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Patrick Minford, Michael Wickens and Yongdeng Xu

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Cardiff Business School
Cardiff University
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
business.cardiff.ac.uk

Testing part of a DSGE model by Indirect Inference

Patrick Minford (Cardiff University and CEPR)*

Michael Wickens (Cardiff University, University of York and CEPR)†

Yongdeng Xu (Cardiff University)‡

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Abstract

We propose a new type of test. Its aim is to test subsets of the structural equations of a DSGE model. The test draws on the statistical inference for limited information models and the use of indirect inference to test DSGE models. Using Monte Carlo experiments on two subsets of equations of the Smets-Wouters model we show that the model has accurate size and good power in small samples. In a test of the Smets-Wouters model on US Great Moderation data we reject the specification of the wage-price but not the expenditure sector, pointing to the first as the source of overall model rejection.

Keywords: sub sectors of models, limited information, indirect inference, testing DSGE models equations, Monte Carlo, power, test size

JEL Classification: C12; C32; C52; E1

1 Introduction

In this paper we show how it is possible to test subsets of the structural equations of a DSGE model using indirect inference (II). Although a key feature of DSGE models is that they represent general equilibrium, implying that the model is a complete structural system, it may be of interest to examine individual structural equations or subsets of equations. One reason is that econometric tests of DSGE models, although rare, commonly lead to their rejection. It may therefore be useful to find which of the structural equations is causing the whole model to be rejected.

*Patrick.minford@btinternet.com; Cardiff Business School, Cardiff University, CF10 3EU, UK

†Cardiff Business School, Cardiff University, CF10 3EU, UK

‡Corresponding author. Tel.: +44 (0)29 208 74150; Email: xuy16@cf.ac.uk; Cardiff Business School, Cardiff University, CF10 3EU, UK

This problem falls under the category of limited information inference. In deriving our test we make use of a key result on statistical inference for limited information models by Godfrey and Wickens(1982). Given a subset of the structural equations of an econometric model, they show that a simple way to estimate and test these equations is to augment them with unrestricted reduced form equations of the endogenous variables not explained but included in the subset. Full information statistical procedures can then be used on the resulting complete model. For example, using full information maximum likelihood estimation would give the standard limited information maximum likelihood estimator. This method can easily be adapted for use in indirect inference and applied to DSGE models.

As explained in Le et al. (2011, 2016), indirect inference is well-suited to estimating and testing DSGE models. It can also be used if, as is common, the model has already been estimated by Bayesian methods. The solution of a (linearised) DSGE model where the exogenous variables can be represented by a VAR is a VAR in the complete set of variables, endogenous and exogenous. The VAR coefficients are functions of the structural parameters of the DSGE model and are therefore restricted. An II test is provided by comparing the VAR estimated using data simulated from the estimated DSGE model with the original data used to estimate the DSGE model. Le et al. (2011, 2016) propose using a Wald test based on the VAR coefficients. Alternatively, the test could be based on other features of the model, such as the associated impulse response functions or the moments - Minford, Wickens and Xu (2016) compare the test with these features and find that mostly the properties are quite similar.

Le et al. (2016) have shown that a Wald test is very powerful; falsifying the DSGE model's coefficients by as little as 7% usually results in a 100% rejection. As the solution of a DSGE model is a restricted VAR, a natural way to modify indirect inference for testing a subset of DSGE structural equations is to augment this subset with unrestricted VAR equations derived from the solution to the the corresponding, but not necessarily specified, complete DSGE model. These unrestricted VAR equations together with the subset of DSGE equations form a new complete, but limited information, DSGE model. The solution to this completed DSGE model will be a VAR that incorporates the restrictions from only the subset of equations. Indirect inference can now be carried out as before by simulating data from the completed DSGE model and comparing the estimates of the unrestricted VAR based on the simulated and the original data.

We apply this test to two subsets of equations of the widely-used Smets and Wouters (2007) DSGE model (hereafter SWUS), namely, the wage-price sector and the expenditure equations, the consumption-investment sector. The model is based on post-war data. There is particular interest in the wage-price subset as II tests of the complete SWUS model by Le et al.(2011) reject the model, but a modified version of the model that specifies greater price flexibility is not rejected for the Great Moderation period. Given that one of the principal aims of the SWUS model is to modify the real business cycle model by including sticky prices on the grounds that this may be why the RBC model is usually rejected by the data, the wage-price sector is a critical part of the SWUS model. We find that the limited information II test confirms our original conclusion that the price-wage sector is misspecified. We also find that although the consumption-investment sector is not rejected during the period of the Great Moderation 1980Q1-2004Q4, it is rejected for the whole sample 1947Q1-2004Q4.

After setting out the relevant theory for the testing of limited information DSGE models, we examine the size and power properties of these tests comparing them with a full information II test of the DSGE model where, ideally, a researcher would test the estimated parameters of the subset of the model, keeping the rest of the model's parameters fixed at their true values. In practice, of course, these true values are unavailable. We find that the size of the proposed test is accurate and that the power of the test for the misspecification of the price-wage sector is higher than that for the consumption-investment sector which suggests that the latter is less important in the model's overall specification.

The remainder of the paper is organized as follows. Section 2 describes the limited information DSGE model consisting of a subsector of the corresponding, but not necessarily specified, full DSGE model. Section 3 explains Indirect Inference and the testing procedure. Section 4 evaluates the performance of the II test for a limited information DSGE model of the wage-price and consumption-investment sectors of the SWUS model using Monte Carlo simulations based on small sample performance. In Section 5 we carry out a test of the SWUS model for US post-war data. Section 6 concludes.

2 The limited information DSGE model

DSGE models (possibly after linearization) have the general form:

$$\begin{aligned} A_0 E_t y_{t+1} &= A_1 y_t + B z_t \\ z_t &= R z_{t-1} + \varepsilon_t \end{aligned} \tag{1}$$

where y_t contains the endogenous variables and z_t the exogenous variables. The exogenous variables may be observable or unobservable. For example, they may be structural disturbances. We assume that z_t may be represented by an autoregressive process with disturbances ε_t that are $NID(0, \Sigma)$. Assuming that the conditions of Fernandez-Villaverde, Rubio-Ramirez, Sargent and Watson (2007) are satisfied, the solution to this model can be represented by a VAR of form

$$\begin{bmatrix} y_t \\ z_t \end{bmatrix} = F \begin{bmatrix} y_{t-1} \\ z_{t-1} \end{bmatrix} + G \begin{bmatrix} \xi_t \\ \varepsilon_t \end{bmatrix}. \tag{2}$$

where ξ_t are innovations.

Consider a subset of the structural equations of this complete DSGE model in which y_{1t} are the endogenous variables that are (partially) determined in this subset and y_{2t} are the remaining endogenous variables where $y_t = (y_{1t}, y_{2t})'$.

The subset of structural equations may be written

$$A_{01}E_t y_{1t+1} + A_{02}E_t y_{2t+1} = A_{11}y_{1t} + A_{12}y_{2t} + B_1 z_t. \tag{3}$$

Where variables are not included in this subset the corresponding elements are zero.

In order to estimate or to test this subset of equations we augment the subset with unrestricted versions of the solution to y_{2t} derived from the full DSGE model for which only the subset of interest is assumed to be specified; the structural equations for y_{2t} are not specified. This gives the completed model

$$\begin{aligned}\begin{bmatrix} A_{01} & A_{02} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} E_t y_{1,t+1} \\ E_t y_{2,t+1} \\ E_t z_{t+1} \end{bmatrix} &= \begin{bmatrix} A_{11} & A_{12} & B_1 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} y_{1t} \\ y_{2t} \\ z_t \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 & 0 \\ F_{21}^U & F_{22}^U & F_{23}^U \\ 0 & 0 & R \end{bmatrix}\begin{bmatrix} y_{1,t-1} \\ y_{2,t-1} \\ z_{t-1} \end{bmatrix} \\ &+ \begin{bmatrix} 0 & 0 & 0 \\ G_{21}^U & G_{21}^U & G_{23}^U \\ 0 & 0 & I \end{bmatrix}\begin{bmatrix} \xi_{1t} \\ \xi_{2t} \\ \varepsilon_t \end{bmatrix}. \end{aligned} \tag{4}$$

where the superscript U denotes that the matrix is unrestricted[1]. This completed model is, in effect, a DSGE model with limited information. The solution to this limited information DSGE model is also a VAR but with coefficient restrictions reflecting only the structural restrictions in the subset of equations.

A special case of the DSGE model is where all of the exogenous variables are unobservable and may be regarded as structural shocks. An example is the SWUS model to be examined below. This case and its solution can be represented as above both for the complete DSGE model and the limited information DSGE model.

3 Indirect inference

A full explanation of how to test a DSGE model using the method of indirect inference is explained in Le et al. (2011, 2016). This can be applied to tests of a subset of the structural equations of a DSGE model by exploiting their representation as a limited information DSGE model. In the case of a fully specified DSGE model, we bootstrap N samples of simulated data from the model. We then estimate the auxiliary model formed from the solution to the DSGE model which we represent as a VAR(1) using both the actual data and the N samples of simulated data. Denoting these respectively by a_T and a_S $(S = 1, ..N)$, where α is a vector of all of the VAR coefficients, we then use a Wald statistic (WS) based on the difference between a_T, the estimates of

[1] In practice, these matrices are estimated by OLS from eq(2).

the VAR coefficients derived from actual data, and $\overline{a_S(\theta_0)}$, the mean of their distribution based on the simulated data. The test statistic is

$$WS = (a_T - \overline{a_S(\theta_0)})' W(\theta_0)(a_T - \overline{a_S(\theta_0)}) \tag{5}$$

where the covariance $W(\theta_0)$ can be calculated directly from the bootstrap samples:

$$W(\theta_0)^{-1} = \frac{1}{N} \Sigma_{k=1}^{N} (a_k - \overline{a_k})'(a_k - \overline{a_k})$$

WS is asymptotically a $\chi^2(r)$ distribution, with the number of restrictions equal to the number of elements in a_T. We carry out the test based on the empirical distribution of WS which can be obtained by bootstrapping. [2]

In applying this test procedure to a subset of structural equations of a DSGE model we simulate the data using the completed limited information DSGE model, equation (5) with unrestricted reduced form equations for y_2 in place of structural equations. We may then carry out the test as above; in practice as explained in Le et al (2016) we use a VAR1 in a limited set of y variables to obtain an appropriate level of power.

4 Evaluating the test in small samples via Monte Carlo Experiments

The size and power properties of this subset test for small samples may be examined using Monte Carlo experiments. The sample size is chosen as 200, which is typical for macro data. We design Monte Carlo simulations following the approach in Le et al (2016). We use the SWUS model as a full DSGE model and create 1000 samples from this model, which is assumed to be true. Then we obtain from these samples the distribution of the Wald statistic by bootstrapping (the bootstrap number is 500) when the model is true. We use this distribution to assess how many times the x% False model is rejected with 95% confidence.

The false models are generated as follows. We keep fixed the VAR coefficients of the endogenous variables that are not of interest and only falsify the coefficients in the structural equations in the subset of interest. We generate the falseness by introducing an increasing degree of numerical mis-specification of the parameters. Thus we construct a model whose parameters are moved x% away from their true values in both directions. [3][4]

We are interested in whether our test a) is accurate as a test of whether the subset model is false b) guides us accurately on whether the subset model

[2] The Appendix in Minford, Wickens and Xu (2016) shows the steps involved in finding the Wald statistic.

[3] See Le et al (2016) section 4.1 for full details of the experiments.

[4] For all the experiments, the eigenvalues of reduced form VAR coefficients are all strictly less than unity in modulus, so the Fernandez-Villaverde, Rubio-Ramirez, Sargent and Watson (2007) condition that the DSGE model has a VAR representation is satisfied.

is causing the whole model to be rejected. For a) we examine how the subset model is rejected by the behaviour of the subset variables alone as it becomes more false: the power of the test against the subset variables' own behaviour. For b) we examine how the subset model is rejected by the behaviour of the main whole-model variables: the power of the test against general macro behaviour. To check the test accuracy in each case we repeat the Monte Carlo analysis using the true parameters in the rest of the model: this MC experiment tells us the true effect of the falsity of submodel parameters.

4.1 Size and power of the subset test.

4.2 Price-wage equations

We begin with the price-wage equation subset of the SWUS model. These are derived under the assumption of Calvo contracts as New-Keynesian Phillips curves, respectively for price- and wage-setting as:

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 E_t \pi_{t+1} - \pi_3(\alpha(k_t^s - l_t) - w_t) + \varepsilon_t^p. \tag{6}$$

$$\begin{aligned} w_t \quad = \quad & w_1 w_{t-1} + (1-w_1)(E_t w_{t+1} + E_t \pi_{t+1}) - w_1 \pi_t + w_3 \pi_{t-1} \\ & -w_4(w_t - (\sigma_l l_t + \frac{1}{1-\lambda/\gamma}(c_t - \lambda/\gamma c_{t-1})) + \varepsilon_t^w. \end{aligned} \tag{7}$$

The price and wage mark up disturbances follow an AR(1) process: $\varepsilon_t^p = \rho_p \varepsilon_{t-1}^p + \eta_t^p$, $\varepsilon_t^w = \rho_w \varepsilon_{t-1}^w + \eta_t^w$.[5]

The two key endogenous variables are π_t and w_t. The other endogenous variables that are presented in the full SWUS model are $y_t, c_t, i_t, l_t, w_t, k_t^s, q_t$. The two exogenous shocks are ε_t^p and ε_t^w. Call this the 'subset model'.

We begin our analysis by testing the subset model on only the two variables (π_t and w_t) from the subset, using a VAR(1) as the auxiliary model. Here we would expect the subset model to be rejected at a fast-rising rate as its parameters are falsified, because it is being tested only on these variables themselves.

Table 1 gives the size and power of the test on wage-price subset equations. We falsify the parameters in price-wage subset and fix the unrestricted VAR for other endogenous variables. We use two types of falseness: +/- x% alternative and +/- x% randomly. We report the power of the test at 5% theoretical size and empirical size. We also report the rejection rate for the "ideal model", i.e. one where we have the true model and then progressively falsify the coefficients of only the subset. Instead of replacing the rest of the model with VAR equations, we use the true parameter values. This tests whether substituting the VAR projection for the rest of the model - the key practical aspect of our method - causes inaccuracy in small samples.

[5] See Smets and Wouters (2007) for the full model and details of the two equations, including an explanation of the parameters.

Table 1: Rejection Rates at 95% level: test 2 variables (p_t, w_t)VAR(1).

Falseness is given by +/- x% alternation								
	0%	1%	3%	5%	7%	10%	15%	20%
Theoretical size	0.080	0.079	0.109	0.253	0.472	0.880	0.977	0.976
Empirical size	0.050	0.047	0.081	0.166	0.370	0.808	0.935	0.948
"Ideal" model	0.050	0.059	0.136	0.370	0.758	0.998	1.000	1.000
Falseness is given by +/- x% randomly								
	0%	1%	3%	5%	7%	10%	15%	20%
Theoretical size	0.079	0.117	0.176	0.574	0.727	0.801	1.000	1.000
Empirical size	0.050	0.070	0.126	0.454	0.630	0.731	0.999	1.000
"Ideal" model	0.050	0.074	0.372	0.402	0.568	0.798	0.999	1.000

The size of the test (when the falseness is 0%) is about 8%, which is very close to the 5% theoretical size. This shows that our proposed test has good size. In practice we can use a Monte Carlo analysis of the test to correct its size; we do this in the second row of the table. As expected, the test has considerable power: the greater the falseness, the higher are the rejection rates. When the falseness is about 10%, the model will be rejected more than 70% of the time.

Most interestingly, the test using our VAR method mirrors rather closely what would occur if we were able to test the submodel with full knowledge of the other equations' true parameters (the "ideal" model).

We now repeat the analysis using the same auxiliary model that we used to test the whole model: a three variable VAR(1) in the key variables y_t(output), π_t(inflation), r_t (interest rate). Here we might expect that increasing falseness would cause the rejection rate to increase more slowly because other variables are included in the auxiliary model.

Table 2: Rejection Rates at 95% level: falseness is given by +/- alternation, test 3 variables (y_t, p_t, r_t) VAR(1).

	0%	1%	3%	5%	7%	10%	15%	20%
Empirical size	0.050	0.044	0.059	0.137	0.283	0.718	0.861	0.865
"Ideal" model	0.050	0.058	0.158	0.455	0.775	0.851	0.930	0.978

The power is lower but still large, indicating that the price/wage equation subset is important in causing the whole model to be rejected. Again we see that the VAR method mirrors reasonably well what the ideal method would give us, though in this case it has less than ideal power at lower levels of falsity.

4.2.1 The consumption-investment equations

The dynamics of consumption are derived from the consumption Euler equation and those of investment from the investment Euler equation, to yield respectively:

$$c_t = c_1 c_{t-1} - (1 - c_1) E_t c_{t+1} + c_2(l_t - E_t l_{t+1}) - c_3(r_t - E\pi_{t+1} + \varepsilon_t^b). \quad (8)$$

$$i_t = i_1 i_{t-1} + (1 - i_1)E_t i_{t+1} + i_2 q_t + \varepsilon_t^i. \quad (9)$$

The consumption and investment mark-up disturbance follows an AR(1) process: $\varepsilon_t^b = \rho_b \varepsilon_{t-1}^b + \eta_t^b$, $\varepsilon_t^i = \rho_i \varepsilon_{t-1}^i + \eta_t^i$. The two key endogenous variables are c_t and i_t. The other endogenous variables that are presented in full SWUS model are $y_t, l_t, w_t, r_t, \pi_t, k_t^s$, q_t, which are generated from the unrestricted VAR.

We begin our analysis by testing the subset model on only the two variables (c_t and i_t) from the subset, using a VAR(1) as the auxiliary model.

Table 3: Rejection Rates at 95% level: test 2 variables (p_t, w_t) VAR(1).

Falseness is given by +/- x% alternation								
	0%	1%	3%	5%	7%	10%	15%	20%
Theoretical size	0.094	0.205	0.071	0.244	0.139	0.194	0.271	0.147
Empirical size	0.050	0.126	0.036	0.156	0.079	0.116	0.154	0.063
"Ideal" model	0.050	0.055	0.057	0.056	0.056	0.069	0.103	0.140
Falseness is given by +/- x% randomly								
	0%	1%	3%	5%	7%	10%	15%	20%
Theoretical size	0.096	0.160	0.231	0.066	0.057	0.378	0.332	0.253
Empirical size	0.050	0.086	0.138	0.035	0.032	0.286	0.215	0.175
"Ideal" model	0.050	0.060	0.059	0.056	0.132	0.072	0.124	0.308

Table 3 gives the size and power of the test on consumption-investment subset equations. The size of the test is 9.5%. Considering the sample size and small sample bias for VAR estimates, this is not far from the 5% theoretical level. The difference from previous experiments is that the power of the test is very weak. When we falsify the parameters in the consumption-investment subset equations, the rejection rates are low and do not increase much with increasing falsity. This is also true for the "ideal" testing procedure which assumes knowledge of the true parameters in the rest of the model. The latter indicates that the consumption-investment subset is not important in determining the performance of the whole model.

We now repeat the analysis using the same auxiliary model as we used to test the whole model: a three variable VAR(1) in the key variables y_t (output), π_t (inflation), r_t (interest rate).

Table 4: Rejection Rates at 95% level: falseness is given by +/- alternation, test 3 variables (y_t, p_t, r_t) VAR(1).

	0%	1%	3%	5%	7%	10%	15%	20%
Empirical size	0.050	0.098	0.036	0.110	0.092	0.118	0.134	0.161
"Ideal" model	0.050	0.061	0.067	0.088	0.111	0.171	0.337	0.511

The power of the test is still very weak: falsifying the parameters of this subset generates low rejection rates that hardly increase with rising falsity. This is also true for our ideal testing procedure where we know the other parameters; when falsity is extreme in the subset parameters, the power does improve. Again this indicates that the consumption-investment subsector of the model would not cause the whole model to be rejected unless it is extremely false (15% or more). Under such extreme falsity our VAR method fails to have ideal power.

5 Applying the method to two subsets on US post-war data

One of the purposes of this test is to judge whether when a DSGE model is rejected it is only parts of the model that are 'causing trouble' and whether this is because they are highly misspecified or only a little misspecified.

In their work evaluating the Smets and Wouters model on US post-war data, Le et al (2011) found that the model as estimated by SWUS with Bayesian methods was decisively rejected by the II test on the full post-war sample. A 'New Classical' version of the model was also decisively rejected. They then considered a compromise version in which a competitive sector coexisted with a sticky-price sector in both the labour and goods markets. They found that, once re-estimated by II, this compromise or 'hybrid' version was not rejected for the Great Moderation period, but was rejected for both earlier periods and for the whole sample. It would therefore seem probable that the wage-price subset in its original form may be a problematic component of the SWUS model. This subsector can be compared with the consumption-investment subset which has not been found to be problematic.

We use the subset test on both sectors for the whole sample where the model seems to have quite general problems and for the period of the Great Moderation. This gave the following results

Table 5: p-values of transformed Wald statistics in indirect inference test.

Great Moderation 1980Q1-2004Q4		Post War 1947Q1-2004Q4	
Wage-price	Consumption-investment	Wage-price	Consumption-investment
0.000	0.140	0.000	0.003

For the Great Moderation we obtain a p-value for the wage-price subset of 0.000 and a p-value of consumption-investment subset of 0.140. These results support our conjecture that the problem did indeed lie with the price-wage sector and not the spending sector.

For the full post-war sample both subsets are strongly rejected: the p-value of price-wage subset is 0.000 and the p-value of the consumption-investment is 0.003. These results suggest that the model may be mis-specified as a representation of the whole period; fixing the wage-price sector seems not to be sufficient.

6 Conclusion

This paper has proposed a new type of test: a test of a subset of equations of a DSGE model. It is not common to test DSGE models, especially if they are estimated using Bayesian methods. Based on the available evidence, for example Le et al. (2011), there is a strong likelihood that if they were tested they would be rejected. This raises the question of whether rejection is due to the whole model being misspecified or just sectors of the model. Le et al. (2011), for example, present evidence that suggests that the wage-price sector of the Smets-Wouters model of the US is misspecified. Given that one of the principal aims of their model is to modify the real business cycle model by including sticky prices on the grounds that this may be why the RBC model is usually rejected by the data, the wage-price sector is a critical part of the SWUS model.

The proposed new test involves testing a subset of the structural equations of a DSGE model using indirect inference. The test is a modification of the II test proposed by Le et al.(2016) for complete DSGE models and draws on the theory of estimating and testing limited information models.

We examined the properties of the test using Monte Carlo experiments on two subsets of equations of the SWUS model, the wage-price and the expenditure sectors. We found that the test has accurate size and good power. We also carried out tests of these two sectors using US post-war data. We found that for the period of the Great Moderation the wage-price sector is rejected and both sectors are rejected using the whole sample. This supports earlier work that suggested that rejection of the SWUS model over the Great Moderation period was due to the wage-price sector in the model having excessive stickiness. We conclude that this test can be a useful tool for analysing empirical weaknesses in DSGE models.

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