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Fixed vs. Flexible Pricing in a Competitive Market

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Abstract: We study the selection and dynamics of two popular pricing policies—fixed price and flexible price—in markets for big ticket items. Our paper extends previous work in marketing, e.g. Desai and Purohit (2004) with a fully competitive and dynamic framework by also taking into account non-economic (behavioral) aspects of bargaining. We construct and analyze a competitive search (directed search) model which allows us to endogenize the expected demand depending on pricing rules and posted prices. Interestingly, when customers get additional satisfaction for negotiating a lower price, we find that a unique equilibrium emerges with full segmentation: Haggler customers avoid fixed-price firms and exclusively shop at flexible firms whereas non-haggler customers do the opposite. We also find that the equilibrium fixed price and the flexible list price both increase in customer satisfaction, implying that sellers take advantage of the positive utility enjoyed by hagglers in the form of higher prices. Remarkably, we also observe a spillover effect in that fixed price sellers, who do not even cater to hagglers, also raise their prices if customer satisfaction goes up. Finally, considering the presence of seasonal cycles with most big ticket items, we analyze a scenario where market demand goes through periodic ups and downs and find that equilibrium prices remain mostly stable despite significant fluctuations in demand. This finding suggests a plausible competition-based explanation for the stability of prices.
Keywords: pricing policy, big ticket items, negotiation, competition, competitive search

1 Introduction

In markets for big ticket items (e.g., houses, apartments, used cars, boats, home furniture, jewelry) fixed and flexible pricing policies often co-exist. That is, while some sellers clearly indicate that they are flexible and open to bargaining (e.g., a homeowner putting "OBO" (or best offer) next to the asking price), other sellers point out fixed-prices by using words such as "sharp price" or "no-negotiations". Some popular used car supermarkets in the UK, such as Cargiant, offer only fixed prices and leave no room for negotiation, whereas it is still a common practice to negotiate in most other used-car dealerships. Similarly, many retailers selling big ticket items who are well-known for fixed-price selling are reported to allow haggling in recent years (Richtel, 2008; Agins and Collins, 2001). Some retailers even go one step further and train their employees in the art of bargaining with customers (Stout, 2013). In addition, consumers vary in their bargaining ability and practice in purchasing such items. According to Consumer Report's recent national survey of American adults on their haggling habits, while a notable portion of individuals report negotiating when they purchased appliances (39%), jewelry (32%), furniture (43%), and collectibles and antiques (48%), with 89% of those who haggled obtained discounts at least once, others simply have not tried bargaining at all (Marks, 2013).

Although the practice of both fixed price and flexible price policies are widespread in markets for big ticket items, and such items are typically the biggest purchases in most consumers' lives, there are surprisingly few studies in marketing literature (see for example Desai and Purohit (2004) for an exception) that investigate strategic drivers and implications of these seemingly contrasting pricing policies. In addition, while competition plays a significant role in most sellers' pricing strategies[1], existing literature primarily focuses on no competition or limited competition settings using Hotelling or Cournot frameworks (Riley and Zeckhauser, 1983; Wang, 1995; Desai and Purohit, 2004; Kuo et al., 2011). Finally, the process of bargaining involves significant psychological dynamics for customers; in particular, haggling may be discomforting and costly due to additional effort or opportunity cost of time, or due to concerns of being perceived as cheap or unclassy (Evans and Beltramini, 1987; Pruitt, 2013). In contrast, the very same process may provide additional non-economic pleasure and satisfaction due to smartshopper feelings (Schindler, 1989) or transaction utility (Thaler, 1985). That is, customers can actually enjoy the bargaining process and get a small amount of additional satisfaction from obtaining a deal. Positive feelings for bargaining "transcends the satisfaction of mere economic gain" with "feelings of competence and mastery" (Sherry, 1990). Indeed, such non-economic pleasure from bargaining has been widely reported and highlighted in the consumer behavior literature (Babin et al., 1994; Pruitt, 2013; Schindler, 1998; Sherry, 1990), but to our knowledge, this subtle element has not been thoroughly analyzed in the existing theoretical marketing research, especially within a fully competitive and dynamic framework.

Our goal in this paper is to understand the dynamics and consequences of fixed and flexible pricing policies in markets for big ticket items, which typically exhibit the following characteristics. First, the majority of these markets are decentralized and operate via search and matching: it takes time and effort to locate an item or to attract a buyer, and depending on the market, a player may have to wait days or even weeks before he buys or sells. Second, sellers typically have limited inventories (which is the case in markets for houses, apartments, used cars or boats, where sellers usually possess a single item, and to a certain extent it is the case in markets for home furniture, jewelry, or antiques); consequently, there may be significant trade frictions in that a product available today may not be available tomorrow. Third, a large number of firms compete with each other in order to attract customers and in doing so they use a range of pricing tools and tactics in an effort to appeal to customers. Fourth, customers are heterogeneous in their ability and willingness to negotiate, so firms' pricing policies may have significant consequences in terms of the kind of customers they

[1] Bitran and Caldentey (2003) point out the importance of market competition in pricing policies and quote an executive suggesting that "price competition among retailers is today the main driver in selection of a particular pricing policy". Similarly, Elmaghraby and Keskinocak (2003) note that "competitors' pricing decisions need to be considered while developing a dynamic pricing policy, an important element missing from the current literature."

attract or dispel. What is more the bargaining process comes with the behavioral features discussed above.

Our modeling approach takes into account all of the aforementioned characteristics, and it is based on a competitive search (directed search) framework. Extending existing work in marketing on fixed and flexible pricing policies (i.e., Desai and Purohit, 2004), our directed search approach allows us to explicitly incorporate full market competition in a decentralized and dynamic setup, which we believe is the first study doing so in the marketing literature.

The model provides several important insights. First, our analysis of the benchmark case, where customers are neutral to the bargaining process, reveals that fixed and flexible pricing policies co-exist in the same marketplace, and each policy comes with its own list price and customer demographics. Flexible firms foresee an eventual surplus loss during negotiations, and they strategically inflate the list price. Such inflated prices however, put off non-haggler customers as they cannot negotiate better deals. As such, flexible firm can attract only haggler customers. Fixed price firms, on the other hand, announce lower prices and attract both types of customers.

Second, we find that when customers get additional pleasure (proxied by the parameter $\varepsilon > 0$) from successfully haggling down the list price, then a unique equilibrium emerges with full segmentation of customers. That is, haggler customers avoid fixed-price firms and exclusively shop at flexible firms whereas non-haggler customers do the opposite. More interestingly, the equilibrium fixed price as well as the equilibrium flexible price both increase with customers' additional satisfaction from bargaining. Surprisingly, even fixed price sellers, who do not cater to haggler customers, benefit from the additional pleasure from bargaining and raise their list price. This is because higher enjoyment of bargaining (larger ε) results in more sellers to adopt flexible pricing, which in turn leads to a fewer number of fixed sellers. Being the only outlets where non-hagglers can shop, fixed-price sellers can then post higher list prices as demand at each one of them increases. Overall, our analytical results indicate that sellers' profit rises with ε because sellers are able to convert customers' enjoyment of the bargaining process (e.g., smartshopper feelings) into higher prices, and therefore higher profits. This suggests that if firms are able to provide an enjoyable experience of bargaining for their customers, they may then reap the benefits in the form of marginally higher prices and profits.

Finally, taking into account the fact that most big ticket item purchases follow seasonal trends, we consider the price dynamics in a long selling period where demand goes through seasonal ups and downs. An interesting finding is that prices do not fluctuate as much as the demand. This observation provides an interesting insight. In explaining the stability of prices, a significant portion of existing literature in marketing highlights price fairness concerns (Xia et al., 2004; Bolton et al., 2003; Anderson and Simester, 2008) which has its origins in the principle of dual entitlement put forward by Kahneman et al. (1986). Our model, on the other hand, provides an additional explanation for price stability that is based on competition in a decentralized and dynamic market.

2 Model

2.1 Description of the Model

Consider a dynamic market that runs for $t = 1, 2, ..., T$ periods and is populated by a continuum of heterogeneous buyers and homogeneous sellers. Each sellers has one unit of a product that he is willing to sell above his reservation price, zero, and each buyer wants to purchase one unit and is willing to pay up to his reservation price, one. Buyers are divided into two according to their bargaining abilities. Low types (non-hagglers) are not skilled in bargaining and never attempt to negotiate the list price. High types (hagglers) on the other hand are skilled in bargaining and negotiate the price whenever it is worthwhile to do so. (In Appendix 2 we extend the model by considering N types.)

The market is decentralized and operates via competitive search. At the beginning of each period sellers simultaneously and independently announce a list price $r_{m,t} \in [0, 1]$ and a declaration $m \in \{\text{firm } (f), \text{ best offer } (b)\}$ indicating whether they are firm with the price or whether they are flexible to accept a counter offer. If the seller is firm then the transaction takes place at the list price. If he is flexible then the transaction may involve bargaining, but if the buyer does not wish to bargain or if two or more buyers are present at the firm then no bargaining takes place and the item is sold at the initially posted price (more on this below).

Buyers observe sellers' announcements and then choose to visit a seller. It is possible that multiple customers show up at the same location, so we let $n = 0, 1, 2, \ldots$ denote the realized demand. If $n \geq 2$ then each buyer has an equal chance $1/n$ of being selected. If transaction occurs at price p_t then the seller obtains $\beta^{t-1} p_t$ and the buyer obtains $\beta^{t-1}(1 - p_t)$, where β is the common discount factor.

The decentralized nature of the market, coupled with sellers' limited inventories, creates trade frictions in that no one is guaranteed of an immediate trade and players may have to try for several periods before they can actually buy or sell. Indeed, if multiple customers show up at the same firm, then some of them walk out empty-handed because of the limited inventory. Similarly, due to the decentralized matching process a seller may well end up with no customer at all. In either case players need to wait for the subsequent period to try again. Waiting, of course, is costly as future payoffs are discounted at rate β.[2]

The market starts with a measure of s_1 sellers and b_1 buyers, of which a fraction $\eta_1 \in (0, 1)$ are low types. At the end of each period, players who transact exit the market while the remaining ones move to the next period to play the same game. Outgoing players may be fully or partially replaced. Specifically, we assume that at the beginning of each period $t = 2, 3...$ a new cohort of b_t^{new} buyers and s_t^{new} sellers enter the market joining the existing players. The measures of buyers and sellers present in the market at time t, denoted by b_t and s_t, depend on the entering cohorts as well as the existing players who are yet to trade. (In Section 6 we discuss how b_t and s_t evolve over time.)

Finally, our model captures the psychological utility (or disutility) associated with bargaining.

[2] In Appendix 2 we explore a variation where unmatched buyers and sellers get to meet for a second time during the same trading period and show that the results remain robust, upto a modification in outside options.

Successfully haggling and purchasing an item below the posted price can be viewed as a psychologically rewarding experience. Or in contrast, it can be viewed as a negative, stressful and trying endeavor. In order to take into account these behavioral considerations, we introduce a parameter, ε, which can be positive, negative or zero depending on how customers perceive the bargaining experience and we explore how this parameter affects the selection of equilibrium pricing rules. Notice that the parameter ε is relevant for haggler customers only. It is inapplicable for non-hagglers, as they always pay the list price due to their lack of bargaining skills.

2.2 Discussion of the Modeling Approach

Markets for big ticket items exhibit the stylized characteristics introduced above. First, these markets are competitive in that a large number of sellers compete with each other for scarce customer and in doing so, they offer a myriad of choices to stand out in the competition e.g. discounts off the posted price, long term payment options including in-house financing, shared ownership options, and so on. These terms, inevitably, determine how much demand each firm gets and what type of customers it attracts. In this paper we focus on, arguably, the most important competition tool: prices, or to be more general, pricing rules.[3]

Second, the aforementioned markets are decentralized and operate via search and matching. It takes time and effort to locate an item or to attract a buyer, and, depending on the market, it takes days or even weeks before one actually gets to buy or sell. Finally, sellers in these markets have limited inventories (typically one in the housing, used car or boats markets); so, if multiple customers turn up at the same time, then availability becomes an issue and some buyers inevitably walk out and search again. These observations suggest that the decentralized competitive search approach that we take in this paper is indeed well-suited to model the aforementioned markets.

Our modelling approach sets us apart from the rest of the literature in several ways. First, while a number of papers in marketing and pricing literatures consider strategic implications of different pricing policies, most of these papers do not consider competition at all. Instead, they focus on a monopolist seller who receives customers exogenously (Riley and Zeckhauser, 1983; Wang, 1995; Kuo et al., 2011, 2013).[4] Second, the small number of papers allowing for competition, portray a rather limited outlook as they consider only two sellers in frictionless Hotelling or Cournot frameworks.[5]

[3]Eliciting views from all major automobile suppliers in the UK on how they set their list prices, the UK Competition Commission (2000) reports (Chapter 4 - Prices): "For all suppliers, the prices and specifications of their competitors' models emerged as the most important factors in setting their list prices (recommended retail prices—RRPs). In general, suppliers appear to set their list prices at about the same level as their competitors, for equivalent cars, and seek to compete through other means such as marketing, specifications and offers."

[4]Riley and Zeckhauser (1983) examine a monopolist seller facing risk neutral customers, and suggest that fixed pricing is optimal in comparison to negotiations. This is because while haggling may be advantageous in terms of price discrimination, the gains from haggling are more than offset when buyers refuse purchasing at higher prices. Wang (1995) create a dynamic model with a monopolist seller, and conclude that bargaining is preferable if it costs the same as fixed pricing, or if the common costs are high enough. Focusing on operations-related questions in a monopoly setting, Kuo et al. (2011) characterize the optimal posted price and the resulting negotiation outcome as a function of inventory and time, and Kuo et al. (2013) focus on pricing policy in a supply chain.

[5]For instance, Wernerfelt (1994) finds that in a duopoly bargaining may be profit maximizing for sellers as it helps them avoid the costly Bertrand competition. Using a Hotelling framework, Gill and Thanassoulis (2016) study strategically chosen stochastic discounts in markets with prior list-price-setting competition.

In a closer work to ours, Desai and Purohit (2004) consider a duopoly setting and use a Hotelling framework to examine the implications of haggling and fixed price policy decisions by two retailers. They show that depending on the parameters, there may exist equilibria in which both firms choose fixed prices, both firms offer haggling, or where one firm offers haggling and the other charges fixed prices. An important finding of theirs is that the benefits of price discrimination in a monopoly setting do not necessarily transfer over to a competitive environment. A limitation of Desai and Purohit (2004) is the limited interaction between posted prices and discount prices in the case where both firms follow haggling policy as haggling customers are not affected by the posted prices since these are never effective outside options. While we, too, consider heterogeneous customers (i.e., hagglers and non-hagglers) and limited inventory firms, our work differs by characterizing a fully competitive and dynamic market, and by endogenizing customer arrivals to firms depending on the pricing policy chosen by firms and their list prices.

Spurred by Burdett et al. (2001), the directed search approach is a recent development in Search Theory and is generally viewed as a natural progression from the classic Mortensen-Pissarides style random search approach. The majority of papers in the directed search literature assume that sellers use a specific selling format, typically fixed pricing (Shimer, 2005; Menzio and Shi, 2010; Galenianos and Kircher, 2012). Only a small number of papers in the literature study pricing mechanism selection (Eeckhout and Kircher, 2010; Virag, 2011), which is what we focus here. We differ from these papers in three dimensions. First they do not consider customer heterogeneity, i.e. the haggler vs. non-haggler distinction. This is important because the pricing mechanism selection coupled with customer heterogeneity leads to the segmentation of customers across firms. Second, the aforementioned papers are based on static one-shot setups where players get to search only once, whereas our setup is dynamic; players who cannot buy or sell are able to try again in subsequent periods. Finally, to the best of our knowledge, the behavioral aspects of bargaining (whether customers enjoy the haggling process, or dislike it or are neutral to it) and how such considerations may affect prices and the selection of pricing rules is not studied at all in the directed search literature.

A unique feature of our model is the incorporation of the behavioral element ε, which refers to buyers' additional pleasure or displeasure from bargaining. A negative value for ε indicates disutility or costs associated with haggling which has been highlighted in a small number of studies as buyers' bargaining disutility (Morton et al., 2011) or haggling costs (Desai and Purohit, 2004). A negative ε suggests that customers may have the ability to bargain down the list price, but nevertheless, they may dislike the bargaining process, say, for the fear of being seen as cheap or unclassy, or due to opportunity cost of time. A positive value for ε is similar to the concept of smartshopper feelings mentioned in the consumer behavior literature (Schindler, 1989). Such positive feelings are increased with consumers perception of being responsible for obtaining the discount (Schindler, 1998). That is, obtaining a lower price through bargaining may give a sense of accomplishment or a thrill of feeling victorious, and may lead consumer to feel proud, smart, or competent (Holbrook et al., 1984; Jones et al., 1997). This is also related to (Thaler, 1985)'s notion of transaction utility, which suggests that consumers derive utility not only from the financial aspects of the exchange (i.e., acquisition utility), but also from the psychological aspects of the transaction (i.e., transaction utility). By incorporating this psychological element into our model, which to our knowledge is the first study doing so in the analytical marketing literature, we offer a rich and realistic account for the emergence of fixed and flexible pricing policies and their implications.

2.3 Bargaining and the Sale Price

We move backwards in the analysis, starting with the determination of the bargained price in a meeting. We, then, turn to buyers' search decisions and explain how the expected demand at each firm is pinned down. Finally, we turn to the sellers' problem and explore how they select prices and pricing rules.

The list price at a flexible firm may be negotiated if the firm has a single customer; if two or more customers are present then no bargaining takes place and the item is sold at the posted price.[6] Let θ denote the bargaining power of high type buyers relative to the seller. The bargaining power of low types is normalized to zero. Similarly, let $u_{h,t+1}$ and π_{t+1} denote, respectively, a high type buyer's and a seller's expected payoff ("value of search") in period $t+1$. These payoffs serve as outside options during negotiations, i.e. in case of disagreement the buyer walks out with payoff $\beta u_{h,t+1}$ and the seller with $\beta\pi_{t+1}$. The negotiated price, y_t, can be found as the solution to the following maximization problem:[7]

$$\max_{y_t\in[0,1]} \left(1-y_t+\varepsilon-\beta u_{h,t+1}\right)^{\theta}\left(y_t-\beta\pi_{t+1}\right)^{1-\theta}$$

The solution yields

$$y_t = 1-\beta u_{h,t+1}-\theta\left(1-\beta u_{h,t+1}-\beta\pi_{t+1}\right)+\varepsilon\left(1-\theta\right). \tag{1}$$

The bargained price y_t falls with θ, i.e. the higher the buyer's negotiation skills, the lower the price.[8] In addition y_t rises with π_{t+1} and falls with $u_{h,t+1}$ i.e. the stronger the seller's outside option the higher the price and the stronger the buyer's outside option the lower the price. As it turns out, outside options depend on λ_{t+1}, λ_{t+2}, .., λ_T, i.e. how competitive the market is expected to be in the next period, in the period after, and so on. Even though bargaining is bilateral and takes place between two players in private, it is still driven by market competition, which filters into the negotiation process via outside options.

Whether or not buyers attempt to renegotiate depends on how y_t compares with the list price $r_{b,t}$ as well as the parameter ε. The case $\varepsilon=0$ is straightforward: buyers opt for bargaining if they can

[6] The assumption that haggling is possible only if there is a single customer in store ($n=1$) is without loss in generality. One can recast the model where haggling may be possible for any $n\geq 1$. This modification, inevitably, causes the analysis to become significantly more cumbersome, yet it does not add any additional insight. Indeed, most of our results do not depend on the closed form expression of the bargained price y_t or whether bargaining takes place with one buyer or multiple buyers. Instead, they depend on the fact that hagglers are able to obtain a higher expected utility at flexible stores than non hagglers, which would hold true irrespective of n. Furthermore, it does not make intuitive sense where the item is sold below the posted price at a time when two or more customers want to buy it.

[7] See Muthoo (2000) Chapter 2 or Osborne and Rubinstein (1990) Chapter 2 for more on Nash bargaining.

[8] Note that $u_{h,t+1}+\pi_{t+1}\leq 1$ since the total payoff in a transaction cannot exceed the maximum surplus, one. Therefore the expression $1-\beta\left(u_{h,t+1}+\pi_{t+1}\right)$ is positive; hence y_t falls in θ

negotiate a better deal than the list price, i.e. if $y_t \leq r_{b,t}$. If, however, $\varepsilon < 0$ then buyers attempt to renegotiate only if the deal they end up getting warrants incurring the negative ε, i.e. if $y_t \leq r_{b,t} + \varepsilon$. We assume that buyers' bargaining power is sufficiently large to ensure that, even after accounting for the negative ε, they would still prefer bargaining over purchasing at the posted price. (The other scenario where they would not even attempt negotiating trivially yields a fixed price equilibrium.) Finally, if $\varepsilon > 0$ then buyers opt for bargaining if $y_t \leq r_{b,t}$. The parameter ε is absent from this condition because the joy of getting a deal, proxied by the positive ε, kicks in only if the item is purchased below the list price.

These conditions require the bargaining power θ to be sufficiently high, which we assume to be the case for now. (Subsequently we will provide the necessary thresholds.) The opposite case where even high types are unable to negotiate a better deal is trivial as the availability of bargaining becomes immaterial and the model collapses to a de-facto fixed price setting.

2.4 Buyer's Problem

Demand Distribution. In the tradition of the competitive search literature we focus on visiting strategies that are symmetric and anonymous (Burdett et al., 2001; Shimer, 2005; Eeckhout and Kircher, 2010). Symmetry requires buyers of the same type use the same visiting strategies. Anonymity, on the other hand, means that visiting strategies ought to depend on what sellers post but not on sellers' identities i.e. sellers posting the same list price $r_{m,t}$ and trading with the same pricing rule m ought to be visited with the same probability.[9]

Given symmetry and anonymity, the number of applications at a firm follows a Poisson distribution. To see why, and to get some intuition on how the matching process works, consider a finite setting with B buyers and S sellers, where the buyer seller ratio equals to $\lambda = B/S$. For a moment ignore the haggler-price taker distinction and suppose that all buyers are price takers. Also suppose that all sellers use fixed pricing and post the same list price, say, $r = 0.5$. Since all sellers compete with the same rule and post the same price, symmetry and anonymity in buyers' visiting strategies imply that the probability that a buyer visits a particular seller is $1/S$. Consequently, the probability that the seller gets n customers equals to

$$\Pr[n] = \binom{B}{n} (1/S)^n (1 - 1/S)^{B-n},$$

i.e. the seller receives customers according to a binomial distribution with parameters B and $1/S$. The expected number of customers, therefore, equals to $B \times 1/S = \lambda$. Now fix λ and let B and S tend to infinity (recall that we have a continuum of buyers and sellers). As the market gets large,

[9] Imposing symmetry and anonymity on visiting strategies is, of course, a restriction; however these assumptions facilitate the characterization of the equilibrium and lead to outcomes which are analytically tractable. As such, with few exceptions the vast majority of the directed search literature restricts attention to such strategies. A notable exception is an extension in Burdett et al. (2001) where they consider a simple 2 by 2 setup with only two buyers and two sellers and construct equilibria supported by non symmetric strategies; however such equilibria require coordination among buyers on who goes where. In a small market with few buyers such coordination may be possible, but in a large market with multiple buyers and sellers such coordination is not feasible. The symmetric equilibrium, on the other hand, requires no coordination.

the binomial distribution converges to the Poisson distribution with arrival rate λ, that is

$$\Pr[n] = \frac{e^{-\lambda}\lambda^n}{n!}.$$

Along this example every firm competes with fixed pricing and posts the same list price, and therefore, the expected demand at each firm equals to λ.[10] If sellers were to post different prices, or pick different pricing rules, then, again because of symmetry and anonymity, the demand distribution at each firm would be still Poisson, but each with a different arrival rate that depends on what the seller posts and how it compares with the rest of the market (Galenianos and Kircher, 2012). For instance, if a seller posts a lower price, say 0.4, while everyone else still posts 0.5, then his expected demand q would be higher than λ (more on this below).

In the full-fledged model the expected demand q depends not only on the list price r, but also on the pricing rule m, the date t and buyers' type i. Specifically, the probability that a firm with the terms $(r_{m,t}, m)$ meets $n = 0, 1, 2...$ customers of type $i = h, l$ is given by

$$\Pr[n] = \frac{e^{-q_{i,m,t}(r_{m,t})} q_{i,m,t}^n (r_{m,t})}{n!} \equiv z_n (q_{i,m,t}). \tag{2}$$

The fact that q is indexed by i indicates that, when thinking about the total demand at a firm, one has to consider arrivals from high types $q_{h,m,t}$ as well as low types $q_{l,m,t}$.

Firms post their prices and pricing rules, and buyers direct their search depending on how attractive these terms are. All else equal, cheaper firms attract more customers and expensive firms attract fewer customers; however price is not the only concern for a buyer when deciding where to shop. Each seller has a limited inventory, so buyers must also take into account the likelihood of not being able to purchase today and having to try again in the next period. In that respect it is easier to purchase at expensive firms as they tend to be less crowded, so customers do not necessarily head straight to the cheapest firm. The expected demands adjust to ensure that buyers are indifferent across all firms posting different prices or pricing rules.[11] In what follows we make these arguments precise.

Expected Utilities. Let $U_{i,m,t}$ denote a type $i = h, l$ buyer's expected utility at a firm trading with rule $m \in \{f, b\}$. Consider a fixed price firm with price $r_{f,t}$. We have

$$U_{i,f,t} = \sum_{n=0}^{\infty} \frac{z_n\left(q_{h,f,t}+q_{l,f,t}\right)}{n+1} (1 - r_{f,t}) + \left[1 - \sum_{n=0}^{\infty} \frac{z_n\left(q_{h,f,t}+q_{l,f,t}\right)}{n+1}\right] \beta u_{i,t+1}. \tag{3}$$

High types and low types arrive at Poisson rates $q_{h,f,t}$ and $q_{l,f,t}$. The distribution of the total demand, therefore, is also Poisson with arrival rate $q_{h,f,t} + q_{l,f,t}$. So, a buyer who finds himself at the firm finds $n = 0, 1, \ldots$ *other* buyers with probability $z_n (q_{h,f,t} + q_{l,f,t})$. He purchases with probability $1/(n + 1)$ and his payoff is $1 - r_{f,t}$. With the complementary probability, given by the expression in square

[10] Even though the ex-ante expected demand at each firm is λ, the ex-post realized demand is uncertain. A firm may well end up getting no customer at all, or it may get more customers than it can serve.

[11] Throughout the text, despite the nuances, we use the words "expected demand", "arrival rate" and "queue length" interchangeably.

brackets, the buyer fails to transact so he moves to the next period, where he expects to earn $\beta u_{i,t+1}$.

Now consider a flexible firm with list price $r_{b,t}$. A low type (non-haggler) buyer always pays the list price $r_{b,t}$; so, his expected utility $U_{l,b,t}$ is similar to above:

$$U_{l,b,t} = \sum_{n=0}^{\infty} \frac{z_n\left(q_{h,b,t}+q_{l,b,t}\right)}{n+1}\left(1 - r_{b,t}\right) + \left[1 - \sum_{n=0}^{\infty} \frac{z_n\left(q_{h,b,t}+q_{l,b,t}\right)}{n+1}\right]\beta u_{l,t+1}. \tag{4}$$

A high type's expected utility $U_{h,b,t}$, on the other hand, is given by

$$\begin{aligned} U_{h,b,t} = \;& z_0\left(q_{h,b,t}+q_{l,b,t}\right)\left(1+\varepsilon-y_t\right) + \sum_{n=1}^{\infty} \frac{z_n\left(q_{h,b,t}+q_{l,b,t}\right)}{n+1}\left(1 - r_{b,t}\right) \\ & + \left[1 - \sum_{n=0}^{\infty} \frac{z_n\left(q_{h,b,t}+q_{l,b,t}\right)}{n+1}\right]\beta u_{h,t+1}. \end{aligned} \tag{5}$$

With probability $z_0\left(q_{h,b,t}+q_{l,b,t}\right)$ the high type buyer is alone at the firm, in which case he bargains and obtains the item paying y_t. Since the transaction involves bargaining the buyer obtains the additional (dis)utility ε. (For now we treat ε generically, i.e. we do not specify whether it is positive, negative or zero.) The second part of the expression is similar to above: with probability $z_n\left(q_{h,b,t}+q_{l,b,t}\right)$ he finds $n = 1, 2\ldots$ competitors; so he purchases with probability $1/(n+1)$ paying the list price $r_{b,t}$ (recall that if multiple customers are present then no bargaining takes place). Finally with the complementary probability he fails to transact and moves to period $t+1$, where he expects to earn $\beta u_{h,t+1}$.

Lemma 1 *We have $\partial U_{i,m,t}/\partial r_{m,t} < 0$ and $\partial U_{i,m,t}/\partial q_{i,m,t} < 0$, where $i = h, l$ and $m = f, b$.*

The proof is skipped as it is based on straightforward algebra. Put simply, the Lemma says buyers dislike expensive or crowded firms (the ones with a high price r or high demand q). The first claim is self-explanatory; the second claim follows from the fact that customers are less likely to purchase at crowded firms.

Let $\bar{U}_{i,t}$ denote the maximum expected utility (*market utility*) a type i customer can obtain in the entire market at time t. For now we treat $\bar{U}_{i,t}$ as given, subsequently it will be determined endogenously.[12] So, consider an individual seller who advertises the price package $(r_{m,t}, m)$ and suppose high and low type buyers respond to this advertisement with arrival rates $q_{h.m.t} \geq 0$ and $q_{l,m,t} \geq 0$. The rates satisfy

$$q_{i,m,t} > 0 \text{ if } U_{i,m,t}\left(r_{m,t}, q_{h,m,t}, q_{l,m,t}\right) = \bar{U}_{i,t} \text{ else } q_{i,m,t} = 0. \tag{6}$$

The indifference condition (6) says that the price package and the arrival rates must generate an expected utility of at least $\bar{U}_{h,t}$ for high type customers, else they will stay away ($q_{h,m,t} = 0$) and at least $\bar{U}_{l,t}$ for low type customers, else they will stay away ($q_{l,m,t} = 0$).

[12] The market utility approach is standard in the directed search literature as it greatly facilitates the characterization of equilibrium (Burdett et al., 2001; Shimer, 2005; Menzio and Shi, 2010). Galenianos and Kircher (2012) provide game theoretic foundations for the use of the market utility paradigm in a variety of directed search setups.

The indifference condition also reveals a "law of demand" in that the expected demand $q_{i,m,t}$ decreases as the list price $r_{m,t}$ increases. In words, cheaper firms attract more customers and expensive firms attract fewer customers. To see why, apply the Implicit Function Theorem to (6) to obtain

$$\frac{dq_{i,m,t}}{dr_{m,t}} = -\frac{\partial U_{i,m,t}/\partial r_{m,t}}{\partial U_{i,m,t}/\partial q_{i,m,t}}.$$

The numerator and the denominator are both negative (Lemma 1); hence $dq_{i,m,t}/dr_{m,t}$ is negative, indicating that if the seller raises r then buyers respond by decreasing q. From a seller's point of view, raising the price brings in more revenue; however it lowers the expected demand. The seller's problem involves finding a balance between these two opposing effects, which we study next.

2.5 Seller's Problem and Definition of Equilibrium

The expected profit of a fixed price seller is given by

$$\Pi_{f,t} = \left[1 - z_0\left(q_{h,f,t} + q_{l,f,t}\right)\right] r_{f,t} + z_0\left(q_{h,f,t} + q_{l,f,t}\right) \beta\pi_{t+1}. \tag{7}$$

The expression in square brackets is the probability of getting at least a customer, in which case the item is sold at list price $r_{f,t}$. With the complementary probability the seller fails to get a customer and moves to the next period where he expects to earn $\beta\pi_{t+1}$, which represents his discounted value of search in period $t+1$. The expected profit of a flexible seller is similar:

$$\begin{aligned} \Pi_{b,t} = \quad & z_0\left(q_{l,b,t}\right) z_1\left(q_{h,b,t}\right) y_t + \left[z_0\left(q_{h,b,t}\right) z_1\left(q_{l,b,t}\right) + \textstyle\sum_{n=2}^{\infty} z_n\left(q_{l,b,t} + q_{h,b,t}\right)\right] r_{b,t} \\ & + z_0\left(q_{l,b,t} + q_{h,b,t}\right) \beta\pi_{t+1}. \end{aligned} \tag{8}$$

With probability $z_0\left(q_{l,b,t}\right) z_1\left(q_{h,b,t}\right)$ the seller gets a single high type customer, who haggles and obtains the item at price y_t. The expression in square brackets is the probability of getting either a single low type customer or getting multiple customers. In either case list price $r_{b,t}$ is charged. The last bit, as above, deals with the possibility of not getting any customer at all. A seller's objective is to maximize the profit subject to the fact that he must provide buyers with their market utilities. Specifically each seller solves

$$\max_{m\in\{f,b\}, r_{m,t}\in[0,1], \left(q_{h,m,t}, q_{l,m,t}\right)\in\mathbb{R}_+^2} \Pi_{m,t} \quad \text{subject to (6).} \tag{9}$$

Indifference constraints in (6) determine expected demands $q_{h,m,t}$ and $q_{l,m,t}$ as functions of the pricing rule m and the list price $r_{m,t}$. Note that the seller faces two indifference constraints, one for high type customers and one for low type customers. If both constraints bind, then the seller is able to attract both types of customers. If a single constraint binds then he attracts one type only. (The case where neither constraint binds, of course, can be ruled out as it implies that the seller gets no customer at all.)

Sellers are free to post any price within $[0,1]$ and they are also free to be fixed or flexible with what they post. Letting $\alpha_{m,t}(r_{m,t})$ denote the fraction of sellers posting $r_{m,t}$ we have

$$\alpha_{m,t}(r_{m,t}) > 0 \text{ only if } \Pi_{m,t}(r_{m,t}, q_{h,m,t}, q_{l,m,t}) = \bar{\Pi}_{m,t} \text{ else } \alpha_{m,t}(r_{m,t}) = 0, \tag{10}$$

where

$$\bar{\Pi}_{m,t} \equiv \max_{r'_{m,t}\in[0,1],(q'_{h,m,t},q'_{l,m,t})\in\mathbb{R}^2_+} \Pi_{m,t}(r'_{m,t}, q'_{h,m,t}, q'_{l,m,t}).$$

Similarly letting $\varphi_{m,t}$ denote the fraction of sellers opting for rule m, we have

$$\varphi_{m,t} > 0 \text{ only if } \bar{\Pi}_{m,t} = \max_{\tilde{m}\in\{f,b\}} \bar{\Pi}_{\tilde{m},t}, \text{ else } \varphi_{m,t} = 0, \tag{11}$$

i.e. rule m is selected only if it delivers the highest expected profit. This does not mean that a unique pricing rule will prevail in equilibrium. It is possible that, and indeed it is the case that, both rules emerge in equilibrium delivering equal profits.

Finally, to close down the model, we need two feasibility conditions to ensure that the weighted sum of expected demands (per seller) consisting of type i buyers equals to the market wide buyer-seller ratio for that particular type. Recall that λ_t is the total buyer-seller ratio in period t and that η_t is the fraction of low type buyers. Letting $\lambda_{l,t} = \eta_t\lambda_t$ and $\lambda_{h,t} = (1-\eta_t)\lambda_t$ we have

$$\varphi_{b,t}\int_0^1 \alpha_{b,t}(r_{b,t})q_{i,b,t}(r_{b,t})dr_{b,t} + \varphi_{f,t}\int_0^1 \alpha_{f,t}(r_{f,t})q_{i,f,t}(r_{f,t})dr_{f,t} = \lambda_{i,t} \text{ for } i = h, l. \tag{12}$$

There are two equations in (12), one for high types and one for low types, and the equations are designed to take into account the possibility of each seller posting a different price. In Lemma 2, however, we prove that sellers competing with the same rule m end up posting the same list price $r_{m,t}$; so, borrowing that result, and noting that $\varphi_{f,t} + \varphi_{b,t} = 1$, the equations in (12) become

$$\varphi_{f,t}q_{i,f,t} + (1-\varphi_{f,t})q_{i,b,t} = \lambda_{i,t} \text{ for } i = h, l. \tag{13}$$

We can now define the equilibrium.

Definition 1 *A competitive search equilibrium ("equilibrium") consists of prices $\mathbf{r}^*_{m,t}$, expected demands $\mathbf{q}^*_{h,m,t}$, $\mathbf{q}^*_{l,m,t}$ and fractions $\boldsymbol{\alpha}^*_{m,t}$, $\boldsymbol{\varphi}^*_{m,t}$ satisfying the demand distribution (2), buyer's indifference (6), profit maximization (9), equal profits (10)-(11) and feasibility (12).*

The evolution of the buyer seller ratio λ_t and the fraction of non-hagglers η_t, also part of the equilibrium, is discussed in Section 6.

3 Characterization of Equilibria: The Benchmark Case

The parameter ε plays an important role in determining the nature of the equilibria. We start with the case where $\varepsilon = 0$, i.e where customers are neutral to the bargaining process.

Proposition 1 *Suppose $\varepsilon = 0$. Depending on how large θ is, the model exhibits two types of equilibria:*

- ***Partial Segmentation Equilibrium (Eq-PS):*** *If $\theta \geq \bar{\theta}_t \equiv z_1(\lambda_t) / [1 - z_0(\lambda_t)]$ then there exists a continuum of payoff-equivalent equilibria, where an indeterminate fraction $\varphi^*_{f,t} \geq \eta_t$ of firms trade via fixed pricing and remaining firms trade via flexible pricing. Fixed and flexible firms post*

$$r^*_{f,t} = 1 - \beta u_{t+1} - \frac{z_1(\lambda_t)}{1 - z_0(\lambda_t)} (1 - \beta u_{t+1} - \beta \pi_{t+1}) \text{ and} \tag{14}$$

$$r^*_{b,t} = 1 - \beta u_{t+1} - \frac{z_1(\lambda_t)(1-\theta)}{1 - z_0(\lambda_t) - z_1(\lambda_t)} (1 - \beta u_{t+1} - \beta \pi_{t+1}), \tag{15}$$

and if negotiations ensue the transaction occurs at price

$$y^*_t = 1 - \beta u_{t+1} - \theta (1 - \beta u_{t+1} - \beta \pi_{t+1}). \tag{16}$$

Prices satisfy $r^*_{b,t} > r^*_{f,t} > y_t$, i.e. flexible firms post a higher price than what fixed price firms post, which in turn, is greater than the bargained price. The inequality in prices leads to a partial segmentation in customer demographics: non-hagglers shop exclusively at fixed price firms whereas hagglers shop anywhere. In any equilibrium sellers and buyers earn

$$\pi_t = 1 - \beta u_{t+1} - [z_0(\lambda_t) + z_1(\lambda_t)] (1 - \beta u_{t+1} - \beta \pi_{t+1}), \tag{17}$$

$$u_t = z_0(\lambda_t) [1 - \beta u_{t+1} - \beta \pi_{t+1}] + \beta u_{t+1} \tag{18}$$

- ***Fixed Price Equilibrium (Eq-FP):*** *If $\theta < \overline{\theta}_t$, i.e. if high type buyers are not skilled enough in negotiations, then the availability of bargaining becomes immaterial and fixed pricing emerges as the unique equilibrium, i.e. all sellers adopt fixed pricing and post $r^*_{f,t}$. Equilibrium payoffs are the same as above, i.e. sellers and buyers earn π_t and u_t.*

The main message of the Proposition is that fixed and flexible pricing rules can co-exist in the same marketplace; however each rule comes with its own list price and customer demographics. Flexible firms announce higher prices and attract high types only. Fixed price firms, on the other hand, announce lower prices and attract both types of customers.

To see why prices are unequal, note that flexible stores factor in the fact that they may end up selling at a discount, so they raise their prices to cover themselves against this contingency. In other words, they strategically inflate the sticker price anticipating the eventual surplus loss during negotiations. Fixed price firms, on the other hand, are committed to charge what they post, so they post moderate prices. While the relationship between flexible pricing and inflated sticker prices may sound intuitive, to our knowledge, this is the first study providing a competition based explanation to such phenomenon with a decentralized market equilibrium approach.

We see an example of this phenomenon when JC Penney reverted back to its old pricing strategy of offering sales and bargains after more than a year of trying and failing "fair and square" (i.e. fixed) prices. Time magazine reports that (Tuttle, 2013):

> In early 2012, JC Penney promised the end of “fake prices”—ones that were inflated

> just so that shoppers could be tricked into thinking the inevitable discounts represented amazing deals. Well, it's already time to welcome back discounts and inflated prices alike. Among other reasons, JC Penney CEO Ron Johnson lost his job recently because customers seemed to hate the no-discounting "fair and square" pricing that was a core part of the retailer's dramatic 2012 makeover. [...] The bargain-hunting website dealnews has since commenced tracking prices at JC Penney. What it's discovered is that the prices of certain items—designer furniture, in particular—have risen by 60% or more at JC Penney almost overnight. One week, a side table was listed at $150; a few days later, the "everyday" price for the same item was up to $245.

The fact that list prices went up as much as 60% after the retailer switched from a fixed pricing regime to flexible pricing is in line with what our model predicts. What is more, the difference in posted prices explains why customers are segmented. Flexible firms post higher prices, however, hagglers are not deterred as they can bargain down those prices anyway. Non-hagglers, on the other hand, have no option but to pay the list price, so they self-select themselves to fixed price firms, where prices are moderate.

The inequality of prices raises the question of whether buyers or sellers may want to pass a potential trading opportunity in the hope of getting a better deal in subsequent periods, and the answer is no. In the proof of the proposition we show that players who are in a match are better off transacting immediately instead of walking away. There are two reasons for this. First, waiting is costly (the discount factor is less than one), so players have a strong incentive to settle a deal as early as possible. And more importantly, second, the market is decentralized and it operates via search and matching, so no-one is guaranteed to find a suitable match in subsequent periods. A seller may not get a customer at all, whereas a buyer may well end up in a crowded firm and walk out empty handed as a result. Therefore, a sure transaction today, even under the worst case scenario—buying at the highest price $r_{b,t}^*$ for a buyer, selling at the lowest price y_t^* for a seller—is still better than walking away and facing the prospect of not being able to buy or sell tomorrow. (In the proof of the proposition we analytically prove these arguments.)

4 Characterization of Equilibria when Customers Dislike Bargaining

The discussion so far revolved around the case $\varepsilon = 0$. If, on the other hand, customers dislike the bargaining experience then the result is remarkably simple.

Proposition 2 *If $\varepsilon < 0$ then fixed pricing emerges as the unique equilibrium. For characterization see item Eq-FP in Proposition 1.*

Recall that if $\varepsilon = 0$ then fixed and flexible pricing are payoff equivalent in equilibrium and sellers are indifferent to select either pricing rule. If ε falls below zero then this indifference no longer holds because the negative ε filters into flexible sellers' profits causing them to earn less than their fixed price competitors. Sellers can avoid the negative impact of ε by switching to fixed pricing, which explains why the fixed price outcome emerges as the unique equilibrium.[13]

It is worth pointing out the fixed price equilibrium arises not because buyers would not bargain anyway (because of the negative ε), but because offering flexible pricing causes sellers to lose on profits, and therefore, in equilibrium no venue offers this option in the first place. Indeed in the proof of the Proposition we consider the out of equilibrium scenario where a firm offers flexible pricing and we assume that high types' bargaining power is sufficiently large to ensure that, even after accounting for the negative ε, they would still prefer bargaining over purchasing at the posted price. (The other scenario where they would not even attempt negotiating trivially yields a fixed price outcome.) We show that along this scenario the negative ε filters into the flexible firm's profit, and the firm is better off by unilaterally switching to fixed pricing.

This finding suggests that for sellers, flexible pricing strategy is not a viable option if customers indeed dislike the haggling process. More specifically, if sellers realize that even potential hagglers might dislike bargaining for their products (e.g. due to the fear of being seen unclassy when haggling for a low ticket item) and they can not effectively reduce or eliminate such displeasure, perhaps due to product characteristics, then they have an incentive to practice fixed pricing.

Finally, we turn the case where customers get a psychological satisfaction if they manage to purchase the item below the posted price.

5 Characterization of Equilibria when Customers Enjoy Bargaining

Proposition 3 *Suppose ε is positive but sufficiently small.*

- ***Full Segmentation Equilibrium (Eq-FS):*** *If $\theta \geq \tilde{\theta}_t$, where*

$$\tilde{\theta}_t \equiv \frac{z_1(q^*_{h,b,t})}{1-z_0(q^*_{h,b,t})} - \frac{\varepsilon z_1(q^*_{h,b,t})q^*_{h,b,t}}{[1-z_0(q^*_{h,b,t})]\left[1-\beta u_{h,t+1}-\beta\pi_{t+1}+\varepsilon\right]} \tag{19}$$

then there exists a unique equilibrium where a fraction $\varphi^*_{f,t} < \eta_t$ of firms choose fixed pricing while the rest opt for flexible pricing. Equilibrium prices are given by

$$r^*_{f,t} = 1 - \beta u_{l,t+1} - \frac{z_1(q^*_{l,f,t})}{1-z_0(q^*_{l,f,t})}\left(1-\beta u_{l,t+1} - \beta\pi_{t+1}\right) \tag{20}$$

$$r^*_{b,t} = 1 - \beta u_{h,t+1} - \frac{z_1(q^*_{h,b,t})(1-\theta)}{1-z_0(q^*_{h,b,t})-z_1(q^*_{h,b,t})}\left[1-\beta u_{h,t+1} - \beta\pi_{t+1} + \varepsilon - \frac{q^*_{h,b,t}\varepsilon}{1-\theta}\right] \tag{21}$$

$$y^*_t = 1 - \beta u_{h,t+1} - \theta\left(1-\beta u_{h,t+1} - \beta\pi_{t+1}\right) + \varepsilon\left(1-\theta\right). \tag{22}$$

The equilibrium is characterized by full segmentation of customers: low types avoid flexible firms and high types avoid fixed price firms. Expected demands satisfy $q^*_{h,b,t} < \lambda < q^*_{l,f,t}$, i.e. flexible

[13] From sellers' point of view the negative ε is an indirect cost. It is incurred by buyers, but nevertheless it bleeds into the sellers' profit functions and thereby induces them to switch to fixed pricing. If an indirect cost can disturb the payoff equivalence between fixed and flexible pricing and cause sellers to turn to fixed pricing, then a direct cost, such as the cost of implementing a flexible price environment, would lead to the same outcome.

firms attract fewer customers than fixed price firms. Equilibrium payoffs are as follows

$$\pi_t = 1 - \beta u_{l,t+1} - [z_0(q^*_{l,f,t}) + z_1(q^*_{l,f,t})]\,(1 - \beta u_{l,t+1} - \beta\pi_{t+1}) \tag{23}$$

$$u_{h,t} = z_0(q^*_{h,b,t})\,(1 - \beta u_{h,t+1} - \beta\pi_{t+1}) + [z_0(q^*_{h,b,t}) - z_1(q^*_{h,b,t})]\varepsilon + \beta u_{h,t+1} \tag{24}$$

$$u_{l,t} = z_0(q^*_{l,f,t})[1 - \beta u_{l,t+1} - \beta\pi_{t+1}] + \beta u_{l,t+1} \tag{25}$$

- ***Fixed Price Equilibrium (Eq-FP):*** *If $\theta < \tilde{\theta}_t$ then fixed pricing emerges as the unique equilibrium. For characterization see item Eq-FP in Proposition 1.*

When compared to the benchmark case $\varepsilon = 0$, the introduction of a positive ε leads to two important results: uniqueness of the equilibrium (instead of a continuum of equilibria) and full segmentation of customers (instead of partial segmentation). The multiplicity of equilibria in the benchmark case is disturbing for two reasons. First, the model loses predictive power as one cannot know how many firms are firm with the price and how many are flexible. Second, and perhaps more worrisome, is the presence of an equilibrium where the fraction of fixed price sellers $\varphi^*_{f,t}$ may, in fact, be equal to 1, i.e. a fixed price outcome where no seller offers flexible pricing, despite the availability of bargaining and despite the fact that high types are sufficiently skilled in negotiations. The introduction of a positive ε eliminates the continuum of equilibria, and instead, yields a unique equilibrium. In the benchmark if sufficiently many sellers pick fixed pricing, then the marginal seller is indifferent between picking either pricing rule, which is why there is a continuum of equilibria where $\varphi^*_{f,t}$ can be anywhere between η_t and 1. But if $\varepsilon > 0$ then the marginal seller is strictly better off picking flexible pricing, because, compared to the benchmark, buyers have a larger appetite for flexible deals, yet there are not sufficiently many sellers offering such deals. The marginal seller can earn more if he deviates to flexible pricing, which explains why the introduction of a positive ε unsettles the aforementioned indifference and yields to a unique equilibrium.

The equilibrium is characterized by full segmentation of customers: low types avoid flexible firms whereas high types avoid fixed price firms. The reason behind the first relationship is the same as in the benchmark: flexible firms post negotiable but high prices, but non-hagglers cannot negotiate; hence they avoid these firms. The second relationship is due to the positive ε. In the benchmark model high types were indifferent between fixed and flexible firms, so they would shop anywhere. The introduction of ε unsettles this indifference in favor of flexible venues because in this setting high types not only are able to bargain down the list price but also get some additional satisfaction from doing so.

The result on self selection and segmentation is indeed important as it shows that the type of demand one gets strategically depends on the pricing rule one selects at the first place. As indicated in the Introduction, most of the existing literature in pricing strategies assume a non-competitive environment, typically a monopolist seller, where heterogenous customers (myopic, strategic etc.) are assumed to arrive at an exogenous rate and irrespective of the pricing rule in place, e.g. Cachon and Swinney (2009), Aviv and Pazgal (2008), Kuo et al. (2011). The segmentation result, however, suggests that in a competitive environment where the exogenous demand assumption is relaxed, then customers may not visit some firms in the first place due to self-selection.

Anecdotal evidence suggests that customers indeed love the feeling of purchasing the item below the posted price and that they inevitably gravitate towards outlets offering such deals. As mentioned earlier, retail giant JC Penney made a bold move in January 2012 by ridding their stores of all discounts, sales and bargains in an effort to establish "fair and square" pricing. Unfortunately, for JC Penney, this strategy did not work as its core consumers, who were accustomed to sales and bargains, began leaving the retailer in droves. At the end, the now ousted CEO Ron Johnson had to confess this(Tuttle, 2013):

> “I thought people were just tired of coupons and all this stuff. The reality is all of the couponing we did, there were a certain part of the customers that loved that. They gravitated to stores that competed that way. So our core customer, I think was much more dependent, and enjoyed coupons more than I understood.”

To understand how equilibrium objects change with respect to ε we simulate prices and the fraction of firms adopting flexible pricing against ε and calendar time t. The simulations are based on a stationary environment where outgoing agents are assumed to be replaced with clones; thus η_t, b_t and s_t remain constant throughout all market activity. The stationarity of the environment ensures that the observed dynamics do not stem from fluctuations in the number or composition of buyers and sellers.[14]

Price trajectories in 1a and 1b reveal that for any given t the equilibrium fixed price and the flexible list price both increase in ε, implying that sellers take advantage of the positive utility enjoyed by hagglers in the form of higher prices. Remarkably fixed price sellers, who do not even cater to hagglers, also raise their prices if ε goes up. The mechanism behind this *spillover effect* is this. As ε goes up, more firms offer flexible pricing (see panel 1c) and fewer firms offer fixed pricing. Since fixed price firms are the only outlets where non-hagglers can shop, the expected demand at fixed price firms goes up. The rising demand, naturally, leads to higher prices. The fact that fixed price firms get more crowded and charge higher prices points to another interesting spillover effect in that non-bargaining customers, who shop only at fixed price firms, end up receiving less utility

[14] The simulations are based on the following parametrization: $b_1 = s_1 = 1$, $\eta_1 = 0.5$, $\theta = 0.6$, $T = 25$ and $\beta = 0.9$. The parameter ε ranges from 0 to 0.05.

when bargaining customers enjoy bargaining.[15]

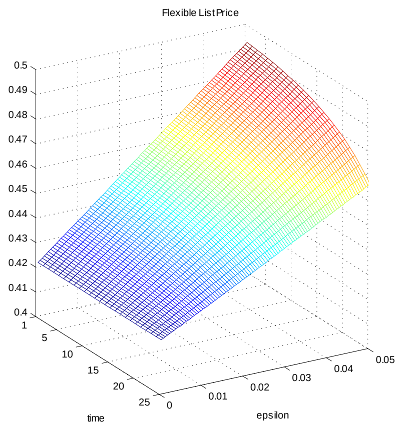

Figure 1a

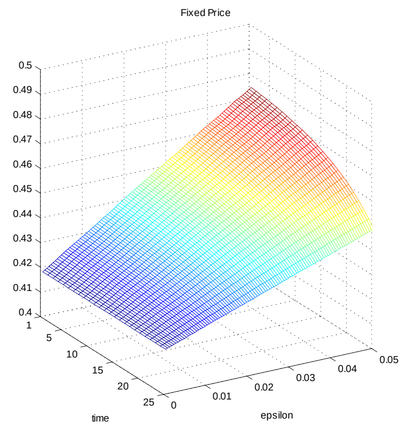

Figure 1b

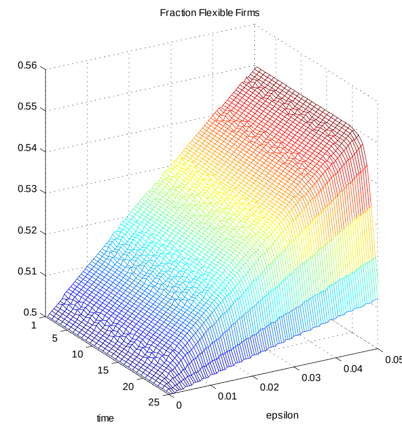

Figure 1c

Second, price trajectories with respect to calendar time reveal that prices are high in the early market (when t is small) and they fall as market activity nears an end. The pattern is more visible for larger values of ε. Indeed, if $\varepsilon \approx 0$ then prices remain rather flat over time, however if $\varepsilon \approx 0.05$ then they clearly exhibit a falling pattern. The reason is this. When t is small, sellers are not worried about not being able to trade as they know the market will remain active for a long while. So, they list higher prices in order to take advantage of the presence of high type buyers and benefit from the positive ε they bring with. Towards the end of the market however, the fear of not being able to trade kicks in, as such, prices start to fall.[16]

Third, the equilibrium percentage of sellers adopting flexible pricing falls in t (panel 1c). To see why, notice that along Eq-FS flexible stores attract, on average, fewer customers than fixed price stores, which means that they are relatively less likely to trade and exit. Initially sellers are not too worried about not being able to sell, so a large number of them remain flexible in an effort to trade with high type customers. But as t grows large, sellers start to switch to fixed pricing to maximize their likelihood of making a sale.

Panel 1c further reveals that the percentage of sellers adopting flexible pricing rises in ε, i.e. a larger value from good feelings results in more sellers opting for flexible pricing. The intuition is simple. Flexible sellers are able to convert a larger ε into higher prices, and thereby, into higher profits. The market is competitive; so, if ε rises then more sellers start operating via flexible pricing in an effort to take advantage of this profit making opportunity. The proposition below summarizes the above discussion analytically (the proof is in Appendix 1).

[15] We thank an anonymous referee for pointing out the second relationship between ε and its impact on the non-bargaining customers' utility.

[16] The drop in prices is only gradual and it does not warrant buyers to delay their purchase. We prove that in equilibrium buyers and sellers are better off trading immediately due to (i) discounting and (ii) trade frictions (not being guaranteed to trade in the subsequent period).

Proposition 4 *The equilibrium profit π_t rises in ε.*

The proposition has two important managerial implications. First, it is notable to observe that sellers can indeed convert customers' enjoyment of the bargaining process (e.g., smartshopper feelings) into higher prices, and therefore higher profits. This implies that if firms are able to raise customers' enjoyment of the bargaining process (e.g., through such actions as better training of the salesforce to be highly courteous during bargaining, providing a relaxing environment for price negotiation, among others), this can then provide returns in the form of higher prices and profits. Second, our results point out that for sellers, there is potential money to be made from people's enjoyment of bargaining. Indeed, sellers who recognize, and successfully assess, whether and to what extent their customers derive additional pleasure from bargaining, can adjust their prices accordingly and obtain higher profits.

6 Price Dynamics

In this section we explore how equilibrium prices respond to fluctuations in expected demand. We proxy the expected demand by the buyer-seller ratio $\lambda_t = b_t/s_t$ since along Eq-PS and Eq-FP the expected demand at each store is exactly equal to λ_t, whereas along Eq-FS expected demands $q^*_{h,b,t}$ and $q^*_{l,f,t}$ are proportional to it (they increase if it increases and they fall if it falls).

To determine the trajectory of λ_t one needs to focus on how the measures of buyers and sellers evolve over time. Recall that the market starts with a measure of s_1 sellers and b_1 buyers, of which a fraction η_1 are low types. At the end of each period, trading players leave the market and the ones who could not trade move to the next period to play the same game. In addition, at the beginning of each period $t = 2, 3...$ a new cohort of s^{new}_t sellers and b^{new}_t buyers, of which a fraction η^{new}_t are low types, enter the market joining the existing players. The proposition below pins down how these measures evolve over time.

Proposition 5 *Along Eq-PS and Eq-FP the measures of buyers and sellers evolve according to*

$$b_t = b^{new}_t + b_{t-1} - s_{t-1}(1 - z_0(\lambda_{t-1})) \text{ and } s_t = s^{new}_t + s_{t-1}z_0(\lambda_{t-1}) \text{ for } t \geq 2. \tag{26}$$

The fraction of non-hagglers, on the other hand, evolves according to

$$\eta_t = [b^{new}_t \eta^{new}_t + b_{t-1}\eta_{t-1} - \eta_{t-1}s_{t-1}(1 - z_0(\lambda_t))]/b_t. \tag{27}$$

Specifically if $\eta^{new}_t = \eta_1$ then $\eta_t = \eta_1$ for all $t \geq 2$. Along Eq-FS we have

$$b_t = b^{new}_t + b_{t-1} - (l_{t-1} + h_{t-1}) \quad \text{and} \quad s_t = s^{new}_t + s_{t-1} - (l_{t-1} + h_{t-1})$$

where $l_{t-1} \equiv s_{t-1}\varphi^*_{f,t-1}[1 - z_0(q^*_{f,t-1})]$ and $h_{t-1} \equiv s_{t-1}(1 - \varphi^*_{f,t-1})[1 - z_0(q^*_{h,t-1})]$. The fraction of non-hagglers evolves according to $\eta_t = [b_{t-1}\eta_{t-1} - l_{t-1} + \eta^{new}_t b^{new}_t]/b_t$.

Given the equations governing b_t and s_t, one can pin down how λ_t evolves over time and then, via reverse engineering, one can impose specific trajectories on λ_t. To see how note that along Eq-PS or Eq-FP we have

$$\lambda_t = \frac{b_t^{new} + b_{t-1} - s_{t-1}(1 - e^{-\lambda_{t-1}})}{s_t^{new} + s_{t-1}e^{-\lambda_t - 1}} \text{ for } t \geq 2. \tag{28}$$

The trajectory is endogenous but it is partly driven by the measures of incoming cohorts $\{b_t^{new}, s_t^{new}\}_{t=2}^{k+1}$, which are exogenous. This means that one can reverse engineer and pick the exogenous numbers in such a way that the trajectory follows a particular pattern one may have in mind. Specifically, we consider seasonal cycles where λ_t starts low in the beginning of the cycle, peaks in the middle of the cycle and subsides towards the end of the cycle and each such cycle lasts, say, k periods, that is $\lambda_t = \lambda_{t+k}$, for some integer k. For instance consider a case with $k = 2$, where the market alternates between episodes of high and low demand. Suppose in odd periods we want to have $\lambda_{odd} = 0.5$ and in even periods $\lambda_{even} = 1$. One can produce such cycles by picking starting values, say, $b_1 = 1$ and $s_1 = 2$ and the new entrants as $b_{t=even}^{new} = 1.7$, $s_{t=even}^{new} = 0.7$, $b_{t=odd}^{new} = 0.3$ and $s_{t=odd}^{new} = 1.3$. Note that in even periods more buyers and fewer sellers enter the market whereas in odd periods the opposite happens.[17]

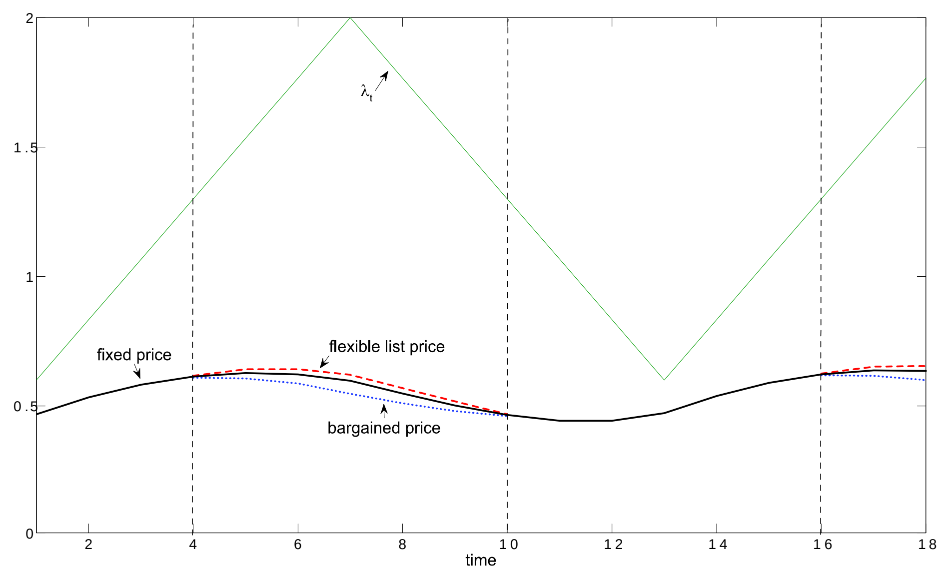

Figure 2

[17] In period 1 the buyer seller ratio equals to $\lambda_1 = b_1/s_1 = 0.5$. At the end of the period $s_1\left(1 - e^{-\lambda_1}\right) = 0.79$ buyers and sellers trade and exit, which means that a measure of 1.21 sellers and 0.21 buyers are unable to trade so they move to the next period (per the discussion above, we do not worry about the fraction of hagglers). At the beginning of period 2, $b_{t=even}^{new} = 1.7$, $s_{t=even}^{new} = 0.7$ enter the market; thus $b_2 = s_2 = 1.9$ and therefore $\lambda_2 = 1$. At the end of period 2, $s_2\left(1 - e^{-\lambda_2}\right) = 1.2$ buyers and sellers trade and exit; hence a measure 0.7 sellers and 0.7 buyers move to period 3. At the beggining of period 3, $b_{t=odd}^{new} = 0.3$, $s_{t=odd}^{new} = 1.3$ join them; thus $b_3 = 1$ and $s_3 = 2$ and therefore $\lambda_3 = 1$. And so on. Observe, however, that this solution is not unique. There is a continuum of other pairs of $\{b_1, s_1\}$ and $\{b_t^{new}, s_t^{new}\}_{t=2}^{k+1}$ producing the same cycle.

In the simulation we pick $k = 12$ and select the entering cohorts in such a way that the expected demand follows a zigzag trajectory: the cycle starts when lambda is at its lowest value 0.6, then it peaks at 2 in the middle of the season, then it declines back to 0.6 and then it starts again (see Figure 2). In addition, for the sake of simplicity we assume that $\eta_t^{new} = \eta_1 = 0.5$ so that η_t remains constant at 0.5 at all times.[18]

There are a few observations that stand out. First, prices seem to follow the same trajectory as the expected demand λ_t; they rise as λ_t rises and they fall as it falls. The intuition is simple. If λ_t goes up then sellers face less competition to attract customers, so they post higher prices. If λ_t falls then they face stiffer competition and cut their prices.

Second, the simulation confirms that the flexible list price is indeed higher than the fixed price. As discussed earlier, flexible sellers understand that they may well end up selling at a lower price than what they initially post, so they inflate the list price upfront to cover themselves against this contingency.

Third, there is a time lag between prices and the expected demand—prices seem to front-run the expected demand by about two periods. Indeed, prices peak around $t = 5$, whereas the expected demand peaks at $t = 7$. Similarly, prices dip at around $t = 11$, which, again, is well before λ_t reaches its own minimum at $t = 13$. To understand why, note that prices depend not only on the current demand, but on the entire sequence $\{\lambda_{t+j}\}_{j=0}^{T}$, as such, if the general outlook of future demand turns negative, then prices start to fall even if demand keeps rising for a short while. For instance at $t = 6$ sellers understand that demand will rise only for one more period, after which it will fall for six consecutive periods until $t = 13$; so, they start cutting prices. By the time λ_t peaks at $t = 7$ prices have already started falling. The opposite happens at the end of the cycle. By the time the demand dips at $t = 13$, prices have already started rising. (We remind the reader that players, due to the decentralized nature of the model, are always willing to transact immediately rather than waiting.)

The final and arguably the most interesting observation is that prices do not fluctuate as much as the expected demand. Even though the demand goes through sharp zigzags, prices follow much smoother trajectories with little fluctuation.[19] The reason is this. Prices depend on the entire demand sequence $\{\lambda_{t+j}\}_{j=0}^{T}$ and if the terminal period T is sufficiently far away then sellers effectively face a market with cyclical demand that goes through periodic ups and downs. (In the simulation we have $T = 360$, which means that the market goes through thirty cycles of twelve periods before it comes to an end.) With cyclical demand, if β is sufficiently large then future total demand is more or less constant because the variation in demand is mostly accounted for; hence prices do not fluctuate as much. Indeed as $\beta \to 1$ price trajectories converge to each other and they start to look

[18] Note that in Eq-PS if $\eta_t^{new} = \eta_1$ then $\eta_t = \eta_1$ for all t. In words, the fraction of low types remains constant at η_1 throughout all market activity, provided that the fraction in the entrant cohorts is also η_1. (This relationship holds in Eq-FP as well, but this is rather immaterial because no one negotiates in Eq-FP.) To see why note that along Eq-PS the expected demand at all firms is equal to λ_t; thus all buyers trade and exit at the same rate. This means that the ratio of hagglers to non hagglers is not disturbed by how fast different types of buyers exit the market. If this ratio is not disturbed externally either then η_t remains constant at η_1 for all $t \geq 2$. This relationship does not hold along Eq-FS because in that equilibrium hagglers are more likely to trade than non hagglers, and they exit the market at a faster rate. Simulations suggest that if we fix $\eta_t^{new} = \eta_1$ then η_t converges to a level slightly above η_1.

[19] In the simulation, the maximum value of λ_t is more than three times its minimum value, but for prices this ratio is less than 1.5. Similarly, the coefficient of variation for λ_t is 0.32, whereas for prices it is less than 0.1.

like flat lines. On the other hand, as $\beta \to 0$ the impact of any lambda beyond, say, the current period becomes ignorable, which leaves the current demand λ_t as the dominant factor driving the prices. Consequently, price trajectories start to follow the trajectory of the current demand λ_t closely, exhibiting a similar zigzag pattern.[20]

The parameter β (inversely) proxies the severity of trade frictions. A small value of β indicates that players who are unable to buy or sell today incur significant waiting costs before trying again in the subsequent period. The discussion above suggests that fluctuations in prices depend on the degree of trade frictions. If trade frictions are severe (i.e. if waiting is costly) then prices move significantly; else, they remain stable even though the demand goes through sharp ups and downs. In the simulation we have $\beta = 0.8$, which is a moderately high value; hence the stable prices. The following proposition summarizes the discussion so far (the proof is in Appendix 1).[21]

Proposition 6 *Both in Eq-PS an in Eq-FP if T is sufficiently large then for all $1 < t \ll T$ we have*

$$\lim_{\beta\to 1} \Delta y_t^* = \lim_{\beta\to 1} \Delta r_{b,t}^* = \lim_{\beta\to 1} \Delta r_{f,t}^* = 0,$$

where $\Delta y_t^* \equiv y_t^* - y_{t-1}^*$ denotes the difference in prices ($\Delta r_{b,t}^*$ and $\Delta r_{f,t}^*$ are likewise).

The observation pertaining price stability is important for the following reason. In marketing the phenomenon of stable prices in the presence of fluctuating demand and supply is predominantly explained with fairness concerns, which originates from the principle of dual entitlement put forward by Kahneman et al. (1986). This principle suggests, among others, that customers have perceived fairness levels for both firm profits and retail prices, and it is 'not fair' for retailers to change the price arbitrarily, or just to increase the firm's existing profit, for example, by taking advantage of excess demand (Xia et al., 2004; Bolton et al., 2003; Anderson and Simester, 2008). However, our model provides a new explanation for the stability of prices which is driven by competition in a decentralized and dynamic market. The idea is simple: if trade frictions are ignorable and players do not mind to search for an extended time, then the variation in future demand is largely accounted for in current prices, so, prices do not fluctuate much. While fairness concerns could be a dominant driver of price stability in many markets, our results suggest that the phenomenon of stable prices can be obtained as a result of market competition with forward looking rational players [22].

[20] We thank the AE for pointing out the interplay between the cyclicality of demand and stable prices.

[21] We restrict the Proposition within Eq-PS and Eq-FP because in the other equilibrium (Eq-FS) expected demands $q_{h,b,t}^*$ and $q_{l,f,t}^*$ have non-trivial closed form solutions rendering an analytic proof elusive. Numerical simulations, however, suggest that along Eq-FS, too, prices tend to remain stable if β and T are large.

[22] Forward looking customers who may strategize over the timing of their purchases has received significant attention in the dynamic pricing literature (Besanko and Winston, 1990; Su, 2007; Cho et al., 2009; Cachon and Swinney, 2009; Yuan and Han, 2011). In addition, price stability, the fact that prices are sticky and that they are not that responsive to changes in costs or demand has also been analyzed in the economics literature by highlighting, inter alia, the role of consumers' loss aversion (Heidhues and Kőszegi, 2008), the risk of antagonizing customers (Anderson and Simester, 2010), and the role of consumer lock-in in forward-looking customer markets (Nakamura and Steinsson, 2011). We differ from these studies with our focus on price mechanism selection in a fully competitive framework with heterogenous buyers. In other words, even though the idea that forward looking agents may facilitate smooth prices is intuitive and plausible, it is not at all obvious whether and under what conditions the smooth price phenomenon would emerge in a competitive and complex setting such as ours.

7 Concluding Remarks

In this paper we develop economic intuition on the selection and dynamics of two popular pricing rules—fixed price and flexible price—in competitive markets for big ticket items using the competitive (or directed) search paradigm. Fixed pricing is plain enough, flexible pricing involves bargaining between the buyer and seller. Despite bargaining being a common practice in many buying selling situations, previous analytical models of bargaining in marketing have mostly focused on business to business and channel relationships (Iyer and Villas-Boas, 2003; Dukes et al., 2006; Guo and Iyer, 2013), leaving room for models investigating the practice of bargaining in business to consumer settings.

Big ticket item purchases are typically the biggest transactions in most consumers' lives, and non-economic dynamics such as pleasure or displeasure associated with bargaining can play a central role during these transactions. As such, our modeling approach incorporates this additional behavioral element into the model. As expected, a small amount of displeasure from the bargaining process yields to a unique fixed price equilibrium. On the other hand, remarkably, a small amount of satisfaction from bargaining results in a unique equilibrium with full segmentation of customers. In addition, in this case, we find that list price in both fixed and flexible sellers increase with customers' additional satisfaction from bargaining.

As fixed and flexible pricing co-exist in many modern day markets, it is important to gain a better conceptual understanding of these pricing strategies. In this paper, we provide a theoretical rationale for firms' selection and strategic implications of fixed and flexible pricing in a fully competitive setting by focusing on big ticket item markets. Fixed and flexible pricing formats, of course, are not exclusive to these markets, and they co-exist in many other marketplaces. For example in many classified advertisement websites such as Craigslist, one observes indicators for both flexible price selling ("OBO" –or best offer) and fixed price selling (e.g., "sharp price") for seemingly similar items. Similarly, on eBay, in addition to the auction setup, individuals typically have two major options to sell the product: (i) using a fixed price (e.g., "Buy It Now") or (ii) using a flexible price in which the seller can either accept the offer, decline it, or respond with a counter offer. While we recognize that our model is stylized and some of our modeling assumptions do not apply to broader product categories or markets, we believe our paper is an important step towards a better understanding of fixed and flexible selling strategies.

Our study has also connections to research on everyday low pricing (EDLP) and promotional (Hi/Lo) pricing strategies employed by retailers (Lal and Rao, 1997; Ho et al., 1998; Ellickson and Misra, 2008). Fixed pricing resembles EDLP, and flexible pricing resembles Hi/Lo pricing in some ways. As such, our setting has some distinctions and similarities with EDLP and promotional pricing. Buyers shop for a single big ticket item in our case, whereas EDLP and Hi/Lo research is mainly concerned with customers shopping for a set of items or product categories. In addition, in our setting sellers announce their pricing rules and advertise prices beforehand but there is uncertainty in the final negotiated price. Such uncertainty is more prevalent in EDLP and Hi/Lo pricing as buyers typically are not aware of promotions before visiting stores. Finally, search and trade frictions play an essential role in our model, whereas these are typically small or negligible for EDLP and Hi/Lo settings.

In addition to the practical results and insights discussed earlier in the paper, our article provides two important methodological contributions to pricing literature in marketing by relaxing two common assumptions. First, the model goes beyond the no-competition (i.e., a monopolist seller) or limited-competition settings (i.e. Bertrand/Cournot/Hotelling models usually with two firms) and it takes into account the fully competitive nature of many buying-selling situations. The directed search framework helps us model the pricing problem under full competition in a rigorous yet tractable way. Second, rather than assuming exogenous arrivals to firms, our framework allows us to endogenize the expected demand depending on the list price and the pricing rules. Overall, we think that incorporating competitive search models into marketing problems could open up a new avenue of research for scholars in this area.

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8 Appendix 1

8.1 Proof of Propositions 1, 2 and 3

The proof is by induction. In what follows we show that the claims in the propositions hold in the terminal period T. Then we establish the inductive step. To start, substitute the terminal payoffs $\pi_{T+1} = u_{h,T+1} = u_{l,T+1} = 0$ into (3), (4) and (5) to obtain

$$\begin{aligned}
U_{h,f,T} &= U_{l,f,T} = \tfrac{1 - z_0\left(q_{h,f,T}+q_{l,f,T}\right)}{q_{h,f,T}+q_{l,f,T}} \left(1 - r_{f,T}\right) \\
U_{l,b,T} &= \tfrac{1 - z_0\left(q_{h,b,T}+q_{l,b,T}\right)}{q_{h,b,T}+q_{l,b,T}} \left(1 - r_{b,T}\right) \\
U_{h,b,T} &= U_{l,b,T} + z_0 \left(q_{h,b,T} + q_{l,b,T}\right) \left(r_{b,T} - y_T + \varepsilon\right).
\end{aligned}$$

A high type buyer requests negotiations if $y_T < r_{b,T} + \varepsilon$. This requires θ to be large enough, which we assume to be the case for now. The fact that $y_T < r_{b,T} + \varepsilon$ implies that $U_{h,b,T} > U_{l,b,T}$. At fixed price firms, on the other hand, we have $U_{h,f,T} = U_{l,f,T}$. It follows that $\bar{U}_{h,T} \geq \bar{U}_{l,T}$. Similarly, substituting $\pi_{T+1} = 0$ into profit functions (7) and (8) and re-arranging yields

$$\Pi_{f,T} = 1 - z_0 \left(q_{h,f,T} + q_{l,f,T}\right) - q_{h,f,T} U_{h,f,T} - q_{l,f,T} U_{l,f,T}, \tag{29}$$

$$\Pi_{b,T} = 1 - z_0 \left(q_{h,b,T} + q_{l,b,T}\right) - q_{h,b,T} U_{h,b,T} - q_{l,b,T} U_{l,b,T} + q_{h,b,T} z_0 \left(q_{h,b,T} + q_{l,b,T}\right) \varepsilon. \tag{30}$$

Lemma 2 *In a competitive search equilibrium all flexible firms post the same list price $r_{b,T}$ and cater to high type buyers only. Similarly, fixed price firms post the same list price $r_{f,T}$, but their customer base depends on ε. If $\varepsilon \leq 0$ then they cater to both types of customers but if $\varepsilon > 0$ then they cater to low types only.*

The Lemma establishes how customer demographics would look like if a competitive search equilibrium were to exist (it does not prove existence). These results greatly facilitate the characterization of the equilibrium, which we accomplish subsequently.

Proof of Lemma 2. The proof consists of the following steps.

- Step 1. Flexible firms cannot attract both types of customers; they attract either the high types (hagglers) or the low types (non-hagglers).

- Step 2. Flexible firms attract high types only.

- Step 3. Fixed price firms cannot attract high types only; they attract either both types or just the low types.

- Step 4a. If $\varepsilon \leq 0$ then fixed price firms attract both types of customers.

- Step 4b. If $\varepsilon > 0$ then fixed price firms attract low types only.

- Step 5. All flexible firms post the same list price $r_{b,T}$ and all fixed price firms post the same list price $r_{f,T}$.

Step 1. We prove that flexible firms cannot attract both types of customers. By contradiction, suppose they do, i.e. consider a flexible firm where expected demands $q_{h,b,T}$ and $q_{l,b,T}$ are both positive. This means that $U_{h,b,T} = \bar{U}_{h,T}$ and $U_{l,b,T} = \bar{U}_{l,T}$. Recall that $U_{h,b,T} > U_{l,b,T}$. It follows that $\bar{U}_{h,T} > \bar{U}_{l,T}$. The seller's profit equals to

$$\begin{aligned}\Pi_{b,T} &= 1 - z_0\left(q_{h,b,T} + q_{l,b,T}\right) - q_{h,b,T}U_{h,b,T} - q_{l,b,T}U_{l,b,T} + q_{h,b,T}z_0\left(q_{h,b,T} + q_{l,b,T}\right)\varepsilon \\ &= 1 - z_0\left(q_{h,b,T} + q_{l,b,T}\right) - \left(q_{h,b,T} + q_{l,b,T}\right)U_{l,b,T} - \Delta,\end{aligned}$$

where $\Delta := q_{h,b,T}z_0\left(q_{h,b,T} + q_{l,b,T}\right)\left(r_{b,T} - y_T + \varepsilon\right)$. Note that Δ is positive as $r_{b,T} - y_T + \varepsilon > 0$. Now suppose that this seller keeps his price intact at $r = r_{b,T}$ but changes the rule from 'flexible' to 'fixed'. We claim that the seller loses all high type customers ($q_{h,f,T} = 0$) but gains new low type customers one-for-one, so that his new expected demand $q_{l,f,T}$ equals to his previous expected demand $q_{h,b,T} + q_{l,b,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$. Since $\bar{U}_{h,T} > \bar{U}_{l,T}$ there are two possibilities:

- $U_{h,f,T} = \bar{U}_{h,T}$ and therefore $U_{l,f,T} > \bar{U}_{l,T}$. This case is impossible since, $U_{l,f,T}$, by definition, cannot exceed the market utility $\bar{U}_{l,T}$.
- $U_{l,f,T} = \bar{U}_{l,T}$ and therefore $U_{h,f,T} < \bar{U}_{h,T}$. This means that $q_{l,f,T}$ is positive and satisfies $U_{l,f,T} = \bar{U}_{l,T}$ while $q_{h,f,T} = 0$ since $U_{h,f,T} < \bar{U}_{h,T}$. This scenario is possible.

Since $U_{l,f,T} = \bar{U}_{l,T}$ and $U_{l,b,T} = \bar{U}_{l,T}$ (from above) we have $U_{l,b,T} = U_{l,f,T}$. This implies that

$$\tfrac{1-z_0\left(q_{h,b,T}+q_{l,b,T}\right)}{q_{h,b,T}+q_{l,b,T}}\left(1-r\right) = \tfrac{1-z_0\left(q_{l,f,T}\right)}{q_{l,f,T}}\left(1-r\right)$$

and therefore $q_{l,f,T} = q_{h,b,T} + q_{l,b,T}$. So, by switching to fixed pricing, the seller indeed keeps his total demand intact. The seller now earns

$$\Pi_{f,T} = 1 - z_0\left(q_{l,f,T}\right) - q_{l,f,T}U_{l,f,T}.$$

Using the equality $q_{l,f,T} = q_{h,b,T} + q_{l,b,T}$ it is easy to show that $\Pi_{f,T} - \Pi_{b,T} = \Delta > 0$, i.e. the seller earns more than he did before; hence the initial outcome could not be an equilibrium.

Step 2. We now show that flexible firms attract high types only. Suppose the opposite is true i.e. they attract low types only (the third scenario where they attract both types is ruled out in Step 1). This means that $U_{l,b,T} = \bar{U}_{l,T}$ and $U_{h,b,T} < \bar{U}_{h,T}$ therefore $q_{l,b,T} > 0$ and $q_{h,b,T} = 0$. Recall that $U_{h,b,T} > U_{l,b,T}$. It follows that $\bar{U}_{h,T} > \bar{U}_{l,T}$. According to our conjecture high types stay away from flexible firms, so they must be shopping at fixed price firms. This means that $U_{h,f,T} = \bar{U}_{h,T}$. Recall, however, that $U_{h,f,T} = U_{l,f,T}$, which implies $U_{l,f,T} > \bar{U}_{l,T}$; a contradiction since $U_{l,f,T} \leq \bar{U}_{l,T}$ by definition.

Step 3. Suppose there is a fixed price firm that caters just to high types. This implies $U_{l,f,T} < \bar{U}_{l,T}$ and $U_{h,f,T} = \bar{U}_{h,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$. It follows that $\bar{U}_{h,T} < \bar{U}_{l,T}$; a contradiction since $\bar{U}_{h,T} \geq \bar{U}_{l,T}$.

Step 4a. We will show that if $\varepsilon \leq 0$ then fixed price firms attract both types of customers. The previous step established that fixed price firms serve either both types of customers or low types only. Below we rule out the second alternative.

By contradiction suppose fixed price firms indeed attract low types only, i.e. suppose that $q_{l,f,T} > 0$ and $q_{h,f,T} = 0$. This implies that $U_{h,f,T} < \bar{U}_{h,T}$ and $U_{l,f,T} = \bar{U}_{l,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$; hence $\bar{U}_{l,T} < \bar{U}_{h,T}$. From Step 2 we know that flexible firms attract high types only, i.e. $q_{h,b,T} > 0$ and $q_{l,b,T} = 0$. This implies that $U_{h,b,T} = \bar{U}_{h,T}$ and $U_{l,b,T} < \bar{U}_{l,T}$. A fixed price firm solves

$$\max_{q_{l,f,T} \in \mathbb{R}_+} 1 - z_0(q_{l,f,T}) - q_{l,f,T} U_{l,f,T} \quad \text{s.t.} \quad U_{l,f,T} = \bar{U}_{l,T}.$$

The first order condition (FOC) implies that

$$z_0(q_{l,f,T}) = \bar{U}_{l,T} \;\Rightarrow\; \Pi_{f,T} = 1 - z_0(q_{l,f,T}) - z_1(q_{l,f,T}).$$

Similarly a flexible firm solves

$$\max_{q_{h,b,T} \in \mathbb{R}_+} 1 - z_0(q_{h,b,T}) - q_{h,b,T} U_{h,b,T} + z_1(q_{h,b,T})\,\varepsilon \quad \text{s.t.} \quad U_{h,b,T} = \bar{U}_{h,T}.$$

The FOC is given by

$$z_0(q_{h,b,T}) + [z_0(q_{h,b,T}) - z_1(q_{h,b,T})]\,\varepsilon = \bar{U}_{h,T}.$$

Thus

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T}) - z_1(q_{h,b,T}) + q_{h,b} z_1(q_{h,b,T})\,\varepsilon.$$

Suppose $\varepsilon = 0$. Then the equal profit condition $\Pi_{b,T} = \Pi_{f,T}$ implies that $q_{l,f,T} = q_{h,b,T}$. Substituting this into the FOCs above we have $\bar{U}_{h,T} = \bar{U}_{l,T}$; a contradiction since $\bar{U}_{l,T} < \bar{U}_{h,T}$. Now Suppose $\varepsilon < 0$. The equal profit condition implies that $q_{l,f,T} < q_{h,b,T}$. To see why fix some $q_{h,b,T}$ and note that $\Pi_{f,T} > \Pi_{b,T}$ even when $q_{l,f,T} = q_{h,b,T}$ because $\varepsilon < 0$. The function $\Pi_{f,T}$ falls if $q_{l,f,T}$ decreases, so if $q_{l,f,T}$ exceeds $q_{h,b,T}$ then $\Pi_{f,T}$ further exceeds $\Pi_{b,T}$. It follows that for equal profits we must have $q_{l,f,T} < q_{h,b,T}$. Recall that $U_{h,T} > U_{l,T}$. This requires

$$\varepsilon \lesseqgtr \frac{z_0(q_{l,f,T}) - z_0(q_{h,b,T})}{z_0(q_{h,b,T})(1 - q_{h,b,T})} \text{ if } 1 \lesseqgtr q_{h,b,T}.$$

Hence, there are two scenarios:

- $1 > q_{h,b,T}$ and $\varepsilon > \frac{z_0(q_{l,f,T}) - z_0(q_{h,b,T})}{z_0(q_{h,b,T})(1 - q_{h,b,T})}$: Recall that $q_{l,f,T} < q_{h,b,T}$. It follows that $z_0(q_{l,f,T}) > z_0(q_{h,b,T})$ which in turn implies that $\varepsilon > 0$; a contradiction since $\varepsilon < 0$.

- $1 < q_{h,b,T}$ and $\varepsilon < \frac{z_0(q_{l,f,T}) - z_0(q_{h,b,T})}{z_0(q_{h,b,T})(1 - q_{h,b,T})}$: This case, too, produces a contradiction. To see why note that the equal profit condition $\Pi_{b,T} = \Pi_{f,T}$ implies that

$$\varepsilon = \left[(1 + q_{h,b,T})\, z_0(q_{h,b,t}) - (1 + q_{l,f,T})\, z_0(q_{l,f,T})\right] / q_{h,b,T}^2.$$

Substituting this into the inequality above we need

$$z_0\left(q_{h,b,T}\right) - z_0\left(q_{l,f,T}\right) > \left(q_{h,b,T} - q_{l,f,T}\right) z_0\left(q_{l,f,T}\right)\left(q_{h,b,T} - 1\right).$$

Since $q_{l,f,T} < q_{h,b,T}$ and $1 - q_{h,b,T} < 0$ the left hand side of the inequality is negative whereas the right hand side is positive; a contradiction.

Step 4b. We show that if $\varepsilon > 0$ then fixed price firms cater to low types only. Step 3 establishes that fixed price firms cannot be catering to high types only. This leaves two possibilities: either they serve both types or they serve low types only. Below we rule out the first alternative, which means that if an equilibrium exits where some sellers compete with fixed pricing, then those sellers must be catering to low types only.

To start, suppose, by contradiction, that there is a fixed price seller who attracts both types of customers, i.e. suppose that $q_{h,f,T}$ and $q_{l,f,T}$ are both positive and satisfy $U_{h,f,T} = \bar{U}_{h,T}$ and $U_{l,f,T} = \bar{U}_{l,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$. It follows that $\bar{U}_{l,T} = \bar{U}_{h,T}$. Letting $q_{f,T} := q_{h,f,T} + q_{l,f,T}$, a fixed price seller solves

$$\max_{q_{f,T}\in\mathbb{R}_+} \Pi_{f,T} = \max_{q_{f,T}\in\mathbb{R}_+} 1 - z_0\left(q_{f,T}\right) - q_{f,T}U_{h,f,T} \quad \text{s.t.} \quad U_{h,f,T} = \bar{U}_{h,T}.$$

After substituting the constraint into the objective function, the FOC is given by

$$z_0\left(q_{f,T}\right) = \bar{U}_{h,T} \Rightarrow \Pi_{f,T} = 1 - z_0\left(q_{f,T}\right) - z_1\left(q_{f,T}\right). \tag{31}$$

We argue that this seller would earn more if he were to switch to flexible pricing. Note that after such a switch he would attract high types only (Steps 1 and 2), i.e. $q_{h,b,T} > 0$ and $q_{l,b,T} = 0$. He solves

$$\max_{q_{h,b,T}\in\mathbb{R}_+} \Pi_{b,T} = 1 - z_0\left(q_{h,b,T}\right) - q_{h,b,T}U_{h,b,T} + z_1\left(q_{h,b,T}\right)\varepsilon \quad \text{s.t.} \quad U_{h,b,T} = \bar{U}_{h,T}.$$

The FOC is given by

$$z_0\left(q_{h,b,T}\right) + \left[z_0\left(q_{h,b,T}\right) - z_1\left(q_{h,b,T}\right)\right]\varepsilon = \bar{U}_{h,T} \tag{32}$$

and therefore

$$\Pi_{b,T} = 1 - z_0\left(q_{h,b,T}\right) - z_1\left(q_{h,b,T}\right) + q_{h,b,T}z_1\left(q_{h,b}\right)\varepsilon.$$

We can now compare expected profits and show that the deviation is profitable, i.e. $\Pi_{b,T} > \Pi_{f,T}$. To start note that expressions (31) and (32) together imply that

$$\varepsilon = \frac{z_0\left(q_{f,T}\right) - z_0\left(q_{h,b,T}\right)}{z_0\left(q_{h,b,T}\right)\left(1 - q_{h,b,T}\right)}.$$

Recall that ε is positive; thus $q_{h,b,T} \neq q_{f,T}$, so we have either $q_{f,T} < q_{h,b,T}$ or $q_{f,T} > q_{h,b,T}$.

- Suppose $q_{f,T} < q_{h,b,T}$. Under this specification we have $\Pi_{f,T} < \Pi_{b,T}$. To see why, fix some

$q_{h,b,T}$ and note that $\Pi_{f,T} < \Pi_{b,T}$ even when $q_{f,T} = q_{h,b,T}$. The function $\Pi_{f,T}$ decreases as $q_{f,T}$ decreases, so if $q_{f,T}$ falls below $q_{h,b,T}$ then $\Pi_{f,T}$ falls further below $\Pi_{b,T}$.

- Suppose $q_{f,T} > q_{h,b,T}$. Let $\Delta := \Pi_{b,T} - \Pi_{f,T}$. We will show that Δ is positive. Substitute ε into $\Pi_{b,T}$, and use the fact that $z_1(q) = qz_0(q)$ to obtain

$$\Delta = (q_{f,T} - q_{h,b,T})\, z_0(q_{f,T}) + \frac{z_0(q_{h,b,T}) - z_0(q_{f,T})}{q_{h,b,T} - 1}.$$

Since $q_{f,T} > q_{h,b,T}$ the first expression on the right hand side is positive. The inequality $q_{f,T} > q_{h,b,T}$ implies that $z_0(q_{h,b,T}) > z_0(q_{f,T})$. For ε to be positive the denominator must be negative, hence we have $q_{h,b,T} > 1$. It follows that the second expression, too, is positive. Hence Δ is positive, which means that the deviation is profitable, i.e. $\Pi_{b,T} > \Pi_{f,T}$.

Step 5. Recall from Step 3 that flexible firms cater to high types only; so, consider such a firm with price $r_{b,T}$ and expected demand $q_{h,b,T}$. From Step 4b we know that its FOC is given by

$$z_0(q_{h,b,T})\,[1 + \varepsilon - \varepsilon q_{h,b,T}] = \bar{U}_{h,T}$$

Solving $U_{h,b,T} = z_0(q_{h,b,T})\,[1 + \varepsilon - \varepsilon q_{h,b,T}]$ for the list price $r_{b,T}$ we have

$$\widehat{r}_{b,T} = 1 - \frac{z_1(q_{h,b,T})(y_T - \varepsilon q_{h,b,T})}{1 - z_0(q_{h,b,T}) - z_1(q_{h,b,T})}.$$

Now consider another flexible firm with price $r'_{b,T}$ and expected demand $q'_{b,T}$. His FOC is given by

$$z_0(q'_{h,b,T})[1 + \varepsilon - \varepsilon q'_{h,b,T}] = \bar{U}_{h,T}.$$

Combining both FOCs we have $q'_{h,b,T} = q_{h,b,T}$. This, in turn, implies that $\widehat{r}'_{b,T} = \widehat{r}_{b,T}$ as the price function above is one-to-one. Going through similar steps one can show that fixed price firms, too, post identical prices. This completes the proof of Lemma 2. $\blacksquare$

Now we can start characterizing the equilibria. There are three cases.

8.1.1 Case 1: $\varepsilon = 0$.

Per Lemma 2 if $\varepsilon = 0$ then flexible firms attract high types, i.e. we have $U_{h,b,T} = \bar{U}_{h,T}$ and $U_{l,b,T} < \bar{U}_{l,T}$ and therefore $q_{h,b,T} > 0$ and $q_{l,b,T} = 0$. Substituting $\varepsilon = 0$ and $q_{l,b,T} = 0$ into (30) yields

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T}) - q_{h,b,T} U_{h,b,T}.$$

The seller's problem is $\max_{q_{h,b,T} \in \mathbb{R}_+} 1 - z_0(q_{h,b,T}) - q_{h,b,T} U_{h,b,T}$ subject to $U_{h,b,T} = \bar{U}_{h,T}$. After substituting the constraint into the objective function, the first order condition (FOC) is given by $z_0(q_{h,b,T}) = \bar{U}_{h,T}$. The second order condition is trivial, hence the solution corresponds to a maximum. Substituting the FOC into $\Pi_{b,T}$ yields

$$\Pi_{b,T} = 1 - z_0\left(q_{h,b,T}\right) - z_1\left(q_{h,b,T}\right). \tag{33}$$

Now consider a fixed price seller. If $\varepsilon = 0$ then fixed price sellers attract both types of customers, i.e. $q_{h,f,T}$ and $q_{l,f,T}$ are both positive and satisfy $U_{h,f,T} = \bar{U}_{h,T}$ and $U_{l,f,T} = \bar{U}_{l,T}$. Since $U_{h,f,T} = U_{l,f,T}$ we have $\bar{U}_{l,T} = \bar{U}_{h,T}$. Letting $q_{f,T} := q_{h,f,T} + q_{l,f,T}$ denote the total demand, the fixed price seller solves

$$\max_{q_{f,T}\in\mathbb{R}_+} \Pi_{f,T} = \max_{q_{f,T}\in\mathbb{R}_+} 1 - z_0\left(q_{f,T}\right) - q_{f,T}U_{h,f,T} \quad \text{s.t.} \quad U_{h,f,T} = \bar{U}_{h,T}.$$

The FOC is given by $z_0\left(q_{f,T}\right) = \bar{U}_{h,T}$. The seller's profit, therefore, is equal to

$$\Pi_{f,T} = 1 - z_0\left(q_{f,T}\right) - z_1\left(q_{f,T}\right). \tag{34}$$

Both FOCs together imply that $q_{h,f,T} + q_{l,f,T} = q_{h,b,T}$, i.e. expected demands at a fixed and flexible firm must be identical. Substituting this equality into the feasibility conditions in (13) and using the fact that $q_{l,b,T} = 0$ one obtains

$$q_{h,b,T} = \lambda_T, \quad q_{h,f,T} = \lambda_T(\varphi^*_{f,T} - \eta_T)/\varphi^*_{f,T} \quad \text{and} \quad q_{l,f,T} = \lambda_T\eta_T/\varphi^*_{f,T},$$

where $\varphi^*_{f,T}$ denotes the fraction of fixed price firms. Note that for any $\varphi^*_{f,T} \in [\eta_T, 1]$ expected demands $q_{h,f,T}$ and $q_{l,f,T}$ are both positive and satisfy the relationship above. This means that $\varphi^*_{f,T}$ is indeterminate, so we have a continuum of equilibria where $\varphi^*_{f,T}$ can be anywhere in between η_T and 1. Furthermore, in any given equilibrium flexible sellers and fixed price sellers have the same expected demand λ_T.

Now we can obtain equilibrium payoffs and list prices. Recall that $\bar{U}_{l,T} = \bar{U}_{h,T} = z_0\left(q_{h,b,T}\right)$. Since $q_{h,b,T} = \lambda_T$ we have $u_T = z_0\left(\lambda_T\right)$. Similarly substituting $q_{h,b,T} = q_{f,T} = \lambda_T$ into (33) and (34) yields sellers' equilibrium profit $\Pi_{f,T} = \Pi_{b,T} \equiv \pi_T = 1 - z_0\left(\lambda_T\right) - z_1\left(\lambda_T\right)$. Given that $u_T = z_0\left(\lambda_T\right)$ one can obtain the equilibrium fixed price by solving $U_{h,f,T} = z_0\left(\lambda_T\right)$ for $r_{f,T}$ and the equilibrium flexible price by solving $U_{h,b,T} = z_0\left(\lambda_T\right)$ for $r_{b,T}$. We have

$$r^*_{f,T}\left(\lambda_T\right) = 1 - \tfrac{z_1(\lambda_T)}{1-z_0(\lambda_T)} \quad \text{and} \quad r^*_{b,T}\left(\lambda_T\right) = 1 - \tfrac{z_1(\lambda_T)(1-\theta)}{1-z_0(\lambda_T)-z_1(\lambda_T)}.$$

Finally substituting $\varepsilon = 0$ and $u_{h,T+1} = 0$ into (1) yields the equilibrium bargained price $y^*_T = 1 - \theta$. Observe that expressions for $r^*_{f,T}$, $r^*_{b,T}$, y^*_T, π_T and u_T can be obtained by substituting $u_{T+1} = \pi_{T+1} = 0$ into expressions (14), (15), (16), (17) and (18) on display in Proposition 1, confirming the validity of the Proposition for the terminal period T.

So far we assumed that high type buyers are sufficiently skilled in bargaining. Now we can put some structure behind this assumption. A buyer negotiates if $y_T \leq r^*_{b,T}$, which, after substituting for $r^*_{b,T}$ and re-arranging, is equivalent to $\theta \geq \overline{\theta}\left(\lambda_T\right)$, where $\overline{\theta}\left(\lambda_T\right) := z_1\left(\lambda_T\right)/\left[1 - z_0\left(\lambda_T\right)\right]$. So, high types negotiate if their bargaining power exceeds threshold $\overline{\theta}$ and purchase at the list price otherwise.

Straightforward algebra reveals that if $\theta > \overline{\theta}(\lambda_T)$ then $r^*_{b,T}(\lambda_T) > r^*_{f,T}(\lambda_T) > y_T$, i.e. flexible firms advertise higher prices than fixed price firms.

The case $\theta < \overline{\theta}(\lambda_T)$ is trivial. Since even hagglers do not find it worthwhile to negotiate the list price, the availability of bargaining becomes immaterial and the model collapses to a fixed price setting. Technically this is equivalent to the outcome where $\varphi^*_t = 1$, i.e. where all firms trade via fixed pricing, post $r^*_{f,T}$ and serve both types of customers. The total demand at each firm equals to λ_T, whereas the equilibrium payoffs are still given by $u_T = z_0(\lambda_T)$ and $\pi_T = 1 - z_0(\lambda_T) - z_1(\lambda_T)$.

8.1.2 Case 2: $\varepsilon < 0$.

In what follows we will show that if $\varepsilon < 0$ then no firm adopts flexible pricing. The proof is by contradiction, i.e. suppose that an equilibrium exists where at least one firm adopts flexible pricing. We will show that this firm earns less than its fixed price competitors. To start recall that if $\varepsilon < 0$ then a flexible firm attracts high types only while low types stay away (Lemma 2) i.e. $U_{h,b,T} = \bar{U}_{h,T}$ and $U_{l,b,T} < \bar{U}_{l,T}$ hence $q_{h,b,T} > 0$ and $q_{l,b,T} = 0$. The flexible firm solves

$$\max_{q_{h,b,T}\in\mathbb{R}_+} 1 - z_0(q_{h,b,T}) - q_{h,b,T}U_{h,b,T} + z_1(q_{h,b,T})\varepsilon \quad \text{s.t.} \quad U_{h,b,T} = \bar{U}_{h,T} \ .$$

The first order condition is given by

$$z_0(q_{h,b,T}) + [z_0(q_{h,b,T}) - z_1(q_{h,b,T})]\varepsilon = \bar{U}_{h,T}.$$

It follows that

$$\Pi_{b,T} = 1 - z_0(q_{h,b,T}) - z_1(q_{h,b,T}) + q_{h,b,T}z_1(q_{h,b,T})\varepsilon.$$

Now consider fixed price firms. Per Lemma 2 they attract both types of customers i.e. $q_{h,f,T} > 0$ and $q_{l,f,T} > 0$ and satisfy $U_{h,f,T} = \bar{U}_{h,T}$ and $U_{l,f,T} = \bar{U}_{l,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$. It follows that $\bar{U}_{l,T} = \bar{U}_{h,T}$. Letting $q_{f,T} := q_{h,f,T} + q_{l,f,T}$, a fixed price seller solves

$$\max_{q_{f,T}\in\mathbb{R}_+} 1 - z_0(q_{f,T}) - q_{f,T}U_{h,f,T} \quad \text{s.t.} \quad U_{h,f,T} = \bar{U}_{h,T}.$$

The FOC is given by $z_0(q_{f,T}) = \bar{U}_{h,T}$; therefore

$$\Pi_{f,T} = 1 - z_0(q_{f,T}) - z_1(q_{f,T}).$$

We will show $\Pi_{f,T} > \Pi_{b,T}$. First note that the FOCs together imply that

$$z_0(q_{h,b,T}) + [z_0(q_{h,b,T}) - z_1(q_{h,b,T})]\varepsilon = z_0(q_{f,T}) \Rightarrow \varepsilon = \frac{z_0(q_{f,T}) - z_0(q_{h,b,T})}{z_0(q_{h,b,T})(1-q_{h,b,T})}.$$

The fact that $\varepsilon < 0$ implies that either we have (i) $q_{f,T} < q_{h,b,T}$ and $q_{h,b,T} > 1$ or we have (ii) $q_{f,T} > q_{h,b,T}$ and $q_{h,b,T} < 1$. Now we can compare profits. Let $\Delta \equiv \Pi_{f,T} - \Pi_{b,T}$. We will show that

$\Delta > 0$. Note that

$$\begin{aligned}\Delta &= z_0\left(q_{h,b,T}\right) - z_0\left(q_{f,T}\right) + z_1\left(q_{h,b,T}\right) - z_1\left(q_{f,T}\right) - q_{h,b,T} z_1\left(q_{h,b,T}\right)\varepsilon \\ &= \frac{z_0\left(q_{h,b,T}\right) - z_0\left(q_{f,T}\right)}{1 - q_{h,b,T}} - z_0\left(q_{f,T}\right)\left(q_{f,T} - q_{h,b,T}\right)\end{aligned}$$

The first step follows after substituting for $\Pi_{f,T}$ and $\Pi_{b,T}$ whereas the second step is obtained after substituting for ε and noting that $z_1(q) = qz_0(q)$. Observe that under condition (i) both terms of Δ are positive; hence $\Delta > 0$. Under condition (ii) the first term is positive but the second one is negative so we need a closer inspection. Fix $q_{h,b,T} < 1$ and note that Δ falls in $q_{f,T}$ under the restrictions of (ii). It follows that Δ reaches a minimum when $q_{f,T} \searrow q_{h,b,T}$ (recall that under (ii) we have $q_{f,T} > q_{h,b,T}$). Note that $\lim_{q_{f,T} \searrow q_{h,b,T}} \Delta = 0$; thus $\Delta > 0$ when (ii) holds.

The inequality $\Delta > 0$ implies that if fixed and flexible sellers compete in the same market then fixed price sellers earn more than flexible sellers; so there cannot be an equilibrium where flexible pricing is adopted by any firm. The implication is that if $\varepsilon < 0$ then the only possible outcome is where all sellers trade via fixed pricing, which we have already characterized in Case 1.

8.1.3 Case 3: $\varepsilon > 0$.

Per Lemma 2 if $\varepsilon > 0$ then flexible stores attract high types only i.e. $q_{h,b,T} > 0$ and $q_{l,b,T} = 0$ satisfying $U_{h,b,T} = \bar{U}_{h,T}$ and $U_{l,b,T} < \bar{U}_{l,T}$. Substitute $q_{l,b,T} = 0$ into the expression of $\Pi_{b,T}$ to obtain

$$\Pi_{b,T} = 1 - z_0\left(q_{h,b,T}\right) - q_{h,b,T} U_{h,b,T} + z_1\left(q_{h,b,T}\right)\varepsilon.$$

The seller's problem is

$$\max_{q_{h,b,T} \in \mathbb{R}_+} 1 - z_0(q_{h,b,T}) - q_{h,b,T} U_{h,b,T} + z_1\left(q_{h,b,T}\right)\varepsilon \;\text{ s.t. }\; U_{h,b,T} = \bar{U}_{h,T}.$$

The FOC is given by

$$z_0\left(q_{h,b,T}\right) + \left[z_0\left(q_{h,b,T}\right) - z_1\left(q_{h,b,T}\right)\right]\varepsilon = \bar{U}_{h,T}. \tag{35}$$

The SOC is satisfied if ε is small enough, which we assume to be case, hence the solution yields a maximum.[23] Substituting this equality into $\Pi_{b,T}$ yields

$$\Pi_{b,T} = 1 - z_0\left(q_{h,b,T}\right) - z_1\left(q_{h,b,T}\right) + q_{h,b,T} z_1\left(q_{h,b,T}\right)\varepsilon. \tag{36}$$

Now consider fixed price sellers. They attract low types only (Lemma 2), i.e. $q_{h,f,T} = 0$ and $q_{l,f,T} > 0$ satisfying $U_{h,f,T} < \bar{U}_{h,T}$ and $U_{l,f,T} = \bar{U}_{l,T}$. Recall that $U_{h,f,T} = U_{l,f,T}$. It follows that $\bar{U}_{h,T} > \bar{U}_{l,T}$. Substituting $q_{h,f,T} = 0$ into the expression of $\Pi_{f,T}$ yields

$$\Pi_{f,T} = 1 - z_0\left(q_{l,f,T}\right) - q_{l,f,T} U_{l,f,T}. \tag{37}$$

[23] The second order condition is satisfied if $-z_0\left(q_{h,b,T}\right) - z_0\left(q_{h,b,T}\right)\left[2 - q_{h,b,T}\right]\varepsilon < 0$. If $q_{h,b,T} \leq 2$ then the inequality is satisfied irrespective of ε. If $q_{h,b,T} > 2$ then we need $\varepsilon < 1/(q_{h,b,T} - 2)$.

The seller solves

$$1 - z_0\left(q_{l,f,T}\right) - q_{l,f,T}U_{l,f,T} \text{ s.t. } U_{l,f,T} = \bar{U}_{h,T}.$$

The FOC implies

$$z_0\left(q_{l,f,T}\right) = \bar{U}_{l,T} \text{ and therefore } \Pi_{f,T} = 1 - z_0\left(q_{l,f,T}\right) - z_1\left(q_{l,f,T}\right). \tag{38}$$

Recall that $\varphi_{f,T}$ denotes the fraction of sellers who compete with fixed pricing. Substituting $q_{h,f,T} = q_{l,b,T} = 0$ into the feasibility conditions in (13) yields

$$q_{l,f,T} = \eta_T \lambda_T / \varphi_{f,T} \quad \text{and} \quad q_{h,b,T} = \left(1 - \eta_T\right)\lambda_T / \left(1 - \varphi_{f,T}\right).$$

We will show that there exists a unique $\varphi^*_{f,T} \in (0, \eta_T)$ satisfying the equal profit condition $\Pi_{f,T} = \Pi_{b,T}$, proving the equilibrium exists and it is unique. Let $\Delta(\varphi_{f,T}) \equiv \Pi_{b,T} - \Pi_{f,T}$. Combining (36) and (38) it is easy to show that

$$\Delta\left(\varphi_{f,T}\right) = z_0(q^*_{l,f,T}) + z_1(q^*_{l,f,T}) - z_0(q^*_{h,b,T}) - z_1(q^*_{h,b,T}) + q^*_{h,b,T} z_1(q^*_{h,b,T})\varepsilon.$$

Note that Δ rises in $q_{l,b,T}$, which in turn rises in $\varphi_{f,T}$, and that Δ falls in $q_{l,f,T}$, which in turn falls in $\varphi_{f,T}$. It follows that $d\Delta/d\varphi_{f,T} > 0$. Furthermore note that $\Delta\left(\eta_T\right) > 0$ and $\Delta\left(0\right) < 0$ if ε is small.[24] The intermediate value theorem implies that there exists a unique $\varphi^*_{f,T} \in (0, \eta_T)$ satisfying $\Delta(\varphi^*_{f,T}) = 0$. Since $\varphi^*_{f,T} < \eta_T$ we have $q^*_{h,b,T} < \lambda_T < q^*_{l,f,T}$ i.e. fixed price firms are more crowded than flexible firms.

The equilibrium payoffs are immediate from the first order conditions (35) and (38). We have

$$u_{h,T} = z_0(q^*_{h,b,T}) + \left[z_0(q^*_{h,b,T}) - z_1(q^*_{h,b,T})\right]\varepsilon, \quad u_{l,T} = z_0(q^*_{l,f,T}), \quad \pi_T = 1 - z_0(q^*_{l,f,T}) - z_1(q^*_{l,f,T}).$$

Given that high and low type buyers earn, respectively, $u_{h,T}$ and $u_{l,T}$ one can obtain the equilibrium flexible price by solving $U_{h,b,T} = u_{h,T}$ for $r_{b,T}$ and the equilibrium fixed price by solving $U_{l,f,T} = u_{l,T}$ for $r_{f,T}$. We have

$$r^*_{b,T} = 1 - \tfrac{z_1(q^*_{h,b,t})(1-\theta)}{1 - z_0(q^*_{h,b,t}) - z_1\left(q_{h,b,t}\right)}\left[1 + \varepsilon - \tfrac{q^*_{h,b,t}\varepsilon}{1-\theta}\right] \quad \text{and} \quad r^*_{f,T} = 1 - \tfrac{z_1(q^*_{l,f,t})}{1 - z_0(q^*_{l,f,t})}.$$

Finally substituting $\pi_{T+1} = u_{h,T+1} = 0$ into (1) yields the bargained price $y^*_T = (1-\theta)(1+\varepsilon)$. Observe that expressions for $r^*_{f,T}$, $r^*_{b,T}$, y^*_T, π_T, $u_{h,T}$ and $u_{l,T}$ can be obtained by substituting $u_{h,T+1} = u_{l,T+1} = \pi_{T+1} = 0$ into (20), (21), (22), (23), (24) and (25) in Proposition 3, confirming the validity of the Proposition for the terminal period T.

A high type buyer negotiates if $y^*_T \leq r^*_{b,T} + \varepsilon$. After substituting for $r^*_{b,T}$ and y^*_T and re-arranging

[24] Note that $\Delta\left(0\right) = -z_0(q) - z_1(q) + qz_1(q)\varepsilon$, where $q = \left(1 - \eta_T\right)\lambda_T$. If follows that $\Delta\left(0\right) < 0$ if $\varepsilon < (1+q)/q^2$. The expression on the right hand side is positive. Since ε is assumed to be positive but sufficiently small the inequality is satisfied.

this condition is equivalent to

$$\theta \geq \tilde{\theta}_T \equiv \frac{z_1(q^*_{h,b,T})}{1-z_0(q^*_{h,b,T})} - \frac{\varepsilon z_1(q^*_{h,b,T})q^*_{h,b,T}}{(1+\varepsilon)\left[1-z_0(q^*_{h,b,T})\right]}.$$

If $\theta < \tilde{\theta}_T$ then even hagglers do not find it worthwhile to negotiate the list price. The availability of bargaining becomes immaterial and the model collapses to a fixed price setting which was characterized earlier in Case 1.

This completes the proof of the terminal period T. Going through a similar analytical process one can establish the inductive step as well. As the analysis is largely the same the inductive step is relegated to Appendix 2.

8.2 Other Proofs

Proof of Proposition 4. In what follows we prove that $d\pi_t/d\varepsilon > 0$ and $du_{l,t}/d\varepsilon < 0$, where π_t is given by (23) and $u_{l,t}$ is given by (25). The proof is by induction, where we start with the terminal period T. Substituting the terminal payoffs $u_{l,T+1} = \pi_{T+1} = 0$ into (23) and (25) yields

$$\pi_T = 1 - z_0(q^*_{l,f,T}) - z_1(q^*_{l,f,T}) \quad \text{and} \quad u_{l,T} = z_0(q^*_{l,f,T}),$$

and therefore

$$\frac{d\pi_T}{d\varepsilon} = z_1(q^*_{l,f,T})\frac{dq^*_{l,f,T}}{d\varepsilon} \quad \text{and} \quad \frac{du_{l,T}}{d\varepsilon} = -z_0(q^*_{l,f,T})\frac{dq^*_{l,f,T}}{d\varepsilon}.$$

Our goal is to show that the first derivative is positive and the second one is negative. Notice that both relationships hold if $dq^*_{l,f,T}/d\varepsilon > 0$, so below we establish that this is indeed the case. Let $\Delta_T \equiv \Pi_{b,T} - \Pi_{f,T}$, where $\Pi_{b,T}$ is given by (36) and $\Pi_{f,T}$ is given by (37), and note that the expected demand $q^*_{l,f,T}$ satisfies $\Delta_T = 0$. By the implicit function theorem we have

$$\frac{dq^*_{l,f,T}}{d\varepsilon} = -\frac{\partial\Delta_T/\partial\varepsilon}{\partial\Delta_T/\partial q^*_{l,f,T}}.$$

Note that Δ_T rises in ε and falls in $q^*_{l,f,T}$. It follows that $dq^*_{l,f,T}/d\varepsilon > 0$. This proves the claim for period T. Now for the inductive step suppose that $d\pi_{t+1}/d\varepsilon > 0$ and $du_{t+1}/d\varepsilon < 0$. We will show that $d\pi_t/d\varepsilon > 0$ and $du_t/d\varepsilon < 0$. Notice that

$$\begin{aligned}\frac{d\pi_t}{d\varepsilon} = &\ -\frac{du_{l,t+1}}{d\varepsilon}\beta\left[1 - z_0(q^*_{l,f,t}) - z_1(q^*_{l,f,t})\right] + \frac{d\pi_{t+1}}{d\varepsilon}\beta[z_0(q^*_{l,f,t}) + z_1(q^*_{l,f,t})] \\ &+ (1 - \beta u_{l,t+1} - \beta\pi_{t+1})\, z_1(q^*_{l,f,t})\frac{dq^*_{l,f,t}}{d\varepsilon}\end{aligned}$$

The first line is positive due to the inductive step. Hence, in order to establish $d\pi_t/d\varepsilon > 0$ it suffices to show that $dq^*_{l,f,t}/d\varepsilon > 0$. Let $\Delta_t \equiv \Pi_{b,t} - \Pi_{f,t}$, where $\Pi_{b,t}$ is given by (47) and $\Pi_{f,t}$ is given by (49), and note that $q^*_{l,f,t}$ satisfies $\Delta_t = 0$. By the implicit function theorem we have

$$\frac{dq^*_{l,f,t}}{d\varepsilon} = -\frac{\partial\Delta_t/\partial\varepsilon}{\partial\Delta_t/\partial q^*_{l,f,t}}.$$

Note that Δ_t rises in ε and falls in $q^*_{l,f,t}$; thus $dq^*_{l,f,t}/d\varepsilon > 0$. This proves the claim $d\pi_t/d\varepsilon > 0$. The other claim can be proved by going through similar steps. ■

Proof of Proposition 5. Consider Eq-PS first. Along this equilibrium path the expected demand at any store at time $t-1$ is equal to λ_{t-1}, so each seller trades with probability $1-z_0(\lambda_{t-1})$. The law of large numbers implies that $s_{t-1}(1-z_0(\lambda_{t-1}))$ sellers trade and exit the market. Each transaction involves one seller and one buyer, so the total number of buyers who trade and exit is also $s_{t-1}(1-z_0(\lambda_{t-1}))$. The number of sellers present in period t is, then, $s_t = s_t^{new} + s_{t-1}z_0(\lambda_{t-1})$, whereas the number of buyers is $b_t = b_t^{new} + b_{t-1} - s_{t-1}(1-z_0(\lambda_{t-1}))$.

Now turn to the proportions of hagglers and non hagglers. In period $t-1$ the total demand at any fixed price firm equals to λ_{t-1} of which $\lambda_{t-1}\eta_{t-1}/\varphi^*_{f,t-1}$ are non-hagglers and $\lambda_{t-1}(\varphi^*_{f,t-1} - \eta_{t-1})/\varphi^*_{f,t-1}$ are hagglers (Proposition 1). Since buyers are equally likely to be selected at the point of transaction, the probability that the purchasing customer is going to be a low type equals to $\eta_{t-1}/\varphi^*_{f,t-1}$. There are $\varphi^*_{f,t-1}s_{t-1}$ fixed price firms present in the market, each seller trades with probability $1-z_0(\lambda_{t-1})$ and each transaction involves one buyer and one seller; so, the number of non-haggler customers who trade and exit equals to

$$\varphi^*_{f,t-1}s_{t-1} \times (1-z_0(\lambda_{t-1})) \times \frac{\eta_{t-1}}{\varphi^*_{f,t-1}} = \eta_{t-1}s_{t-1}(1-z_0(\lambda_{t-1})).$$

Remaining buyers move to period t. The number of non-hagglers present in period t, given by $\eta_t b_t$, equals to

$$\eta_t b_t = b_t^{new}\eta_t^{new} + b_{t-1}\eta_{t-1} - \eta_{t-1}s_{t-1}(1-z_0(\lambda_t)).$$

It follows that η_t is given by expression (27), on display in Proposition 5. This completes the discussion on Eq-PS. Along Eq-FP, as in Eq-PS, the expected demand at any store at time $t-1$ is equal to λ_{t-1} so b_t and s_t evolve as in (26). The proportion of hagglers, too, evolves as in (27), but this is rather irrelevant because along Eq-FP buyers do not negotiate anyway.

Now consider the final scenario, Eq-FS, where non hagglers shop at fixed price stores and hagglers shop at flexible stores. The number of fixed price sellers trading and exiting the market at time $t-1$ is equal to $s_{t-1}\varphi^*_{f,t-1}(1-z_0(q^*_{f,t-1})) \equiv l_{t-1}$ whereas the number flexible sellers trading and exiting the market is equal to $s_{t-1}(1-\varphi^*_{f,t-1})(1-z_0(q^*_{h,t-1})) \equiv h_{t-1}$. Each transaction involves one buyer and one seller; thus $s_t = s_{t-1} - (l_{t-1}+h_{t-1}) + s_t^{new}$ and $b_t = b_{t-1} - (l_{t-1}+h_{t-1}) + b_t^{new}$. Finally note that there are $b_{t-1}\eta_{t-1}$ non-hagglers in the market at $t-1$, of which l_{t-1} exit the market while the rest move to period t. Therefore $\eta_t = [b_{t-1}\eta_{t-1} - l_{t-1} + \eta_t^{new}b_t^{new}]/b_t$. This completes the proof.■

Proof of Remark 6. If $\varepsilon \leq 0$ then $r^*_{f,t}$, π_t and u_t are given by (14), (17) and (18). Letting $x_t \equiv 1-\beta u_t - \beta\pi_t$ these expressions can be re-written as follows:

$$\pi_t = 1-\beta u_{t+1} - [z_0(\lambda_t)+z_1(\lambda_t)]\,x_{t+1};\;\; u_t = \beta u_{t+1} + z_0(\lambda_t)\,x_{t+1};\;\;\; r^*_{f,t} = 1-\beta u_{t+1} - x_{t+1}\frac{z_1(\lambda_t)}{1-z_0(\lambda_t)}$$

Letting $\Delta r^*_{f,t} \equiv r^*_{f,t} - r^*_{f,t-1}$ and noting that $x_t = 1 - \beta + z_1(\lambda_t)\beta x_{t+1}$ we have

$$\Delta r^*_{f,t} = (1-\beta)\left[\tfrac{z_1(\lambda_{t-1})}{1-z_0(\lambda_{t-1})} - \beta u_{t+1}\right] + x_{t+1}\left[\tfrac{\beta z_1(\lambda_t) z_1(\lambda_{t-1})}{1-z_0(\lambda_{t-1})} + \beta z_0\left(\lambda_t\right) - \tfrac{z_1(\lambda_t)}{1-z_0(\lambda_t)}\right].$$

Our goal is to show that $\lim_{\beta\to 1}\Delta r^*_{f,t} = 0$. It is clear that if $\beta \to 1$ then the first term, which is a multiplicative of $1-\beta$, will vanish; however the second term, which is a multiplicative of x_{t+1} needs some inspection. The equation $x_t = 1-\beta + z_1(\lambda_t)\beta x_{t+1}$ pins down the relationship between x_t and x_{t+1}. Iteration on t yields

$$x_{t+1} = (1-\beta)\times\left[1 + \sum_{i=1}^{s-1}\beta^i \prod_{j=1}^{i} z_1\left(\lambda_{t+j}\right)\right] + \underbrace{\beta^s \prod_{j=1}^{s} z_1\left(\lambda_{t+j}\right)\times x_{t+1+s}}_{O(s)},$$

where $s \in \mathbb{N}_+$ is an arbitrary integer. The terms $z_1\left(\lambda_{t+j}\right)$ are all strictly less than 1. Since T is large, one can pick s large enough to ensure that $O\left(s\right) \approx 0$; hence

$$x_{t+1} \approx (1-\beta)\times\left[1 + \sum_{i=1}^{s-1}\beta^i \prod_{j=1}^{i} z_1\left(\lambda_{t+j}\right)\right].$$

Consequently we have $\lim_{\beta\to 1} x_{t+1} = 0$; and therefore $\lim_{\beta\to 1}\Delta r^*_{f,t} = 0$. This completes the proof for $\Delta r^*_{f,t}$. The remaining cases pertaining Δy^*_t and $\Delta r^*_{b,t}$ can be proved similarly. ■

9 Appendix 2 – Not intended for publication

9.1 Inductive Step

Our goal in this section is to establish that the claims in Propositions 1, 2 and 3 hold true in period t, assuming they are true in period $t+1$. We start by re-arranging the expected payoffs for buyers and sellers. Noting that $\sum_{n=0}^{\infty} \frac{z_n(q)}{n+1} = \frac{1-z_0(q)}{q}$, the expression for $U_{i,f,t}$, given by (3), can be re-written as

$$U_{i,f,t} = \tfrac{1-z_0\left(q_{h,f,t}+q_{l,f,t}\right)}{q_{h,f,t}+q_{l,f,t}} \left(1 - r_{f,t} - \beta u_{i,t+1}\right) + \beta u_{i,t+1}. \tag{39}$$

Similarly we have

$$\begin{aligned} U_{l,b,t} = &\quad \tfrac{1-z_0\left(q_{h,b,t}+q_{l,b,t}\right)}{q_{h,b,t}+q_{l,b,t}} \left(1 - r_{b,t} - \beta u_{l,t+1}\right) + \beta u_{l,t+1} \\ U_{h,b,t} = &\quad \tfrac{1-z_0\left(q_{h,b,t}+q_{l,b,t}\right)}{q_{h,b,t}+q_{l,b,t}} \left(1 - r_{b,t} - \beta u_{h,t+1}\right) + z_0\left(q_{h,b,t}+q_{l,b,t}\right)\left(r_{b,t}+\varepsilon-y_t\right) + \beta u_{h,t+1} \end{aligned} \tag{40}$$

Note that

$$U_{h,b,t} = U_{l,b,t} + z_0\left(q_{h,b,t}+q_{l,b,t}\right)\left(r_{b,t}-y_t+\varepsilon\right) + \left[1 - \tfrac{1-z_0\left(q_{h,b,t}+q_{l,b,t}\right)}{q_{h,b,t}+q_{l,b,t}}\right]\beta\left(u_{h,t+1}-u_{l,t+1}\right). \tag{41}$$

Using these expressions we can now rewrite $\Pi_{f,t}$ and $\Pi_{b,t}$. Equation (39) implies that

$$\left[1 - z_0\left(q_{h,f,t}+q_{l,f,t}\right)\right] r_{f,t} = \left[1 - z_0\left(q_{h,f,t}+q_{l,f,t}\right)\right]\left(1-\beta u_{i,t+1}\right) + \beta u_{i,t+1} - \left(q_{h,f,t}+q_{l,f,t}\right) U_{i,f,t}$$

Substituting this relationship into (7) yields

$$\begin{aligned} \Pi_{f,t} = 1 &- z_0\left(q_{h,f,t}+q_{l,f,t}\right)\left(1-\beta\pi_{t+1}\right) - \left(q_{h,f,t}+q_{l,f,t}\right)\left(U_{i,f,t}-\beta u_{i,t+1}\right) \\ &- \left[1 - z_0\left(q_{h,f,t}+q_{l,f,t}\right)\right]\beta u_{i,t+1}. \end{aligned} \tag{42}$$

Similarly combining (40), (41) with (8) yields

$$\begin{aligned} \Pi_{b,t} = 1 - \beta u_{h,t+1} &- z_0\left(q_{h,b,t}+q_{l,b,t}\right)\left(1-\beta\pi_{t+1}-\beta u_{h,t+1}\right) - q_{h,b,t}\left(U_{h,b,t}-\beta u_{h,t+1}\right) \\ - q_{l,b,t}\left(U_{l,b,t}-\beta u_{l,t+1}\right) &+ q_{h,b,t} z_0\left(q_{h,b,t}+q_{l,b,t}\right)\varepsilon + \tfrac{1-z_0\left(q_{h,b,t}+q_{l,b,t}\right)}{q_{h,b,t}+q_{l,b,t}} q_{l,b,t}\beta\left(u_{h,t+1}-u_{l,t+1}\right) \end{aligned} \tag{43}$$

We can now start characterizing the equilibria. There are three cases: $\varepsilon = 0$, $\varepsilon < 0$ and $\varepsilon > 0$.

9.1.1 Case 1: $\varepsilon = 0$.

Per the inductive assumption we have $u_{h,t+1} = u_{l,t+1} = u_{t+1}$. Substituting $u_{h,t+1} = u_{t+1}$ into (1) yields the expression for the bargained price y_t^*, which is on display in Proposition 1 (equation (16)). For now we assume that $y_t^* \leq r_{b,t}$, which requires θ to be sufficiently large. Furthermore we conjecture that players prefer to transact immediately rather than waiting (verified below).

One can show that flexible firms post the same list price $r_{b,t}$ and cater to high types while fixed price firms post the same list price $r_{f,t}$ and cater to both types if $\varepsilon \leq 0$ and cater to low types if $\varepsilon > 0$. In other words, Lemma 2, which was valid in the terminal period T, is also valid in period t. The proof is almost identical to the proof of Lemma 2; hence it is skipped here.

Since $u_{h,t+1} = u_{l,t+1}$ we have $U_{h,f,t} = U_{l,f,t}$. In addition $U_{h,b,t} > U_{l,b,t}$ since $r_{b,t} > y_t$. Now consider a flexible firm. Since flexible firms attract high types only we have $U_{h,b,t}(r_{b,t}) = \bar{U}_{h,t}$ and $U_{l,b,t}(r_{b,t}) < \bar{U}_{l,t}$, and thus $q_{h,b,t} > 0$ and $q_{l,b,t} = 0$. Substituting these into (43) we have

$$\Pi_{b,t} = \quad 1 - \beta u_{t+1} - z_0\left(q_{h,b,t}\right)\left(1 - \beta\pi_{t+1} - \beta u_{t+1}\right) - q_{h,b,t}\left(U_{h,b,t} - \beta u_{h,t+1}\right)$$

A flexible firm solves $\max_{q_{h,b,t}\in\mathbb{R}} \Pi_{b,t}$ s.t. $U_{h,b,t}(r_{b,t}) = \bar{U}_{h,t}$. The FOC is given by

$$z_0\left(q_{h,b,t}\right)\left[1 - \beta u_{t+1} - \beta\pi_{t+1}\right] = \bar{U}_{h,t} - \beta u_{t+1}. \tag{44}$$

The SOC is trivial, hence the solution to the FOC yields a maximum.

Fixed price firms attract both types of customers, i.e. $U_{h,f,t}(r_{f,t}) = \bar{U}_{h,t}$ and $U_{l,f,t}(r_{f,t}) = \bar{U}_{l,t}$ thus $q_{h,f,t} > 0$ and $q_{l,f,t} > 0$. Note that $u_{h,t+1} = u_{l,t+1} = u_{t+1}$ and that $U_{h,f,t} = U_{l,f,t}$. Thus $\Pi_{f,t}$, given by the expression in (42), becomes

$$\Pi_{f,t} = 1 - \beta u_{t+1} - z_0\left(q_{h,f,t} + q_{l,f,t}\right)\left(1 - \beta\pi_{t+1} - \beta u_{t+1}\right) - \left(q_{h,f,t} + q_{l,f,t}\right)\left(U_{h,f,t} - \beta u_{t+1}\right)$$

A fixed price firms solves $\max_{q_{h,f,t},\, q_{l,f,t}\in\mathbb{R}^2_+} \Pi_{f,t}$ s.t. $U_{h,f,t}(r_{f,t}) = \bar{U}_{h,t}$ and $U_{l,f,t}(r_{f,t}) = \bar{U}_{l,t}$. (It appears that the seller faces two separate constraints, one for high types and one for low types. Recall, however, that $U_{h,f,t} = U_{l,f,t}$, which, in turn, implies that $\bar{U}_{l,t} = \bar{U}_{h,t}$; thus both constraints are identical.) The FOC implies that

$$z_0\left(q_{h,f,t} + q_{l,f,t}\right)\left[1 - \beta u_{t+1} - \beta\pi_{t+1}\right] = \bar{U}_{h,t} - \beta u_{t+1}. \tag{45}$$

FOCs (44) and (45) together imply that $q_{h,f,t} + q_{l,f,t} = q_{h,b,t}$, i.e. expected demands at all firms, fixed or flexible, should be identical. Substitute $q_{l,b,t} = 0$ into the feasibility constraint (13) and use the fact that $q_{h,f,t} + q_{l,f,t} = q_{h,b,t}$ to obtain

$$q_{h,b,t} = \lambda_t,\ q_{h,f,t} = \lambda_t(\varphi^*_{f,t} - \eta_t)/\varphi^*_{f,t} \text{ and } q_{l,f,t} = \lambda_t\eta_t/\varphi^*_{f,t}.$$

Note that, $\varphi^*_{f,t}$ is indeterminate and can take any value within $[\eta_t, 1]$; hence, there is a continuum of equilibria where any fraction $\varphi^*_{f,t} \geq \eta_t$ of sellers compete via fixed pricing while the rest compete via flexible pricing. In addition, note that in any equilibrium the total expected demand at each firm equals to λ_t.

Now we can characterize prices. Combining the FOC (44) with indifference constraint $U_{h,b,t}(r_{b,t}) = \bar{U}_{h,t}$ yields

$$z_0(\lambda_t)\left[1 - \beta u_{t+1} - \beta\pi_{t+1}\right] = U_{h,b,t}(r_{b,t}) - \beta u_{t+1},$$

where $U_{h,b,t}$ is given by (40). Solving this equality for $r_{b,t}$ yields expression (15), on display in Proposition 1. Similarly the FOC (45) along with $U_{h,f,t}(r_{f,t}) = \bar{U}_{h,t}$ implies

$$z_0(\lambda_t)[1 - \beta u_{t+1} - \beta \pi_{t+1}] = U_{h,f,t}(r_{f,t}) - \beta u_{t+1},$$

where $U_{h,f,t}$ is given by (39). Solving this equality for $r_{f,t}$ yields expression (14), on display in Proposition 1. High type buyers negotiate if $r^*_{b,t} \geq y^*_t$, which, after substituting for $r^*_{b,t}$ and y^*_t, is equivalent to $\theta \geq \overline{\theta}_t \equiv z_1(\lambda_t)/[1 - z_0(\lambda_t)]$. Given the expressions for $r^*_{f,t}$ and $r^*_{b,t}$ one can verify that the equilibrium payoffs π_t and u_t are indeed as in Proposition 1 (equations (17) and (18)). In addition note that if $\theta > \overline{\theta}_t$ then $r^*_{b,t} > r^*_{f,t} > y_t$.

If $\theta < \overline{\theta}_t$ then $r^*_{b,t} < y_t$; thus no bargaining takes place as the list price $r^*_{b,t}$ falls below the bargained price y^*_t. In this parameter region the model collapses to a fixed-price setting where $\varphi^*_t = 1$, i.e. where all sellers trade via fixed pricing and post $r^*_{f,t}$ serving both types of customers. The equilibrium demand at each firm is λ_t and the expected payoffs for buyers and sellers remain the same as in (17) and (18).

Transact Now or Wait? The inequality in prices raises the issue of whether players should keep searching for better deals. Below we prove that they are better off trading immediately instead of waiting. There are two cases: (i) $\theta \geq \overline{\theta}_t$ and (ii) $\theta < \overline{\theta}_t$.

Eq-PS: If $\theta \geq \overline{\theta}_t$ then fixed and flexible stores coexist in the same market and prices satisfy $r^*_{b,t} > r^*_{f,t} > y^*_t$. The worst case scenario for a buyer is buying at the highest price $r^*_{b,t}$ whereas the worst case scenario for a seller is selling at the lowest price y^*_t. If players transact at these prices then they clearly would transact at more favorable prices.

Consider a buyer who contemplates trading at $r^*_{b,t}$. He purchases if $1 - r^*_{b,t} > \beta u_{t+1}$, i.e. if the immediate surplus is greater than the present value of search in the next period. After substituting for $r^*_{b,t}$ the inequality is satisfied if $1 - \beta u_{t+1} - \beta \pi_{t+1} > 0$. One can verify that the expression on the left hand side is positive; hence the buyer is better off purchasing at $r^*_{b,t}$ rather than waiting.[25] Since the buyer is willing to transact in this worst case scenario, it is clear that he is ready to transact at lower prices $r^*_{f,t}$ and y^*_t as well.

Now consider a seller. The worst case scenario for him is to sell at y^*_t. He agrees to transact if $y^*_t > \beta \pi_{t+1}$, which, after substituting for y^*_t, is equivalent to $1 - \beta u_{t+1} - \beta \pi_{t+1} > 0$. We know this inequality holds, so the seller, too, wishes to sell instead of walking away. Since he is willing to sell at y^*_t, it is clear that he is ready to sell at higher prices $r^*_{f,t}$ and $r^*_{b,t}$ as well.

Eq-FP: If $\theta \geq \overline{\theta}_t$ then all sellers compete via fixed pricing and post $r^*_{f,t}$. A buyer transacts if $1 - r^*_{f,t} > \beta u_{t+1}$, which after substituting for $r^*_{f,t}$ is equivalent to

$$(1 - \beta u_{t+1} - \beta \pi_{t+1}) \times z_1(\lambda_t)/[1 - z_0(\lambda_t)] > 0.$$

Since the term $1 - \beta u_{t+1} - \beta \pi_{t+1}$ is positive the inequality holds. Similarly the seller transacts if

[25] To see why use the expressions for u_t and π_t to obtain $x_t = 1 - \beta + \beta z_1(\lambda_t) x_{t+1}$, where $x_t \equiv 1 - \beta(u_t + \pi_t)$. We want to show that x_t is positive for all $t = 1, 2, ...T$. Note that if $x_{t+1} > 0$ then $x_t > 0$. Substituting the terminal conditions $u_{T+1} = \pi_{T+1} = 0$ yields $x_T = 1$, which, of course, is positive. Hence x_t is positive for all $t < T$.

$r^*_{f,t} > \beta\pi_{t+1}$, which is equivalent to

$$(1 - \beta u_{t+1} - \beta\pi_{t+1}) \times [1 - z_1(\lambda_t)/[1 - z_0(\lambda_t)]] > 0$$

Both expressions inside the parentheses are positive hence the inequality holds.

9.1.2 Case 2: $\varepsilon < 0$.

As in the terminal period, we will show that if $\varepsilon < 0$ then there cannot be an equilibrium where firms adopt flexible pricing. The proof is by contradiction, i.e. suppose that there is an equilibrium where a firm adopts flexible pricing. We will show that this firm earns less than its fixed price competitors. Recall that if $\varepsilon < 0$ then a flexible firm attracts high types only while low types stay away i.e. $U_{h,b,t} = \bar{U}_{h,t}$ and $U_{l,b,t} < \bar{U}_{l,t}$ hence $q_{h,b,t} > 0$ and $q_{l,b,t} = 0$. Substituting $q_{l,b,t} = 0$ along with the fact that $u_{h,t+1} = u_{l,t+1} = u_{t+1}$ (inductive step) into expression (43) we have

$$\Pi_{b,t} = \quad 1 - \beta u_{t+1} - z_0\left(q_{h,b,t}\right)\left(1 - \beta\pi_{t+1} - \beta u_{t+1}\right) - q_{h,b,t}\left(U_{h,b,t} - \beta u_{t+1}\right) + z_1\left(q_{h,b,t}\right)\varepsilon$$

A flexible firm solves $\max_{q_{h,b,t}\in\mathbb{R}_+} \Pi_{b,t}$ s.t. $U_{h,b,t} = \bar{U}_{h,t}$. The FOC is given by

$$z_0\left(q_{h,b,t}\right)\left(1 - \beta\pi_{t+1} - \beta u_{t+1}\right) + \left[z_0\left(q_{h,b,t}\right) - z_1\left(q_{h,b,t}\right)\right]\varepsilon = \bar{U}_{h,t} - \beta u_{t+1}.$$

The second order condition is trivial since $\varepsilon < 0$. It follows that

$$\Pi_{b,t} = 1 - \beta u_{t+1} - \left(z_0\left(q_{h,b,t}\right) + z_1\left(q_{h,b,t}\right)\right)\left(1 - \beta\pi_{t+1} - \beta u_{t+1}\right) + q_{h,b,t} z_1\left(q_{h,b,t}\right)\varepsilon$$

Now consider fixed price firms. They attract both types of customers i.e. $q_{h,f,t} > 0$ and $q_{l,f,t} > 0$ and satisfy $U_{h,f,t} = \bar{U}_{h,t}$ and $U_{l,f,t} = \bar{U}_{l,t}$. Since $u_{h,t+1} = u_{l,t+1} = u_{t+1}$ we have $U_{h,f,t} = U_{l,f,t}$; and therefore $\bar{U}_{l,t} = \bar{U}_{h,t}$. It follows that $\Pi_{f,t}$, given by (42), becomes

$$\Pi_{f,t} = 1 - \beta u_{t+1} - z_0\left(q_{h,f,t} + q_{l,f,t}\right)\left(1 - \beta\pi_{t+1} - \beta u_{t+1}\right) - \left(q_{h,f,t} + q_{l,f,t}\right)\left(U_{h,f,t} - \beta u_{t+1}\right),$$

Letting $q_{f,t} \equiv q_{h,f,t} + q_{l,f,t}$, a fixed price firm solves $\max_{q_{f,t}\in\mathbb{R}_+} \Pi_{f,t}$ s.t. $U_{h,f,t}\left(r_{f,t}\right) = \bar{U}_{h,t}$. The FOC implies that

$$z_0\left(q_{f,t}\right)\left(1 - \beta u_{t+1} - \beta\pi_{t+1}\right) = \bar{U}_{h,t} - \beta u_{t+1}.$$

Hence

$$\Pi_{f,t} = 1 - \beta u_{t+1} - \left(z_0\left(q_{f,t}\right) + z_1\left(q_{f,t}\right)\right)\left(1 - \beta\pi_{t+1} - \beta u_{t+1}\right).$$

We will show $\Pi_{f,t} > \Pi_{b,t}$. First note that the FOCs together imply that

$$\varepsilon = \tfrac{z_0\left(q_{f,t}\right) - z_0\left(q_{h,b,t}\right)}{z_0\left(q_{h,b,t}\right)\left(1 - q_{h,b,t}\right)}\left(1 - \beta u_{t+1} - \beta\pi_{t+1}\right).$$

Observe that $1 - \beta u_{t+1} - \beta\pi_{t+1}$ is positive; thus the inequality $\varepsilon < 0$ implies that either we have (i) $q_{f,t} < q_{h,b,t}$ and $q_{h,b,t} > 1$ or we have (ii) $q_{f,t} > q_{h,b,t}$ and $q_{h,b,t} < 1$.

Now, let $\Delta \equiv \Pi_{f,t} - \Pi_{b,t}$. We will show that $\Delta > 0$. Note that

$$\begin{aligned} \Delta &= \left[z_0(q_{h,b,t}) - z_0(q_{f,t}) + z_1(q_{h,b,t}) - z_1(q_{f,t})\right](1 - \beta u_{t+1} - \beta\pi_{t+1}) - q_{h,b,t} z_1(q_{h,b,t})\varepsilon \\ &= \left\{\frac{z_0(q_{h,b,t}) - z_0(q_{f,t})}{1 - q_{h,b,t}} - z_0(q_{f,t})(q_{f,t} - q_{h,b,t})\right\}(1 - \beta u_{t+1} - \beta\pi_{t+1}) \end{aligned}$$

The first step follows after substituting for $\Pi_{f,t}$ and $\Pi_{b,t}$ whereas the second step is obtained after substituting for ε and noting that $z_1(q) = qz_0(q)$. The term $1 - \beta u_{t+1} - \beta\pi_{t+1}$ is positive; thus focus on the expression inside the curly brackets (call it Ω). Under condition (i) both terms of Ω are positive; hence $\Delta > 0$. Under condition (ii) the first term of Ω is positive but the second one is negative so it needs a closer inspection. Fix $q_{h,b,t} < 1$ and note that Ω falls in $q_{f,t}$ under the restrictions of (ii). It follows that Ω reaches a minimum when $q_{f,t} \searrow q_{h,b,t}$ (recall that under (ii) we have $q_{f,t} > q_{h,b,t}$). Note that $\lim_{q_{f,t} \searrow q_{h,b,t}} \Omega = 0$. Hence $\Omega > 0$ and therefore $\Delta > 0$ in the region $q_{f,t} > q_{h,b,t}$.

The fact that $\Delta > 0$ implies that fixed price sellers earn more than flexible sellers; hence there cannot be an equilibrium where flexible pricing is adopted. The implication is that if $\varepsilon < 0$ then the only possible outcome is the one where all sellers adopt fixed pricing (Eq-FP), which we have already characterized in Case 1.

9.1.3 Case 3: $\varepsilon > 0$.

If $\varepsilon > 0$ then flexible firms cater to high types only i.e. $U_{h,b,t}(r_{b,t}) = \bar{U}_{h,t}$ and $U_{l,b,t}(r_{b,t}) < \bar{U}_{l,t}$ thus $q_{h,b,t} > 0$ and $q_{l,b,t} = 0$. Substitute $q_{l,b,t} = 0$ into $\Pi_{b,t}$, given by (43), and use the fact that $z_1(q) = qz_0(q)$ to obtain

$$\Pi_{b,t} = 1 - \beta u_{h,t+1} - z_0(q_{h,b,t})(1 - \beta u_{h,t+1} - \beta\pi_{t+1}) - q_{h,b,t}(U_{h,b,t} - \beta u_{h,t+1}) + z_1(q_{h,b,t})\varepsilon$$

A flexible firm's problem is $\max_{q_{h,b,t} \in \mathbb{R}_+} \Pi_{b,t}$ s.t. $U_{h,b,t}(r_{b,t}) = \bar{U}_{h,t}$. The FOC is given by

$$z_0(q_{h,b,t})(1 - \beta u_{h,t+1} - \beta\pi_{t+1}) + \left[z_0(q_{h,b,t}) - z_1(q_{h,b,t})\right]\varepsilon = \bar{U}_{h,b,t} - \beta u_{h,t+1} \tag{46}$$

The SOC is satisfied if ε is small enough, hence the solution yields a maximum.[26] It follows that

$$\Pi_{b,t} = 1 - \beta u_{h,t+1} - \left[z_0(q_{h,b,t}) + z_1(q_{h,b,t})\right](1 - \beta u_{h,t+1} - \beta\pi_{t+1}) + q_{h,b,t} z_1(q_{h,b,t})\varepsilon \tag{47}$$

Now consider fixed price sellers. Recall that they attract low types only, i.e. $U_{h,f,t} < \bar{U}_{h,t}$ and $U_{l,f,t} = \bar{U}_{l,t}$; hence $q_{h,f,t} = 0$ and $q_{l,f,t} > 0$. Substituting $q_{h,f,t} = 0$ into $\Pi_{f,t}$, given by (42), yields

$$\Pi_{f,t} = 1 - \beta u_{l,t+1} - z_0(q_{l,f,t})\left[1 - \beta u_{l,t+1} - \beta\pi_{t+1}\right] - q_{l,f,t}(U_{l,f,t} - \beta u_{l,t+1})$$

[26] The second order condition is satisfied if $-z_0(q_{h,b,t})\left[1 - \beta u_{h,t+1} - \beta\pi_{t+1}\right] - \varepsilon\left(2z_0(q_{h,b,t}) - z_1(q_{h,b,t})\right) < 0$. If $2z_0(q_{h,b,t}) > z_1(q_{h,b,t})$, i.e. if $2 > q_{h,b,t}$ then the inequality is satisfied irrespective of ε. If $2 < q_{h,b,t}$ then we need $\varepsilon < (1 - \beta u_{h,t+1} - \beta\pi_{t+1})/(q_{h,b,t} - 2)$. The expression on the right hand side is positive. Since ε is assumed to be positive but sufficiently small the inequality is satisfied.

The seller solves $\max_{q_{l,f,t}\in\mathbb{R}_+} \Pi_{f,t}$ s.t. $U_{l,f,t}(r_{b,t}) = \bar{U}_{l,t}$. The FOC is given by

$$z_0(q_{l,f,t})[1-\beta u_{l,t+1}-\beta\pi_{t+1}] = \bar{U}_{l,f,t} - \beta u_{l,t+1}. \tag{48}$$

The SOC is trivial; hence the solution corresponds to a maximum. It follows that

$$\Pi_{f,t} = 1-\beta u_{l,t+1} - [z_0(q_{l,f,t}) + z_1(q_{l,f,t})](1-\beta u_{l,t+1}-\beta\pi_{t+1}) \tag{49}$$

Recall that $\varphi_{f,t}$ denotes the fraction of sellers who compete with fixed pricing. Substituting $q_{h,f,t} = q_{l,b,t} = 0$ into the feasibility conditions in (13) yields

$$q_{l,f,t} = \eta_t\lambda_t/\varphi_{f,t} \quad \text{and} \quad q_{h,b,t} = (1-\eta_t)\lambda_t/\left(1-\varphi_{f,t}\right).$$

We will show that there exists a unique $\varphi^*_{f,t} \in (0,\eta_t)$ satisfying the equal profit condition $\Pi_{f,t} = \Pi_{b,t}$, proving the equilibrium exists and it is unique. Let $\Delta(\varphi_{f,t}) \equiv \Pi_{b,t} - \Pi_{f,t}$ and note that Δ rises in $q_{h,b,t}$, which in turn rises in $\varphi_{f,t}$, and that Δ falls in $q_{l,f,t}$, which in turn falls in $\varphi_{f,t}$. It follows that $d\Delta/d\varphi_{f,t} > 0$. Furthermore note that $\Delta(\eta_t) > 0$ as $u_{h,t+1} > u_{l,t+1}$ (from the inductive step) whereas $\Delta(0) < 0$ if ε is small.[27] By the intermediate value theorem there exits a unique $\varphi^*_{f,t} \in (0,\eta_t)$ satisfying $\Delta(\varphi^*_{f,t}) = 0$. Since $\varphi^*_{f,t} < \eta_t$ we have $q^*_{h,b,t} < \lambda_t < q^*_{l,f,t}$ i.e. fixed price firms are more crowded than flexible firms.

Now we can obtain equilibrium prices and payoffs. Substituting $q^*_{l,b,t} = 0$ into the expression for $U_{h,b,t}$, given by (40), yields

$$U_{h,b,t} = \tfrac{1-z_0(q^*_{h,b,t})}{q^*_{h,b,t}}(1-r_{b,t}-\beta u_{h,t+1}) + z_0(q^*_{h,b,t})(r_{b,t}+\varepsilon-y_t) + \beta u_{h,t+1}.$$

Solving $U_{h,b,t} = \bar{U}_{h,t}$, where $\bar{U}_{h,t}$ is given by (46), for $r_{b,t}$ yields the expression for $r^*_{b,t}$, which is on display in Proposition 3 (equation ((21))). Similarly substituting $q^*_{h,f,t} = 0$ into $U_{l,f,t}$, given by (40), and solving the equation $U_{l,f,t} = \bar{U}_{l,t}$, where $\bar{U}_{l,t}$ is given by (48), for $r_{f,t}$ yields the expression for $r^*_{f,t}$ (equation (20)). Equilibrium payoffs π_t, $u_{h,t}$ and $u_{l,t}$ are immediate from the first order conditions (46) and (48). Finally the equilibrium bargained price y^*_t is obtained by substituting $u_{h,t+1}$ into (1).

High type buyers bargain if $y^*_t \leq r^*_{b,t} + \varepsilon$. After substituting for $r^*_{b,t}$ and y^*_t and re-arranging this condition is equivalent to

$$\theta \geq \tilde{\theta}_t \equiv \tfrac{z_1(q^*_{h,b,t})}{1-z_0(q^*_{h,b,t})} - \tfrac{\varepsilon z_1(q^*_{h,b,t})q^*_{h,b,t}}{[1-z_0(q^*_{h,b,t})]\left[1-\beta u_{h,t+1}-\beta\pi_{t+1}+\varepsilon\right]}.$$

If $\theta < \tilde{\theta}_t$ then even high types would not opt for bargaining; thus the availability of bargaining becomes immaterial and the model collapses to a fixed price setting, characterized earlier (Eq-FP).

[27] Basic algebra reveals that $\Delta(0) < 0$ if

$$\varepsilon < \frac{\beta(u_{h,t+1}-u_{l,t+1})}{qz_1(q)} + \frac{1+q}{q^2}[1-\beta u_{h,t+1}-\beta\pi_{t+1}], \text{ where } q = (1-\eta_t)\lambda_t.$$

The expression on the right hand side is positive. The inequality holds as ε is positive but sufficiently small.

Transact Now or Wait? We have already established that players are better off trading immediately along Eq-FP (see Case 1 above). What remains to be done is to establish this claim for the other outcome, i.e. Eq-FS. Along this equilibrium high types shop at flexible stores and low types shop at fixed price stores. Start with flexible stores. The worst case scenario for a high type buyer is to purchase at $r^*_{b,t}$ (the alternative is buying at the bargained price y^*_t, which is less than $r^*_{b,t}$). The buyer purchases if $1 - r^*_{b,t} > \beta u_{h,t+1}$. After substituting for $r^*_{b,t}$ the condition is equivalent to

$$\varepsilon < \frac{\left(1-\beta u_{h,t+1}-\beta\pi_{t+1}\right)(1-\theta)}{q^*_{h,b,t}-1+\theta}$$

The expression on the right hand side is positive.[28] Since ε is assumed to positive but sufficiently small the inequality holds, i.e. the buyer is better off purchasing instead of waiting. Now consider the seller, whose worst case scenario is selling at y^*_t. The seller agrees to trade if $y^*_t > \beta\pi_{t+1}$. Substituting for y^*_t the condition is equivalent to $(1-\theta)\left[1-\beta u_{h,t+1}-\beta\pi_{t+1}+\varepsilon\right] > 0$. Both expressions are positive; thus the inequality holds.

Now consider a fixed price firms, where low types shop. A low type buyer purchases if $1 - r^*_{f,t} > \beta u_{l,t+1}$. After substituting for $r^*_{f,t}$ the condition is equivalent to

$$(1-\beta u_{l,t+1}-\beta\pi_{t+1}) \times z_1(q^*_{l,f,t})/[1-z_0(q^*_{l,f,t})] > 0.$$

Since $1-\beta u_{l,t+1}-\beta\pi_{t+1} > 0$ the inequality holds. Similarly the seller trades if $r^*_{f,t} > \beta\pi_{t+1}$, i.e. if

$$(1-\beta u_{l,t+1}-\beta\pi_{t+1}) \times [1-z_1(q^*_{l,f,t})/[1-z_0(q^*_{l,f,t})]] > 0$$

Expressions inside the brackets are positive; hence the inequality holds. This completes the proof. ■

[28] The numerator is positive, but the sign of the denominator, $q^*_{h,b,t}-1+\theta$, needs inspection. Recall that along Eq-FS we have $\theta \geq \tilde{\theta}_t$ and note that $\tilde{\theta}_t \geq z_1(q^*_{h,b,t})/[1-z_0(q^*_{h,b,t})]$. The expression $q^*_{h,b,t}-1+\theta$ is increasing in θ; thus in order to show that it is positive it sufficies to show that $q^*_{h,b,t}-1+z_1(q^*_{h,b,t})/[1-z_0(q^*_{h,b,t})] > 0$. It is easy to verify that this inequality holds true for all values of $q^*_{h,b,t}$, which means that $q^*_{h,b,t}-1+\theta$ is also positive.

9.2 Model with $N \geq 2$ Types of Buyers

In the main text buyers are divided into two types according to their bargaining abilities. Here we consider a setting with N types, where type 1 buyers are the the least skilled in bargaining ("non-hagglers) and type N buyers are the most skilled. Our goal is to check if the results in the main text remain robust to this variation. As this exercise is a robustness check, rather than a full blown analysis, we focus on the one shot game with $\varepsilon = 0$ and then elaborate on what would happen if $\varepsilon < 0$ or $\varepsilon > 0$.

The Outcome of Bargaining. Letting $\theta_i \in [0, 1)$ denote the bargaining power of type $i = 1, 2, ..N$ buyers, we fix $\theta_1 = 0$ and assume that negotiation skills increase in i, that is $\theta_{i+1} > \theta_i$. As in the benchmark, bargaining may ensue only if there is a single customer at the store. If two or more customers are present then the item is necessarily sold at the list price. Furthermore we assume that a buyer's negotiation skill manifests itself at the bargaining table, i.e. once negotiations start the seller can tell how skilled his customer is and correctly identify the parameter θ_i.[29] So, consider the negotiation process between a seller and a type i buyer. The bargained price y_i can be found as the solution to the following maximization problem:

$$\max_{y_i \in [0,1]} (1 - y_i)^{\theta_i} \, y_i^{1-\theta_i}$$

The solution yields $y_i = 1 - \theta_i$. Since $\theta_{i+1} > \theta_i$ we have $y_{i+1} < y_i$, i.e. higher types bargain lower prices. Since $\theta_1 = 0$, type 1 never bargains. We assume that θ_2 is sufficiently large to ensure that $r_b \geq y_2$, i.e. type 2 buyers are skilled enough to obtain a lower price than the posted price. (Otherwise the model collapses to a setting with $N - 1$ types, where type 1 and type 2 buyers are the non-hagglers.) Clearly, if type 2 is skilled enough to ask for bargaining then the higher types $(3, 4, .., N)$ are more than capable of doing so.

Expected Payoffs. Let $q_{i,m}$ denote the expected demand consisting of type i buyers at a store trading via rule m and let

$$q_m \equiv \sum_{i=1}^{N} q_{i,m}, \quad \text{where } m = f, b \text{ and } i = 1, 2, .., N$$

denote the total demand at that store. It follows that the expected utility of a type i buyer at a fixed price store is given by

$$U_{i,f} = \frac{1 - z_0(q_f)}{q_f} (1 - r_f), \text{ for } i = 1, 2, .., N.$$

[29] Notice that identifying who is the most/least skilled among multiple customers is not an issue as the item is sold at the posted price under that contingency. If the model is recast in a way that the price may be negotiated even when there are multiple customers then identification might become an issue. Under that scenario, instead of Nash bargaining, one can use an alternating-offers bargaining scheme, in the tradition of Fudenberg and Tirole, to pin down the bargaining equilibrium; however such an undertaking is beyond the scope of a robustness check.

At a flexible store, on the other hand, we have

$$U_{1,b} = \frac{1 - z_0(q_b)}{q_b}(1 - r_b) \quad \text{and}$$

$$U_{i,b} = z_0(q_b)(1 - y_i) + \sum_{n=1}^{\infty} \frac{z_n(q_b)}{n+1}(1 - r_b), \ i = 2, 3...N.$$

The first line is the expected utility of a type 1 buyer (they never negotiate), whereas the second line is the expected utility of a type i buyer, who would negotiate if he is the sole customer at the store. These expressions are similar to their counterparts in the baseline model and can be interpreted similarly. Basic algebra reveals that

$$U_{i,b} = U_{1,b} + z_0(q_b)(r_b - y_i) \quad \text{and} \quad U_{i+1,b} = U_{i,b} + z_0(q_b)(y_i - y_{i+1}) \text{ for } i = 2, 3, ..N \tag{50}$$

Since $r_b > y_2$ and $y_i > y_{i+1}$ we have $U_{i+1,b} > U_{i,b}$. Now turn to sellers. A fixed price seller expects to earn

$$\Pi_f = [1 - z_0(q_f)] r_f.$$

The expression for Π_f is the same as its counterpart in the benchmark model; however flexible sellers' expected profit is slightly more cumbersome, because potentially they face the prospect of meeting all types of customers and each type negotiates a different price. We have

$$\Pi_b = \sum_{i=2}^{N} \prod_{j=1, j \neq i}^{N} z_0(q_{j,b}) z_1(q_{i,b}) y_i + \left[\prod_{j=2}^{N} z_0(q_{j,b}) z_1(q_{1,b}) + \sum_{n=2}^{\infty} z_n(q_b) \right] r_b$$

To understand the first term note that with probability $\prod_{j=1, j \neq i}^{N} z_0(q_{j,b}) z_1(q_{i,b})$ the seller gets exactly one type i customer, in which case he charges the bargained price y_i (recall that the seller can identify the type of the customer during the negotiation process). To account for all types, the expression needs to be summed over all i, but the summation starts from $i = 2$ because type 1 customers never negotiate. The second expression inside the brackets represent the probability of getting exactly one type 1 customer or getting more than one customer, regardless of the type. In either case the seller charges the posted price r_b. Noting that $\prod_{j=1}^{N} z_0(q_{j,b}) = z_0(q_b)$ and that $x z_0(x) = z_1(x)$ one can show that

$$\Pi_m = 1 - z_0(q_m) - \sum_{i=1}^{N} q_{i,m} U_{i,m}, \text{ where } m = f, b.$$

Now we can state the main result of this section.

Proposition 7 *If $\theta_N \geq \bar{\theta} \equiv z_1(\lambda) / [1 - z_0(\lambda)]$ then there exists a continuum of equilibria, where an indeterminate fraction $\varphi^* \geq \max\{\eta_1, \eta_2, ..., \eta_N\}$ of sellers trade via fixed pricing and remaining sellers trade via flexible pricing. The equilibria are characterized by partial segmentation: Everyone* *but type N customers shop exclusively at fixed price firms whereas type N customers shop anywhere. The expected demand at each store equals to λ. Fixed and flexible price sellers post, respectively*

$$r_f^*(\lambda) = 1 - \frac{z_1(\lambda)}{1 - z_0(\lambda)} \quad \textit{and} \quad r_b^*(\lambda) = 1 - \frac{z_1(\lambda)(1-\theta_N)}{1 - z_0(\lambda) - z_1(\lambda)}$$

The equilibria are payoff-equivalent: in any realized equilibrium sellers and buyers earn $\pi = 1 - z_0(\lambda) - z_1(\lambda)$ and $u = z_0(\lambda)$ no matter which rule sellers compete with and no matter which seller's rule buyers join in. If $\theta_N < \overline{\theta}$, i.e. if type N customers are not skilled enough in negotiations then the availability of flexible pricing becomes immaterial and fixed pricing emerges as the unique equilibrium.

The proposition largely resembles its counterpart in the main text (Proposition 1), which indicates that the results remain rather robust. The key insight in here is that competition among sellers dictates bargaining deals to be designated for the most skilled type, which is why in equilibrium only the most skilled negotiators hunt for bargaining deals and everyone else shops at fixed price venues. An outcome where a firm attracts two different types of customers fails to exist, because along that scenario the lower type ends up with a lower market utility, which is incompatible with profit maximization under competition. An outcome where a firm caters exclusively to a lesser type fails to exist for similar reasons.

In what follows we prove the proposition. Steps 1, 2 and 3, reminiscent of Lemma 2 in the main text, establish how customer demographics pan out along a competitive search equilibrium. We, then, characterize the equilibrium.

- Step 1. A flexible store cannot attract two (or more) different types of customers at the same time. It must be attracting a single type only.

We will show that the store cannot attract two different types at the same time. The fact that it cannot attract more than two types is a corollary. To start, suppose, by contradiction, a flexible store attracts types k and $k+1$, i.e. suppose that $q_{k,b}$ and $q_{k+1,b}$ are both positive whereas $q_{i,b} = 0$ for all $i \neq k, k+1$. The fact that $q_{k,b}$ and $q_{k+1,b}$ are both positive implies that $U_{k,b} = \bar{U}_k$ and $U_{k+1,b} = \bar{U}_{k+1}$. Recall that $U_{k+1,b} > U_{k,b}$. It follows that $\bar{U}_{k+1} > \bar{U}_k$. In addition the fact that $q_{k+j} = 0$, where $j \geq 2$, implies that $U_{k+j,b} < \bar{U}_{k+j}$. Since $U_{k+j,b} > U_{k,b} = \bar{U}_k$ we have $\bar{U}_{k+j} > \bar{U}_k$. In words all types who are better negotiators than type k must have a higher market utility than type k. The seller's profit equals to

$$\begin{aligned}\Pi_b &= 1 - z_0(q_{k,b} + q_{k+1,b}) - q_{k,b}U_{k,b} - q_{k+1,b}U_{k+1,b} \\ &= 1 - z_0(q_{k,b} + q_{k+1,b}) - (q_{k,b} + q_{k+1,b})U_{k,b} - \Delta,\end{aligned}$$

where $\Delta := q_{k+1,b}z_0(q_{k,b} + q_{k+1,b})(y_k - y_{k+1}) > 0$. The second line follows from (50) and note that Δ is positive because $y_k > y_{k+1}$.

Below we show that if this seller switches from flexible pricing to fixed pricing and provides his customers with market utility $\bar{U}_k$ then he could keep his expected demand intact yet he would earn higher profits, rendering the above outcome a non-equilibrium. To start, note that if the seller switches to fixed pricing then all buyers, regardless of their bargaining ability, earn the same expected payoff

$$U_f = \frac{1 - z_0(q_f)}{q_f}(1 - r_f)$$

at his firm. If the seller provides customers with market utility $\bar{U}_k$ then types $k+1$ and above will not visit that store because $\bar{U}_{k+j} > \bar{U}_k$ for all $j \geq 1$ (see above). It follows that the seller will be visited by types k or below. The fact that the seller provides his customers with market utility $\bar{U}_k$ implies that $U_f = \bar{U}_k$. Recall that $U_{k,b} = \bar{U}_k$. It follows $U_f = U_{k,b}$, i.e

$$\Delta = \frac{1 - z_0(q_f)}{q_f}(1 - r_f) - \frac{1 - z_0(q_b)}{q_b}(1 - r_b) - z_0(q_b)(r_b - y_k) = 0.$$

Fix r_b and q_b and note that, per the Intermediate Value Theorem, there exits a unique $\hat{r}_f \in (0, r_b)$ ensuring that $q_f = q_b$ while satisfying $\Delta = 0$. In words if the seller posts $\hat{r}_f$ then he can provide his customers with market utility $\bar{U}_k$ while keeping his expected demand intact. Recall that his prior expected demand was q_b; by posting $\hat{r}_f$ the seller ensures that his new expected demand q_f is the same as q_b. This equality will be useful when we compare expected profits below.

The seller's expected profit under fixed pricing is equal to

$$\Pi_f = 1 - z_0(q_f) - q_f U_f.$$

Since $q_b = q_f$ it is easy to show that

$$\Pi_f - \Pi_b = \Delta > 0.$$

I.e. the seller earns higher profits than he did before; hence the initial outcome could not be an equilibrium. This completes the proof. ■

Step 1 establishes that a flexible store can only attract a single type. This raises the question of whether a flexible store attracts, say, type k while another flexible store attracts type $k+1$. I.e. whether a separating equilibrium where different flexible stores may attract different types could exist. Below we rule out this possibility.

- Step 2. There cannot be an outcome where different flexible stores attract different types of customers. All flexible stores must attract the same type.

Consider two flexible stores, say store A and B. Suppose store A attracts type k only store B attracts type $k+1$ only (Step 1 ruled out the possibility of a store attracting more than one type). So for store A we have $q_k^A > 0$ and $q_i^A = 0$ for all $i \neq k$ and for store B we have $q_{k+1}^B > 0$ and $q_i^B = 0$ for all $i \neq k+1$.

Note that type $k+1$ could shop at store A and obtain a better deal than type k as they are more skilled, but the fact that they stay away from store A indicates that their market utility is higher, i.e. $\bar{U}_{k+1} > \bar{U}_k$. Technically at store A we have $U_{k,b}^A = \bar{U}_k$. The fact that $q_{k+1}^A = 0$ indicates that $U_{k+1,b}^A < \bar{U}_{k+1}$. Recall that $U_{k+1,b}^A > U_{k,b}^A$. It follows that $\bar{U}_{k+1} > \bar{U}_k$.

Store A solves

$$\max_{q^A_{k,b}} \Pi^A_b = \max_{q^A_{k,b}} 1 - z_0\left(q^A_{k,b}\right) - q^A_{k,b}U^A_{k,b} \text{ s.t. } U^A_{k,b} = \bar{U}_k$$

The FOC implies $z_0(q^A_{k,b}) = \bar{U}_k$; hence

$$\Pi^A_b = 1 - z_0\left(q^A_{k,b}\right) - z_1\left(q^A_{k,b}\right).$$

Store B's problem is similar, thus

$$\Pi^B_b = 1 - z_0\left(q^B_{k+1,b}\right) - z_1\left(q^B_{k+1,b}\right).$$

Stores must earn equal profits; thus $\Pi^A_b = \Pi^B_b$. This implies that $q^A_{k,b} = q^B_{k+1,b}$, which in turn implies that $\bar{U}_k = \bar{U}_{k+1}$; a contradiction. ■

- Step 3. Flexible stores must be attracting type N only.

Suppose they attract some other type, say type $k < N$. The fact that type k buyers visit flexible stores while type N buyers stay away indicates that $U_{N,b} < \bar{U}_N$ and $U_{k,b} = \bar{U}_k$. Recall that $U_{N,b} > U_{k,b}$, thus $\bar{U}_N > \bar{U}_k$. Since type N buyers stay away from flexible firms, they must be shopping at fixed price firms. This means that $U_{N,f} = \bar{U}_N$. Recall however that $U_{i,f}$ is the same for all i, thus $U_{k,f} = U_{N,f}$. It follows that $U_{k,f} > \bar{U}_k$; a contradiction since by definition $U_{k,f}$ cannot exceed the market utility $\bar{U}_k$. ■

Characterization of Equilibrium. Flexible stores attract no one but type N, i.e. $q_{N,b} > 0$ and $q_{i,b} = 0$ for all $i \neq N$. It follows that

$$\Pi_b = 1 - z_0\left(q_{N,b}\right) - q_{N,b}U_{N,b}$$

A flexible seller solves

$$\max_{q_{N,b}} 1 - z_0\left(q_{N,b}\right) - q_{N,b}U_{N,b} \quad \text{s.t.} \quad U_{N,b} = \bar{U}_N$$

The FOC implies $z_0\left(q_{N,b}\right) = \bar{U}_N$; hence

$$\Pi_b = 1 - z_0\left(q_{N,b}\right) - z_1\left(q_{N,b}\right).$$

The fact that flexible stores attract no one but type N indicates that types $1, 2, ..., N-1$ must be shopping at fixed price stores. So, let $q_f = \sum_{i=1}^{N} q_{i,f}$ denote the total demand of a fixed price store consisting of type $1, 2, ..., N-1$, and possibly of type N, customers. The fixed price seller solves

$$\max_{q_f \in \mathbb{R}_+} 1 - z_0\left(q_f\right) - q_f U_f \quad \text{s.t.} \quad U_f = \bar{U},$$

where $\bar{U}$ is a generic level or market utility (as it turns out this will be equal to $\bar{U}_N$). The FOC is given by $z_0(q_f) = \bar{U}$. The seller's profit, therefore, is equal to

$$\Pi_f = 1 - z_0(q_f) - z_1(q_f).$$

Both sellers must earn equal profits; i.e. $\Pi_b = \Pi_f$. This indicates that $q_{N,b} = q_f$, i.e. expected demands at fixed and flexible stores must be identical. This means that $\bar{U} = \bar{U}_N$, indicating that all buyers must earn the same market utility and that type N, too, may shop at fixed price stores i.e. $q_{N,f}$ may indeed be positive. Letting φ_f denote the fraction of fixed price sellers and η_i the fraction of type i buyers in the market, with $\sum_{i=1}^{N} \eta_i = 1$, we have

$$\varphi_f q_{i,f} + (1-\varphi_f) q_{i,b} = \lambda \eta_i \text{ for } i = 1, 2, .., N.$$

The feasibility condition is similar to its counterpart in the main text (compare with (13)). Noting that $q_{i,b} = 0$ for $i < N$ we have

$$\underbrace{\varphi_f \sum_{i=1}^{N} q_{i,f}}_{\equiv q_f} + \left(1-\varphi_f\right) q_{N,b} = \lambda \sum_{i=1}^{N} \eta_i = \lambda.$$

Recall that $q_f = q_{N,b}$; hence

$$q_{N,b}^* = \lambda, \quad q_{N,f}^* = \frac{\lambda(\eta_N - 1 + \varphi_f^*)}{\varphi_f^*}, \ q_{i,f}^* = \frac{\lambda \eta_i}{\varphi_f^*} \text{ for } i < N.$$

Note that for any $\varphi_f^* \geq \max\{\eta_1, \eta_2, ..., \eta_N\}$ expected demands $q_{i,f}^*$ are positive and satisfy the relationship above. This means that φ_f^* is indeterminate and we have a continuum of equilibria where $\varphi_f^* \in [\max\{\eta_1, \eta_2, ..., \eta_N\}, 1]$. Note that if $\varphi_f^* \geq \max\{\eta_1, \eta_2, ..., \eta_N\}$ then $\sum_{i=1}^{N} q_{i,f}^* = q_{N,b}^* = \lambda$, i.e. in any given equilibrium flexible sellers and fixed price sellers have the same expected demand λ. To complete the proof we need to pin down the equilibrium payoffs and prices; but this is a rather mechanic task and it can be accomplished by going through the steps outlined in the proof of Proposition 1; hence it is skipped here. ■

What if $\varepsilon \neq 0$? First, if $\varepsilon < 0$ then, as in the benchmark, no seller would offer flexible pricing. To see why, notice that if $\varepsilon = 0$ then sellers are indifferent between fixed and flexible pricing. If, however, ε falls below zero then this indifference would no longer hold because the negative ε would filter into flexible sellers' profits causing them to earn less than fixed price stores. Sellers can avoid this negative effect by switching to fixed pricing. This claim can be proved by repeating the steps in the proof of Proposition 2 because the key in that proof is the fact that a negative ε hurts flexible sellers' profits, which would remain true irrespective of whether there are two types of customer or N types of customers.

If $\varepsilon > 0$ then we expect Proposition 3 to go through with the above caveat—that flexible stores attract type N customers and that everyone else shops at fixed price stores. To establish this claim one needs to prove Steps that are analogous to Step 1, 2 and 3 above. A close look at their proofs reveals that the key factor driving the results is the inequality $U_{i+1,b} > U_{i,b}$—the fact that higher types earn more at a bargaining store than lower types. With $\varepsilon > 0$ expected utilities $U_{i,b}$ would have different closed form expressions, but, nevertheless the inequality $U_{i+1,b} > U_{i,b}$ would remain as the parameter ε is orthogonal to the bargaining ability θ_i. As such, the claims in Steps 1, 2 and 3 would go through even if $\varepsilon > 0$. Once customer demographics are settled (that flexible stores attract type N customers and fixed price stores attract everyone else), the characterization of the equilibrium can, then, be accomplished by virtually repeating the same steps as in the proof of Proposition 3.

9.3 Second Round Matching

In the main text buyers who are unable to get an offer from a firm or firms who are unable to receive a customer need to wait until the next trading period before they can try again. Here we study a variation where unmatched players may be costlessly re-matched with trading counterparts before moving to the next period—a process we refer as *second round matching*. In what follows we reconstruct the equilibria under this modification and show that the results of the benchmark model remain unchanged, subject to a modification in outside options. Since this exercise is a robustness check rather than a full blown analysis, we analyze the case $\varepsilon = 0$ in detail and then elaborate on what would happen if $\varepsilon < 0$ or $\varepsilon > 0$.

We assume that in each trading period two rounds of meetings take place. The first one is the matching process in the benchmark model. At the end of this round, inevitably, some buyers and sellers remain unmatched, so these players costlessly enter into a second round, where they are randomly matched with one another. One can specify a number of ways on how this may work, but to keep things simple and tractable we remain agnostic about the matching process, and simply assume that each buyer, regardless of his type, gets to trade with probability $\omega_{B,t}$ whereas each seller, regardless of whether he was fixed or flexible with the list price, gets to trade with probability $\omega_{S,t}$. The key observation is that, even in the second round players are not guaranteed to trade, i.e. the matching function may assign multiple buyers to a seller, in which case some buyers will be unable to buy, or it may assign no buyers to a seller, in which case the seller will have no choice but to wait for the next period. For now we take $\omega_{B,t}$ and $\omega_{S,t}$ as given, but at the end of this section we show how they might be tied to the fundamentals of the model, for example, via a standard urn-ball matching function.

Another issue that needs to be addressed is how the transaction in the second round is settled. This can be done in a number of ways, e.g. a fifty-fifty split, trading at the initially posted price and so on. Again, we remain agnostic about this mechanism, and instead assume that after a transaction in the second round the seller obtains payoff $p_t \in [\underline{p}_t, \overline{p}_t]$ and the buyer obtains $1 - p_t$. For now we take the boundaries of p_t as given but subsequently they will be pinned down endogenously.

Proposition 8 *Fix some $p_t \in [\beta\pi_{t+1}, 1 - \beta u_{t+1}]$. If $\theta > \bar{\theta}_t$ then there exists a continuum of equilibria, where an indeterminate fraction $\varphi^*_{f,t} \geq \eta_t$ of sellers trade via fixed pricing and remaining sellers trade via flexible pricing. Sellers post*

$$
\begin{aligned}
r^*_{f,t} &= 1 - \mu_{B,t} - \frac{z_1(\lambda_t)}{1 - z_0(\lambda_t)} \left(1 - \mu_{B,t} - \mu_{S,t}\right) \\
r^*_{b,t} &= 1 - \mu_{B,t} - \frac{z_1(\lambda_t)(1-\theta)}{1 - z_0(\lambda_t) - z_1(\lambda_t)} \left(1 - \mu_{B,t} - \mu_{S,t}\right)
\end{aligned}
$$

where

$$\mu_{B,t} = \omega_{B,t}(1 - p_t) + (1 - \omega_{B,t})\beta u_{t+1} \quad \text{and} \quad \mu_{S,t} = \omega_{S,t} p_t + (1 - \omega_{S,t})\beta\pi_{t+1}$$

In case negotiations ensue transaction occurs at price

$$y_t^* = 1 - \mu_{B,t} - \theta\left(1 - \mu_{B,t} - \mu_{S,t}\right)$$

The expected demand at each store equals to λ_t, however the equilibria are characterized by partial segmentation of customers: non-hagglers shop exclusively at fixed price firms whereas hagglers shop anywhere. In any equilibrium sellers and buyers earn

$$\begin{aligned}
\pi_t &= 1 - \mu_{B,t} - \left[z_0\left(\lambda_t\right) + z_1\left(\lambda_t\right)\right]\left(1 - \mu_{B,t} - \mu_{S,t}\right) \\
u_t &= z_0\left(\lambda_t\right)\left[1 - \mu_{B,t} - \mu_{S,t}\right] + \mu_{B,t}.
\end{aligned}$$

If $\theta < \overline{\theta}_t$ then fixed pricing emerges as the unique equilibrium: all sellers post $r_{f,t}^*$ and serve both types of customers. The total demand at each firm equals to λ_t and the equilibrium payoffs remain the same as above.

In the main text a buyer's outside option is βu_{t+1}, which is the present value of his expected payoff in the next period. With the prospect of second round meetings, his outside option is $\mu_{B,t} = \omega_{B,t}\left(1 - p_t\right) + \left(1 - \omega_{B,t}\right)\beta u_{t+1}$, which is a weighted average: with probability $\omega_{B,t}$ the buyer gets to trade in the second round and obtains $1 - p_t$ and with the complementary probability $1 - \omega_{B,t}$ he is unable to trade even in the second round, so he walks away with βu_{t+1}. Sellers' outside option $\mu_{S,t}$ can be interpreted similarly. A comparison between this proposition and its counterpart in the main text, Proposition 1, reveals that they are virtually identical if one updates the outside options with their current form in here, which indicates that the results remain robust.

The second round meeting gives customers and firms another chance to transact without incurring additional costs, as such, it diminishes trade frictions and improves everyone's outside options (one can show that $\mu_{B,t} > \beta u_{t+1}$ and $\mu_{S,t} > \beta\pi_{t+1}$). This effect is similar to raising the discount factor in the benchmark model. Indeed in the benchmark model βu_{t+1} and $\beta\pi_{t+1}$ can be improved simultaneously by raising β, which lowers waiting costs for everyone and renders trade frictions less biting.

An important question is whether players would like to trade immediately rather than waiting. Although we address this issue more technically in the proof of the proposition, the answer is yes—both in the first round as well as in the second round players are better off transacting whenever they have an opportunity to do so. Recall that in the second round buyers get payoff $1 - p_t$ and sellers get p_t. The fact that $p_t \in \left[\beta\pi_{t+1}, 1 - \beta u_{t+1}\right]$ ensures that both the firms and their customers are willing to trade during the second round meetings instead of waiting for the next period. If p_t falls outside these boundaries then either the firm or the customer will walk away, rendering second round meetings immaterial and causing the model to collapses to its version in the main text. The crucial question is, then, whether players would want to transact in the first round instead of waiting for the second round. The answer is, still yes. Trade frictions may be lessened by the prospect of second round meetings, but they are not completely wiped out as no one is guaranteed to a sure trade, and therefore players are better off trading immediately instead of waiting. It is worth pointing out that the prospect of second round meetings filters into the equilibrium objects, i.e. the prices and payoffs are determined taking into consideration the the new version of outside options, which convinces buyers and sellers to trade at those prices instead of waiting.

As mentioned above, the analysis is based on the case $\varepsilon = 0$; however given the results so far we can speculate on what would happen if $\varepsilon > 0$ or $\varepsilon < 0$. A detailed comparison between the proof of Proposition 8 and the proof of its counterpart in the main text, Proposition 1, reveals that both proofs follow virtually identical steps if one replaces the outside options in the benchmark with their current form. The parameter ε is orthogonal to the determination of outside options, as such we expect Propositions 2 and 3 , which correspond to cases $\varepsilon < 0$ and $\varepsilon > 0$, to go through in similar fashion.

Proof of Proposition 8. The proof is by induction; however the analysis of the terminal period is quite similar to the analysis of the inductive step; hence skipping it we directly analyze the inductive step pertaining period t.

Bargaining. The Nash product in this version of the model is given by

$$\max_{y_t \in [0,1]} \left(1 - y_t - \mu_{B,t}\right)^{\theta} \left(y_t - \mu_{S,t}\right)^{1-\theta}.$$

The solution yields

$$y_t = 1 - \mu_{B,t} - \theta \left(1 - \mu_{B,t} - \mu_{S,t}\right).$$

We assume that $y_t < r_{b,t}$, which requires θ to be sufficiently large, i.e. hagglers have sufficient bargaining power to negotiate the list price.

Expected payoff. We construct an equilibrium under the conjecture that non hagglers shop at fixed price stores whereas hagglers shop at both types of stores. Furthermore we conjecture that players transact immediately instead of waiting. We will verify both of these conjectures once we pin down equilibrium prices and payoffs. Along our conjecture, the expected utility of a high type buyer, who shops at a best offer store, is given by

$$U_{h,b,t} = z_0\left(q_{h,b,t}\right)\left(1 - y_t\right) + \frac{1 - z_0\left(q_{h,b,t}\right) - z_1\left(q_{h,b,t}\right)}{q_{h,b,t}}\left(1 - r_{b,t}\right) + \frac{q_{h,b,t} - 1 + z_0\left(q_{h,b,t}\right)}{q_{h,b,t}}\mu_{B,t}. \quad (51)$$

With probability $z_0\left(q_{h,b,t}\right)$ the buyer is alone at the store and purchases the item through negotiations at price y_t. With probability $z_n\left(q_{h,b,t}\right)$ he encounters $n = 1, 2, ..$ other buyers, and his probability of being able to buy is

$$\sum_{n=1}^{\infty} \frac{z_n\left(q_{h,b,t}\right)}{n+1} = \frac{1 - z_0\left(q_{h,b,t}\right) - z_1\left(q_{h,b,t}\right)}{q_{h,b,t}}.$$

If he manages to purchase, then he pays the list price $r_{b,t}$. Finally with the complementary probability he is unable to buy in the first round, so he obtains $\mu_{B,t}$. A flexible seller's profit is given by

$$\Pi_{b,t} = z_1\left(q_{h,b,t}\right) y_t + \left[1 - z_0\left(q_{h,b,t}\right) - z_1\left(q_{h,b,t}\right)\right] r_{b,t} + z_0\left(q_{h,b,t}\right) \mu_{S,t}$$

If there is a single customer then the transaction occurs at price y_t, if there are more than one customer then the transaction occurs at $r_{b,t}$ and if the seller does not get a customer then he obtains $\mu_{S,t}$. Given the expression for $U_{h,b,t}$ we can rewrite the profit function as follows

$$\Pi_{b,t} = 1 - z_0\left(q_{h,b,t}\right) - q_{h,b,t}U_{h,b,t} + \left[q_{h,b,t} - 1 + z_0\left(q_{h,b,t}\right)\right]\mu_{B,t} + z_0\left(q_{h,b,t}\right)\mu_{S,t}.$$

Now consider a fixed price store. Letting $q_{f,t} \equiv q_{h,f,t} + q_{l,f,t}$ denote the total expected demand, both types of buyers obtain the same expected utility at the fixed price store, where

$$U_{h,f,t} = U_{l,f,t} \equiv U_{f,t} = \frac{1 - z_0\left(q_{f,t}\right)}{q_{f,t}}\left(1 - r_{f,t}\right) + \frac{q_{f,t} - 1 + z_0\left(q_{f,t}\right)}{q_{f,t}}\mu_{B,t}. \tag{52}$$

The expression is similar to $U_{h,b,t}$ except for the fact that the transaction occurs at the fixed price $r_{f,t}$ even if there is a single customer at the store. A fixed price seller's profit is equal to

$$\Pi_{f,t} = \left[1 - z_0\left(q_{f,t}\right)\right]r_{f,t} + z_0\left(q_{f,t}\right)\mu_{S,t},$$

which can be rewritten as

$$\Pi_{f,t} = 1 - z_0\left(q_{f,t}\right) - q_{f,t}U_{f,t} + \left[q_{f,t} - 1 + z_0\left(q_{f,t}\right)\right]\mu_{B,t} + z_0\left(q_{f,t}\right)\mu_{S,t}.$$

Characterization of the Equilibrium. Recall that non hagglers shop at fixed price stores whereas hagglers shop at both types of stores. This means that $U_{l,f,t} = \bar{U}_{l,t}$ and $U_{h,f,t} = U_{h,b,t} = \bar{U}_{h,t}$. Since $U_{h,f,t} = U_{l,f,t}$ we have $\bar{U}_{h,t} = \bar{U}_{l,t} \equiv \bar{U}_t$. A flexible seller maximizes $\Pi_{b,t}$ subject to $U_{h,b,t} = \bar{U}_t$. Substituting the constraint into the objective function, the first order condition is given by

$$z_0\left(q_{h,b,t}\right) - \bar{U}_t + \left[1 - z_0\left(q_{h,b,t}\right)\right]\mu_{B,t} - z_0\left(q_{h,b,t}\right)\mu_{S,t} = 0.$$

It follows that

$$\Pi_{b,t} = 1 - z_0\left(q_{h,b,t}\right) - z_1\left(q_{h,b,t}\right) - \left[1 - z_0\left(q_{h,b,t}\right) - z_1\left(q_{h,b,t}\right)\right]\mu_{B,t} + \left[z_0\left(q_{h,b,t}\right) + z_1\left(q_{h,b,t}\right)\right]\mu_{S,t}.$$

Similarly a fixed price seller maximizes $\Pi_{f,t}$ subject to $U_{f,t} = \bar{U}_t$. The first order condition is given by

$$z_0\left(q_{f,t}\right) - \bar{U}_t + \left[1 - z_0\left(q_{f,t}\right)\right]\mu_{B,t} - z_0\left(q_{f,t}\right)\mu_{S,t} = 0,$$

which implies that

$$\Pi_{f,t} = 1 - z_0\left(q_{f,t}\right) - z_1\left(q_{f,t}\right) - \left[1 - z_0\left(q_{f,t}\right) - z_1\left(q_{f,t}\right)\right]\mu_{B,t} + \left[z_0\left(q_{f,t}\right) + z_1\left(q_{f,t}\right)\right]\mu_{S,t}.$$

In equilibrium sellers must earn equal profits, i.e. $\Pi_{f,t} = \Pi_{b,t}$; thus $q_{h,b,t} = q_{f,t} = q_{h,f,t} + q_{l,f,t}$. It follows that

$$q_{h,b,t} = \lambda_t,\ q_{h,f,t} = \lambda_t(\varphi^*_{f,t} - \eta_t)/\varphi^*_{f,t} \text{ and } q_{l,f,t} = \lambda_t\eta_t/\varphi^*_{f,t},$$

where $\varphi^*_{f,t}$ denotes the equilibrium fraction of fixed price sellers. Note that, $\varphi^*_{f,t}$ is indeterminate in that any value within $[\eta_t, 1]$ satisfies the equalities above; hence, there is a continuum of equilibria where any fraction $\varphi^*_{f,t} \geq \eta_t$ of sellers compete via fixed pricing while the rest compete via flexible pricing. Notice, however, in any equilibrium, the total expected demand at each firm equals to λ_t.

Now we can obtain expressions for equilibrium prices. Combining the first order condition of flexible sellers with indifference constraint $U_{h,b,t} = \bar{U}_t$ yields

$$z_0(\lambda_t) + [1 - z_0(\lambda_t)]\, \mu_{B,t} - z_0(\lambda_t)\, \mu_{S,t} = U_{h,b,t},$$

where $U_{h,b,t}$ is given by (51). Solving this equality for $r_{b,t}$ yields the expression for $r^*_{b,t}$ in the body of the proposition. The equilibrium fixed price $r^*_{f,t}$ is obtained likewise. The first order condition of fixed price sellers along with the indifference constraint $U_{f,t} = \bar{U}_{h,t}$ implies

$$z_0(\lambda_t) + [1 - z_0(\lambda_t)]\, \mu_{B,t} - z_0(\lambda_t)\, \mu_{S,t} = U_{f,t}$$

where $U_{f,t}$ is given by (52). Solving this equality for $r_{f,t}$ yields the expression for $r^*_{f,t}$ in the body of the proposition. High type buyers negotiate if $r^*_{b,t} \geq y^*_t$, which, after substituting for $r^*_{b,t}$ and y^*_t, is equivalent to $\theta \geq \overline{\theta}_t = z_1(\lambda_t) / [1 - z_0(\lambda_t)]$. Given the expressions for $r^*_{f,t}$ and $r^*_{b,t}$ one can verify that the equilibrium payoffs are as follows $\Pi_{b,t} = \Pi_{f,t} = \pi_t$ and $U_{h,b,t} = U_{f,t} \equiv u_t$, where π_t and u_t are given in the body of the proposition.

If $\theta < \overline{\theta}_t$ then $r^*_{b,t} < y_t$; thus no bargaining takes place as the list price $r^*_{b,t}$ is already below the bargained price y^*_t. As in the benchmark model, in this parameter region, the model collapses to a fixed-price setting.

Proof of Conjecture 1: Players transact immediately rather than waiting.

If $p_t \in [\beta\pi_{t+1}, 1 - \beta u_{t+1}]$ then sellers and buyers would be willing to trade in the second round instead of waiting for the next period. Indeed if $p_t \geq \beta\pi_{t+1}$ then the seller is better off transacting at p_t instead of waiting for period $t+1$ and obtaining $\beta\pi_{t+1}$. Similarly if $p_t \leq 1 - \beta u_{t+1}$ then the buyer is better off purchasing instead of waiting for the next period and getting βu_{t+1}.

Now consider the first round. It is straightforward to show that if $\theta \geq \overline{\theta}_t$ then $r^*_{b,t} > r^*_{f,t} > y^*_t$. From a sellers' perspective the worst case scenario is transacting at y^*_t, which is the lowest price. Similarly for a buyer the worst case scenario is purchasing at $r^*_{b,t}$. If they agree to transact under these worst case scenarios then they would agree to transact under more favorable prices.

Consider a buyer who contemplates buying at $r^*_{b,t}$. He would transact if $1 - r^*_{b,t} \geq \mu_{B,t}$, i.e. if his immediate surplus $1 - r^*_{b,t}$ exceeds his outside option $\mu_{B,t}$ associated with walking away at the end of of round 1. Basic algebra reveals that this inequality is satisfied if

$$\tfrac{z_1(\lambda_t)(1-\theta)}{1-z_0(\lambda_t)-z_1(\lambda_t)} \left(1 - \mu_{B,t} - \mu_{S,t}\right) > 0.$$

The left hand side is positive; hence the inequality holds. Now consider a seller, whose worst case scenario is selling at y^*_t. The seller transacts if $y^*_t \geq \mu_{S,t}$, i.e. if his immediate surplus y^*_t exceeds his outside option $\mu_{S,t}$ associated with walking away at the end of round 1. Basic algebra reveals that this inequality is satisfied if

$$(1-\theta)\left(1-\mu_{B,t}-\mu_{S,t}\right)>0.$$

Again, both expressions are positive; hence the seller, too, is willing to transact immediately.

Proof of Conjecture 2. Low types strictly prefer fixed price stores and high types are indifferent. A low type's expected utility at a best offer store is given by

$$U_{l,b,t}=\tfrac{1-z_0(\lambda_t)}{\lambda_t}\left(1-r^*_{b,t}\right)+\tfrac{\lambda_t-1+z_0(\lambda_t)}{\lambda_t}\mu_{B,t}$$

Substituting for $r^*_{b,t}$ it is easy to show if $\theta>\theta_t$ then $U_{l,b,t}<u_t$; confirming indeed that low types are better off staying away from best offer stores. To show that high types are indifferent between fixed and flexible stores we need to show that along the equilibrium path we have $U_{h,b,t}=U_{f,t}$. Substituting $r^*_{b,t}$ and $r^*_{f,t}$ it is a matter of basic algebra to verify that indeed this equality holds, confirming the validity of the conjecture. This completes the proof of the proposition. ■

Matching Function. Here we show how $\omega_{S,t}$ and $\omega_{B,t}$ may derived from the fundamentals of the model if one assumes that second round meetings are governed by "urn-ball matching", where all unmatched buyers (balls) and all unmatched sellers (urns) enter into a random matching process (see Petrongolo and Pissarides (2001)). Matching frictions are due to the random nature of the process—some urns receive several balls and others none. Given the process one can pin down the probabilities $\omega_{B,t}$ and $\omega_{S,t}$ as follows. Along the equilibrium outlined in the Proposition, at the end of the first round $s_t\left(1-z_0\left(\lambda_t\right)\right)$ sellers are matched. Players transact immediately and each transaction takes one buyer and one seller. This implies that $b_t-s_t\left(1-z_0\left(\lambda_t\right)\right)$ buyers and $s_t z_0\left(\lambda_t\right)$ sellers are not matched.[30] The buyer-seller ratio in the second round is equal to

$$\lambda'_t=\frac{b_t-s_t\left(1-z_0\left(\lambda_t\right)\right)}{s_t z_0\left(\lambda_t\right)}=\frac{\lambda_t-1+z_0\left(\lambda_t\right)}{z_0\left(\lambda_t\right)}.$$

The second equation follows from the fact that $s_t=b_t\lambda_t$. An unmatched seller's chance of being able to transact, $\omega_{S,t}$, is equal to the probability of meeting at least a buyer, i.e.

$$\omega_{S,t}=1-z_0\left(\lambda'_t\right).$$

Similarly, an unmatched buyer's chance of transacting in the second round is equal to

$$\omega_{B,t}=\sum_{n=0}^{\infty}\frac{z_n\left(\lambda'_t\right)}{n+1}=\frac{1-z_0\left(\lambda'_t\right)}{\lambda'_t}.$$

With probability $z_n\left(\lambda'_t\right)$ he encounters $n=0,1,2,\ldots$ other buyers there (recall that due to randomness

[30] The payoff in the second round is the same for all buyers, thus we do not need to keep track of high and low types during this process. As an aside, however, note that along the equilibrium in Proposition 8 high and low types trade at the same rate, thus, the ratio of high types to low types remains intact among unmatched buyers; see the analysis in the main text for a formal proof for this argument

of the process a seller may get more than one buyer), in which case he has a probability of $\frac{1}{n+1}$ obtaining the item (each buyer has an equal chance). The second equality follows from the facts that $z_{n+1}(x) = \frac{x z_n(x)}{n+1}$ and that $\sum_{n=0}^{\infty} z_n(x) = 1$. Notice that if $\lambda_t' > 1$ then $\omega_{S,t} > \omega_{B,t}$, i.e. if there are more buyers in the pool than sellers, then a seller is more likely to meet a trading partner than a buyer. The opposite is true if $\lambda_t' < 1$.

9.4 Sellers' Implementation Cost of Bargaining

In our model firms do not incur any implementation costs to sell via bargaining. However given the results on how the nature of equilibria respond to ε we can predict what would happen if sellers were to incur such a cost. Recall that if $\varepsilon = 0$ then both pricing rules are payoff equivalent and sellers are indifferent to pick either fixed pricing or flexible pricing. If, however, ε turns negative then the payoff equivalence breaks down and fixed pricing emerges as the unique outcome. From sellers' point of view the negative ε is an indirect cost. It is incurred by buyers, but nevertheless it bleeds into the sellers' profit functions and thereby induces them to switch to fixed pricing. The implication is that if an indirect cost can disturb the payoff equivalence between fixed and flexible pricing then a direct cost will result in the same outcome, i.e. introducing a cost of implementing bargaining into the setting $\varepsilon = 0$ would cause flexible sellers to earn less, and thereby, lead to a fixed price equilibrium. Needless to say, introducing such a cost into a setting with $\varepsilon < 0$ will only reinforce the fixed price outcome.

If, however, $\varepsilon > 0$ then the outcome is less clear because along Eq-FS sellers are able to convert the positive ε into higher prices and, thereby, earn higher profits compared to a fixed price equilibrium. So, if one inserts an implementation cost into the framework with $\varepsilon > 0$ then whether or not sellers would revert back to fixed pricing depends on how this cost compares with the difference in profits. If the cost is prohibitively large then we would expect a fixed price equilibrium to emerge and if the cost is sufficiently small then Eq-FS should survive, albeit with fewer flexible stores (compared to the benchmark model with no cost).

9.5 Game with Infinite Horizon

In our model the market runs for a finite number of periods, i.e. $T < \infty$. Under this specification one can solve the model recursively by substituting the terminal payoffs $u_{T+1} = \pi_{T+1} = 0$ into the equilibrium conditions to obtain payoffs for period T, which then can be substituted to obtain payoffs for period $T-1$, and so on. The method is straightforward, but more importantly, one does not need to worry about how market demand fluctuates over time, driven by the tuple $\{b_t^{new}, s_t^{new}, \eta_t^{new}\}_{t=2}^{T}$.

If $T = \infty$ then one can prove existence of equilibrium and analytically characterize a solution if the market exhibits some cyclicality, i.e. if agents face the same outlook, say, every k periods. The cyclical nature of the model would allow us to prove analogous versions of Propositions 1, 2 and 3 using induction and then, again, exploiting cyclicality we can pin down equilibrium payoffs and prices. As an example focus on the setting with $\varepsilon = 0$ and consider the simplest possible scenario where the environment is fully stationary in that outgoing agents are replaced by incoming agents one for one (In other words incoming agents are clones of the outgoing agents.) With perfect replacement the number of buyers and sellers, and therefore the expected demand λ_t, remains constant at all times. Since players face the same market outlook irrespective of the calendar time, equilibrium payoffs π_t and u_t, and thereby, equilibrium prices are also time independent, which allows us to solve the model analytically. (To prove existence of the equilibrium one needs to virtually repeat the steps outlined in the proof of Proposition 1). Dropping the time subscripts from equations (17) and (18), we have

$$\pi = 1 - \beta u - [z_0(\lambda) + z_1(\lambda)](1 - \beta u - \beta\pi) \quad \text{and} \quad u = z_0(\lambda)[1 - \beta u - \beta\pi] + \beta u.$$

This is a simple system with two equations and two unknowns (π and u), which can be solved easily. Once π and u are pinned down, the equilibrium prices and probabilities readily follow. This solution concept is rather straightforward, but as λ_t starts to fluctuate the system of equations grows rapidly. For instance if λ_t is high in even periods and low in odd periods then we would have a system of four equations and four unknowns (π_{odd}, π_{even}, u_{odd}, u_{even}) to deal with. In general if the cycles lasts k periods then one needs to solve a system of $2k$ equations and unknowns. Needless to say, as k grows large an analytic solution becomes elusive.

If the model cannot be solved analytically then one can fix T at some large value and pick some arbitrary values for terminal payoffs u_{T+1} and π_{T+1} and solve the model via the aforementioned recursive method. The solution would be accurate for $t << T$ because the impact of terminal payoffs vanishes if t is sufficiently far away from T (because of discounting). Our simulations seem to confirm this insight. We fixed $T = 360$, $\beta = 0.95$ and ran simulations for a number of arbitrary values of $u_{T+1} \in [0, 1]$ and $\pi_{T+1} \in [0, 1]$ and saw no impact of the terminal payoffs on equilibrium objects (prices and payoffs) for $t < 350$ or so. In other words, the terminal payoffs seemed to have an influence only for the last ten periods or so, indicating that the numerical solutions were accurate in the preceding periods. Needless to say, the accuracy can be extended by picking a larger T or a smaller β.