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Parental Altruism, Missing Credit Markets and Growth

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Abstract

Parental transfers towards the education of children are non-trivial, especially in countries, characterized by both imperfect credit markets and high economic growth rates. In this paper, we analyze the role of parental altruism on economic growth and dynamic efficiency, especially when credit markets for education loans are missing. We demonstrate conditions under which missing or imperfect credit markets increase economic growth and do not hinder dynamic efficiency. We also show that a newly constructed index of parental altruism, orthogonal to income effects, exhibits high cross-country correlations with model-implied measures of parental altruism at different levels of credit market development.

JEL Classification Codes: I25, O16, O41
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1 Introduction

In most real-world economies, well-developed credit markets for financing educational investments do not exist. This raises the question of how private transfer arrangements between parents and children influence human capital accumulation and economic growth. A generally accepted view suggests that if young individuals are unable to finance educational investment then the competitive equilibrium of the economy will be dynamically inefficient

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as the ratio of physical to human capital will be too high, and the return on physical capital will be too low. Conventional wisdom suggests that the growth rate of the economy will also be negatively affected. Empirical evidence indicates that private transfers from parents to children for the purpose of education funding are a non-trivial share of household incomes, but vary drastically across countries. Such transfers are particularly high in some developing economies where credit market imperfections are, perhaps, pervasive.

In this study we analyze the role of parental altruistic motives on economic growth and dynamic efficiency across different levels of credit market development.[1] For this purpose, we set out an overlapping generations model of endogenous growth in which altruistic parental transfers contribute to the development of the human capital of the young. We investigate how the parental transfer motive interacts with economic growth and dynamic efficiency in a model that nests imperfect and 'missing' credit markets for educational loans. Contrary to conventional wisdom, we show that economies where markets for education loans are absent may have higher growth rates and a dynamically efficient balanced growth path (BGP). [2]

We demonstrate that economies with missing credit markets exhibit higher growth rates than economies with complete credit markets along the balance growth path, either when both parental altruism towards children's education and intergenerational correlation of human capital are relatively low or when they are relatively high. In both cases, the ratio of

[1]Mukherjee (2018) provides evidence of parent's altruistic behavior towards their children using US data. Specifically, it is shown that parents provide sources to their children without expectations for reciprocal caregiving.

[2]In the empirical literature there is no clear consensus on the impact of financial development on economic growth (see Levine, 2005). In the model of Boldrin and Montes (2005) the absence of credit markets implies dynamic inefficiency: the ratio of physical-to-human capital is too high. De Gregorio and Guidotti (1995) provide empirical evidence that financial development matters for economic growth through an efficiency of investment channel. Law and Singh (2014) provide evidence that financial development exerts a positive impact on growth below a certain threshold, before turning negative.

physical to human capital is strictly higher in economies with complete credit markets than in economies with missing credit markets. High capital ratios undermine economic growth and occur either due to over-saving or under-borrowing relative to the growth-optimal levels. For instance, when the levels of parental altruism and intergenerational correlation of human capital are higher than certain thresholds and markets are complete, parental transfers are high and young individuals choose to save part them in financial intermediaries, rather than exploiting the high level of intergenerational persistence of human capital to generate more human capital for future generations. Likewise, we demonstrate that dynamic efficiency occurs at relatively low levels of parental altruism in economies with complete credit markets and relatively high levels of parental altruism in economies with missing credit markets. In both economies dynamic inefficiency is due to higher ratios of physical to human capital than the socially optimal levels. [3] Under complete markets, the latter occurs due to the fact that high levels of parental altruism imply levels of parental transfers which exceed the socially optimum levels of investment in human capital. As a result, young individuals invest the excess amount in the financial market and the economy ends up over-accumulating physical capital. When credit markets are missing, low levels of parental altruism lead to a high ratio of physical to human capital as young individuals are prevented from borrowing to fund their education which relies solely on parental transfers. Similar results are obtained when the credit market for education loans exists but young individuals have limited access to it. Hence, the absence of well-developed credit markets may not hinder efficiency and economic growth. These results provide a possible explanation for the high growth rates of several

[3] In other words, there is room for increasing welfare via the reallocation of capital.

emerging East Asian economies with very high levels of parental investment but relatively undeveloped credit markets.[4]

Such differences in altruistic motives across countries could be a factor behind the mixed findings in the empirical literature on borrowing constraints and growth: Japelli and Pagano (1994) show that borrowing constraints are associated with higher growth, whereas De Gregorio (1996) finds borrowing constraints negatively affect human capital accumulation and growth. Our results suggest that at certain levels of parental altruism, over-investment in physical capital reduces growth relative to the growth of an otherwise identical but credit constrained economy due to a misallocation between physical and human capital. Our results are also consistent with the observation, highlighted in Coeurdacier et al. (2015), that savings in emerging Asian markets with high growth rates are higher than savings in advanced economies and lower growth rates. Thus, parental altruism may be a factor behind cross-country differences in saving and growth that the literature has hitherto had difficulty explaining.

To evaluate the extent to which the model exhibits a realistic cross-country behavior, we compute correlations between model-implied measures of parental altruism and corresponding data measures. In doing so, we construct a cross-country measure of parental altruism towards children's education and a credit measure which allows us to classify the countries according to their credit market development. The former is approximated by an *attitude-based* measure using information from the World Values Survey (WVS), while the latter is

[4] Seth (2002) discusses the cultural roots of high parental investment in education in South Korea, known as 'education fever'. Anderson and Kohler (2013) argue that education fever is a factor behind low fertility rates in East Asia. The high levels of parental investment in education in East Asian economies have also received mainstream media attention (BBC, 2013; The Economist, 2013).

constructed using information on credit and savings. The measures of parental altruism and credit confirm the theoretical findings that the relationship between real per capita GDP growth and parental altruism differs across levels of credit market development. We also find that the model-implied measures of parental altruism are highly correlated with the *attitude-based* measure at different levels of credit market development. In particular, the benchmark correlation is 0.72 for unconstrained (complete markets) sample, 0.71 for the constrained (no credit market) sample and 0.45-0.77 for the sample of countries with limited access to credit markets (borrowing limit). Sensitivity analysis suggests that these results are robust to different values of key parameters and different thresholds for credit market development.

Our model is related to a large literature on credit market development and economic growth. Galor and Zeira (1993) show that the combination of credit market imperfections and initial wealth differentials can drive persistent differences in economic development through the impact on human capital accumulation. Much of the literature has focused specifically on borrowing constraints and growth. Using an endogenous growth model, Japelli and Pagano (1994) show that borrowing constraints raise economic growth due to higher accumulation of physical capital that drives productivity growth. This result depends on the absence of human capital investment. If there are borrowing constraints which hinder investment in human capital, the positive relationship between credit constraints and growth may be reversed (De Gregorio, 1996), though this need not be the case (de la Croix and Michel, 2007; Kitaura, 2012).[5] Here, we show parental altruism has important implications

[5] In a similar vein, Caballe (1995) shows that the efficient growth rate in an economy with dynastic altruism (see Barro, 1974) may be lower than in the case of a non-operative bequest motive.

for this debate: its effects on growth and dynamic efficiency are not monotonic, but depend crucially on the level of credit market development.

We model altruistic motives using *parental altruism* where parents care about the education of their children and contribute towards the development of their human capital. Specifically, parents derive utility from the amount transferred in order to fund their children's development of human capital. These joy-of-giving preferences differ from bequests in that middle-aged parents make transfers to their young children, who in turn use the transfer primarily to finance education.[6] This intuitive assumption makes altruistic models more tractable than under the dynastic approach of Barro (1974) and thus allows us to provide a full characterization of the impact of parental altruism on growth at different levels of credit market development.

The remainder of the paper proceeds as follows. Section 2 provides background, including cross-country evidence on education expenditures and parental transfers toward children's education. Section 3 introduces the economic environment and derives a number of results relating to growth and dynamic efficiency. Section 4 discusses the construction of cross-country indicators of parental altruism and credit market imperfections that we use in the empirical evaluation of the theory. Finally, Section 5 concludes.

[6] Examples of the bequest version of joy-of-giving (or 'warm glow') preferences include Yaari (1964) and Galor and Zeira (1993), among many others.

2 Education Expenditures and Parental Transfers

Cross-country evidence suggests household expenditures on education as a percentage of total consumption, varies significantly both across countries and across broader regions. The share of household expenditure on education is relatively high in East Asia and the Pacific (6%) and relatively low in the western world such as the European Union (1.1%) and the USA (2.4%).[7] The data suggest that the share of household expenditures on education in China is about 6 times higher than the corresponding EU share, while the share of education expenditures in South Korea is almost triple the corresponding USA share. High levels of household expenditure on education are not confined only to the East Asia and the Pacific region. The share of education expenditures in Latin America and the Caribbean is 4.1% and more than three times the corresponding share in the EU.[8] This evidence is consistent with the claims of 'education fever' mentioned in the Introduction.

It is reasonable to expect that relatively higher household expenditure on education is supported, at least to some extent, by relatively higher parental saving or parental day-to-day income, especially if credit is expensive or restricted as in many developing economies. In fact, this is somewhat corroborated by the existing, but rather limited, evidence on cross-country parental transfers toward children's education. Evidence from 15 countries and territories reported in *The Value of Education* research study, commissioned by HSBC,

[7] The source of the data on household education expenditures and total consumption expenditures is Eurostat (2015) and Global Consumption Database (2010). Household consumption expenditure on goods and services include indirect taxes such as VAT and excise duties.

[8] Brazil has an expenditure share of 3.5%, which is more than triple the share in the EU. Japan as well as the broader regions such as the Middle East & North Africa and Sub-Saharan Africa exhibit shares which are, at least, twice as high as the share in the EU. Within the EU, the share of expenditure on education is lowest in Sweden, Finland and Belgium (0.3-0.4%) and is highest in Ireland, Cyprus and Greece (3.5%, 2.6% and 2.4%, respectively).

suggests that 86% of parents are paying for their children's education, while 84% of those with a child in university or college are paying towards their education.[9] The survey also reports that, on average, 63% of parents pay for private tuition for their children, the highest percentages being reported in developing and East-Asian countries and the lowest in developed countries.[10] According to the survey, the highest proportions (22-39%) of university students paying their own education costs reside in developed countries, while the lowest (< 1-5%) reside in developing and East Asian countries.[11] Table 2 reproduces the average total amount spent on a childs education by parents across 15 (widely-spread across all continents) countries, as reported in HSBC's Value of Education survey (2017).[12] Although the sample of countries is small, the table is indicative as it covers countries with very different characteristics. The first column corresponds to all per capita parental payments, from primary to undergraduate level education, while the second column corresponds only to per capita parental payments for college or university education. The table demonstrates clearly the high variation of parental transfers across highly heterogeneous countries. It highlights the relatively large inter-vivos parental transfers in countries in East Asia and Middle East such as Hong Kong, Singapore and the UAE, and the relatively small transfers in countries of the western world such as France and Canada. Although other studies (e.g. Zissimopoulos

[9]The survey represents the views of 8481 parents in 15 countries and territories. All information reported from the survey is reproduced with permission from The Value of Education Foundations for the Future, published in 2016 & 2017 by HSBC Holdings.

[10]The highest percentages are in China (93%), Indonesia (91%), Egypt (88%), Hong Kong (88%), India (83%), Singapore (82%) and Malaysia (81%), and the lowest in France (32%), Canada (31%), Australia (30%) and the UK (23%). Given that, on average, 22% of parents interviewed, admitted to not knowing how much they were spending on their childrens education (Financial Times, June 29, 2017), it is likely that the reported financial contribution of parents is underestimated.

[11]The highest proportions are in Canada (39%), USA (37%) and Australia (22%), and the lowest in Egypt (< 1%), India (1%), Hong Kong (4%) and Singapore (5%).

[12]This survey was conducted online in February 2017

and Smith (2009) and Alessie et al (2014)) also provide evidence on cross-country inter-vivos parental transfers, they are not solely focused on transfers for educational purposes.

Table 1 - Parental transfers toward their children's education - *HSBC* survey

	parental transfers				
country	**Education**	**College/Uni**	**country**	**Education**	**College/Uni**
Hong Kong	132,161	16,182	UK	24,862	6,566
UAE	99,378	18,360	Mexico	22,812	3,807
Singapore	70,939	15,623	Canada	22,602	5,990
USA	58,464	14,678	India	18,909	3,211
Taiwan	56,424	8,180	Indonesia	18,422	2,655
China	42,892	5,718	Egypt	16,863	1,210
Australia	36,402	5,146	France	16,708	5,465
Malaysia	25,479	8,720			

All amounts are expressed in 2016 USD

In the following section, we investigate theoretically the role of parental transfers towards children's education in economic growth and dynamic efficiency, especially in economies where credit markets for financing educational investment are imperfect or nonexistent. In section 4, we investigate the extent to which model-implied measures of parental altruism toward children's education resemble corresponding data measures, taking into account the degree of credit market imperfections. Although, the measure of parental altruism derived in the theory coincides with parental spending on children's education per unit of parental consumption, data limitations prevent us from constructing a direct *spending-based* index for a reasonable cross-sectional length, covering countries with different credit market development.[13] In section 4, we propose an alternative, *attitude-based*, indicator of *relative*

[13] Although the HSBC survey does not provide information about parental income or parental consumption, other surveys such as the Survey of Health, Ageing and Retirement in Europe (SHARE), do. One issue with those surveys is the short cross-section and the limited variety of the countries included in the survey (e.g. SHARE includes only 11 European countries). Given that our aim is to construct an index of reasonable length that allows us to examine separately, and compare, countries with different credit market conditions, the information extracted from current surveys is restrictive.

parental altruism using data from the World Values Survey (WVS) which approximates the spending-based indicator of the theoretical model. We show that cross-country correlations between model-implied and *WVS-based* measures of parental altruism are impressivley high, despite the relatively short samples.

3 Economic Environment

3.1 Complete markets

We consider an overlapping generations (OLG) economy, populated by agents who live for three periods.[14] Within each generation, the agents are homogeneous and population increases at the rate $n \geq 0$. An agent draws utilty from consumption, $c_{t,m} > 0$, when middle age, consumption, $c_{t+1,o} > 0$, when old age and the amount of transfers, $\omega_t > 0$, that the agent provides when middle aged to each of his children for the development of their human capital.[15] The consumption of the young is assumed to be incorporated in the consumption of the middle aged. The lifetime utility of a young agent born in period $t-1$ is defined as $U(c_{t,m}, c_{t+1,o}, \omega_t) = u(c_{t,m}) + \beta u(c_{t+1,o}) + \gamma u(\omega_t)$, where $\beta > 0$, $\gamma > 0$ and $u(\cdot)$ is an incresing and twice differentiable function with $u''(\cdot) < 0$.[16] Young agents born in period

[14]The model has a similar structure to that of Boldrin and Montes (2005) with the difference that parents are altruistic and care about their children's development of human capital while the children have the option to use the financial market not only as a credit market for funding their education but also as a financial investment opportunity.

[15]The literature distinguishes between (i) *bequests*, which are transfers made upon death and which may be accidental; and (ii) *inter-vivos* transfers, which are made between living people. The empirical literature has found that inter-vivos transfers are a substantial fraction of total transfers from parents to children (Gale and Scholz, 1994; Cox and Raines, 1985).

[16]Lambrecht et al. (2005) assume that parents pay for the education of their children but derive utility from the total income of their children. In addition, children cannot borrow to fund their education and rely solely on parents. An alternative way to model altruism is to assume that the utility function of the children is an argument of the utility function of the parents (see Barro, 1974). Rangazas (2000) however, finds that

$t-1$, are endowed with $h_{t-1}^{y} > 0$ units of human capital that are invested in the production of next period human capital, along with additional resources denoted by $d_{t-1} > 0$. An agent's human capital, $h_t > 0$, evolves according to a smooth, homogeneous of degree one function $h\left(d_{t-1}, h_{t-1}^{y}\right)$. Aggregate output Y_t is produced in a perfectly competitive market which comprises of large number of homogeneous firms, each producing output using human and physical capital according to a smooth, concave and constant returns to scale production function F. The latter enables us to write $Y_t = F(H_t, K_t)$, where H_t and K_t correspond to aggregate human and physical capital, respectively. Firms maximize profits by taking as given the price of human capital, w_t, and the price of physical capital R_t^k,while the price of output is normalized to unity. This implies that w_t and R_t^k correspond to the marginal product of human and physical capital, respectively.

There is a frictionless and perfectly competitive financial market which serves as an intermediary between agents and firms, enabling them to borrow and lend (invest) at the same gross interest rate, R_t, as in Boldrin and Montes (2005). Due to perfect foresight, a simple arbitrage argument suggests that the gross interest rate, R_t must be equal to the return of physical capital R_t^k. Specifically, a young individual born in period $t-1$, will either borrow $\overline{b}_{t-1} > 0$ from the financial market, if the optimal investment in human capital, d_{t-1}, exceeds parental transfers that is, $\overline{b}_{t-1} = d_{t-1} - \omega_{t-1}$ or save (invest) $-\overline{b}_{t-1} > 0$ if parental transfers exceed the optimal investment in human capital that is $-\overline{b}_{t-1} = \omega_{t-1} - d_{t-1}$.[17] A middle age individual saves $s_{t-1} > 0$ for his retirement while firms borrow from the credit

a standard infinitely lived neoclassical model with Barro-type altruistic preferences is inconsistent with the data. This finding motivates us to specify parental altruism in terms of warm-glow preferences.

[17]Galor and Zeira (1993) have a similar setting where agents can borrow to invest in human capital if the parental endowment is small enough.

market in order to invest, I_{t-1}^k, in next period's physical capital. It is assumed that one unit of investment in physical capital corresponds to one unit of physical capital that is, $I_{t-1}^k = K_t$.[18]

In the second period of his life, a middle aged individual supplies labor in a perfectly competitive labor market at the wage rate w_t, per unit of human capital, and receives the revenue from his investment in the financial market (if $\overline{b}_{t-1} < 0$) or pays off the loan of the previous period (if $\overline{b}_{t-1} > 0$) at the gross interest rate R_t. Then, he contributes to his childrens' education and makes further personal consumption-saving decisions. In particular, the middle age agent transfers ω_t to each of his $1+n$ children and saves s_t in the financial market for his retirement. Since agents within each generation are homogeneous, the aggregate savings of the middle aged, the aggregate borrowing (saving) of the young and the aggregate human capital can be written as $S_t = (1+n)^{t-1} s_t$, $\overline{B}_t = (1+n)^t \overline{b}_t$, and $H_t = (1+n)^{t-1} h_t$, respectively. The total assets held by financial intermediaries must be equal to the total liabilities recorded in their balance sheets that is, $S_t = B_t + K_{t+1}$, where $B_t = \overline{B}_t/(1+n)$. The latter expressed per middle aged individual is $s_t = b_t + k_{t+1}$, where $b_t = \overline{b}_t/(1+n)$, $k_{t+1} = (1+n)\widetilde{k}_{t+1}$ and $\widetilde{k}_{t+1}$ is physical capital per middle aged individual in period $t+1$.[19] Given that F is homogeneous of degree one, we can also express input prices as a function of $x_t = \widetilde{k}_t/h_t$ that is, $w_t = f(x_t) - x_t f'(x_t)$ and $R_t = f'(x_t)$, where $f(x_t) = F(1, x_t)$. Thus, the ratio of the gross interest rate to the wage rate can be written as a decreasing function of the factor intensity ratio that is, $R_t/w_t = \kappa(x_t)$. Finally, in old

[18]Full depreciation of physical capital is a reasonable assumption, and empirically plausible for this model as the period may correspond to 30-40 actual years.

[19]In Boldrin and Montes (2005), b_t is replaced with d_t which is restricted to always be non-negative.

age, the agent consumes all his wealth. It follows that the budget constraints, respectively, of an agent in middle age and old age are the following: $c_{t,m}+ s_t + R_t\bar{b}_{t-1}+ (1+n)\,\omega_t = w_t h_t$ and $c_{t+1,o} = R_{t+1}s_t$. Then, the problem for an agent born in period $t-1$ is

$$\max_{d_{t-1},s_t,\omega_t}\{u\left(w_t h\left(d_{t-1}, h^y_{t-1}\right) - s_t - R_t\left(d_{t-1}-\omega_{t-1}\right) - (1+n)\,\omega_t\right)$$
$$+\beta u\left(R_{t+1}s_t\right) + \gamma u\left(\omega_t\right)\}.$$

Notice that production can be expressed in terms of output per middle aged agent of period t that is, $y_t = F\left(h_t, \widetilde{k}_t\right)$. In the analysis that follows, we consider the following parametric version of the economy where $u\left(\theta\right) = \ln\left(\theta\right)$, $F\left(h_t, \widetilde{k}_t\right) = Ah_t^{\delta}\widetilde{k}_t^{1-\delta}$, $h\left(d_{t-1}, h^y_{t-1}\right) = B\left(d_{t-1}\right)^{\zeta}\left(h^y_{t-1}\right)^{1-\zeta}$, $h^y_{t-1} = \mu h_{t-1}$, with $A \geq 1$, $B \geq 1$, $\mu > 0$, $\delta \in (0,1)$ and $\zeta \in (0,1)$.

Equilibrium: *Given initial conditions* $\{d_{-1},\ b_{-1},\ h_0,\ k_0\}$, *there are sequences of prices* $\{R_t, w_t\}_{t=0}^{\infty}$ *and quantities* $\{d_t,\ \bar{b}_t,\ h_{t+1},\ k_{t+1},\ \omega_t\}_{t=0}^{\infty}$ *that satisfy the optimal conditions of the problem of agents, such that the resource constraint,* $F\left(h_t, \widetilde{k}_t\right) = c_{t,m} + \frac{c_{t,o}}{1+n} + s_t + (1+n)\,\omega_t$ *and the balance sheet of financial intermediaries,* $s_t = b_t + k_{t+1}$, *hold for* $t \geq 0$.

Note that the optimality conditions indicate that γ corresponds to parental transfers towards the education of children per unit of parental consumption that is, $\gamma = (1+n)\omega_t/c_{t,m}$. Manipulating the optimality conditions, it can be shown that $k_{t+1} = \widetilde{\Psi}\left(\gamma\right) = \Psi A^{-1} y_t$, $s_t = (1-\delta)^{-1}\left[1-\delta(1-\zeta)\right]k_{t+1}$, $d_t = \delta\zeta\Psi\left[A(1+n)(1-\delta)\right]^{-1} y_t$, $\omega_t = \gamma\left[\delta\zeta\left(\beta+\gamma\right)\right]^{-1}\left[1-\delta\left(1-\zeta\right)\right]d_t$

and $h_{t+1} = \Phi h_t^{1-\zeta(1-\delta)} \widetilde{k}_t^{\zeta(1-\delta)}$, where

$$\Psi = \frac{(1-\delta)[\delta\beta(1-\zeta)+\gamma]A}{[1-\delta(1-\zeta)]\,(1+\beta+\gamma)} \text{ and } \Phi = \left(\frac{B^{\frac{1}{\zeta}}\mu^{\frac{1-\zeta}{\zeta}}\delta\zeta\Psi}{(1-\delta)(1+n)}\right)^{\zeta}$$

Proposition 1: *There exists $\gamma^* > 0$ such that when $\gamma < \gamma^*$ then, $\overline{b}_t > 0$; when $\gamma > \gamma^*$ then, $\overline{b}_t < 0$; when $\gamma = \gamma^*$ then, $\overline{b}_t = 0$, where $\gamma^* \equiv \beta\delta\zeta(1-\delta)^{-1}$.*

Proof. *It follows from the fact that $\overline{b}_{t-1} = [\beta\delta\zeta - \gamma\,(1-\delta)][\delta\zeta\,(\beta+\gamma)]^{-1} d_{t-1}$ and $d_{t-1} > 0$.*

■

Given the characteristics of the economy, proposition 1 establishes the condition under which young agents borrow from the credit market (i.e. $\overline{b}_t > 0$) in order to fund their education. The proposition indicates that the young agents will borrow from the credit market in order to develop their human capital only if the level of altruism of parents is below a certain threshold. In other words, if parents' level of altruism is sufficiently high, young agents find it optimal to invest part of their endowment in the credit market. The intuition for this result is as follows. If parental altruism is below the threshold value, parents transfer to their children not enough to cover their children's desired level of investment in education, and thus there is need for them to borrow from the credit market. If parents are more altruistic than this (i.e. γ exceeds the threshold value), they end up transferring more to their children than they would like to invest in education, and so the children place what is left over as savings in a financial intermediary.

It is straightforward to show that[20]

$$x_t = \left(\frac{x_0 (1+n)(1+g_{h0})}{\Psi} \right)^{(1-\delta)^t} \prod_{i=0}^{t} \left[\frac{\Psi}{(1+n)(1+g_{ht-i})} \right]^{(1-\delta)^i}. \qquad (1)$$

As in Boldrin and Montes (2005), the only rest point of (1) is the origin. In other words, when $g_{ht} = g_h$, the only case where $x_t = x$ for all $t \geq 1$ is when $x = x_0$.

Definition 1 *At the Balanced Growth Path (BGP), there are constants x, g and g_y such that, $\frac{K_t}{H_t} = x$, $\frac{K_t}{K_{t-1}} = \frac{H_t}{H_{t-1}} = \frac{Y_t}{Y_{t-1}} = 1 + g_y$, and $\frac{k_t}{k_{t-1}} = \frac{y_t}{y_{t-1}} = \frac{h_t}{h_{t-1}} = 1 + g$ where $1 + g_y = (1+n)(1+g)$.*

It follows that the ratio of physical to human capital and the growth rate at the unique BGP are given by,

$$x = \left[\frac{\Psi}{(1+n)(1+g)} \right]^{\frac{1}{\delta}} = \left(\frac{\Psi}{\Phi(1+n)} \right)^{\frac{1}{\delta+\zeta(1-\delta)}} \quad \text{and} \quad 1+g = \left(\frac{\Psi}{1+n} \right)^{\frac{\zeta(1-\delta)}{\delta+\zeta(1-\delta)}} \Phi^{\frac{\delta}{\delta+\zeta(1-\delta)}},$$

respectively. The latter implies that g is a monotonically increasing function of γ.

As noted by Abel et al. (1989), an equilibrium path is dynamically inefficient if the economy is consistently investing more in capital than it earns in profit. The BGP is dynamically efficient for a given level of investment in capital, if there is no other BGP that satisfies the economy's resource constraint with a higher level of welfare for one or more generations. In other words, the BGP is dynamically efficient, for a given level of investment, if there is no other investment allocation that a social planner can achieve for a non-negative gain

[20] Notice that $k_{t+1} = \Psi A^{-1} y_t$ implies $x_t = [(1+n)(1+g_{ht})]^{-1} \Psi x_{t-1}^{1-\delta}$, where g_{ht} is the time t growth rate of human capital. Then the latter can be solved backwards and be reduced to (1).

in welfare for all generations living in the new BGP. Following Del Rey and Lopez-Garcia (2013, 2016), we assume that the social planner preserves the functional form of individual preferences, while treating generations equally across time. Along the BGP, the objective of the social planner is to pick stationary values for $\widehat{c}_m = c_{t,m}/h_t$, $\widehat{c}_o = c_{t+1,o}/h_t$ and $\widehat{\omega} = \omega_t/h_t$ that maximize the utility function, given by $U(\widehat{c}_m, \widehat{c}_o, \widehat{\omega}) = u(\widehat{c}_m) + \beta u(\widehat{c}_o) + \gamma u(\widehat{\omega})$, subject to the balanced growth version of the resource constraint, as demonstrated in the proof of proposition 2.

Definition 2 *A BGP is dynamically inefficient if a reduction in x increases the welfare, as measured by $U(\widehat{c}_m, \widehat{c}_o, \widehat{\omega})$, of generations living on the new BGP. Otherwise the BGP is dynamically efficient.*[21]

Proposition 2: *The complete markets BGP, is dynamically efficient if $R \geq (1+n)(1+g)$ or, equivalently, $\widetilde{\Psi}(\gamma) \leq 1-\delta$ or, equivalently, $\gamma \in \Omega^c \equiv \{\gamma > 0; \gamma \leq \gamma^c\}$, where $\gamma^c \equiv [[1-\delta(1-\zeta)](1+\beta) - \delta\beta(1-\zeta)][\delta(1-\zeta)]^{-1} > 0$, with $\Omega^c \neq \varnothing$ only if $\delta < \delta^c \equiv (1+\beta)(1+2\beta)^{-1}(1-\zeta)^{-1}$.*

Proof. *See the appendix.* ■

Proposition 2 establishes that the BGP is dynamically efficient under complete markets if the level of parental altruism is below a certain threshold. It suggests that for relatively high levels of parental altruism, parents give an inefficiently large transfer to their children that exceeds the amount necessary for optimal investment in human capital. As a result,

[21] Note that although $\widehat{d}$ is also an investment variable, the social planner of our model sets $\widehat{d} = \widehat{\omega}$. Since $\widehat{\omega}$ is an argument of the utility function, there is no clear cut condition for dynamic efficiency in terms of $\widehat{\omega}$. Thus, contrary to Del Rey and Lopez-Garcia (2016), we focus solely on x when examining dynamic efficiency of the BGP, following Boldrin and Montes (2005).

young agents save the remainder, or over-invest in the financial market in a socially inefficient manner. The latter leads to an over-accumulation of physical capital since x exceeds the maximum socially optimal level. In other words, when $\gamma > \gamma^c$, the allocation (k, h) is such that x exceeds the maximum socially optimal level and the interest rate is strictly smaller than the growth rate of aggregate output. An alternative allocation of physical and human capital where output is reallocated from physical to human capital, increases the interest rate relative to the growth rate and the welfare of all generations living on the BGP. Propositions 1 and 2 imply that under complete markets, (i) $\bar{b}_t \geq 0$ is a necessary condition for dynamic efficiency of the BGP if $\gamma \leq \gamma^c < \gamma^*$ and a sufficient condition if $\gamma \leq \gamma^* \leq \gamma^c$ and (ii) the BGP is dynamically efficient when $\bar{b}_t < 0$ only if $\gamma^* < \gamma \leq \gamma^c$. The latter demonstrates that strictly positive investments in the financial market by the young need not always lead to dynamic inefficiency.[22]

3.2 Incomplete Markets

3.2.1 No Credit Market for Education Loans

First, we consider the case where the credit market for education loans is absent. Thereby young agents cannot borrow in order to fund educational investment, i.e. $b_t = 0$ for all $t \geq 0$. As a result, investment in education will be funded entirely by transfers, i.e. $d_t = \omega_t$. The firms continue to have access to credit. It follows that the problem solved by an agent born

[22]It is straightforward to show that it is possible that the BGP is dynamically efficient when $\bar{b}_t < 0$ only when $0 < \beta < (1-\delta)[1-\delta(1-\zeta)]\overline{\beta}^{-1}$, where $\overline{\beta} = \delta(1-\zeta)(1+\delta) - [1-\delta(1-\zeta)](1-\delta)$, which requires that $0 < \zeta < 1-\delta-(1-\delta)(2\delta)^{-1}$, where the latter holds only when $\delta > 1/2$. In other words, a necessary condition that the BGP is dynamically efficient when $\bar{b}_t < 0$ is that production is human capital intensive. The latter suggests that when most of the output is produced by human capital, it is possible for young agents to place a significant part of the transfer in a financial intermediary, without violating dynamic efficiency.

in period $t-1$ is identical to the case of complete markets, except that d_{t-1} is no longer a choice variable while $s_t \equiv k_{t+1}$. The optimality conditions for this problem imply that $k_{t+1} = s_t = \overline{\Psi} A^{-1} y_t$, $d_t = \omega_t = \gamma \overline{\Psi} \left[\beta A(1+n)\right]^{-1} y_t$ and $h_{t+1} = \overline{\Phi} h_t^{1-(1-\delta)\zeta} \widetilde{k}_t^{(1-\delta)\zeta}$, where $\overline{\Psi} = \beta\delta A \left(1+\beta+\gamma\right)^{-1}$ and $\overline{\Phi} = \left[B^{\frac{1}{\zeta}} \mu^{\frac{1-\zeta}{\zeta}} \gamma \overline{\Psi} \left[\beta(1+n)\right]^{-1}\right]^{\zeta}$. As in the case of complete markets, it can be shown that the ratio of physical to human capital, $\overline{x}_t$, satisfies (1), where Ψ is replaced with $\overline{\Psi}$ and the only rest point is the origin, i.e. $\overline{x}_t = \overline{x} = \overline{x}_0$ for all $t \geq 1$.[23] The definition of the BGP is the same as definition 1 as well as the functional forms of the ratio of physical to human capital and the the growth rate along the BGP are the same as those in the case of complete markets with $\overline{x}$, $\overline{g}$, $\overline{g}_y$ and $\overline{\Psi}$, replacing x, g, g_y and Ψ. Contrary to the case of complete markets, the growth rate of the BGP may be either increasing, decreasing or unchanged in response to an increase in the level of parental altruism. Specifically, $\partial\overline{g}/\partial\gamma > 0$ if $\gamma < \gamma^{\overline{g}}$, $\partial\overline{g}/\partial\gamma = 0$ if $\gamma = \gamma^{\overline{g}}$ and $\partial\overline{g}/\partial\gamma < 0$ if $\gamma > \gamma^{\overline{g}}$, where $\gamma^{\overline{g}} \equiv \delta(1-\delta)^{-1}(1+n)^2(1+\beta)$. Contrary to the case of complete markets, the ratio of investment in physical capital to output in incomplete markets, $\widetilde{\overline{\Psi}}(\gamma)$, is a monotonically decreasing function of γ. Then, proposition 3 follows from definition 2.

Proposition 3: *With no credit market, the BGP is dynamically efficient if* $R \geq (1+n)(1+\overline{g})$ *or, equivalently,* $\widetilde{\overline{\Psi}}(\gamma) \leq 1-\delta$ *or, equivalently,* $\gamma \in \Omega^{in} \equiv \{\gamma > 0;\, \gamma \geq \gamma^{in}\} \neq \varnothing$, *where* $\gamma^{in} \equiv \left[\beta\delta - (1-\delta)(1+\beta)\right](1-\delta)^{-1}$.[24]

Proof. *See the appendix.* ■

[23] Using the equations for the saving rates of sections 3.1 and 3.2.1 and letting $[s/y]^{MC}$ and $[s/y]^{CM}$ denote the saving rates in the economies with missing and complete markets, respectively, it can be shown that for any $\gamma < \gamma^*$, $[s/y]^{MC} > [s/y]^{CM}$, whereas for any $\gamma > \gamma^*$ $(\gamma = \gamma^*)$, $[s/y]^{MC} < [s/y]^{CM}$ $\left([s/y]^{MC} = [s/y]^{CM}\right)$.

[24] $\Omega^{in} \neq \varnothing$ even when $\gamma^{in} \leq 0$ or equivalently $\delta \leq \delta^{in} \equiv (1+\beta)(1+2\beta)^{-1}$.

Proposition 3, establishes that the BGP will be dynamically efficient under no credit markets if the degree of parental altruism exceeds a threshold value. Intuitively, since the young cannot borrow to fund human capital investment, dynamic efficiency can be achieved only if parental transfers are large enough to ensure that human capital is not under-accumulated in a socially inefficient manner. This, in turn, requires a high degree of altruism (i.e. high enough γ). In other words, when $\gamma < \gamma^{in}$, parental transfers are relatively small and since young individuals are credit constrained, the allocation (k, h) is such that x exceeds the maximum socially optimal level and the interest is strictly smaller than the growth rate of aggregate output. As in the case of complete markets, an alternative allocation of physical and human capital where output is reallocated from physical to human capital, increases the interest rate relative to the growth rate and the welfare of all generations living on the BGP. The result implies that economies with sufficiently high levels of altruism might accumulate human capital in an efficient manner despite the absence of credit markets for funding investment in education. Therefore, proposition 3 suggests that missing credit markets might not lead to inefficiencies, contrary to conventional wisdom.

Proposition 4: *There exist $0 < \delta^s < 1$, such that $\gamma^c > \gamma^{in}$, if $\delta < \delta^s$, $\gamma^c = \gamma^{in}$, if $\delta = \delta^s$ and $\gamma^c < \gamma^{in}$, if $\delta > \delta^s$, where $\delta^s \equiv (1+\beta)[1+(2-\zeta)\beta]^{-1}$ and $\gamma^c = \gamma^{in} = \gamma^s = \zeta(1+\beta)(1-\zeta)^{-1} > 0$.*

Proof. *See the appendix.* ■

Proposition 4 implies that $\{\gamma \in \Omega^{in} \wedge \gamma \notin \Omega^c\} \neq \varnothing$, $\{\gamma \notin \Omega^{in} \wedge \gamma \in \Omega^c\} \neq \varnothing$, $\{\gamma \in \Omega^{in} \wedge \gamma \in \Omega^c\} \neq \varnothing$ and $\{\gamma \notin \Omega^{in} \wedge \gamma \notin \Omega^c\} \neq \varnothing$. In other words, there is a threshold level of human capital intensity in production, δ^s, above which, any level of parental altruism with a dynamically efficient BGP in the economy without credit, implies a dynamically inefficient BGP in an otherwise identical economy with perfect credit markets. Notice that any $\gamma \in \{\gamma \in \Omega^{in} \ \wedge \ \gamma \notin \Omega^{c}\}$, must be sufficienty large, since $\gamma^{c} < \gamma^{in} \leq \gamma$. If human capital intensity is less or equal than the threashold level, δ^{s}, as long as $\gamma^{in} \leq \gamma \leq \gamma^{c}$, both the complete and the no-credit market economies have dynamically efficient BGPs.

Proposition 5: *There are thresholds $\overline{\gamma} > 1$ and $\underline{\gamma} < 1$, where $0 < \underline{\gamma} < \gamma^{*} < \overline{\gamma}$ and $\widetilde{\zeta} \equiv \delta(1+\delta)^{-1}$ such that:* ***(a)*** *If $\zeta < \widetilde{\zeta}$ then* ***(i)*** *$g = \overline{g}$ if $\gamma = \gamma^{*}$ or $\gamma = \overline{\gamma}$,* ***(ii)*** *$g < \overline{g}$ if $\gamma^{*} < \gamma < \overline{\gamma}$ and* ***(iii)*** *$g > \overline{g}$ if $\gamma < \gamma^{*}$ or $\gamma > \overline{\gamma}$.* ***(b)*** *If $\zeta > \widetilde{\zeta}$ then* ***(i)*** *$g = \overline{g}$ if $\gamma = \underline{\gamma}$* or *$\gamma = \gamma^{*}$,* ***(ii)*** *$g < \overline{g}$ if $\underline{\gamma} < \gamma < \gamma^{*}$ and* ***(iii)*** *$g > \overline{g}$ if $\gamma < \underline{\gamma}$ or $\gamma > \gamma^{*}$.* ***(c)*** *If $\zeta = \widetilde{\zeta}$ then* ***(i)*** *$g = \overline{g}$ if $\gamma = \gamma^{*}$ and* ***(ii)*** *$g > \overline{g}$ if $\gamma < \gamma^{*}$ and if $\gamma > \gamma^{*}$.*

Proof. *See the appendix.* ■

Proposition 5 establishes that (i) it is possible for the growth rate under no credit markets to exceed that under complete markets, and (ii) that whether it does can be linked to the degree of parental altruism. The first part of the result is interesting in itself because it indicates that the absence of credit markets need not imply lower steady-state growth. This is again a result that goes contrary to conventional wisdom on the role of credit markets. The second part of the result is also interesting because it suggests that economies with sufficiently low or high levels of parental altruism but highly imperfect credit markets could outperform those with well-developed credit markets in terms of growth performance.[25]

The proposition suggests that an economy with a missing credit market for education

[25] Let MC and CM denote the economy with a missing credit market and the economy with a complete market. Then note that proposition 5 also implies that $\overline{g} > g$ and $[s/y]^{MC} > [s/y]^{CM}$ if $\underline{\gamma} < \gamma < \gamma^{*}$ and $\zeta > \widetilde{\zeta}$, whereas $\overline{g} > g$ and $[s/y]^{MC} < [s/y]^{CM}$ if $\gamma^{*} < \gamma < \overline{\gamma}$ and $\zeta < \widetilde{\zeta}$.

loans outperforms, in terms of growth, along the BGP an otherwise identical economy with complete credit markets either when the level of intergenerational correlation in human capital, $(1-\zeta)$, and the level of parental altruism are relatively high or when they are both relatively low. In the first case, under complete markets and a specific range of high levels of parental altruism, young agents choose to save part of the parental transfer than exploiting the relatively high intergenerational persistence in human capital to generate more human capital for future generations. Higher savings by the young lead to an over-accumulation of physical capital relative to human capital along the BGP. The economy avoids the latter when the young are restricted from accessing the financial market. In the second case, under complete markets and a range of low levels of parental altruism, the young use the financial market to borrow a relatively large amount to complement the parental transfer in investing in their human capital development, rather than saving part of the transfer, allowing the economy to generate a growth-optimal ratio of physical to human capital. When the young are restricted from accessing the credit market for education loans, excess borrowing is prevented and so the economy avoids over-accumulation of human capital. Notice that threshold $\widetilde{\zeta}$ is positively related to the intensity of the use of labor in production. It follows that as the degree of labor intensity increases, the more likely it becomes that the economy with a missing credit market outperforms an economy with complete markets at relatively high levels of parental altruism than low levels.[26]

These findings might help to explain the mixed results that have been found in empir-

[26] This result can be compared to the finding of de la Croix and Michel (2007) who demonstrate that the maximum growth rate is achieved in a borrowing constrained regime as long as the elasticity of earnings to education is high enough. The elasticity of earnings to education (assuming that d captures the level of education) corresponds to $\delta\zeta$.

ical analyses of the effects of borrowing constraints on growth. For instance, while Japelli and Pagano (1994) found that the presence of borrowing constraints tend to raise growth rates, the opposite result was found by De Gregorio (1996). Proposition 5, offers a possible explanation for the fact that several East Asian economies with exremely high level of parental investment in education (e.g. South Korea, China and Taiwan) have been able to grow at fast rates over recent decades despite the absence of well-developed credit markets for education. A numerical illustration of the findings is displayed in Figure 1. Here, we set $\beta = 0.3$, $\zeta = 0.60$, $\delta = 2/3$, and $n = 1/2$. The scaling parameters A and B are set at 10 and 2.5. These parameter choices are similar to the one we use in the empirical part of the paper (see Section 4.2). As indicated by the analytical results, the growth rate under complete

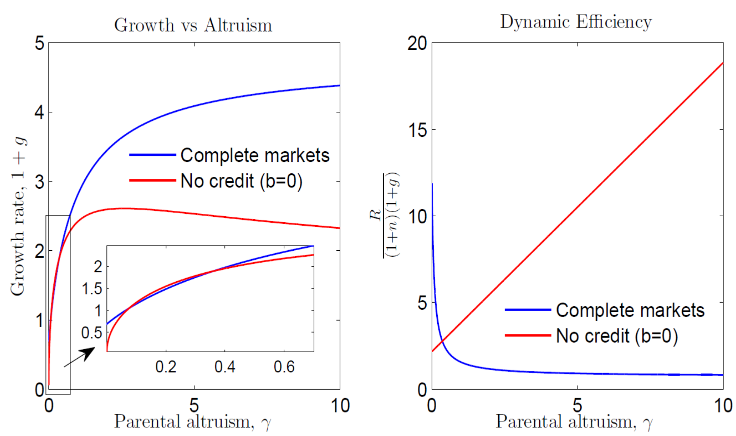

Figure 1: Growth and efficiency vs the parental altruism motive (Case: $h_p > \widetilde{h}_p$)

markets increases monotonically with γ, while in the no-credit market case growth initially increases with γ before reaching a maximum and declining – as explained later these results are consistent with the pattern of the index of parental altruism displayed in Figure 3. Notice that the growth rate with no-credit market exceeds that under complete markets for a range of γ values, consistent with Proposition 5. Since $\zeta < \tilde{\zeta}$ under the above calibration, we are in the first case in Proposition 5: the growth rates intersect when $\gamma = \gamma^*$ (= 0.15) and when $\gamma = \overline{\gamma}$ (=0.63), and for $\gamma \in (\gamma^*, \overline{\gamma})$ the growth rate is higher under no-credit market. We also see from the second panel of Figure 1 that the required condition for dynamic efficiency, $R \geq (1+n)(1+g)$, is always met for $\gamma > 0$ in the case of no credit. This is consistent with Proposition 3 given that $\gamma^{in} = -0.7$ under the above calibration. By contrast, the balanced growth path is dynamically efficient under complete markets only when $\gamma \leq \gamma^c$ (= 1), as implied by Proposition 2.

3.2.2 Credit Market with Limited Access

We now consider the intermediate case where young households may borrow up to a limit in order to invest in education. As Coeurdacier et al. (2015), we assume, in particular, that they can borrow up to a fraction $\lambda > 0$ of the present value of their labor income when middle aged, or $\bar{b}_{t-1} \leq \lambda w_t h_t / R_t$.[27] The Kuhn-Tucker conditions with a binding borrowing constraint imply that $k_{t+1} = A^{-1}\overline{\overline{\Psi}} y_t$, $s_t = (1-\delta)^{-1}\left[1-\delta(1-\lambda)\right] k_{t+1}$, $d_t = \overline{\overline{\Psi}}[\gamma\left[1-\delta(1-\lambda)\right] + \beta\delta\lambda][A\beta(1-\delta)(1+n)]^{-1} y_t$, $\omega_t = \overline{\overline{\Psi}}[1-\delta(1-\lambda)][A\beta(1-\delta)]^{-1} y_t$ and $h_{t+1} = \overline{\overline{\Phi}} h_t^{1-(1-\delta)\zeta} \widetilde{k}_t^{(1-\delta)\zeta}$ where[28]

$$\overline{\overline{\Psi}} = \frac{\beta\delta(1-\delta)(1-\lambda)A}{(1+\beta+\gamma)\left[1-\delta(1-\lambda)\right]} \quad \text{and} \quad \overline{\overline{\Phi}} = \left[\frac{B^{\frac{1}{\zeta}} \mu^{\frac{1-\zeta}{\zeta}} \overline{\overline{\Psi}} \left[\gamma\left[1-\delta(1-\lambda)\right] + \beta\delta\lambda\right]}{\beta(1-\delta)(1+n)}\right]^{\zeta}.$$

[27]Contrary to the case of no-credit market, $\bar{b}_{t-1} < 0$ is assumed to be unconstrained, as in the case of complete markets.

[28]It is worth noting that the equation for the savings rate implies that the savings rate is higher for economies with tighter borrowing constraints (smaller λ). Since the borrowing constraint binds for relatively small γ, this result is consistent with the result on savings rate of section 3.2.1 - see footnote 23.

For the reasons stated in the previous sections, the functional forms of the ratio of physical to human capital and the growth rate at the unique BGP with a binding borrowing limit are the same as those in the case of complete markets with $\overline{\overline{x}}$, $\overline{\overline{g}}$, $\overline{\overline{g_y}}$ and $\overline{\overline{\Psi}}$, replacing x, g, g_y and Ψ, respectively.

Proposition 6: *Along the BGP, for any $\lambda > 0$ such that $\lambda < \zeta$, there exists $\Omega^{bin} \equiv \{\gamma > 0; \gamma < \gamma^{bin}\} \neq \varnothing$ such that the borrowing limit is binding only if $\gamma \in \Omega^{bin}$, where $\gamma^{bin} \equiv [\beta\delta(\zeta - \lambda)] [1 - \delta(1 - \lambda)]^{-1}$, while for any $\lambda \geq \zeta$, $\Omega^{bin} = \varnothing$ and the borrowing limit does not bind.*

Proof. *See the appendix.* ■

Proposition 6 states that for a high enough level of parental altruism ($\gamma \geq \gamma^{bin}$), the young will receive large enough parental transfers that the borrowing limit is not binding. Then the first-order conditions collapse to those under complete markets, and hence the economy is on the complete markets BGP, i.e. $g = \overline{\overline{g}}$. As long as the intergenerational correlation of human capital, $1 - \zeta$, is lower than threshold $(1 - \lambda)$, the borrowing constraint binds if and only if parental altruism is relatively small. Proposition 6 also demonstrates that the threshold γ^{bin}, below which the borrowing constraint binds, is strictly smaller than the threshold γ^*, below which the young borrow in order to fund their education under complete markets. This implies that even if $\lambda < \zeta$, for any $\gamma \in \left\{\gamma > 0;\ \gamma^{bin} < \gamma < \gamma^*\right\}$, the borrowing constraint does not have any adverse effect on young agents as they can borrow the same amount they would have borrowed if markets were complete. Notice also that $\overline{\overline{g}}$ is always increasing in γ under reasonable parameter values.

Having identified the conditions under which the borrowing constraint is binding, we now proceed to an analysis of growth rates and dynamic efficiency. The main results are summarized in Propositions 7 and 8.

Proposition 7: *When the borrowing limit is binding, the BGP is dynamically efficient if* $R \geq (1+n)\left(1+\overline{\overline{g}}\right)$ *or, equivalently, if* $\gamma \in \Omega^{\overline{in}} \equiv \{\gamma > 0;\ \gamma^{\overline{in}} \leq \gamma < \gamma^{bin}\}$, *where* $\gamma^{\overline{in}} \equiv [\beta\delta(1-\lambda) - [1-\delta(1-\lambda)](1+\beta)]\,[1-\delta(1-\lambda)]^{-1}$, *and* $\Omega^{\overline{in}} \neq \varnothing$ *only if* $\zeta > 1-(\beta\delta)^{-1}(1-\delta(1-\lambda))(1+\beta)$. [29]

Proof. *See the appendix.* ■

As in the no-credit market case, when the young are restricted from borrowing (i.e. the borrowing limit binds), the level of parental altruism must be above a certain threshold to ensure adequately large parental transfers, preventing under-accumulation of human capital.

Proposition 8: *For* $\lambda < \zeta$, ***(a)*** *if* $\lambda \geq \widetilde{\lambda}$, *then* ***(i)*** $g = \overline{\overline{g}}$ *if* $\gamma \geq \gamma^{bin}$ *and* ***(ii)*** $g < \overline{\overline{g}}$ *if* $\gamma < \gamma^{bin}$; ***(b)*** *if* $\lambda < \widetilde{\lambda}$ *then for* $\zeta \leq \widetilde{\zeta}^{*}$, ***(i)*** $g = \overline{\overline{g}}$ *if* $\gamma \geq \gamma^{bin}$ *and* ***(ii)*** $g > \overline{\overline{g}}$ *if* $\gamma < \gamma^{bin}$, *while for* $\zeta > \widetilde{\zeta}^{*}$, *there exists* $\underline{\gamma}_2 \in \Omega^{bin}$, *as long as* $\lambda < \lambda_\gamma$, *such that* ***(i)*** $g = \overline{\overline{g}}$ *either if* $\gamma = \underline{\gamma}_2$ *or* $\gamma \geq \gamma^{bin}$, ***(ii)*** $g < \overline{\overline{g}}$ *if* $\underline{\gamma}_2 < \gamma < \gamma^{bin}$ *and* ***(iii)*** $g > \overline{\overline{g}}$ *if* $\gamma < \underline{\gamma}_2$, *where* $\widetilde{\lambda} = (1-\delta)(1-\zeta)\,[1-\delta(1-\zeta)]^{-1}$, $\widetilde{\zeta}^{*} = \delta(1-\delta)(1-\lambda)\,[1-\delta^2(1-\lambda)]^{-1}$ *and* $\lambda_\gamma = (1-\zeta)\,[1-\delta(1-\lambda)]$.

Proof. *See the appendix.* ■

Proposition 8 suggests that an economy with limited access to education loans outperforms, in terms of growth, an otherwise identical economy with complete credit markets

[29] When the borrowing limit is not binding, the BGP is dynamically efficient according to Proposition 2.

along the BGP either when (i) credit constraints are loose or (ii) credit constraints are tight while the degree intergenerational correlation of human capital, $(1-\zeta)$, is relatively low and parental altruism falls within a range of relatively high values. In both cases, under complete markets, young agents end up over-accumulating human capital relative to physical capital. Specifically, in the first case, the loose credit constraints prevent young individuals from over-borrowing and over-investing in human capital, keeping the ratio of physical to human capital at a growth-optimal level, relative to that in complete markets. The latter is too low as the young over-borrow to invest in human capital. In the second case, under complete markets, young individuals choose to borrow and complement the parental transfer for the development of their human capital. However, since the degree of intergenerational correlation of human capital is low, they end up over-accumulating human capital relative to physical capital.

Empirical studies (De Gregorio, 1996 and Aghion et al., 2010) suggest that the relationship between growth and credit constraints is mainly negative. However, as shown in section 4, there are cases of countries with undeveloped credit markets exhibiting high economic growth rates. For instance, economic growth in China has proceeded at a fast rate despite the absence of well developed credit markets. Proposition 8 provides a possible explanation of these observations that relates the restrictions to borrowing with the intergenerational correlation of human capital and the level of parental altruism.[30] Figure 2 provides a numerical illustration of the theoretical results. The calibration is unchanged, except that to illustrate

[30] Using the growth rate equations along the BGP, it is straightforward to show that there exists a threshold $\tilde{\tilde{\gamma}} > 0$ which depends on λ with $\lim_{\lambda \to 0} \tilde{\tilde{\gamma}}(\lambda) = 1$, such that for any $\gamma \in \Omega^{bin}$, $\overline{\overline{g}} > \overline{g}$ if $\gamma > \tilde{\tilde{\gamma}}(\lambda)$, $\overline{\overline{g}} < \overline{g}$ if $\gamma < \tilde{\tilde{\gamma}}(\lambda)$ and $\overline{\overline{g}} = \overline{g}$ if $\gamma = \tilde{\tilde{\gamma}}(\lambda)$. In other words, the BGP growth rate in the economy with a missing market is strictly greater than the growth rate in the economy with limited access to credit, at low levels of parental altruism.

two different cases we consider two values for ζ and fix the borrowing constraint parameter at $\lambda = 0.1$. In Case 1 (upper panels of Figure 2), we set $\zeta = 0.60$ as in the empirical part of the paper – this corresponds to case (b) of Proposition 8. As expected, growth is higher for $\gamma < \gamma^{bin}$ (=0.25) in the economy with borrowing limit and coincides with the complete markets growth rate for $\gamma \geq \gamma^{bin}$. In Case 2 (lower panels of Figure 2), we set $\zeta = 0.90$, which produces the final sub-case of Proposition 8. The growth rate starts out higher in the economy with borrowing limit but then falls below the growth rate in the complete markets economy. Once $\gamma \geq \gamma^{bin}$ (=0.40), the growth rates in the complete markets and borrowing limit cases coincide because the borrowing limit ceases to be binding. In both cases the economy is dynamically efficient for the range of γ values considered in Figure 2. Only for somewhat higher values of γ do we get dynamic inefficiency.

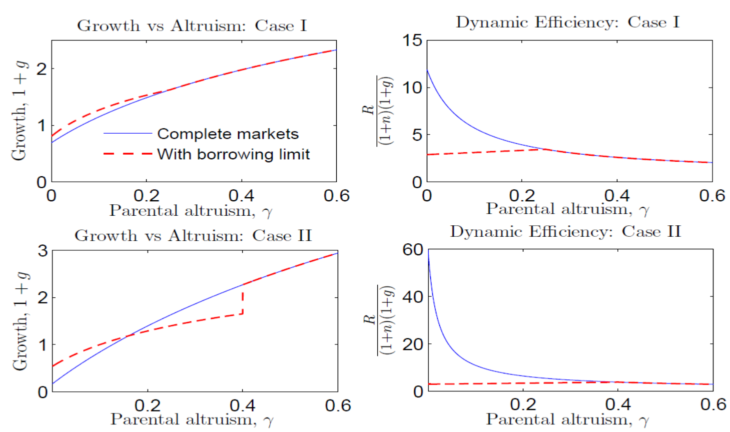

Figure 2: Growth and efficiency vs the parental altruism motive

Several studies examine the relationship between borrowing constraints, the savings rate and economic growth.[31] The long-run gross growth rate in our model can be expressed as

[31] In a model without altruistic motives, Jappelli and Pagano (1994) show that borrowing constraints increase the savings rate and thereby raise economic growth whereas De Gregorio (1996) shows that once

$(1+g) = \varkappa(s/y)(x)^{-\delta}$ where where $\varkappa$ is a constant. Thus, growth depends positively on the savings effect, s/y, and negatively on the capital composition effect.[32] The dominance of either the savings effect or the capital composition effect hinges on a number of factors, among which the parental altruism motive and the degree of credit markets imperfection. Our theoretical results indicate a non-monotonic relationship between borrowing constraints and growth that depends on the parental altruism motive. Among others, our model provides theoretical justification of the finding of Jappelli and Pagano (1994) that the positive effect of liquidity constraints on the savings rate cannot be rejected by the data.[33] Our model also demonstrates conditions, which relate to the degree of intergenerational correlation of human capital and the degree of parental altruism, that justify the observation of Coeurdacier et al. (2015) that emerging Asian economies exhibit higher growth rates and savings than those of advanced economies. Our results suggest that the latter may occur either under the existence of an imperfect credit market for education loans or when credit markets are completely absent.[34]

In summary, this section has presented several findings that challenge the conventional

investment in human capital is introduced, borrowing constraints lower economic growth because of the negative effect on human capital accumulation. In the latter borrowing constraints affect human capital only indirectly via the incentive to work.

[32] In the model of Jappelli and Pagano (1994), only the savings effect is present (raising the savings rate, borrowing constraints raise growth) while in the model of De Gregorio (1996) the growth rate is related to the capital composition effect (borrowing constraints raise the ratio of physical to human capital which lowers economic growth).

[33] As noted in footnote 23, the savings rate in the economy with a missing credit market is strictly greater than the savings rate in the economy with complete markets for relatively low levels of parental altruism. Moreover, the savings rates of sections 3.1 and 3.2 along with proposition 6, imply that that for any $\gamma \in \Omega^{bin}$ along the BGP , $[s/y]^{MC} > [s/y]^{LABC} > [s/y]^{CM}$, where $[s/y]^{LABC}$ denotes the savings rate in the economy with limited access to the credit market for education loans.

[34] Specifically, proposition 5 suggests that $\overline{g} > g$ and $[s/y]^{MC} > [s/y]^{CM}$ if $\underline{\gamma} < \gamma < \gamma^*$ and $\zeta > \widetilde{\zeta}$, while proposition 8 suggests that for any $\gamma \in \Omega^{bin}$, $[s/y]^{LABC} > [s/y]^{CM}$ and $\overline{\overline{g}} > g$ if either $\lambda \geq \widetilde{\lambda}$ or $\lambda < \widetilde{\lambda}$ and $\zeta > \widetilde{\zeta}^*$ and $\gamma > \underline{\gamma}_2$.

wisdom that the absence of credit markets or imperfections in credit markets are likely to hinder efficiency and growth.[35] Consequently, the results may help to shed light on empirical observations which relate credit constraints and growth, as well as the surprisingly strong growth performance of several East Asian economies. In the following section we investigate the extent to which our model exhibits realistic cross-country behavior.

4 Parental Altruism: Data vs Model

As noted in Section 2, we would ideally like to construct a cross-country measure of γ that is defined as parental spending on their children's education per unit of parental consumption. Unfortunately, the existing micro data are inadequate to enable us to construct a direct measure of γ. We therefore use the World Values Survey (WVS), a nationally representative survey that is conducted in more than 50 countries around world using a common questionnaire. The main advantage of the WVS data is that it allows us to construct a cross-country attitude-based proxy of γ that covers a fairly wide sample of countries.

We utilize responses to Question V182 from Wave 6 of the World Values Survey (2010-2014). The question is worded as follows: *To what degree are you worried about the following situations? ... Not being able to give my children a decent education.* Representative groups of respondents in 60 countries were asked this question. We use country-level responses to assign altruism scores to each country. We score countries in ascending order according to the total percentage of respondents who answered 'Very much' or a 'A great deal', which we

[35] If we instead considered a small open economy that takes the world real interest rate as given, as in De Gregorio (1996), our key theoretical results relating to *relative* growth rates and dynamic efficiency remain intact. These results are available upon request.

interpret as altruistic responses.[36] From the total sample of 60 countries we removed three outliers with negative average growth rates (Kuwait, Kyrgyzstan, Ukraine), one economy for which growth data was not available (Libya), and eight economies in which credit market data was not available (Iraq, Morocco, Palestine, Qatar, Taiwan, Trinidad and Tobago, Tunisia and Uzbekistan), leaving a final sample of 48.[37] We refer to this as the 'WVS Score'.

The answer of a respondent in the WVS may be heavily influenced by the level of his or her income. That is, a low-income respondent may report that he is more worried about providing a good education for his children relative to a high-income respondent, despite the fact that the two may care the same about educating their children. This claim is reinforced by the high cross-country correlation coefficient between the survey ranking indicator and per capita real income. To address this issue, we regress the WVS score on a constant and per-capita real income to obtain a set of income-adjusted survey scores, i.e. we obtain an income adjusted WVS score, denoted by $\widehat{\gamma}$, net of the estimated income component (the fitted value of the WVS score due to income). In this way, we ensure that our our proxy for γ is orthogonal to income effects and hence comparable across countries with different levels of economic development.

As our measure of credit market imperfections, we construct a Credit Index that enables us to classify each country either as *Credit Unconstrained*, *Highly Credit Constrained*, or *Credit Constrained with limited access to credit.* These classifications correspond to the three

[36]The other responses were: 'Not much', 'Not at all' and various non-responses: 'Inappropriate', 'Not applicable', 'No answer', and 'Don't know'. To compute the total percentage of altruistic responses in each country we excluded non-responses from the sample and computed the 'WVS score' of altruistic responses as a percentage of total responses minus non-responses. In this case the categories 'Very much', 'A great deal', 'Not much', 'Not at all' sum to 100%.

[37]The number of respondents to the survey exceeded 1,000 in all countries except Poland (966 respondents) and New Zealand (841 respondents).

theoretical models of section 3, the complete markets model (labeled as *Unconstrained*), the model with no credit market for education loans (labeled as *No Credit*) and the model with limited access to credit for education loans (labeled as *Borrowing Limit*), respectively. Given that proper data on cross-country education credit is unavailable, we assume that economies with ample credit opportunities also imply ample credit opportunities for education. Our Credit Index is based on the following four indicators of credit provision and financial development:[38] (i) Account at a financial institution;[39] (ii) Borrowing from a financial institution in the past year;[40] (iii) Credit card ownership;[41] (iv) Domestic Credit to the Private Sector / GDP (Average 1990-2010, %).[42] For each indicator, we set thresholds reflecting a minimal level of credit provision / financial development.[43] Our chosen thresholds for the above indicators are 40%, 10%, 10%, 25%, respectively. These thresholds occur at fairly similar percentiles of 33.3%, 35.9%, 31.9%, 34.7%, respectively. Our Credit Index is then defined as:

$$\text{Credit Index}_i \equiv 1 - \frac{1}{4}\sum_{j=1}^{4} FD_{i,j}$$

where $FD_{i,j}$ is a dummy variable equal to 1 if indicator j in country i is below the threshold.

[38] Couerdacier et al. (2015) also rely on indicators of financial development and household credit to assess borrowing constraints across countries (namely, the US and China).

[39] % of respondents age 15+ with an account (self or together with someone else) at a bank, credit union, another financial institution (e.g., cooperative, microfinance institution), or the post office (if applicable) including respondents who reported having a debit card.

[40] % (age 15+) of respondents who report borrowing any money from a bank or another type of financial institution in the past 12 months. Average value is computed based on years 2011 and 2014.

[41] % (age 15+) respondents who report having a credit card. Average value based on years 2011 and 2014.

[42] Domestic Credit to the Private Sector includes financial resources provided to the private sector, including loans, purchases of nonequity securities, and trade credits and other accounts receivable, that establish a claim for repayment. For some countries these claims include credit to public enterprises. Average value from 1990-2010 is used.

[43] When missing values were encountered, those years were excluded from the calculation of the average. In our final sample of 48 countries, no country had more than 6 missing values, and 34 had no missing values.

We classify economies as *Unconstrained* if Credit Index$_i = 1$, as *No Credit* if Credit Index$_i \leq 1/4$, and otherwise as *Borrowing Limit*. In our final sample of 48 countries, 19 economies are classified as Unconstrained, 12 as No Credit, and 17 as Borrowing Limit. Table 2 displays the credit market classification of the countries in our sample and the country specific education-based measure of parental altruism.[44]

Table 2 – Credit market classification and the measure of parental altruism

Credit Unconstrained				No Credit		Borrowing Limit			
country	$\widehat{\gamma}$	**country**	$\widehat{\gamma}$	**country**	$\widehat{\gamma}$	**country**	$\widehat{\gamma}$	**country**	$\widehat{\gamma}$
Australia	85,4	Poland	68,4	Algeria	88,7	Argentina	69	Romania,	71,8
Bahrain,	76,3	Singapore	115,9	Armenia	95,8	Brazil	101,2	Russia	84,2
Chile	88,1	Slovenia	68	Azerbaijan	87,8	Belarus	79,1	Thailand	75,9
Cyprus	87,7	S. Africa	80,5	Egypt	95,9	China	93,4	Turkey	92,6
Estonia	85,5	Spain	87	Ghana	95,7	Colombia	103,9	Uruguay	84,4
Germany	80,1	Sweden	67,3	Jordan	70,9	Ecuador	87,5		
Kazakhstan	92,1	US	85,9	Mexico	107,1	Georgia	89,9		
S. Korea	117,2			Nigeria	86,8	Hong Kong	106,9		
Lebanon	105,3			Pakistan	80,8	India	72,7		
Malaysia	112,3			Rwanda	94,5	Japan	106,3		
Netherlands	67,9			Zimbabwe	84,6	Peru	98,1		
New Zealand	67,4			Yemen	89,6	Philippines	96		

Figure 3 displays the relationship between our indicator of parental altruism and economic growth from 1970-2010.[45] The first panel displays the whole sample of countries and suggests an overall positive relationship between parental altruism and growth. The theory suggests that the direction of the relationship between growth and altruism for all three models of section 3 is mainly differentiated in relatively high levels of parental altruism. Given that there is a limited number of observations for high levels of parental altruism the full sample

[44] To save on space, we do not list the country credit scores on table 2. They are available upon request.

[45] Growth is defined as the average annual growth rate of real GDP per person, at constant 2011 national prices, from 1970-2010, based on the Penn World Table, Version 9 (see Feenstra et al., 2015). For eight former Soviet Union economies, the period 1990-2010 was used for data availability reasons.

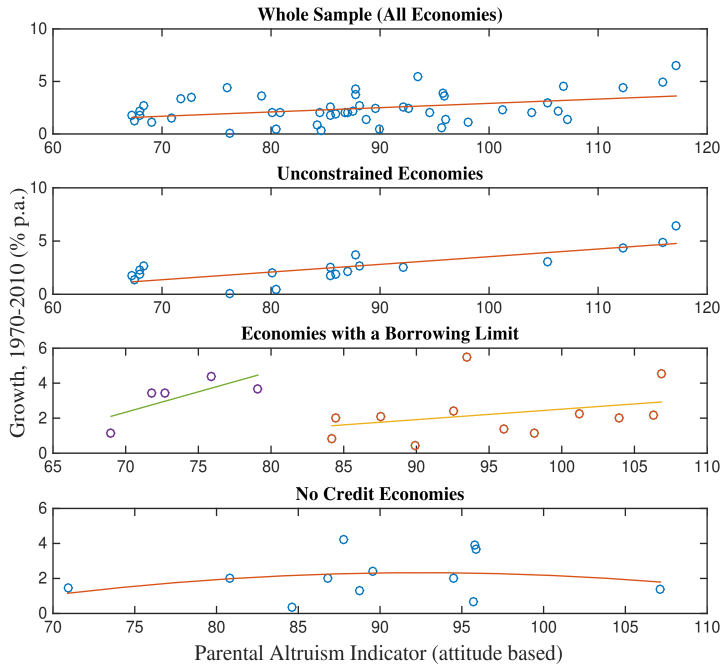

Figure 3: Economic growth vs parental transfer motive

does not allow us to identify the three models. The latter is also statistically evident in the first row (full sample) of table 3. The relationship is clearly differentiated across countries with different credit market status when countries are classified according to their credit market score. In countries with highly-developed credit markets (2nd panel), increases in parental altruism are associated with higher rates of economic growth, as predicted by the Complete Markets model. We see the same positive relationship for economies classified as having a Borrowing Limit (3rd panel). In this case however, we identify two distinct sub-groups of countries. This could be due to differences in structural parameters that we cannot identify due to data limitations and lack of information. Finally, despite the limited number of observations, No Credit economies (4th panel) display the hump-shaped relationship between growth and parental altruism that the theory suggest. Therefore, panels 2-4 suggest that parental altruism influence growth differently depending on credit market status.

To evaluate the model, we compute cross-country correlations between the model-implied measures of parental altruism and the corresponding attitude-based measures from the WVS. To compute the former, we take a heuristic approach by assuming that all characteristics are the same across countries, other than the level of parental altruism, population growth, economic growth and the credit market for education loans. We make this assumption because we either do not have adequate cross-country information about the rest of the parameters or do not have convincing information that these parameters are significantly different across countries. In particular, we test the model's ability to correctly rank countries in terms of altruism, subject to the constraint that each county's γ be chosen to minimize the distance between their growth rate in the data and their model-implied growth rate. Formally, we solve the following minimization problem for each country i:

$$\min_{\gamma_i \in [\underline{\gamma}, \overline{\gamma}]} \left\{ \left[g_i \left(\gamma_i, n_i \right) - g_i^* \right]^2 \right\},$$

where $g_i(\gamma_i, n_i)$ is the model-implied growth rate of country i with population growth rate n_i, and g_i^* is the corresponding growth rate in the data. We solve the above problem numerically using a grid search procedure that updates γ by small increments, starting from a lower bound $\underline{\gamma} > 0$ and ending at an upper bound $\overline{\gamma}$.[46] In the case of the Complete Markets model, $g_i(\gamma_i, n_i)$ is monotonically increasing so for each country there will be a unique $\gamma_i^* \in [\underline{\gamma}, \overline{\gamma}]$ that minimizes the residual function. In the model with Borrowing Limit,

[46] In the empirical applications that follow we set $\underline{\gamma} = 1 \times 10^{-5}$, $\overline{\gamma} = 500$ and use increments of 0.001.

by contrast, $g(\gamma_i, n_i)$ need not be monotonically increasing. Indeed, once γ_i is high enough ($\geq \gamma^{bin}$), the borrowing constraint becomes slack, which can cause a discontinuity in the growth rate $g(\gamma_i, n_i)$, making it possible for two values of γ_i to be consistent with a given growth rate, which lies within a specific range. Similarly, in the No Credit model, $g(\gamma_i, n_i)$ is a smooth concave function, so there may be 'low' and a 'high' values of γ_i such that $g(\gamma_i, n_i)$ matches the growth rate in the data, g_i^*. Whenever the issue of two γ_i for a given g_i^* arises, we search for two zeros of $g_i(\gamma_i, n_i)$.[47]

To provide a check of the predictions of the theory, we compute the correlation between the model-implied altruism ranking and the survey ranking implied by the income-adjusted score vector $\widehat{\gamma}$.[48] In this context, a perfect positive correlation of +1 would mean that the model ranks all countries in the same order as the survey measure, whereas a perfect negative correlation of -1 means the two rankings are 'mirror images' (i.e. the most altruistic country in the survey is the least altruistic according to the model etc). Hence, positive correlations provide support for the theory. To address the issue of two γ_i that arises in the models with imperfect credit markets, we simulate the model a number of times and in each simulation randomly assign one of the two roots γ_i when we come across cases of multiplicity. We then choose the simulation that gives the highest correlation between the data and model prediction. In this way, we give the models the best possible chance to match the data.[49]

In calibrating the models, we assume each period in the OLG model lasts 30 years. The

[47] We checked the effectiveness of the search procedure by examining visual plots for the residual function.

[48] We rank the cross-country parental altruism measure from the survey in ascending order and then rank the model-implied γ's so that the latter preserves the same order of countries as the former. Our ranking of $\widehat{\gamma}$ assigns 1 to the country with highest altruism, 2 to the country with second highest, and so up to the sample size of N. In cases of ties (e.g. countries for which the endpoints $\underline{\gamma}$ or $\overline{\gamma}$ minimize distance to the data), countries are assigned the same ranking.

[49] No multiple γ_i were found in the model with borrowing limit, in stark contrast to the no-credit model.

population growth rate of each country, n_i, is calculated from the *annual* growth rate over the period 1970-2010, using the formula $n_i = (1 + \overline{n}_i^{ann})^{30} - 1$.[50] The discount factor, β is set at 0.30, consistent with a quarterly value of 0.99. The share of labor income in output, δ is equal to 2/3, which implies 1/3 of output goes to physical capital. We set $\zeta = 0.60$ so that intergenerational persistence in human capital is $1-\zeta = 0.40$, in line with the empirical literature (see Chusseau et al. 2012, Table 8.4). In the model with borrowing limit we set the fraction of present value labor income that can be borrowed at $\lambda = 0.10$. This matches one of the values in Kitaura (2012).[51] Finally, we fix B at 2.5 and A at 10 so that growth rates generated by model are of the same sign and order of magnitude as those in the data. In what follows, we report correlations for different versions of the model as well as sensitivity analysis.

Table 3 reports the correlations for the three versions of the model studied in the theory part of the paper. All three models have positive correlations of around 0.4 on the full sample. These relatively modest correlations are not surprising as the theory suggests one-size-fits-all model is not appropriate for countries with very different credit market conditions. The correlations increase once we separate countries into subgroups by credit market status. For instance, the Unconstrained and No Credit models have correlations of around 0.7 with their respective subsamples. In the case of the Borrowing Limit model, the correlations also rise, but the sample was separated into two groups; Figure 3 suggests such treatment is necessary. Overall, the results are supportive of the theory; the data and model-implied

[50] The population measure is taken from the Penn World Tables, Version 9

[51] In a quantitative OLG model with a similar type of borrowing constraint, Couerdacier et al. (2015) use values of 0.16 (US) and 0.01 (China). Our calibration of 0.10 is close to the average of 0.085.

parental altruism rankings are positively correlated, and each model performs better on samples with the designated credit market status than on the full sample.[52]

Table 3 – Correlation of the WVS measure and model rankings

Sample	Complete Markets	Borrowing Limit	No Credit	*N=obs*
Full Sample	0.43	0.43	0.39	48
Unconstrained	**0.72**	-	-	19
Limit (γ^{low}; γ^{high})	-	**0.77**, **0.45**	-	6, 11
No Credit	-	-	**0.71**	12

We now investigate the sensitivity of these results. To do so, we perform two checks: a parameter sensitivity analysis and loosening / tightening of cut-off values in the Credit Index. For the former we consider 'high' and 'low' values of the discount factor β, the labour income share δ, the parameter ζ, which determines intergenerational persistence in human capital $(1-\zeta)$, and the borrowing limit parameter λ. Table 4 suggests that most correlations take on similar values. The only marked changes are in the correlations of the No Credit model, which falls to 0.284 for the case of the full sample (low β calibration) and to 0.639 for the No Credit subsample (high δ calibration). In addition, one of the Borrowing Limit correlations falls from 0.77 to 0.60, though this is driven by a relatively small change in the model-implied altruism ranking. For the Credit Index sensitivity check we moved the thresholds in the Credit Index up by one-tenth ('loose') or down by one-tenth ('tight').[53] This leads to variation in the classification of countries as Unconstrained, Borrowing Limit or No Credit, as shown in the final column of Table 4. In most cases the correlations are very

[52] We do not report off-diagonal correlations in the lower part of Table 3 to avoid clutter. However, these results also offer some support. For instance, the performance of the No Credit model on the Unconstrained sample is somewhat worse than its performance on the No Credit sample (and vice versa).

[53] For example, in the 'tight' case the cut-off values changed from 40%, 10%, 10% and 25% to 36%, 9%, 9% and 22.5%.

Table 4 – Sensitivity analysis

Correlation of WVS and model rankings: Parameter sensitivity analysis				
β=0.40, β=0.20	**Complete Markets**	**Borrowing Limit**	**No Credit**	**N**
Full Sample	0.43, 0.43	0.43, 0.43	0.28, 0.41	48
Unconstrained	**0.72, 0.72**	-	-	19
Limit (γ^{low}; γ^{high})	-	**0.77, 0.45; 0.77, 0.44**	-	6,11
No Credit	-	-	**0.69, 0.70**	12
δ=0.700, δ=0.633				
Full Sample	0.43, 0.43	0.43, 0.43	0.34, 0.39	48
Unconstrained	**0.72, 0.72**	-	-	19
Limit (γ^{low}; γ^{high})	-	**0.77, 0.45; 0.77, 0.42**	-	6,11
No Credit	-	-	**0.69, 0.64**	12
ζ=0.70, ζ=0.50				
Full Sample	0.44, 0.42	0.44, 0.42	0.35, 0.33	48
Unconstrained	**0.73, 0.76**	-	-	19
Limit (γ^{low}; γ^{high})	-	**0.60, 0.42; 0.77, 0.46**	-	6,11
No Credit	-	-	**0.71, 0.75**	12
λ=0.15, λ=0.05				
Full Sample	-	0.43, 0.43	-	48
Limit (γ^{low}; γ^{high})	-	**0.77, 0.45; 0.77, 0.44**	-	6,11
Correlations of WVS and model rankings: Credit classification checks				
Unconstrained Loose	0.75	-	-	20
Unconstrained Tight	0.90	-	-	14
Limit Loose (γ^{low}; γ^{high})	-	0.90, 0.83	-	5, 13
Limit Tight (γ^{low}; γ^{high})	-	0.70, 0.36	-	5, 12
No Credit Loose	-	-	0.69	16
No Credit Tight	-	-	0.74	11

high. The exception is the correlation of 0.36 for the one Borrowing Limit subsample under the tight thresholds case. This happens because the composition of the sample is somewhat different to the baseline case: countries who were previously classified as No Credit become Borrowing Limit while some who were previously Borrowing Limit are now classified as Unconstrained.

5 Conclusion

We have presented an OLG model of endogenous growth driven by human capital accumulation. The model features altruistic transfers from parents to children for education purposes and nests 'missing credit markets' for education loans. Using the model, we establish conditions under which missing or imperfect credit markets increase economic growth and do not hinder dynamic efficiency. The parental altruism motive plays a key role in the results; however its implications for growth and dynamic efficiency are not monotonic, but depend crucially on the extent of credit market development.

We find support for the theory using a newly-constructed attitude-based cross-country index of parental altruism. We show that the latter is positively correlated with the predicted altruism index from model specifications that are disciplined by real GDP and population growth rates. Moreover, when we condition on credit market development using a Credit Index we find strong positive correlations that are supportive of the non-monotonic role of parental altruism on growth. We thus argue that parental altruism may be a factor behind cross-country differences in growth which the literature has hitherto had difficulty explaining.

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Appendix: Proofs

Proof of Proposition 2. Since $s_t = b_t + k_{t+1} = (1+n)\bar{b}_t + k_{t+1} = (1+n)d_t - (1+n)\omega_t + k_{t+1}$, savings per efficient labor at the balanced growth path reduce to $\widehat{s} = (1+n)\widehat{d} - (1+n)\widehat{\omega} + (1+n)xB\mu^{1-\zeta}\widehat{d}^{\zeta}$, where $\widehat{d} = d_t/h_t = \widehat{\omega} + \widehat{\bar{b}}$, $\widehat{\omega} = \omega_t/h_t$, $\widehat{\bar{b}} = \bar{b}/h_t$ and $B\mu^{1-\zeta}\widehat{d}^{\zeta} = 1+g$. The balanced growth version of the resource constraint is obtained by dividing all terms of (6) by h_t: $Ax^{1-\delta} = \widehat{c}_m + \frac{\widehat{c}_o}{B\mu^{1-\zeta}\widehat{d}^{\zeta}}\frac{1}{1+n} + (1+n)\widehat{d} + (1+n)xB\mu^{1-\zeta}\widehat{d}^{\zeta}$. The social planner maximizes $U(\widehat{c}_m, \widehat{c}_o, \widehat{\omega})$ subject to the latter. Note that the $\widehat{\bar{b}}$ terms cancel out in the BGP resource constraint which implies that the planner will set $\widehat{d} = \widehat{\omega}$. It follows that the planner's choice variables reduce to $\{\widehat{c}_m,\ \widehat{c}_o,\ x,\ \widehat{\omega}\}$. The optimal condition for x is $(1-\delta)Ax^{-\delta} = (1+n)\,(1+g)$. According to definition 2, the BGP is dynamically inefficient if a reduction in x induces a strictly positive change in $U(\widehat{c}_m, \widehat{c}_o, \widehat{\omega})$ of current generations as well as generations of transient periods. Using BGP resource constraint, it can be shown that $\frac{\partial \widehat{c}_m}{\partial x}\Big|_{\widehat{c}_o, \widehat{\omega}} = (1-\delta)Ax^{-\delta} - (1+n)B\mu^{1-\zeta}\widehat{\omega}^{\zeta} \equiv R - (1+n)(1+g)$. The latter and the optimal condition for x imply the necessary and sufficient condition for dynamic efficiency of the BGP that is, $R \geq (1+n)\,(1+g)$. In addition, using the equation of x at the BGP and the fact that R is equal to the marginal product of capital, $R \geq (1+n)\,(1+g)$ reduces to $\Psi \leq (1-\delta)A$, which further reduces to $\gamma \leq \frac{[1-\delta(1-\zeta)](1+\beta)-\delta\beta(1-\zeta)}{\delta(1-\zeta)} \equiv \gamma^c$. Therefore, the complete markets BGP is dynamically efficient if $\gamma \in \Omega^c \equiv \{\gamma > 0; \gamma \leq \gamma^c\}$, *where* $\Omega^c \neq \{\emptyset\}$ if $\gamma^c > 0$, which holds when $\delta < \delta^c \equiv (1+\beta)(1+2\beta)^{-1}(1-\zeta)^{-1}$. ■

Proof of Proposition 3. Using the BGP equation of the ratio of physical to human capital and the fact that g is replaced with $\overline{g}$ and that R is now a function of $\overline{x}$, the necessary and sufficient condition for dynamic efficiency of the BGP path, $R \geq (1+n)(1+g)$ (see proof of proposition 2), reduces to $\overline{\Psi} \leq (1-\delta)A$, which further reduces to $\gamma \geq [\beta\delta - (1-\delta)(1+\beta)](1-\delta)^{-1} \equiv \gamma^{in}$. Therefore, the incomplete markets BGP is dynamically efficient if $\gamma \in \Omega^{in} \equiv \{\gamma > 0; \gamma \geq \gamma^{in}\} \neq \emptyset$. ■

Proof of Proposition 4: . Given that $\partial\gamma^c/\partial\delta = -2\delta^{-3}(1+\beta)(1-\zeta)^{-1} < 0$, $\partial\gamma^{in}/\partial\delta = \beta(2-\delta)(1-\delta)^{-2} > 0$, $\lim_{\delta\to 0}\gamma^c = +\infty$, $\lim_{\delta\to 0}\gamma^{in} = -(1+\beta)$ and $\gamma^{in}|_{\delta=\delta^{in}} = \gamma^c|_{\delta=\delta^c} = 0$, where $\delta^{in} = (1+\beta)(1+2\beta)^{-1} < (1+\beta)[(1-\zeta)(1+2\beta)]^{-1} = \delta^c$, there must be, at least one, intersection point between γ^c and γ^{in}. It is then straightforward to show that the intersection point is unique and occurs at the point where $\gamma^s = \gamma^c = \gamma^{in} = \zeta(1+\beta)(1-\zeta)^{-1} > 0$ and $\delta = \delta^s$ with $0 < \delta^s \equiv (1+\beta)[1+(2-\zeta)\beta]^{-1} < 1$. It follows that $\gamma^c > \gamma^{in}$ if $\delta < \delta^s$; $\gamma^c = \gamma^{in}$ if $\delta = \delta^s$; $\gamma^c < \gamma^{in}$ if $\delta > \delta^s$. ■

Proof of Proposition 5. Along the BGP, we would like to examine the conditions under which $g = \overline{g}$, $g > \overline{g}$ and $g < \overline{g}$. The latter is equivalent to $\Upsilon_1(\xi) = \Upsilon_2(\xi)$, $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ and $\Upsilon_1(\xi) < \Upsilon_2(\xi)$, respectively, where $\xi = \gamma(1-\delta)(\beta\delta\zeta)^{-1}$, $\Upsilon_1(\xi) = [1-\delta(1-\zeta)]^{-1}(1-\zeta)(1-\delta) + \zeta[1-\delta(1-\zeta)]^{-1}\xi$ and $\Upsilon_2(\xi) = \xi^{\delta}$, using the equations for $(1+g)$ and $(1+\overline{g})$ of sections 3.1 and 3.2.1. Notice that $\Upsilon_1(\xi)$ is a linear and increasing function of ξ with $\lim_{\xi\to 0^+}\Upsilon_1(\xi) = [1-\delta(1-\zeta)]^{-1}(1-\zeta)(1-\delta)$, while $\Upsilon_2(\xi)$ is a concave and increasing function of ξ with $\lim_{\xi\to 0^+}\Upsilon_2(\xi) = 0^+$ and $\lim_{\xi\to+\infty}\Upsilon_2(\xi) = +\infty$. Since $\lim_{\xi\to 0}\Upsilon_1(\xi) > \lim_{\xi\to 0}\Upsilon_2(\xi)$, $\Upsilon_1(\xi)$

and $\Upsilon_2(\xi)$ do not intersect for any value of ξ if and only if $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for all values of ξ. The latter is the case only if $\Upsilon_1(\xi^*) > \Upsilon_2(\xi^*)$, where $\partial\Upsilon_1(\xi^*)/\partial\xi = \partial\Upsilon_2(\xi^*)/\partial\xi$ which implies $\xi^* = [\delta(1/\zeta)[1-\delta(1-\zeta)]]^{\frac{1}{1-\delta}}$. In other words, $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for all values of ξ only if $X^{1-\delta} > 1 - \left[\delta^{\frac{1}{1-\delta}} - \delta^{\frac{\delta}{1-\delta}}\right] X$, where $X = [(1/\zeta)[1-\delta(1-\zeta)]]^{\frac{1}{1-\delta}}$. Let the left hand side of the $X-$inequality be denoted by X_1 and the right hand side by X_2. Notice that it cannot be the case that $X \leq 1$ because that would imply that $\zeta \geq 1$. X_1 is an increasing and concave function of X which starts *almost* (since $X > 0$) from the origin. X_2 is a linear and decreasing function of X with $\lim_{X\to 0^+} X_2(X) = 1$. It follows that the $X-$inequality may hold only in the region on the right of the intersection point of X_1 and X_2. In this region, $X_2 < 1$ which implies that $\left[\delta^{\frac{1}{1-\delta}} - \delta^{\frac{\delta}{1-\delta}}\right] X > 0$. Given that the term in brackets is negative, the only way the latter holds is when $X < 0$ which cannot hold since $X > 0$. Thus, it cannot be the case that $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for all values of ξ. It follows that $\Upsilon_1(\xi)$ and $\Upsilon_2(\xi)$ have at least one, and at most two intersection points. At least one of the points of intersection is the point where $\xi = 1$ since $\Upsilon_1(1) = \Upsilon_2(1) = 1$. It can be shown that there are three feasible cases. In *case 1*, the slope of $\Upsilon_2(\xi)$ is greater than the slope of $\Upsilon_1(\xi)$ at $\xi = 1$, i.e. $\zeta < \delta(1+\delta)^{-1}$. Then, there exist $\overline{\xi} > 1$ such that $\Upsilon_1(\overline{\xi}) = \Upsilon_2(\overline{\xi})$ and $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for $\xi < 1$ and $\xi > \overline{\xi}$ while $\Upsilon_1(\xi) < \Upsilon_2(\xi)$ for $1 < \xi < \overline{\xi}$. Therefore, under case 1 the relationship between the growth rates can be summarized, as follows: (i) $g = \overline{g}$ *if* $\xi = 1$ *or* $\xi = \overline{\xi}$ ($\gamma = \gamma^*$ or $\gamma = \overline{\gamma}$); (ii) $g < \overline{g}$ *if* $1 < \xi < \overline{\xi}$ ($\gamma^* < \gamma < \overline{\gamma}$); (iii) $g > \overline{g}$ *if* $\xi < 1$ *or* $\xi > \overline{\xi}$ ($\gamma < \gamma^*$ or $\gamma > \overline{\gamma}$). In *case 2*, the slope of $\Upsilon_2(\xi)$ is smaller than the slope of $\Upsilon_1(\xi)$ at $\xi = 1$, i.e. $\zeta > \delta(1+\delta)^{-1}$. Then, there exist $\underline{\xi} < 1$ such that $\Upsilon_1(\underline{\xi}) = \Upsilon_2(\underline{\xi})$ and $\Upsilon_1(\xi) > \Upsilon_2(\xi)$ for $\xi < \underline{\xi}$ and $\xi > 1$ while $\Upsilon_1(\xi) < \Upsilon_2(\xi)$ for $\underline{\xi} < \xi < 1$. Therefore, under case 2, the relationship between the growth rates can be summarized, as follows: (i) $g = \overline{g}$ *if* $\xi = \underline{\xi}$ *or* $\xi = 1$ ($\gamma = \underline{\gamma}$ or $\gamma = \gamma^*$); (ii) $g < \overline{g}$ *if* $\underline{\xi} < \xi < 1$ ($\underline{\gamma} < \gamma < \gamma^*$); (iii) $g > \overline{g}$ *if* $\xi < \underline{\xi}$ *or* $\xi > 1$ ($\gamma < \underline{\gamma}$ or $\gamma > \gamma^*$). In *case 3*, the slope of $\Upsilon_2(\xi)$ is equal to the slope of $\Upsilon_1(\xi)$ at $\xi = 1$, i.e. $\zeta = \delta(1+\delta)^{-1}$. In this case, $\xi = 1$ is the single point of contact between $\Upsilon_1(\xi)$ and $\Upsilon_2(\xi)$ while in all other cases, $\Upsilon_1(\xi) > \Upsilon_2(\xi)$. Therefore, under case 3 the relationship between the growth rates can be summarized, as follows: (i) $g = \overline{g}$ *if* $\xi = 1$ ($\gamma = \gamma^*$); (ii) $g > \overline{g}$ *if* $\xi < 1$ *and if* $\xi > 1$ ($\gamma < \gamma^*$ and if $\gamma > \gamma^*$). Note that the relationships in brackets are due to the fact that for $\overline{\xi}$ and $\underline{\xi}$, there are unique thresholds $\overline{\gamma}$ and $\underline{\gamma}$ such that $\beta\delta\zeta\overline{\xi} = \overline{\gamma}(1-\delta)$ and $\beta\delta\zeta\underline{\xi} = \underline{\gamma}(1-\delta)$. Since $\overline{\xi} > 1$ and $\underline{\xi} < 1$, the latter implies that $\overline{\gamma} > 1$, $\underline{\gamma} < 1$ and $0 < \underline{\gamma} < \gamma^* < \overline{\gamma}$. ■

Proof of Proposition 6 . Complementary slackness conditions imply that the Lagrange multiplier on the borrowing constraint, $\overline{\mu}_t > 0$, and $\overline{b}_{t-1} = \lambda w_t h_t / R_t$ when the borrowing constraint binds. Along the BGP, the optimality condition $w_t h_1\left(d_{t-1}, h^y_{t-1}\right) - R_t = \overline{\mu}_t$ reduces to $\zeta\left(1+\beta+\gamma\right)(1+\frac{\delta\lambda}{(1-\delta)})\overline{\overline{\Psi}} > (1-\delta)A(1-\lambda)(\gamma+(\beta+\gamma)\frac{\delta\lambda}{1-\delta})$, which then collapses to $\gamma < \gamma^{bin} \equiv \gamma^*\left[\frac{(1-\delta)(\zeta-\lambda)}{[1-\delta(1-\lambda)]\zeta}\right] = \frac{\beta\delta(\zeta-\lambda)}{1-\delta(1-\lambda)}$. It follows that if $\lambda < \zeta$ then $\Omega^{bin} \equiv \{\gamma > 0; \gamma < \gamma^{bin}\} \neq \varnothing$ and the borrowing constraint binds only if $\gamma \in \Omega^{bin}$. If $\lambda \geq \zeta$, then $\gamma^{bin} \leq 0$ and thus $\Omega^{bin} = \varnothing$ since it violates the assumption that $\gamma > 0$, and the borrowing constraint does not bind. ■

Proof of Proposition 7 . When $\lambda < \zeta$ and $\gamma \in \Omega^{bin}$, necessary and sufficient condition for dynamic efficiency of the BGP path as shown in the proof of proposition 2, reduces to $\overline{\overline{\Psi}} \leq (1-\delta)A$, which further reduces to $\gamma \geq [\beta\delta(1-\lambda) - (1-\delta(1-\lambda))(1+\beta)][1-\delta(1-\lambda)]^{-1}$ $\equiv \gamma^{\overline{in}}$. Therefore, the BGP with binding borrowing limit is dynamically efficient if $\gamma \in$

$\Omega^{\overline{in}} \equiv \{\gamma > 0;\ \gamma^{\overline{in}} \leq \gamma < \gamma^{bin}\}$, where $\Omega^{\overline{in}} \neq \varnothing$ *if* $\zeta > 1 - (\beta\delta)^{-1}(1 - \delta(1-\lambda))(1+\beta)$. The former condition ensures that $\gamma^{bin} > \gamma^{\overline{in}}$. If the borrowing constraint is slack, the complete markets BGP and associated condition for dynamic efficiency apply (Proposition 2). ■

Proof of Proposition 8 . Along the BGP, when $\lambda < \zeta$ and $\gamma \in \Omega^{bin}$, we would like to examine the conditions under which $g = \overline{\overline{g}}$, $g > \overline{\overline{g}}$ and $g < \overline{\overline{g}}$. The latter is equivalent to $\overline{\Upsilon}_1\left(\overline{\xi}\right) = \overline{\Upsilon}_2(\overline{\xi})$, $\overline{\Upsilon}_1\left(\overline{\xi}\right) > \overline{\Upsilon}_2(\overline{\xi})$ and $\overline{\Upsilon}_1\left(\overline{\xi}\right) < \overline{\Upsilon}_2(\overline{\xi})$, respectively, where $\overline{\xi} = [\gamma\,[1-\delta(1-\lambda)] + \beta\delta\lambda]/\beta\delta\zeta$, $\overline{\Upsilon}_1\left(\overline{\xi}\right) = \frac{(1-\zeta)[1-\delta(1-\lambda)]-\lambda}{(1-\lambda)[1-\delta(1-\zeta)]} + \left(\frac{\zeta}{(1-\lambda)[1-\delta(1-\zeta)]}\right)\overline{\xi}$ and $\overline{\Upsilon}_2(\overline{\xi}) = \overline{\xi}^{\delta}$, using the equations for $(1+g)$ and $(1+\overline{\overline{g}})$ of sections 3.1 and 3.2.2. Notice that $\overline{\Upsilon}_1(\overline{\xi})$ is a linear and increasing function of $\overline{\xi}$, while $\overline{\Upsilon}_2(\overline{\xi})$ is a concave and increasing function of $\overline{\xi}$, with $\lim_{\overline{\xi}\to 0^+} \overline{\Upsilon}_2(\overline{\xi}) = 0^+$ and $\lim_{\overline{\xi}\to +\infty} \Upsilon_2(\overline{\xi}) = +\infty$. Since $\overline{\xi}$ is a function of γ, $\overline{\Upsilon}_1$ and $\overline{\Upsilon}_2$ can be written as $\overline{\Upsilon}_1(\gamma)$ and $\overline{\Upsilon}_2(\gamma)$. The properties of $\overline{\Upsilon}_1\left(\overline{\xi}\right)$ and $\overline{\Upsilon}_2\left(\overline{\xi}\right)$ imply that there might be either zero or, at most, two intersection points between $\overline{\Upsilon}_1\left(\overline{\xi}\right)$ and $\overline{\Upsilon}_2\left(\overline{\xi}\right)$. A sufficient condition for no intersection points between $\overline{\Upsilon}_1\left(\overline{\xi}\right)$ and $\overline{\Upsilon}_2\left(\overline{\xi}\right)$, i.e. $\overline{\Upsilon}_1\left(\overline{\xi}\right) > \overline{\Upsilon}_2\left(\overline{\xi}\right)$ for all values of $\overline{\xi}$, is that $\overline{\Upsilon}_1\left(\overline{\xi}^*\right) > \overline{\Upsilon}_2\left(\overline{\xi}^*\right)$ for $\overline{\xi}^*$ such that $\partial\overline{\Upsilon}_1\left(\overline{\xi}^*\right)/\partial\overline{\xi} = \partial\overline{\Upsilon}_2\left(\overline{\xi}^*\right)/\partial\overline{\xi}$. The latter reduces to $\overline{\xi}^* = (\delta(1-\lambda)[1-\delta(1-\zeta)]/\zeta)^{\frac{1}{1-\delta}}$. Thus, $\overline{\Upsilon}_1\left(\overline{\xi}^*\right) > \overline{\Upsilon}_2\left(\overline{\xi}^*\right)$ imply that $\overline{X}^{\frac{\delta}{1-\delta}} < \frac{(1-\zeta)[1-\delta(1-\lambda)]-\lambda}{(1-\delta)(1-\lambda)[1-\delta(1-\zeta)]}$, where $\overline{X} = \delta(1-\lambda)[1-\delta(1-\zeta)]/\zeta$. Since, $\overline{X} > 0$, it must be the case that $\lambda < (1-\zeta)[1-\delta(1-\lambda)]$ which also implies that $\zeta < 1-\delta(1-\zeta)$ since $\lambda > 0$. Then, it follows that $\lambda < 1-\delta(1-\zeta)$ and thus, $\lambda < (1-\zeta)[1-\delta(1-\lambda)] < (\lambda-\zeta)/\lambda$. Since $\lambda > 0$, the latter can hold only if $\lambda > \zeta$, which cannot be the case since $\lambda < \zeta$. Therefore, it cannot be the case that $\overline{\Upsilon}_1\left(\overline{\xi}\right)$ and $\overline{\Upsilon}_2\left(\overline{\xi}\right)$ have no intersection points. In what follows, we focus on the cases where there is either one or two intersection points. At least one of the points of intersection is the point where $\overline{\xi} = 1$ since $\overline{\Upsilon}_1(1) = \overline{\Upsilon}_2(1) = 1$. Following the proof of proposition 5, it can be shown that there are four feasible cases. In *case 1*, there is a single intersection point only when the intercept of $\overline{\Upsilon}_1\left(\overline{\xi}\right)$ is negative, i.e. $\lambda \geq (1-\delta)(1-\zeta)[1-\delta(1-\zeta)]^{-1} \equiv \widetilde{\lambda}$. Note that the unique intersection point must be 1. Since $\overline{\xi}=1$ implies that $\gamma=\gamma^{bin}$, it follows that (i) $g=\overline{\overline{g}}$ if $\gamma \geq \gamma^{bin}$ and (ii) $g<\overline{\overline{g}}$ *if* $\gamma < \gamma^{bin}$. For *cases 2-4*, the intercept $\overline{\Upsilon}_1\left(\overline{\xi}\right)$ is strictly positive, i.e. $\lambda < \widetilde{\lambda}$. For *case 2*, recall that when $\overline{\xi}=1$, $\gamma=\gamma^{bin}$. Thus, when the slope of $\overline{\Upsilon}_2\left(\overline{\xi}\right)$ is greater than the slope of $\overline{\Upsilon}_1\left(\overline{\xi}\right)$ at $\overline{\xi}=1$, i.e. $\zeta < \delta(1-\delta)(1-\lambda)[1-\delta^2(1-\lambda)]^{-1} \equiv \widetilde{\zeta}^{*}$, then $\overline{\Upsilon}_1\left(\overline{\xi}\right) \geq \overline{\Upsilon}_2\left(\overline{\xi}\right)$ for any $\overline{\xi} \leq 1$ or equivalently, (i) $g=\overline{\overline{g}}$ *if* $\gamma \geq \gamma^{bin}$ and (ii) $g>\overline{\overline{g}}$ *if* $\gamma < \gamma^{bin}$, since the borrowing constraint will not bind if $\gamma \geq \gamma^{bin}$ and the economy will behave as in the case of complete markets. In *case 3*, the slope of $\overline{\Upsilon}_2\left(\overline{\xi}\right)$ is smaller than the slope of $\overline{\Upsilon}_1\left(\overline{\xi}\right)$ at $\overline{\xi}=1$, i.e. $\zeta > \delta(1-\delta)(1-\lambda)[1-\delta^2(1-\lambda)]^{-1}$. Then, there exist $0<\underline{\xi}^{*}<1$ such that $\overline{\Upsilon}_1\left(\underline{\xi}^{*}\right)=\overline{\Upsilon}_2\left(\underline{\xi}^{*}\right)$, $\overline{\Upsilon}_1\left(\overline{\xi}\right) > \overline{\Upsilon}_2\left(\overline{\xi}\right)$ for $\overline{\xi}<\underline{\xi}^{*}$, and $\overline{\Upsilon}_1\left(\overline{\xi}\right) < \overline{\Upsilon}_2\left(\overline{\xi}\right)$ for $\underline{\xi}^{*}<\overline{\xi}<1$. For any $\overline{\xi}>1$, the borrowing constraint will not bind and thus it will behave as in the case of complete markets. It follows that there is $\underline{\gamma}_2$, as long as $\lambda < (1-\zeta)[1-\delta(1-\lambda)] = \lambda_{\gamma}$, such that $\underline{\gamma}_2 < \gamma^{bin}$ Therefore, the relationship between g and $\overline{\overline{g}}$ is summarized, as follows: (i) $g=\overline{\overline{g}}$ if $\gamma = \underline{\gamma}_2$ or $\gamma \geq \gamma^{bin}$, (ii) $g<\overline{\overline{g}}$ *if* $\underline{\gamma}_2 < \gamma < \gamma^{bin}$ and (iii) $g>\overline{\overline{g}}$ *if* $\gamma < \underline{\gamma}_2$. Finally, in *case 4*, the slope of $\overline{\Upsilon}_2\left(\overline{\xi}\right)$ is equal to the slope of $\overline{\Upsilon}_1\left(\overline{\xi}\right)$ at $\overline{\xi}=1$, i.e. $\zeta = \delta(1-\delta)(1-\lambda)[1-\delta^2(1-\lambda)]^{-1}$. In this case, $\overline{\xi}=1$ is the single point of contact between $\overline{\Upsilon}_1\left(\overline{\xi}\right)$ and $\overline{\Upsilon}_2\left(\overline{\xi}\right)$ while in all other cases, $\overline{\Upsilon}_1\left(\overline{\xi}\right) > \overline{\Upsilon}_2\left(\overline{\xi}\right)$. Therefore, the relationship between g and $\overline{\overline{g}}$ is summarized, as follows: (i) $g=\overline{\overline{g}}$ *if* $\gamma \geq \gamma^{bin}$ and (ii) $g>\overline{\overline{g}}$ *if* $\gamma < \gamma^{bin}$. ■