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Gains from Variety? Product Differentiation and the Possibility of Losses from Trade under Cournot Oligopoly with Free Entry

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Gains from Variety?

Product Differentiation and the Possibility of Losses from Trade under Cournot Oligopoly with Free Entry

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Abstract

In a free-entry Cournot oligopoly model with a quadratic utility function that yields differentiated products, it is shown that there are losses from trade when the trade cost is close to the prohibitive level. Although the total number of varieties increases, there is a reduction in consumer surplus. This occurs because trade leads to an increase in imported varieties where consumer surplus is low due to the high trade cost and a decrease in domestically-produced varieties where consumer surplus is high. This result is in contrast with results from the free-entry Cournot oligopoly models with homogeneous products of Brander and Krugman (1983) and Venables (1985); the monopolistic competition models such as Krugman (1980) and Venables (1987), and heterogeneous firm models such as Melitz (2003) and Melitz and Ottaviano (2008).

Keywords: Gains from Trade, Trade Liberalisation, Free Entry, Cournot Oligopoly, Product Variety.

JEL Classification: F12.

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1. Introduction

An important source of gains from trade identified by the *new* trade theory is the increase in product variety available to consumers. The gains from increased variety were first demonstrated by Krugman (1979) using a monopolistic competition model with a general *love of variety* utility function and by Krugman (1980) using a CES utility function that became standard in the monopolistic competition literature. In a monopolistic competition model with trade costs, Venables (1987) showed that there were unambiguously gains from trade due to increased variety. The magnitude of the gains from increased product variety were measured by Broda and Weinstein (2006) and found to be significant. In the *new new* trade theory with heterogeneous firms started by Melitz (2003), which was an extension of Krugman (1980), there were aggregate productivity gains as well as the gains from increased variety. Melitz and Ottaviano (2008) used a quadratic utility function rather than the usual CES utility function and found that there were gains from increased variety as well as the productivity gains. Although, according to Arkolakis et al. (2008) and Arkolakis, Costinot, and Rodríguez-Clare (2012) these new sources of the gains from trade have not affected the magnitude of the estimated gains from trade.

In parallel, the literature on trade under oligopoly started by Brander (1981) has generally concentrated on the case of homogeneous products with Cournot competition and an exogenous number of firms. In such models trade has a pro-competitive effect leading to gains from increased competition if trade costs are fairly low and the possibility of losses from trade if trade costs are close to the prohibitive level. With free entry and exit of firms and trade costs, Brander and Krugman (1983) and Venables (1985) demonstrated that there were always gains from trade as trade lowers the market price and increases consumer surplus. In a model with differentiated products, using a quadratic utility function, but with no trade costs, Bernhofen

(2001) showed that there are always gains from trade due to the pro-competitive effect and the effect of increased product variety.

In this note, international trade will be analysed in a free-entry Cournot oligopoly model with a quadratic utility function that yields differentiated products, and it is shown that there are losses from trade when trade costs are close to the prohibitive level. Although the total number of varieties increases as a result of international trade, there is a loss of consumer surplus due to the decrease in domestically-produced varieties that outweighs the gains in consumer surplus from the additional imported varieties as consumption of these varieties is infinitesimally small when the trade cost is close to the prohibitive level.

2. The Cournot Oligopoly Model

Suppose that there are two symmetric countries, a home country labelled as A and a foreign country labelled as B. In each country, there is an imperfectly competitive industry producing a differentiated products and a perfectly competitive industry producing a homogeneous good using a constant returns to scale technology. The imperfectly competitive industry consists of identical firms that each have a constant marginal cost c and a fixed cost F. Free entry and exit of firms ensures that profits are equal to zero in equilibrium and this determines the number of firms n in each country. There is also a per-unit trade cost t, which may be a real trade cost, such as a transport cost, or an import tariff that generates revenue for the country. Since the two countries are symmetric and all the firms are identical, the equilibria in the two countries will be symmetric. Therefore, the analysis will derive the equilibrium in the home country then exploit the symmetry between the two countries to work out the equilibrium number of firms and the welfare effects of international trade. The quantity sold by the ith oligopolistic firm from country $C = A, B$ in the home country is x_{Ci}, and the price it receives is p_{Ci}. The quantity of the numeraire good sold in the home country is z and its price is normalised at unity, $p_z = 1$. It is assumed that there is a representative consumer in each country with quasi-linear preferences that can be represented by a quadratic utility function, adapted from Vives (1985):

$$u(\mathbf{x}, z) = \alpha \sum_{C=A}^{B} \sum_{i=1}^{n} x_{Ci} - \frac{\beta}{2} \left[\sum_{C=A}^{B} \sum_{i=1}^{n} x_{Ci}^2 + 2\phi \left(\sum_{i=1}^{n} \sum_{j=1}^{n} x_{Ai} x_{Bj} + \sum_{C=A}^{B} \sum_{i \neq j} x_{Ci} x_{Cj} \right) \right] + z \quad (1)$$

where $\alpha > c > 0$, $\beta > 0$ and $\phi \in [0,1]$. Note that $1/\beta$ is a measure of the size of the market and ϕ is the degree of product substitutability for the differentiated products ranging from zero when the products are independent to one when the products are perfect substitutes. Also, it should be stressed that this utility function exhibits the same *love of variety* effect as the CES utility function widely used in monopolistic competition and heterogeneous firm models. This can be seen by fixing total consumption of the differentiated products at X then, assuming symmetry, consumption of each variety is $x = X/n$, and it can be shown that utility is increasing in the number of varieties, $\partial u/\partial n = \beta(2-\phi)x^2/2 > 0$.

Utility maximisation by the representative consumer yields the inverse demand facing the ith oligopolistic firm from each of the two countries in the home country:

$$\begin{aligned} p_{Ai} &= \alpha - \beta \left[x_{Ai} - \phi \left(\sum_{j \neq i} x_{Aj} + \sum_{j=1}^{n} x_{Bj} \right) \right] \\ p_{Bi} &= \alpha - \beta \left[x_{Bi} - \phi \left(\sum_{j \neq i} x_{Bj} + \sum_{j=1}^{n} x_{Aj} \right) \right] \end{aligned} \quad (2)$$

Consumer surplus will be required for the welfare analysis of trade liberalisation. Since the utility function is quasi-linear, consumer surplus is a valid measure of consumer welfare that in the home country is given by:

$$CS = u - \sum_{C=A}^{B} \sum_{i=1}^{n} p_{Ci} x_{Ci} - z = \frac{\beta}{2} \left[\sum_{C=A}^{B} \sum_{i=1}^{n} x_{Ci}^2 + 2\phi \left(\sum_{i=1}^{n} \sum_{j=1}^{n} x_{Ai} x_{Bj} + \sum_{C=A}^{B} \sum_{i \neq j} x_{Ci} x_{Cj} \right) \right] \quad (3)$$

Since the foreign oligopolistic firms face the per-unit trade cost t when they supply the market in the home country, the gross profits in the home country of the *i*th firm from each of the two countries are:

$$\pi_{Ai} = \left(p_{Ai} - c\right)x_{Ai} \qquad \pi_{Bi} = \left(p_{Bi} - c - t\right)x_{Bi} \tag{4}$$

The gross profits of the firms in the market of the foreign country are defined analogously except that the home firms now face the trade cost. The total net profits of the *i*th firm in each of the two countries will be the gross profits in each of the two countries minus the fixed cost. The equilibrium number of firms will be determined by free entry and exit so that total net profits are equal to zero in equilibrium.

3. The Free-Entry Cournot Equilibrium under Autarky

Under autarky, there is no trade with the foreign country so the home country only consumes the goods produced by the home firms. The demand facing the *i*th oligopolistic firm in the home country under autarky is given by setting $x_{Bj} = 0$ for $j = 1, \ldots, n$ in the first inverse demand function in (2). Hence, differentiating the first equation for profits in (4), yields the first-order conditions for the Cournot-Nash equilibrium:

$$\frac{\partial \pi_{Ai}}{\partial x_{Ai}} = \alpha - 2\beta x_{Ai} - \phi\beta \sum_{j \neq i} x_{Aj} - c = 0 \qquad i = 1, \ldots, n \tag{5}$$

By symmetry, since all firms are identical they will all produce the same quantity in the Cournot-Nash equilibrium, $x_{Ai} = x_{Aj} = x_A$, receive the same price, $p_{Ai} = p_{Aj} = p_A$, and earn the same profits $\pi_{Ai} = \pi_{Aj} = \pi_A$. Solving for the Cournot-Nash equilibrium, taking the number of firms as given, yields the output, price and profits of each firm:

$$x_A = \frac{\alpha - c}{\beta\left(2 + (n-1)\phi\right)} \qquad p_A = c + \frac{(\alpha - c)}{\left(2 + (n-1)\phi\right)} \qquad \pi_A = \frac{(\alpha - c)^2}{\beta\left(2 + (n-1)\phi\right)^2} \tag{6}$$

Under autarky, the home firms do not export to the foreign market so their total net profits are the gross profits made in the home market minus the fixed cost, $\Pi_A = \pi_A - F$. With free entry and exit, total net profits will be equal to zero and this will determine the equilibrium number of firms. Solving for the equilibrium number of firms under autarky, while ignoring the integer constraint yields:

$$n_A^N = \frac{\Omega}{\phi\sqrt{\beta F}} \qquad \text{where} \quad \Omega \equiv (\alpha - c) - (2-\phi)\sqrt{\beta F} > 0 \tag{7}$$

Assume that the fixed cost is sufficiently low so that the denominator is positive, $\Omega > 0$ or $F < (\alpha - c)^2 / \beta(2-\phi)^2$, which ensures that entry occurs into the imperfectly competitive industry. Substituting the equilibrium number of firms (7) into (6) yields the free-entry Cournot-Nash equilibrium output and price under autarky:

$$x_A^N = \sqrt{\frac{F}{\beta}} \qquad p_A^N = c + \sqrt{\beta F} \tag{8}$$

Welfare of the home country under autarky is given by the sum of consumer surplus plus the profits of the home firms, but profits are zero in equilibrium, therefore welfare is just given by consumer surplus. Using symmetry in (3) then using (7) and (8) yields consumer surplus under autarky:

$$W_A^N = CS_A^N = \frac{\beta}{2}\left[n_A^N\left(x_A^N\right)^2 + \phi n_A^N\left(n_A^N - 1\right)\left(x_A^N\right)^2\right] = \frac{\Omega}{2\phi\beta}\left(\alpha - c - \sqrt{\beta F}\right) \tag{9}$$

4. The Free-Entry Cournot Equilibrium with International Trade

If the two countries open up to international trade multilaterally then firms in the home country can export their products to the foreign country and consumers in the home country can buy imported products from the foreign firms. The inverse demand functions facing the firm are given by (2) and the gross profits of the firms are given by (4) in the market of the home country. Note that the foreign firm has to pay the per-unit trade cost t on its exports to the home country. Hence, the first-order conditions for a Cournot-Nash equilibrium with international trade in the home country are:

$$\begin{aligned}\frac{\partial \pi_{Ai}}{\partial x_{Ai}} &= \alpha - 2\beta x_{Ai} - \phi\beta\left(\sum_{j\neq i} x_{Aj} + \sum_{j=1}^{n} x_{Bj}\right) - c = 0 \\ \frac{\partial \pi_{Bi}}{\partial x_{Bi}} &= \alpha - 2\beta x_{Bi} - \phi\beta\left(\sum_{j=1}^{n} x_{Aj} + \sum_{j\neq i} x_{Bj}\right) - c - t = 0\end{aligned} \tag{10}$$

By symmetry, since all the home firms are identical they will all produce the same quantity in the Cournot-Nash equilibrium, $x_{Ai} = x_{Aj} = x_A$, receive the same price, $p_{Ai} = p_{Aj} = p_A$, and earn the same profits $\pi_{Ai} = \pi_{Aj} = \pi_A$ in the market of the home country. Similarly, since all the foreign firms are identical: $x_{Bi} = x_{Bj} = x_B$, $p_{Bi} = p_{Bj} = p_B$ and $\pi_{Bi} = \pi_{Bj} = \pi_B$ in the market of the home country. Also, symmetry implies that the number of firms in each country will be identical in the two countries in equilibrium. Solving for the Cournot-Nash equilibrium, taking the number of firms as given, yields the output of each firm:

$$x_A = \frac{(2-\phi)(\alpha-c) + n\phi t}{\beta\left[(2-\phi)(2+(2n-1)\phi)\right]} \qquad x_B = \frac{(2-\phi)(\alpha-c) - (2+(n-1)\phi)t}{\beta\left[(2-\phi)(2+(2n-1)\phi)\right]} \tag{11}$$

As usual in a Cournot oligopoly, it can be shown that the mark-ups of the home firms and the foreign firms are: $p_A - c = \beta x_A$ and $p_B - c - k = \beta x_B$. Hence, the gross profits of the home firms and the foreign firms, respectively, are: $\pi_A = \beta x_A^2$ and $\pi_B = \beta x_B^2$. The total net profits of a home firm are equal to the sum of gross profits in the two countries minus the fixed cost, $\Pi_A = \pi_A + \pi_A^* - F$, where π_A^* is the gross profits of a home firm from exporting to the foreign country and, by symmetry, this is equal to the gross profits of a foreign firm from exporting to the home country, $\pi_A^* = \pi_B$. Therefore, the total net profits of a home firm are:

$$\Pi_A = \frac{1}{2\beta}\left[\frac{\left(2(\alpha-c)-t\right)^2}{\left(2+(2n-1)\phi\right)^2}+\frac{t^2}{(2-\phi)^2}\right]-F \tag{12}$$

Free entry and exit of firms will drive total net profits to zero, $\Pi_A = 0$, which will determine the equilibrium number of firms. Solving for the equilibrium number of firms, which is the same in both countries, while ignoring the integer constraint, yields:

$$n^T = \frac{(2-\phi)\left[\left(2(\alpha-c)-t\right)\sqrt{\Phi}-\Phi\right]}{2\Phi\phi} \qquad \text{where} \quad \Phi \equiv 2(2-\phi)^2\beta F - t^2 > 0 \tag{13}$$

Therefore, substituting (13) into (11) yields the free-entry Cournot-Nash equilibrium outputs with international trade:

$$x_A^T = \frac{\sqrt{\Phi}+t}{2\beta(2-\phi)} \qquad x_B^T = \frac{\sqrt{\Phi}-t}{2\beta(2-\phi)} \tag{14}$$

Note that the trade cost will be prohibitive so that there will be no trade, $x_B^T = 0$, if $t \geq \sqrt{\Phi} > 0$, and hence, using the definition of Φ from (13), the prohibitive trade cost is $\overline{t} = (2-\phi)\sqrt{\beta F}$. Also, by symmetry, the total output of a home (or foreign) firm is $x_A^T + x_B^T = \sqrt{\Phi}/\beta(2-\phi)$, which is decreasing in the trade cost since $\partial\Phi/\partial t < 0$.

5. Welfare Effects of International Trade

The welfare effects of international trade can now be analysed by comparing the equilibrium with free trade with the equilibrium under autarky. Two cases will be considered: (i) when the trade cost is a real trade cost such as a transport cost, and (ii) when the trade cost is an import tariff that generates revenue. In the first case, since the profits of the domestic firms are equal to zero in equilibrium and the trade cost generates no revenue for the government, the welfare of the home country is just the consumer surplus of the home country.

Using symmetry together with (13) and (14), welfare (consumer surplus) of the home country with international trade is:

$$\begin{aligned} W_A^T(t) &= \frac{\beta}{2}\left[n^T\left(x_A^T\right)^2 + n^T\left(x_B^T\right)^2 + \phi\left(2\left(n^T\right)^2 x_A^T x_B^T + n^T\left(n^T-1\right)\left(\left(x_A^T\right)^2+\left(x_B^T\right)^2\right)\right)\right] \\ &= \frac{1}{8\Phi\phi\beta}\left[\left(2(\alpha-c)-t\right)\sqrt{\Phi}-\Phi\right]\left[\left(2(\alpha-c)-t\right)\sqrt{\Phi}-\Phi+2(1-\phi)(2-\phi)\beta F\right] \end{aligned} \tag{15}$$

Consider the case of totally free trade where the trade cost is equal to zero, $t = 0$, considered by Bernhofen (2001). It is straightforward to show that with a move from autarky to free trade there is a decrease in the quantity sold by each home (foreign) firm in the home (foreign) market, but an increase in the total quantity sold by each firm in the two markets, and consequently prices are lower since the firms move down their average cost curve. There is a decrease in the number of firms in each country, but there is an increase in the total number of varieties available to the consumers as they consume products from both the home and foreign firms. As a result, due to the usual *love of variety* effect, there is an increase in consumer surplus in both the countries and gains from trade. The welfare (consumer surplus) of the home country is given by setting the trade cost equal to zero, $t = 0$, in (15), which yields:

$$W_A^T(0) = \frac{1}{4\beta\phi}\left(\sqrt{2}(\alpha-c)-\sqrt{\beta F}\right)\left(\sqrt{2}(\alpha-c)-\sqrt{\beta F}(2-\phi)\right) > W_A^N \tag{16}$$

Now consider the case when the trade cost is positive and, in particular, the case when the trade cost is close to the prohibitive level. Welfare with international trade (relative to welfare under autarky) is plotted in figure one as a function of the trade cost (for the parameter values: $\alpha = 50$, $\beta = 1$, $\phi = 1/3$, $c = 14$, and $F = 12$). Obviously, when the trade cost is prohibitive, $t \geq \bar{t}$, the equilibrium in each country will be exactly the same as under autarky and therefore welfare (consumer surplus) will be exactly the same as under autarky, $W_A^T(\bar{t}) = W_A^N$. A reduction in the trade cost evaluated at the prohibitive level, $t = \bar{t}$, will lead to a reduction in the number of firms in both countries since, using (13), $\partial n^T / \partial t = \Omega / \phi(2-\phi)\beta F > 0$. The effect on the welfare (consumer surplus) of the home country is obtained by differentiating (15) and evaluating at the prohibitive trade cost:

$$\left.\frac{\partial W_A^T}{\partial t}\right|_{t=\bar{t}} = \frac{(1-\phi)\Omega}{2\phi\beta(2-\phi)} \geq 0 \tag{17}$$

This is strictly positive if products are differentiated, $\phi \in [0,1)$, and equal to zero if products are homogeneous, $\phi = 1$, as in Venables (1985). Therefore, with differentiated products, welfare is upward-sloping at the prohibitive trade cost as shown in figure one and there will be a range of values for the trade cost, $t \in [\underline{t}, \bar{t}]$, where welfare with international trade is lower than welfare under autarky so there are losses from trade. The explanation is that the increased competition with international trade reduces the profits of the home firms and leads to the exit of some firms. As a result, consumers have fewer domestically-produced varieties, and the imported varieties provide little consumer surplus as consumption of these varieties is infinitesimally small. Since the model is symmetric, both countries will lose from trade if the trade cost is close to the prohibitive level. These results lead to the following proposition:

Proposition: *When products are differentiated,* $\phi \in [0,1)$*, and the trade cost is close to the prohibitive level,* $t \in [\underline{t}, \bar{t}]$*, there are losses from trade for both countries.*

Now consider the case when the trade cost is an import tariff that generates revenue for the government. In this case, the only difference with the previous case is that the welfare of the home country is the sum of consumer surplus and the revenue generated by the import tariff, $\tilde{W}_A^T(t) = CS + tn^T x_B^T = W_A^T(t) + tn^T x_B^T$. Welfare with international trade (relative to welfare under autarky) is plotted in figure one as a function of the import tariff. Obviously, the difference between $\tilde{W}_A^T(t)$ and $W_A^T(t)$ is the revenue generated by the import tariff, and they are equal under totally free trade, $t=0$, $W_A^T(0)=\tilde{W}_A^T(0)$. When the import tariff is at the prohibitive level, the effect on the welfare of the home country of a (multilateral) change in the import tariff is:

$$\left.\frac{\partial W_A^T}{\partial t}\right|_{t=\bar{t}} = -\frac{(3-\phi)\Omega}{2\phi\beta(2-\phi)} < 0 \tag{18}$$

This is unambiguously negative therefore welfare is downward-sloping at the prohibitive import tariff as shown in figure one. In this case, the loss of consumer surplus from domestically-produced varieties is outweighed by the revenue generated by the import tariff. Therefore, there is no range of values for the import tariff where there are losses from trade, and a multilateral reduction of import tariffs will increase welfare in both countries.

6. Conclusions

This note has demonstrated the possibility of losses from trade in a free-entry Cournot oligopoly model with differentiated products when the trade cost is close to the prohibitive level despite an increase in the total number of varieties available to consumers. The gains from the availability of imported varieties are outweighed by the loss of domestically-produced varieties as consumption of each of the imported varieties is infinitesimally small when the trade cost is close to the prohibitive level. Since the model is symmetric, both countries will lose from trade if the trade cost is close to the prohibitive level.

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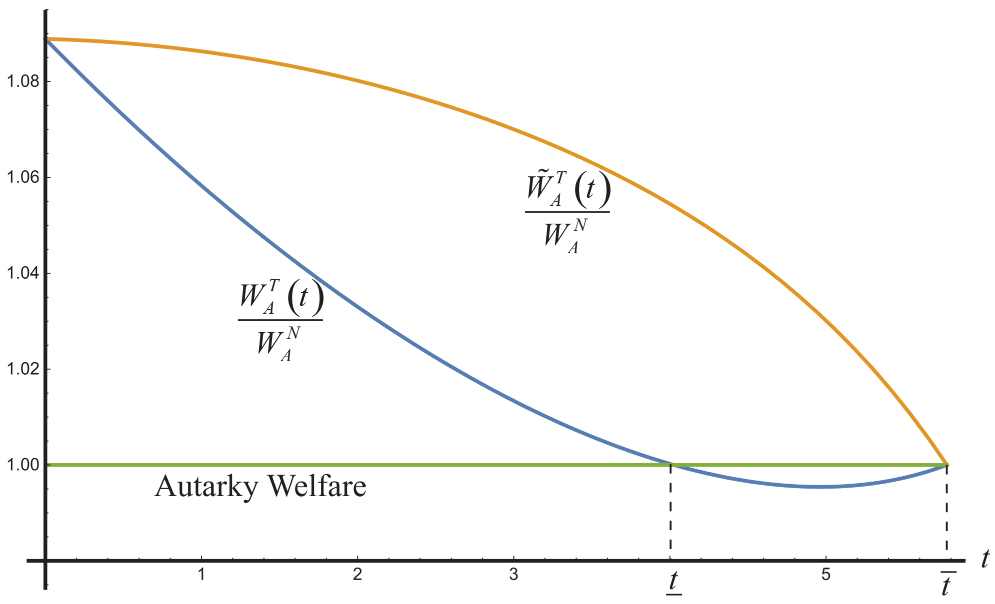

Figure 1: Welfare Effects of International Trade