}$, $\frac{Y}{C}$ and $\frac{G}{C}$ are calibrated to UK post-war averages. $\frac{X}{C}$ and $\frac{M}{C}$ are then set to be consistent with those values.

The baseline calibration is given in Table 3, column 3. The implied AR(1) coefficients for the stationary exogenous variables are given in column 3 of Table 4. Analysis of impulse response functions show real business cycle behaviour consistent with Meenagh et al. (2010); impulse responses for a one-off policy shock are likewise as expected – see section 4.7 below.

Labour share in output, α	0.7
Quarterly discount factor, β	0.97
Quarterly capital depreciation rate, δ	0.0125
K/C	0.196
Y/C	1.732
M/C	0.369
X/C	0.361
G/C	0.442
X/Y	0.208
M/Y	0.213
Y/K	0.333

TABLE 1
Fixed structural parameters

The macroeconometric literature does not offer a strong prior for b_1, the impact of the subsidy shock on next period's TFP, in terms of sign or magnitude. Estimates for the impact of R&D on TFP and of direct subsidies on TFP or output growth vary across different regression models and estimators, for different samples and for different measures of R&D or of the policy environment. The same holds for c_1; compare e.g. Falk (2006) to Westmore (2013). Lacking a compelling rationale for calibrating this model from the existing literature, starting values chosen for these are 0.1 and 0.06 respectively, and we search around these values in the estimation procedure.[14] A preliminary to the estimation is to set bounds on the parameter space; these are set at 30% either side of the baseline calibration. If the parameter starting value is inappropriate, the estimation process will move towards one of the initial bounds, indicating that bounds should be shifted.

The addition of the policy-driven TFP process leads the model to be rejected by the test with this structural calibration (Table 3, column 3). This is true when the policy shock is included in TFP with both a 1-quarter lag and 4-quarter lag; in each case the test statistic falls in the 100th percentile of the bootstrapped Wald distribution. However, when the model is estimated by Indirect Inference a structural parameter set is found such that the model is not rejected by the test. For the parameters listed in Table 3, column 4, the test statistic falls in the 77th percentile of the distribution, signifying a comfortable non-rejection. Several coefficients have moved some way from their starting values.

Assuming a 4 quarter lag for the impact of the subsidy shock yields a borderline non-

[14] A small starting value for c_1 is preferred since the labour supply effects induced by policy change should plausibly be small.

	r	Output	Labour	Q	NFA	TFP
e_r	0.169	0.002	0.009	0.012	0.031	0
e_A	0.231	0.350	0.228	0.300	0.012	0.371
e_N	0.031	0.002	0.015	0.001	0.001	0
e_K	0.160	0.025	0.045	0.014	0.012	0
e_w	0.122	0.020	0.162	0.006	0.005	0
e_X	0.034	0.010	0.103	0.145	0.653	0
e_M	0.028	0.001	0.014	0.013	0.054	0
e_{subs}	0.142	0.589	0.406	0.470	0.096	0.629
C^F	0.005	0.002	0.016	0.036	0.131	0
r^F	0.071	0.000	0.001	0.004	0.004	0
G	0.005	0.000	0.002	0.000	0.000	0

TABLE 2
Variance decomposition for key endogenous variables based on estimated parameter set 1. NFA is Net Foreign Assets. Q is the real exchange rate.

rejection with a Wald percentile of 94.48, obtained from a different structural parameter set (Table 3, col. 5). This is a much weaker result.

4.5. Variance Decomposition

A variance decomposition for key variables with this coefficient set is reported in Table 2.[15] See Appendix E for the full variance decomposition (all endogenous variables); here we pick out output and TFP due to their relevance for the growth question, as well as labour supply (impacted by the subsidy) and key open economy variables: the real interest rate, real exchange rate and net foreign assets. The identified subsidy shock generates considerable variability across all endogenous variables and accounts for 62.8% of total variance in TFP in the estimated model, more than the independent shock to TFP.[16] The estimated value of b_1 is clearly large enough to distinguish this model clearly from an exogenous productivity growth model.

[15] We bootstrap the model and calculate the variance of the simulated endogenous variables generated by each of the eleven shocks, taken one at a time. For each column, the cell values indicate the proportion of the total model variance for that endogenous variable generated by each exogenous variable; columns of Table 2 sum to unity.

[16] The subsidy shock and shocks to $e_{A,t}$ are bootstrapped independently.

		Starting calibration	Estimates I	Estimates II (4-quarter lag)
CRRA coefficient (C_t)	ρ_1	1.0	0.9712	0.7156
CRRA coefficient (x_t)	ρ_2	1.2	1.5198	1.0674
Preference weight on C_t	θ_0	0.5	0.5267	0.5657
Home bias in consumption	ω	0.7	0.5431	0.7988
Foreign equivalent of ω	ω^F	0.7	0.8819	0.7344
Import demand elasticity	σ	1.0	0.7676	0.9380
Elasticity of substitution (C_t^{d*}, C_t^{f*})	σ^F	0.7	0.8522	0.5009
Capital equation coefficients[17]	$\zeta_1, \zeta_2, \zeta_3, \zeta_4$	$0.51, 0.47, 0.02, 0.25$	$0.63, 0.35, 0.02, 0.24$	$0.61, 0.37, 0.02, 0.20$
$\frac{\partial R\&D_t}{\partial s_t'}$	c_1	0.06	0.0632	0.0480
$\frac{\partial [d \ln A_{t+1}]}{\partial s_t'}$	b_1	0.1	0.0901	0.0969
Wald percentile		100	77.04	94.48

TABLE 3
Structural Model Parameters

Exogenous variable	AR coefficient	Starting calibration	Estimated Model 1	Estimated Model 2
Shock to real interest rate	ρ_r	0.860	0.858	0.839
Shock to TFP	ρ_A	0.589	0.577	0.588
Shock to labour demand	ρ_N	0.897	0.897	0.913
Shock to capital demand	ρ_K	0.765	0.951	0.950
Shock to real wage	$\rho_{\tilde{w}}$	0.879	0.837	0.943
Shock to export demand	ρ_X	0.939	0.939	0.938
Shock to import demand	ρ_M	0.848	0.832	0.854
Shock to R&D subsidy	ρ_S	0.974	0.974	0.971
Shock to foreign consumption demand	ρ_{CF}	0.939	0.939	0.953
Shock to foreign real interest rate	ρ_{rF}	0.851	0.851	0.837
Shock to government consumption	ρ_G	0.972	0.972	0.951

TABLE 4
AR coefficients for stationary exogenous variables

Extent of falseness (absolute), θ	TRUE	0.50%	1.00%	1.50%	2.00%	2.50%	3.00%
Rejection rate	5%	5.80%	8.82%	26.58%	84.68%	100.0%	100.0%

TABLE 5
Rejection rates, all coefficients falsified together

4.6. Power Exercise

The small sample properties of Indirect Inference have been investigated elsewhere (see Le et al. 2016 for references). However, Monte Carlo results are difficult to generalise from one context to another so we check the power of the Indirect Inference test for our particular setup. To do this, we introduce falseness into the structural parameters, θ, moving them away from predefined true values by a certain percentage (randomly in either a positive or negative direction). Using a bootstrapped Wald distribution based on the misspecified model, we see whether the Indirect Inference test as implemented above will correctly reject this model given a sample from the true (correctly specified) model. The rate at which the test statistic falls in the 95th-100th percentile range of the distribution, for a particular degree of falseness, gives a sense of how reliable the procedure is. Rejection rates are given in Table 5. The results of the exercise indicate that the testing method applied in the study is powerful. Coefficients just 2.5% away from their true values will result in a certain rejection.

The above power function holds when all parameters are falsified together to the same degree. We would most of all like to know whether the addition of the R&D subsidy is appropriate. This policy affects the model via parameters b_1 and c_1. We therefore investigate the power of the test when these coefficients alone are misspecified, holding all other coefficients to their true values. The results are reported in Table 6. When just two coefficients are falsified the power of the test is reduced. However, the test rejects over 99% of the time when the coefficients b_1 and c_1 are 50% away from their true values. This furnishes us with a confidence interval for our estimates.

4.7. Policy Reform and Growth Episode

A temporary shock to the detrended R&D subsidy has the effect in the model of increasing the level of TFP permanently and also generates a long-lasting TFP growth episode, with knock-on effects on the rest of the economy. Impulse responses to a one-off, 1 percentage point increase in s_0' are shown in Figure 3. The simulation is based on the estimated structural

Falseness (%)	**0**	**1**	**3**	**5**	**7**	**10**
Rejection rate (%)	5.0	5.34	5.98	7.22	8.88	12.84
Falseness (%)	**15**	**20**	**30**	**40**	**50**	**60**
Rejection rate (%)	23.62	40.72	75.94	94.22	99.18	99.88
Falseness (%)	**0**	**-1**	**-3**	**-5**	**-7**	**-10**
Rejection rate (%)	5.0	4.92	4.72	4.90	5.32	6.40
Falseness (%)	**-15**	**-20**	**-30**	**-40**	**-50**	**-60**
Rejection rate (%)	10.54	18.64	55.92	93.52	99.94	100.0

TABLE 6
Rejection rates when just two structural coefficients are falsified

parameter set found above. After 70 quarters the loglevel of output is 2 percentage points higher than its no shock state (note, balanced growth has been removed here). The average annual growth increase over the 17.5 year episode is therefore 0.11 percentage points per annum.

How confident can we be in these results? The power exercise above shows that the Indirect Inference test (exactly as applied in this paper) is robust against misspecification in the model's structural parameters. There is the further issue of identification. Work checking the identification of rational expectations DSGE models finds that they generally are overidentified (the notable exception is models featuring sunspots); see Le et al. 2017. It is a priori likely that the model we use here is identified since models of this type routinely pass identification tests, but in particular we would like to show that the reduced form of this model could not be confused with a model in which R&D subsidies respond endogenously to TFP. To check identification we apply the numerical identification test developed by Le et al (2017).[18] For this test a 5 variable VARX(4) is used as the auxiliary model. We find that when the structural parameters are falsified by 0.3% together (randomly, up or down), false models are rejected 100% of the time. Therefore the VAR distribution implied by the true model is clearly distinguishable from that implied by other models, even those with parameters in the near neighbourhood of the 'true' set.

4.7.1. Robustness checks

Robustness checks show that results are invariant to the interpolation technique (quadratic versus constant match) and to the way in which missing values were supplied for years 1982

[18] The idea is to check whether any other structural model could generate the model's reduced form by creating a large number of data samples of large size from the true model, and testing whether any possible alternative model is rejected for these samples at the same 5% rate as the true model itself.

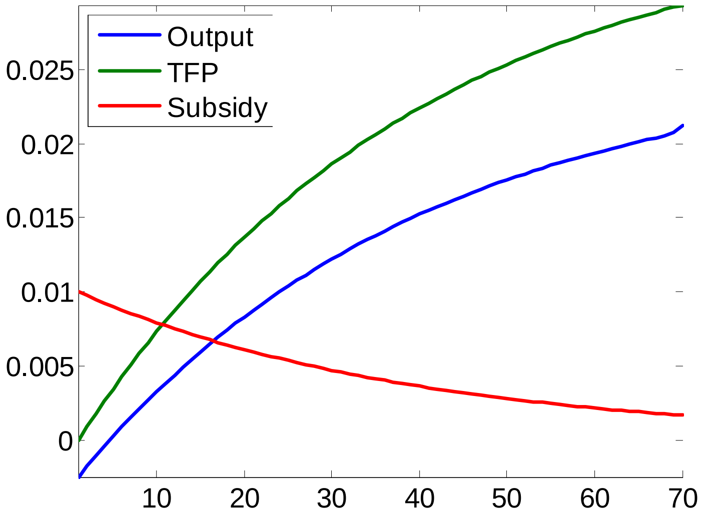

FIG. 3 Impulse Responses for a 1 pc point increase in R&D subsidies; 70 quarters.

and 1984.[19] We also check the detrending method for the R&D subsidy variable. Though direct subsidies have fallen steadily since the 1980s, from the late 1990s the trend slows. We therefore try removing the trend using i) the HP filter and ii) a quadratic time trend (Fig. 4). The earlier results are robust for the HP filtered series – the coefficient set reported in Table 3, column 4, is firmly not rejected with a test statistic in the 84th percentile of the Wald distribution. With the quadratic time trend the model is rejected at the 5% significance level.

5. CONCLUSION

We have written down a DSGE model particularly suited to the UK open economy in which productivity is driven systematically by direct subsidies to private sector R&D. Structural models of this kind are valuable tools for policymakers, but it is increasingly recognised that the value of their quantitative implications rests on the credibility of the structural parameters. Taking our cue from the incipient literature estimating and evaluating DSGE models, we test and estimate the model by Indirect Inference for the period 1981 – 2010. Our test focuses on whether the model can explain output and productivity as endogenous variables, and we find that it does. The estimated impact of current direct subsidies to private R&D on total

[19] Missing values were calculated as i) the average of two contiguous values, ii) equal to previous value, iii) equal to following value. The Wald test result was similar for all three.

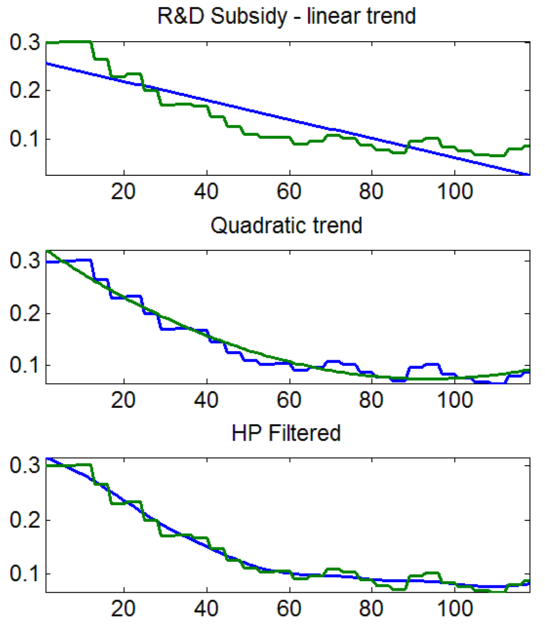

FIG. 4 R&D subsidy variable - various detrending methods

factor productivity growth one-quarter ahead, b_1, is 0.09, signifying that in this sample a 1 percentage point increase in the detrended ratio of government funded BERD to total BERD raises productivity by 0.09 percent over the quarter, with permanent effects on the level. Given the estimated structural model, we conduct a simulated policy reform experiment. A one-off, one percentage point increase in direct subsidies dying out gradually generates a transitional growth episode in TFP lasting nearly two decades. This translates into an increase in the average annual growth rate of output of 0.2 percentage points over those decades. Our results thus strongly suggest that there is a role for R&D subsidies in promoting growth in the medium term.

The power exercise we conduct lends significant robustness to these conclusions on the role of R&D subsidies. In our Monte Carlo study, the introduction of 2.5% misspecification in the structural parameters leads to a 100% rejection rate. When falseness is introduced *only* into the two coefficients particularly related to subsidies (b_1 , c_1) the test is still sure to reject the model when these two parameters stray further than 50% from their true values. This allows us to construct uncertainty bounds around our estimates (and hence around the predicted growth episode): in the case of b_1, the 'worst case' interval is $(0.045,\ 0.13)$.

We also apply the numerical model identification test proposed in Le et al. (2017), and confirm that the model is identified. This is a key strength of the approach, as there is no ambiguity in the causality running between policy shocks and economic growth. A model in which growth causes policy, for example, would be clearly distinguishable from the model we have tested here. Therefore this study adds empirical support for a causal impact of R&D policy on transitional growth in the UK since the 1980s.

Finally, the study fits within a wider research agenda on the role of R&D policy in economic growth in industrialised countries. The model abstracts heavily from the processes surrounding the R&D investment decision and the way that direct subsidies enter it in practice. A more elaborate model of the R&D channel could give greater insight into exactly how direct subsidies drive TFP at the level of microfoundations. In this study we provide evidence of the positive direction of the subsidy impact and the extent of that effect on the macroeconomy, findings which are certainly of interest to policymakers and which seem to be of first order importance; future work can build on this.

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APPENDIX A: MODEL DERIVATIONS, CONT.

A.1. First order condition for z

The first order condition for z_t is:

$$\frac{dL}{dz_t} = 0 = -\beta^t \lambda_t w_t + \beta^t \lambda_t s_t + E_t \sum_{i=1}^{\infty} \beta^{t+i} \lambda_{t+i}.\frac{d\ d_{t+i}}{dz_t}$$

At the (N_t, z_t) margin, the optimal choice of z_t trades off the impacts of a small increase dz_t on labour earnings, subsidy payments, and expected dividend income. Here, $\frac{dA_{t+i}}{dA_{t+i-1}} = \frac{A_{t+i}}{A_{t+i-1}}$ and hence for $i \geq 1$, $\frac{d\ A_{t+i}}{dz_t} = \frac{d\ A_{t+i}}{dA_{t+i-1}}.\frac{d\ A_{t+i-1}}{dA_{t+i-2}}.....\frac{d\ A_{t+2}}{dA_{t+1}}.\frac{d\ A_{t+1}}{dz_t} = A_{t+i}\frac{A_t}{A_{t+1}}a_1$. In turn, $\frac{dd_{t+i}}{dz_t} = \frac{Y_{t+i}}{A_{t+i}}A_{t+i}\frac{A_t}{A_{t+1}}a_1$. It may be objected that dz_t enhances output directly through its effect on productivity (holding inputs fixed), and also induces the firm to hire more capital to exploit its higher marginal product (similarly for labour). We assume the effect of dz_t on the future dividend ($d_{t+i} = \pi_{t+i}$) is simply its direct effect via higher TFP, on the basis that any effects on input demands are second order. Therefore the expected change in the dividend stream is based on forecasts for choice variables (set on other first order conditions) that are assumed independent of the agent's own activities in context of price forecasts; she anticipates only the effect of z_t on the level of output that can be produced with given inputs from $t+1$ onwards. With substitution from 28, the rearranged first order condition is:

$$\beta^t \gamma_t C_t^{-\rho_1} w_t = \frac{a_1}{a_0 + a_1 z_t + u_t}.E_t \sum_{i=1}^{\infty} \beta^{t+i} \gamma_{t+i} C_{t+i}^{-\rho_1} Y_{t+i} + \beta^t \lambda_t s_t$$

On the left hand side is the return on the marginal unit of N_t, the real consumer wage; on the right is the present discounted value of the expected increase in the dividend stream as a result of a marginal increase in z_t, plus time t subsidy incentives attached to R&D activity.[20] Substituting again from 28 for z_t yields

$$\frac{A_{t+1}}{A_t} = a_1.\frac{E_t \sum_{i=1}^{\infty} \beta^i \gamma_{t+i} C_{t+i}^{-\rho_1} Y_{t+i}}{\gamma_t C_t^{-\rho_1}(w_t - s_t)}$$

The preference shock to consumption, γ_t, is an AR(1) stationary process $\gamma_t = \rho_\gamma \gamma_{t-1} + \eta_{\gamma,t}$. Setting $\rho_1 \simeq 1$, $\frac{C_t}{Y_t}$ is approximated as a random walk, so $E_t \frac{Y_{t+i}}{C_{t+i}} = \frac{Y_t}{C_t}$ for all $i > 0$.[21] The expression becomes

$$\frac{A_{t+1}}{A_t} = a_1.\frac{\frac{\beta\rho_\gamma}{1-\beta\rho_\gamma}.\frac{Y_t}{C_t}}{\frac{w_t}{C_t}(1 - s_t')}$$

where $\frac{s_t}{w_t} \equiv s_t'$; see main model discussion.

A.2. The labour supply response to subsidies

Taking the total derivative of the time endowment in 3 gives $dx_t = -dN_t - dz_t$, or $\frac{dx_t}{x_t} = \frac{-dN_t - dz_t}{x_t}$. Taking $\bar{N} \approx \bar{x} \approx \frac{1}{2}$ in some initial steady state with approximately no z activity implies $\frac{dx_t}{\bar{x}} = d\ln x_t \approx -d\ln N_t - \frac{dz_t}{\bar{N}} = -d\ln N_t - 2dz_t$. Substituting into the loglinearised intratemporal condition using

[20] The non-policy cost of generating new productivity via z_t is assumed to be zero.

[21] Although in balanced growth $\frac{C}{Y}$ is constant, in the presence of shocks the ratio will move in an unpredictable way (see Meenagh et al. 2007 for discussion).

this an 24, we obtain

$$d\ln N_t - 2c_1 ds'_t = -\frac{1}{\rho_2} d\ln\xi_t + \frac{1}{\rho_2} d\ln\gamma_t - \frac{\rho_1}{\rho_2} d\ln C_t + \\ \frac{1}{\rho_2}\left[k + d\ln\tilde{w}_t - \frac{1}{\rho}\left[\frac{1-\omega}{\omega}\right]^{\sigma} d\ln\varsigma_t - \left[\frac{1-\omega}{\omega}\right]^{\sigma} d\ln Q_t\}\right]$$

Integrating and rearranging for the log of the real unit cost of labour to the firm, $\ln\tilde{w}_t$, gives expression $\ln\tilde{w}_t = const_4 + \rho_2\ln N_t + \rho_1\ln C_t + \left[\frac{1-\omega}{\omega}\right]^{\sigma}\ln Q_t - \rho_2 2c_1 s'_t + e_{w,t}$ (see main text).

APPENDIX B: THE LINEARISED SYSTEM

The linearised system of optimality conditions and constraints solved numerically is given below. Each equation is normalised on one of the endogenous variables (constants are suppressed in the errors). Variables are in natural logs except where already expressed in percentages. For clarity, $\ln(C_t^d)^*$ and $\ln C_t^f$ are denoted $\ln EX_t$ and $\ln IM_t$.

$$r_t = \rho_1\left(E_t\ln C_{t+1} - \ln C_t\right) + e_{r,t} \tag{42}$$

$$\ln Y_t = \alpha\ln N_t + (1-\alpha)\ln K_t + \ln A_t \tag{43}$$

$$\ln N_t = \ln Y_t - \tilde{w}_t + e_{n,t} \tag{44}$$

$$\ln K_t = \zeta_1\ln K_{t-1} + \zeta_2\ln K_{t+1} + \zeta_3\ln Y_t - \zeta_4 r_t + e_{k,t} \tag{45}$$

$$\ln C_t = \frac{\bar{Y}}{\bar{C}}\ln Y_t - \frac{EX}{\bar{C}}\ln EX_t + \frac{IM}{\bar{C}}\ln IM_t - \frac{\bar{K}}{\bar{C}}\ln K_t + (1-\delta-\gamma_k)\frac{\bar{K}}{\bar{C}}\ln K_{t-1} - \frac{\bar{G}}{\bar{C}}\ln G_t \tag{46}$$

$$\ln\tilde{w}_t = \rho_2\ln N_t + \rho_1\ln C_t + \left[\frac{1-\omega}{\omega}\right]^{\sigma}\ln Q_t + \rho_2 2c_1 s'_t + e_{wh,t} \tag{47}$$

$$\ln w_t = \ln\tilde{w}_t - \left[\frac{1-\omega}{\omega}\right]^{\sigma}\ln Q_t + e_{w,t} \tag{48}$$

$$\ln EX_t = \ln C_t^* + \sigma^F\frac{1}{\omega}\ln Q_t + e_{X,t} \tag{49}$$

$$\ln IM_t = \ln C_t - \sigma\ln Q_t + e_{M,t} \tag{50}$$

$$\ln Q_t = E_t\ln Q_{t+1} + r_t^f - r_t \tag{51}$$

$$\Delta\hat{b}_{t+1}^f = \frac{\widetilde{b}^f}{1+g}r_t^f + \frac{\widetilde{r}^f}{1+g}\hat{b}_t^f + \left(\frac{1}{1+g}\right)\left(\begin{array}{c}\frac{EX}{\bar{Y}}\ln EX_t - \frac{EX}{\bar{Y}}\frac{1}{\omega}\ln Q_t \\ -\frac{IM}{\bar{Y}}\ln IM_t\end{array}\right) \tag{52}$$

$$\ln A_t = \ln A_{t-1} + b_1 s'_{t-1} + e_{A,t} \tag{53}$$

$$\ln C_t^* = \rho_{C^*}\ln C_{t-1}^* + \eta_{C^*,t} \tag{54}$$

$$\ln G_t = \rho_G\ln G_{t-1} + \eta_{G,t} \tag{55}$$

$$r_t^f = \rho_{rf} r_{t-1}^f + \eta_{rf,t} \tag{56}$$

$$s'_t = \rho_s s'_{t-1} + \eta_{s,t} \tag{57}$$

APPENDIX C: DATA DEFINITIONS AND SOURCES

Most UK data are sourced from the UK Office of National Statistics (ONS); others from the International Monetary Fund (IMF), Bank of England (BoE), UK Revenue and Customs (HMRC) and Organisation for Economic Cooperation and Development (OECD). All data seasonally adjusted and in constant prices unless specified otherwise.

Symbol	Variable	Definition and Description	Source
Y	Output	Gross Domestic Product; constant prices.	ONS
N	Labour	Ratio of total employment to 16+ working population[1]	ONS
K	Capital Stock	Calculated from investment data (I) using Eqn.21	(na)
I	Investment	Gross fixed capital formation + changes in inventories	ONS
C	Consumption	Household final consumption expenditure by households	ONS
A	Total Factor Productivity	Calculated as the Solow Residual in Eqn. 18	(na)
G	Government Consumption	General government, final consumption expenditure	ONS
IM	Imports (also C_t^f)	UK imports of goods and services	ONS
EX	Exports (also C_t^{d*})	UK exports of goods and services	ONS
Q	Terms of Trade	Calculated from $\frac{E.P_F}{P}$	(na)
E	Exchange Rate	Inverse of Sterling effective exchange rate	ONS
P_F	Foreign Price Level	Weighted av. of CPI in US (0.6), Germany (0.19) & Japan (0.21)	IMF
P	Domestic General Price Level	Ratio, nominal to real consumption	ONS
b_F	Net Foreign Assets	Ratio of nominal net foreign assets (NFA) to nominal GDP [2]	ONS
w	Consumer Real Wage	Average Earnings Index [3] divided by P_t	ONS
$\widetilde{w}$	Unit cost of labour	Average Earnings Index [3] divided by GDP deflator	ONS
r	Real Interest Rate, Domestic	Nominal interest rate minus one period ahead inflation.	(na)
R	Nominal Interest Rate, Domestic	UK 3 month treasury bill yield	BoE
r_F	Real Interest Rate, Foreign	R_F minus one-period ahead inflation (year-on-year change in P_F)	(na)
R_F	Nominal Interest Rate, Foreign	Weighted av., 3-month discount rates, US, Germany & Japan [4]	IMF
C_F	Foreign Consumption Demand	World exports in goods and services	IMF
s'	R&D Subsidy, Direct	BERD funded by government as proportion of total BERD	OECD

TABLE 7
Data Description

APPENDIX D: AUXILIARY MODEL

The full linearised structural model, comprising a p x 1 vector of endogenous variables y_t, a r x 1 vector of expected future endogenous variables $E_t y_{t+1}$, a q x 1 vector of non-stationary variables x_t and a vector of i.i.d. errors e_t, can be written in the general form

$$A(L)y_t = BE_t y_{t+1} + C(L)x_t + D(L)e_t \tag{58}$$

$$\Delta x_t = a(L)\Delta x_{t-1} + d + b(L)z_{t-1} + c(L)\epsilon_t \tag{59}$$

x_t is a vector of unit root processes, elements of which may have a systematic dependency on the lag of z_t , itself a stationary exogenous variable (this variable is subsumed into the shock below). ϵ_t is an i.i.d., zero mean error vector. All polynomials in the lag operator have roots outside the unit circle. Since y_t is linearly dependent on x_t it is also non-stationary. The general solution to this system is of the form

$$y_t = G(L)y_{t-1} + H(L)x_t + f + M(L)e_t + N(L)\epsilon_t \tag{60}$$

where f is a vector of constants. Under the null hypothesis of the model, the equilbrium solution for the endogenous variables is the set of cointegrating relationships (where Π is p x p)[22]:

$$\begin{aligned} y_t &= [I - G(1)]^{-1}[H(1)x_t + f] && (61)\\ &= \Pi x_t + g && (62)\end{aligned}$$

though in the short run y_t is also a function of deviations from this equilibrium (the error correction term η_t):

$$y_t - (\Pi x_t + g) = \eta_t \tag{63}$$

In the long run, the level of the endogenous variables is a function of the level of the unit root variables, which are in turn functions of all past shocks.

$$\begin{aligned} \bar{y}_t &= \Pi \bar{x}_t + g && (64)\\ \bar{x}_t &= [1 - a(1)]^{-1}[dt + c(1)\xi_t] && (65)\\ \xi_t &= \Sigma_{s=0}^{t-1}\varepsilon_{t-s} && (66)\end{aligned}$$

Hence the long-run behaviour of $\bar{x}_t$ can be decomposed into a deterministic trend part $\bar{x}_t^D = [1 - a(1)]^{-1}dt$ and a stochastic part $\bar{x}_t^S = [1-a(1)]^{-1}c(1)\xi_t$, and the long run behaviour of the endogenous variables is dependent on both parts. Hence the endogenous variables consist of this trend and of deviations from it; one could therefore write the solution as this trend plus a VARMA in deviations from it. An alternative formulation is as a cointegrated VECM with a mixed moving average error term

$$\begin{aligned} \Delta y_t &= -[I - G(1)](y_{t-1} - \Pi x_{t-1}) + P(L)\Delta y_{t-1} + Q(L)\Delta x_t + f + \omega_t && (67)\\ \omega_t &= M(L)e_t + N(L)\varepsilon_t && (68)\end{aligned}$$

which can be approximated as

$$\Delta y_t = -K[y_{t-1} - \Pi x_{t-1}] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + h + \zeta_t \tag{69}$$

or equivalently, since $\bar{y}_{t-1} - \Pi\bar{x}_{t-1} - g = 0$,

$$\Delta y_t = -K[(y_{t-1} - \bar{y}_{t-1}) - \Pi(x_{t-1} - \bar{x}_{t-1})] + R(L)\Delta y_{t-1} + S(L)\Delta x_t + m + \zeta_t \tag{70}$$

considering ζ_t to be i.i.d. with zero mean. Rewriting equation 69 as a levels VARX(1) we get

$$y_t = [I - K]y_{t-1} + K\Pi x_{t-1} + n + \phi t + q_t \tag{71}$$

where the error q_t now contains the suppressed lagged difference regressors, and the time trend is

[22] In fact the matrix Π is found when we solve for the terminal conditions on the model, which constrain the expectations to be consistent with the structural model's long run equilibrium.

included to pick up the deterministic trend in $\bar{x}_t$ which affects both the endogenous and exogenous variables. x_{t-1} contains unit root variables which must be present to control for the impact of past shocks on the long run path of both x and y. This VARX(1) approximation to the reduced form of the model is the basis for the unrestricted auxiliary model used throughout the estimation.

APPENDIX E: VARIANCE DECOMPOSITION (ALL VARIABLES)

	r	Y	N	K	C	w	$\tilde{w}$	X	M	Q	b^F	A	$d(A)$
Shock to r	0.16915	0.00197	0.00946	0.01392	0.04238	0.03864	0.00063	0.01232	0.06000	0.01156	0.03056	0	0
Shock to TFP	0.23125	0.34965	0.22750	0.21159	0.23919	0.11268	0.35403	0.31997	0.14627	0.30031	0.01221	0.37125	0.90164
Shock to N	0.03092	0.00198	0.01529	0.00104	0.00167	0.02777	0.01385	0.00073	0.00006	0.00069	0.00074	0	0
Shock to K	0.16036	0.02454	0.04456	0.41922	0.02423	0.01442	0.01927	0.01438	0.00731	0.01350	0.01233	0	0
Shock to w	0.12176	0.01992	0.16159	0.01146	0.01462	0.03621	0.00125	0.00653	0.00058	0.00613	0.00491	0	0
Shock to X	0.03448	0.01006	0.10313	0.00026	0.16463	0.44235	0.00103	0.10006	0.37763	0.14460	0.65353	0	0
Shock to M	0.02802	0.00123	0.01439	0.00062	0.02004	0.03956	0.00022	0.01404	0.09832	0.01318	0.05384	0	0
Subsidy shock	0.14243	0.58885	0.40561	0.33892	0.41954	0.15387	0.60936	0.50078	0.17308	0.47001	0.09621	0.62875	0.09836
C^F shock	0.00527	0.00159	0.01643	0.00006	0.06412	0.11844	0.00017	0.02681	0.12097	0.03590	0.13144	0	0
r^F shock	0.07095	0.00009	0.00053	0.00285	0.00928	0.01594	0.00016	0.00423	0.01577	0.00397	0.00392	0	0
G shock	0.00539	0.00012	0.00150	0.00006	0.00030	0.00014	0.00002	0.00013	0.00001	0.00013	0.00031	0	0

TABLE 8
Variance Decomposition, Subsidy Model 1