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Information-Revelation and Coordination Using Cheap Talk in a Game with Two-Sided Private Information*

Chirantan Ganguly[†] and Indrajit Ray[‡]

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Abstract

We consider a Bayesian game, namely the Battle of the Sexes with private information, in which each player has two types, High and Low. We allow cheap talk regarding players' types before the game and prove that the unique fully revealing symmetric cheap talk equilibrium exists for a low range of prior probability of the High-type. This equilibrium has a desirable *type-coordination property*: it fully coordinates on the ex-post efficient pure Nash equilibrium when the players' types are different. Type-coordination is also obtained in a partially revealing equilibrium in which only the High-type is not truthful, for a medium range of prior probability of the High-type.

Keywords: Battle of the Sexes, Private Information, Cheap Talk, Coordination, Full Revelation.

JEL Classification Numbers: C72.

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1 INTRODUCTION

In games with multiple (Nash) equilibria, players need to coordinate their actions in order to achieve one of the equilibrium outcomes. Such a coordination problem is more severe in situations where none of the equilibria can be naturally selected, such as in the Battle of the Sexes (BoS, hereafter) game. As we already know, coordination in such games can be obtained using pre-play cheap talk (Farrell 1987).¹ Parallel to the theory, the experimental literature also shows that cheap-talk and any pre-play non-binding communication can significantly improve coordination in games like BoS (Cooper *et al* 1989; Crawford 1998; Costa-Gomes 2002; Camerer 2003; Burton *et al* 2005).

The coordination problem in the BoS type games may be more complicated with incomplete information, where each player has private information about the “intensity of preference” for the other player’s favorite outcome. It is not clear whether coordination using cheap talk (as in the theoretical and the experimental literature with the complete information BoS) would extend to a Bayesian game; moreover, it is not obvious at all whether truthful revelation and thereby separation of players’ types can be achieved in a cheap talk equilibrium.

To analyze the above two issues, we use the simplest possible version of the BoS (as in Banks and Calvert 1992) with two types (“High” and “Low”) for each player regarding the payoff from the other player’s favorite outcome. Apart from its applications,² the BoS with private information is clearly of interest to theorists and experimentalists.

The question we ask is whether in the above $2 \times 2 \times 2$ Bayesian game, players will reveal their types in a direct cheap talk equilibrium and also coordinate on Nash equilibrium outcomes in different states of the world. With incomplete information, efficiency and coordination do not necessarily go together; however, one might find it desirable to coordinate on the (ex-post) efficient outcome when the two players are of different types, in which the compromise is made by the player who suffers a smaller loss in utility.

The structure of the game we consider here has an in-built tension for each player between the desire to compromise in order to avoid miscoordination and the desire to force coordination on one’s preferred Nash equilibrium outcome. This contrasts with the Hawk-Dove game studied in Baliga and Sjöström (2012) and the Cournot game in Goltsman and Pavlov (2014) where a player’s preference over the other player’s action does not depend on his type or action. In this game, it is not a priori

¹In a seminal paper, Farrell (1987) showed that rounds of cheap talk regarding the intended choice of play reduces the probability of miscoordination; the probability of coordination on one of the two pure Nash equilibria increases with the number of rounds of communication (although, at the limit, may be bounded away from 1). Park (2002) identified conditions for achieving efficiency and coordination in a similar game with three players.

²The complete information BoS has many economic applications (see the Introduction in Cabrales *et al* 2000); the corresponding game of incomplete information is not just a natural extension but is also relevant in many of these economic situations where the intensity of preference and its prior probability are important factors.

clear at all whether either full information revelation or coordination can be achieved.

Prior to our current paper, Ganguly and Ray (2009) have analysed this game to see if a truthful cheap talk equilibrium, in which the players reveal their types truthfully before playing, exists at all and compared it with the mediated equilibrium of Banks and Calvert (1992). We improve upon their result; we formally characterise a class of cheap talk equilibria for this Bayesian game augmented by just one stage of direct unmediated cheap talk in which both players make simultaneous announcements on their possible types only. Banks and Calvert (1992) also studied unmediated communication in a similar game allowing more general message spaces (i.e., not restricted to only two types) in the communication phase. Banks and Calvert identified conditions (Proposition 2, Section 4 in their paper) under which the outcome of an ex-ante efficient incentive compatible mediated mechanism can be achieved as the equilibrium of an unmediated communication process. In contrast, the focus of our current paper is to identify conditions under which (full) revelation occurs at the cheap talk stage and some form of coordination property holds. Obviously, these objectives are different from those studied in Banks and Calvert (1992) and may be satisfied even when overall ex-ante efficiency cannot be achieved by unmediated communication.

The main contributions of this paper are thus two-fold. We first revisit the previous work of Ganguly and Ray (2009) regarding the fully revealing equilibrium. We actually correct their result and we here prove that there exists a unique fully revealing symmetric cheap talk equilibrium of this game in which the players announce their types truthfully (Theorem 1).³ The allowable range of the prior probability of the High-type for the fully revealing equilibrium to exist has to be moderately low - the upper bound being strictly less than $\frac{1}{2}$ - suggesting that full revelation is not a cheap talk equilibrium when the probability of a player being High-type is too high or too low.

Second and perhaps more important, following Farrell (1987), we check if the desirable coordination in this context is achieved in a (fully or partially revealing) cheap talk equilibrium outcome. Our unique fully revealing cheap talk equilibrium has the desirable *type-coordination property*: when the players' types are different, it fully coordinates on the ex-post efficient pure Nash equilibrium.⁴ We then ask whether the type-coordination can be achieved in a partially revealing cheap talk equilibria (where the players are not truthful in their cheap talk) particularly when the fully revealing equilibrium does not exist. Keeping the spirit of the fully revealing equilibrium, we consider a class of partially revealing cheap talk equilibria in which only the High-type is not truthful, while the Low-type is truthful. We

³Considering symmetric communication processes seems reasonable here (otherwise, one can always generate trivial asymmetric equilibria where only one player communicates). Since the players are identical, facing an identical symmetric situation ex-ante, we study (type and player) symmetric cheap talk equilibria, following the tradition in the literature (as in Farrell 1987 and Banks and Calvert 1992).

⁴It is easy to check that the fully efficient symmetric outcome in all four states, involving a lottery over the Nash outcomes when players' types are identical, cannot be obtained as a fully revealing cheap talk equilibrium.

analyse this particular type of partial revelation because under the type-coordination property, the High-type is expected to compromise and coordinate on his less preferred outcome when the other player claims to be of Low-type. We identify the unique partially revealing cheap talk equilibrium with the type-coordination property in this set of equilibria⁵ and prove its existence based on the prior probability of the High-type being within a range that turns out to be non-overlapping and higher than that for the fully revealing equilibrium (Theorem 2). We illustrate all these results using a running numerical example in different sections of the paper.

Following the seminal paper by Crawford and Sobel (1982), much of the cheap talk literature has focused on the sender-receiver framework whereby one player has private information but takes no action and the other player is uninformed but is responsible for taking a payoff-relevant decision. There indeed is a small but growing literature on games where both players have private information and can send cheap talk messages to each other.⁶ We have contributed to this literature by analysing symmetric cheap talk equilibria in a game with two-sided information and two-sided cheap talk.

2 MODEL

2.1 The Game

We consider a version of the BoS with incomplete information as given below, in which each of the two players has two strategies, namely, A and B . The payoffs are as in the following table, in which the value of t_i is the private information of player i , $i = 1, 2$, with $0 < t_1, t_2 < 1$. Also, the payoffs to both players from the miscoordinated outcome is normalised to 0, while the payoff to the player 1 (player 2) from (A, A) ((B, B)) is normalised to 1.

		Player 2	
		A	B
Player 1	A	$1, t_2$	$0, 0$
	B	$0, 0$	$t_1, 1$

We assume that t_i is a discrete random variable that takes only two values L and H (where, $0 < L < H < 1$), whose realisation is only observed by player i . For $i = 1, 2$, we henceforth refer to the

⁵We also characterised the complete set of partially revealing cheap talk equilibria in which only the High-type is not truthful while the Low-type is truthful (Proposition 2 in this paper).

⁶Examples of information transmission using two-sided cheap talk under two-sided incomplete information can be found in Farrell and Gibbons (1989), Matthews and Postlewaite (1989), Baliga and Morris (2002), Doraszelski *et al* (2003), Baliga and Sjöström (2004), Chen (2009), Goltsman and Pavlov (2014) and Horner *et al* (2015). Two-sided cheap talk using multiple stages of communication where only one of the players has incomplete information has also been studied by Aumann and Hart (2003) and Krishna and Morgan (2004).

values of t_i as player i 's type (Low, High). We further assume that each player's type is independently drawn from the set $\{L, H\}$ according to a probability distribution with $Prob(t_i = H) = p \in [0, 1]$.

The unique symmetric Bayesian-Nash equilibrium⁷ of this game can be characterised by $\sigma_i(s_i | t_i)$, the probability that player i of type t_i plays the pure strategy s_i .

Proposition 1 *The unique symmetric Bayesian Nash equilibrium of the BoS with incomplete information is given by the following strategy for player 1 (player 2's strategy is symmetric and is given by $\sigma_1(A|t) = \sigma_2(B|t)$, $t = H, L$):*

$$\begin{aligned} \sigma_1(A|H) &= 0 \text{ and } \sigma_1(A|L) = \frac{1}{(1-p)(1+L)} \text{ when } p < \frac{L}{1+L}, \\ \sigma_1(A|H) &= 0 \text{ and } \sigma_1(A|L) = 1 \text{ when } \frac{L}{1+L} \leq p \leq \frac{H}{1+H}, \\ \sigma_1(A|H) &= 1 - \frac{H}{p(1+H)} \text{ and } \sigma_1(A|L) = 1 \text{ when } p > \frac{H}{1+H}. \end{aligned}$$

The proof is straightforward and hence has been omitted here.

Example 1 *To illustrate our game, consider the following numerical example, in which the payoff of the H-type of player 1 from (B, B) is $\frac{2}{3}$, twice that of the L-type as described in the following table.*

	A	B		A	B		A	B		A	B
A	1, $\frac{2}{3}$	0, 0	A	1, $\frac{1}{3}$	0, 0	A	1, $\frac{2}{3}$	0, 0	A	1, $\frac{1}{3}$	0, 0
B	0, 0	$\frac{2}{3}, 1$	B	0, 0	$\frac{2}{3}, 1$	B	0, 0	$\frac{1}{3}, 1$	B	0, 0	$\frac{1}{3}, 1$
	Types: HH		Types: HL			Types: LH			Types: LL		

Suppose, in this example the (independent) prior probability of the H-type, $Prob(t_i = \frac{2}{3})$ is $\frac{1}{5}$. Then, the unique symmetric Bayesian Nash equilibrium of this game is given by the following symmetric strategy profile: player 1 plays A with probability $\frac{15}{16}$ when the type is Low and plays the pure strategy B when the type is High (player 2's strategy is symmetric and is B with probability $\frac{15}{16}$ when the type is Low and A when the type is High) which generates the following distribution over the outcomes for different type profiles (states of the world).

	A	B		A	B		A	B		A	B
A	0	0	A	0	0	A	$\frac{15}{16}$	0	A	$\frac{15}{256}$	$\frac{225}{256}$
B	1	0	B	$\frac{1}{16}$	$\frac{15}{16}$	B	$\frac{1}{16}$	0	B	$\frac{1}{256}$	$\frac{15}{256}$
	Types: HH		Types: HL			Types: LH			Types: LL		

⁷The corresponding game with complete information with commonly known values t_1 and t_2 , has two pure Nash equilibria, (A, A) and (B, B), and a mixed Nash equilibrium in which player 1 plays A with probability $\frac{1}{1+t_2}$ and player 2 plays B with probability $\frac{1}{1+t_1}$.

2.2 Cheap Talk

We study an extended game in which the players are first allowed to have a round of simultaneous canonical cheap talk intending to reveal their private information before they play the above BoS. In the first (cheap talk) stage of this extended game, each player i simultaneously chooses a costless and nonbinding announcement τ_i from the set $\{L, H\}$. Then, given a pair of announcements (τ_1, τ_2) , in the second (action) stage of this extended game, each player i simultaneously chooses an action s_i from the set $\{A, B\}$.

An announcement strategy in the first stage for player i is a function $a_i : \{L, H\} \rightarrow \Delta(\{L, H\})$, where $\Delta(\{L, H\})$ is the set of probability distributions over $\{L, H\}$. We write $a_i(H | t_i)$ for the probability that strategy $a_i(t_i)$ of player i with type t_i assigns to the announcement H . Thus, the announcement τ_i of player i with type t_i is a random variable drawn from $\{L, H\}$ according to the probability distribution with $Prob(\tau_i = H) = a_i(H | t_i)$.

In the second (action) stage, a strategy for player i is a function $\sigma_i : \{L, H\} \times \{L, H\} \times \{L, H\} \rightarrow \Delta(\{A, B\})$, where $\Delta(\{A, B\})$ is the set of probability distributions over $\{A, B\}$. We write $\sigma_i(A | t_i; \tau_1, \tau_2)$ for the probability that strategy $\sigma_i(t_i; \tau_1, \tau_2)$ of player i with type t_i assigns to the action A when the first stage announcements are (τ_1, τ_2) . Thus, player i with type t_i 's action choice s_i is a random variable drawn from $\{A, B\}$ according to a probability distribution with $Prob(s_i = A) = \sigma_i(A | t_i; \tau_1, \tau_2)$. Given a pair of realised action choices $(s_1, s_2) \in \{A, B\} \times \{A, B\}$, the corresponding outcome is generated. Thus, given a strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$, one can find the players' actual payoffs from the induced outcomes in the type-specific payoff matrix of the BoS and hence, the (ex-ante) expected payoffs.

As the game is symmetric, in our analysis, we maintain the following notion of symmetry in the strategies, for the rest of the paper.

Definition 1 *A strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$ is called announcement-symmetric (in the announcement stage) if $a_i(H | t_i) = a_{-i}(H | t_i)$; a strategy profile is called action-symmetric (in the action stage) if $\sigma_i(A | t; \tau_1, \tau_2) = \sigma_{-i}(B | t; \tau_2, \tau_1)$, for all t, τ_1, τ_2 . A strategy profile is called symmetric if it is both announcement-symmetric and action-symmetric.*

Note that Definition 1 preserves symmetry for both players and the types for each player. We consider the following standard notion of equilibrium in this two-stage cheap talk game.

Definition 2 *A symmetric strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$ is called a symmetric cheap talk equilibrium if (i) given the announcement strategies (a_1, a_2) , the action strategies (σ_1, σ_2) constitute a Bayesian-Nash equilibrium in the simultaneously played second stage BoS with the posterior beliefs*

generated by (a_1, a_2) and (ii) the announcement strategies (a_1, a_2) are Bayesian-Nash equilibrium of the simultaneous announcement game given the action strategies (σ_1, σ_2) to be followed.

Definition 2 suggests that a symmetric cheap talk equilibrium can be characterised by a set of (symmetric) equilibrium constraints (2 for the announcement stage and another possible 8 for the action stage).

3 RESULTS

We present some symmetric cheap talk equilibria in this section. We first consider the possibility of full revelation of the types as a result of our canonical cheap talk.

3.1 Fully Revealing Equilibrium

We consider a specific class of strategies in this subsection where we impose the property that the cheap talk announcement should be fully revealing.

Definition 3 A symmetric strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$ is called fully revealing if the announcement strategy a_i reveals the true types with certainty, i.e., $a_i(H|H) = 1$ and $a_i(H|L) = 0$.

We now characterise the fully revealing symmetric cheap talk equilibrium. We first consider a specific fully revealing (separating) strategy profile that we call $S_{separating}$, influenced by the equilibrium action profile in Farrell (1987) for the complete information version of this game. In this strategy profile, the players announce their types truthfully and then in the action stage, they play the mixed Nash equilibrium strategies of the complete information BoS when both players' types are identical and they play (B, B) $((A, A))$, when only player 1's type is H (L).

We state and prove our first result below.⁸

Theorem 1 $S_{separating}$ is the unique fully revealing symmetric cheap talk equilibrium and it exists only for $\frac{L^2 + L^2 H}{1 + L + L^2 + L^2 H} \leq p \leq \frac{LH + LH^2}{1 + L + LH + LH^2}$.

We have postponed the proof of the above theorem to the Appendix of this paper.

3.1.1 Comments and Illustration

Claim 1 The ex-ante expected payoff for any player from $S_{separating}$ is given by $EU_{separating} = p^2 \frac{H}{1+H} + p(1-p)(1+H) + (1-p)^2 \frac{L}{1+L}$, which is increasing over the range of p where it exists.

⁸Theorem 1 in this paper corrects and thus improves upon the main result presented in Ganguly and Ray (2009).

Claim 2 The upper bound for p in Theorem 1, $\frac{LH+LH^2}{1+L+LH+LH^2}$ is always $< \frac{1}{2}$, since $\frac{1}{2} - \frac{HL+H^2L}{1+L+HL+H^2L} = \frac{(1+L-LH-LH^2)}{2(1+L+HL+H^2L)} > 0$, as long as $L < H < 1$.

To understand why p must lie in such a low range for this equilibrium to exist, consider the incentives for deviations by player 1 at the announcement stage. By deviating and claiming to be an L -type, player 1(H -type) gains $1 - \frac{H}{1+H} = \frac{1}{1+H}$ when player 2 is a H -type (with probability p) and loses⁹ $H - \frac{H}{1+L} = \frac{HL}{1+L}$ when player 2 is a L -type (with probability $1 - p$). Since $\frac{1}{1+H} - \frac{HL}{1+L} = \frac{(1+L-LH-LH^2)}{(1+H)(1+L)} > 0$, the gain from the deviation when playing against player 2(H -type) is bigger than the loss when playing against player 2(L -type). If a H -type is equally or more likely than a L -type, then player 1(H -type) will obviously deviate and truthful revelation will not be an equilibrium. So, p must be $< \frac{1}{2}$. Indeed, p needs to be small enough to make the above deviation unattractive and the precise value of p for which this holds is $\frac{HL+H^2L}{1+L+HL+H^2L}$ or less. However, p cannot be too close to 0 either. This is because of incentives for deviations by player 1(L -type). By deviating and claiming to be an H -type, player 1(L -type) gains $L - \frac{L}{1+L} = \frac{L^2}{1+L}$ when player 2 is a L -type (with probability $1 - p$) and loses $1 - \frac{H}{1+H} = \frac{1}{1+H}$ when player 2 is a H -type (with probability p). Since $\frac{1}{1+H} - \frac{L^2}{1+L} = \frac{(1+L-HL^2-L^2)}{(1+H)(1+L)} > 0$, the loss from the deviation when playing against player 2(H -type) is bigger than the gain when playing against player 2(L -type). The expected gain will outweigh the expected loss only if a L -type is much more likely than a H -type (and L is bigger than 0). Hence, player 1(L -type) would deviate at the cheap talk stage only if p is too close to 0.

One may illustrate this equilibrium for the game presented in Example 1.

Example 2 Let us again take $L = \frac{1}{3}$, $H = \frac{2}{3}$. For these values, the range of the prior p for which $S_{separating}$ exists is $\frac{5}{41} (\simeq 0.12) \leq p \leq \frac{5}{23} (\simeq 0.22)$. When $p = \frac{5}{23}$, the payoff from $S_{separating}$ is $\frac{241}{529} (\simeq 0.46)$. This equilibrium generates the following distribution over the outcomes for our example.

	A	B	A	B	A	B	A	B
A	$\frac{6}{25}$	$\frac{9}{25}$	A	0	0	A	1	0
B	$\frac{4}{25}$	$\frac{6}{25}$	B	0	1	B	0	0
	Types: HH		Types: HL		Types: LH		Types: LL	

3.2 Coordination

As demonstrated above, $S_{separating}$ features a specific form of coordination in which the players play (B, B) $((A, A))$ when only player 1's type is H (L), that is, when the players' types are different,

⁹Note that after deviating in the cheap talk stage, player 1 (H -type) may deviate at the action stage as well. In fact, the optimal deviation strategy for player 1 (H -type) is to play action B when player 2 is a L -type.

players fully coordinate on a pure Nash equilibrium outcome that generate the ex-post efficient payoffs of 1 and H . We call this property “type-coordination”.

Definition 4 *A strategy profile is said to have the type-coordination property if the induced outcome is (A, A) and (B, B) , when the players’ true type profile is (L, H) and (H, L) , respectively.*

Note that the type-coordination property can be achieved in other kinds of equilibria. Indeed, as mentioned in the previous section, the Bayesian Nash equilibrium (mentioned in Proposition 1) also satisfies type-coordination property when the prior p is between $\frac{L}{1+L}$ and $\frac{H}{1+H}$ (for example, between $\frac{1}{4}$ and $\frac{2}{5}$ for the parameters $L = \frac{1}{3}$, $H = \frac{2}{3}$).

Although the type-coordination property can be obtained in the fully revealing cheap talk equilibrium or in the Bayesian Nash equilibrium, one might still be interested in exploring other (cheap talk) equilibria. In particular, one might ask whether it is possible to obtain type-coordination for values of p that lie outside the range of p for which type-coordination can be achieved either by $S_{separating}$ or the Bayesian Nash equilibrium.

This motivates us to consider cheap talk equilibria where players do not reveal their types truthfully with probability 1. In the next subsection, we will see how this feature can be obtained in such a partially revealing equilibrium.

3.3 Coordination by a Partially Revealing Equilibrium

In this subsection, we aim to achieve the type-coordination property in a cheap talk equilibrium without full revelation. Keeping the spirit of the unique fully revealing equilibrium, we consider a class of partially revealing symmetric strategy profiles of the above cheap talk game in which only the L -type truthfully reveals while the H -type does not (as under the type-coordination property, the H -type is expected to compromise, when the opponent is of L -type).

Formally, we consider a symmetric announcement strategy profile in which the H -type of player i announces H with probability r and L with probability $(1 - r)$ and the L -type of player i announces L with probability 1, i.e., $a_i(H|H) = r$ and $a_i(H|L) = 0$. Clearly, after the cheap talk phase, the possible message profiles (τ_1, τ_2) that the H -type of player 1 may receive are (H, H) , (H, L) , (L, H) or (L, L) while the L -type of player 1 may receive either (L, H) or (L, L) .

Let us denote an action-strategy of player 1 by $\sigma_1(A|H; H, H) = q_0$, $\sigma_1(A|H; H, L) = q_1$, $\sigma_1(A|H; L, H) = q_2$, $\sigma_1(A|H; L, L) = q_3$, $\sigma_1(A|L; L, H) = q_4$ and $\sigma_1(A|L; L, L) = q_5$. By symmetry, a partially revealing symmetric strategy profile $((a_1, \sigma_1), (a_2, \sigma_2))$ in our set-up can thus be identified by $(r, q_0, q_1, q_2, q_3, q_4, q_5)$.

First note that, on receiving the message profile (H, H) , the players know the true types and hence in any such partially revealing symmetric cheap talk equilibrium, q_0 has to correspond to the mixed

Nash equilibrium of the complete information BoS with values H and H . Thus, $q_0 = \frac{1}{1+H}$. Note also that for the type-coordination property to hold, we need profiles satisfying $q_1 = 0$, $q_3 = 0$ and $q_5 = 1$. Using symmetry, for player 2(H -type), we then must have $\sigma_2(A|H; L, H) = 1 - q_1 = 1$. This implies that in any such profile, $q_2 = 1$ and $q_4 = 1$. Thus, a candidate equilibrium profile with the type-coordination property must have $q_0 = \frac{1}{1+H}$, $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$. We now state our second main result.

Theorem 2 *In a partially revealing symmetric cheap talk equilibrium, in which only the L -type is truthful, that satisfies the type-coordination property ($q_0 = \frac{1}{1+H}$, $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$) r must be $\frac{H+H^2}{1+H+H^2}$; this equilibrium exists only when $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$.*

The proof of the above theorem has been postponed to the Appendix.

3.3.1 Comments and Illustration

Let the equilibrium profile stated in Theorem 2 be called $S_{pooling}$.

Claim 3 *The ex-ante expected payoff for any player from the equilibrium $S_{pooling}$ is given by $EU_{pooling} = \frac{p(1+H)(1+H+H^2-p-H^2p)}{1+H+H^2}$.*

Claim 4 *The upper bound of the range for p in Theorem 2 does not involve L , is increasing in H and is bounded by $\frac{3}{4}$.*

To understand why p must lie within such a range for $S_{pooling}$ to be an equilibrium, consider the incentives for deviations by player 1 at the action stage. According to the above strategy profile, on receiving the message profile (L, L) , player 1(H -type) needs to play B . Given that player 2(H -type) plays A and player 2(L -type) plays B , player 1(H -type) will indeed play B only if he believes that player 2 is more likely to be an L -type than an H -type. This means that player 1(H -type)'s posterior belief about player 2 being an H -type should not be too high. If we denote this posterior belief by p' , then $p' = P(t_i = H | \tau_i = L) = \frac{p-rp}{1-rp}$. Since this posterior p' is an increasing function of the prior p ($\frac{\partial}{\partial p}(\frac{p-rp}{1-rp}) = \frac{(1-r)}{(1-rp)^2} > 0$), the constraint that p' should not be too high implies that the prior p cannot be very high either.¹⁰ Hence, there is an upper bound for p that is strictly less than 1. Similarly, according to the above strategy profile, after receiving the message profile (L, L) , player 1(L -type) needs to play A . Again, given that player 2(H -type) plays A and player 2(L -type) plays B , player 1(L -type) will play A only if the posterior p' is not too small which explains the lower bound on p .

We illustrate $S_{pooling}$ below in our running example.

¹⁰If p were to be equal to 1, i.e., player 2 were certainly an H -type, player 1(H -type) would then definitely have preferred playing A , not B .

Example 3 We use the parameter values $L = \frac{1}{3}$, $H = \frac{2}{3}$ with p between $\frac{19}{46}$ ($\simeq 0.41$) and $\frac{38}{65}$ ($\simeq 0.58$). For such a game, $S_{pooling}$ exists in which the L -type is truthful but the H -type partially reveals his true type with probability $\frac{10}{19}$ ($\simeq 0.53$). The corresponding equilibrium distribution over the outcomes, indicating type-coordination, is as follows.

	A	B		A	B		A	B		A	B
A	$\frac{141}{361}$	$\frac{36}{361}$	A	0	0	A	1	0	A	0	1
B	$\frac{97}{361}$	$\frac{141}{361}$	B	0	1	B	0	0	B	0	0

Types: HH Types: HL Types: LH Types: LL

When $p = \frac{38}{65}$, the payoff from such an equilibrium also turns out to be $\frac{38}{65}$ ($\simeq 0.58$).

Following our Theorem 2 above, one may indeed characterise the whole set of partially revealing symmetric cheap talk equilibria in which only the L -type is truthful. As mentioned already, $(r, q_1, q_2, q_3, q_4, q_5)$ fully characterises such an equilibrium. To characterise this set, one has to consider all the usual equilibrium conditions. Using the equilibrium conditions, one can prove that the following profiles constitute this equilibrium set.

Proposition 2 *The following profiles are the only partially revealing symmetric cheap talk equilibria in which only the L -type is truthful:*

- (i) $q_1 = \frac{1}{1+H}$, $q_2 = q_3 = \frac{p+Hp-rp-H}{p+Hp-rp-Hrp}$, $q_4 = q_5 = 1$ with any $0 < r \leq 1 - \frac{H(1-p)}{p}$; exists when $p > \frac{H}{1+H}$,
- (ii) $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$, $q_5 = \frac{1}{1+L+LH+LH^2-p-Lp-LHp-LH^2p}$ and $r = \frac{LH+LH^2}{p+Lp+LHp+LH^2p}$; exists when $\frac{LH+LH^2}{1+L+LH+LH^2} < p < \frac{L+LH+LH^2}{1+L+LH+LH^2}$,
- (iii) $q_1 = 0$, $q_2 = 1$, $q_3 = \frac{p+Hp+H^2p+H^3p-H-H^2-H^3}{p+Hp+H^2p+H^3p-H^2-H^3}$, $q_4 = q_5 = 1$ and $r = \frac{H^2}{p+H^2p}$; exists when $p > \frac{H+H^2+H^3}{1+H+H^2+H^3}$,
- (iv) $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$, $q_5 = 1$ and $r = \frac{H+H^2}{1+H+H^2}$; exists when $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$,
- (v) $q_1 = \frac{1}{1+H}$, $q_2 = q_3 = 0$, $q_4 = q_5 = 1$ and $r = \frac{p+Hp-H}{p}$; exists when $\frac{H}{1+H} < p < 1$.

We are not presenting the details of the proof of Proposition 2 which can be found in a previous discussion paper version of this paper (Ganguly and Ray 2013). Note that the profile given in (iv) is the same as that in Theorem 2 above.

3.4 Comparing Cheap-Talk and Bayesian-Nash Equilibria

Our cheap talk equilibria, $S_{separating}$ and $S_{pooling}$ both satisfy the type-coordination property. Note that for a fixed value of H and L , the lower bound $\frac{L+LH+LH^2}{1+L+LH+LH^2}$ for the existence of $S_{pooling}$ is bigger

than the upper bound for p for the existence of $S_{separating}$. Thus, these two different equilibria with the type-coordination property exist for distinct values of p . For $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$, when $S_{pooling}$ exists as an equilibrium, $S_{separating}$ is not an equilibrium because the H -type does not want to truthfully reveal his information. Allowing the H -type to reveal his information partially in the cheap talk stage helps sustain the partially revealing equilibrium.

As noted earlier, it is possible to achieve type-coordination in the unique symmetric Bayesian Nash equilibrium of the BoS (without the cheap talk stage) itself when $\frac{L}{1+L} \leq p \leq \frac{H}{1+H}$ (see Proposition 1).

It is also conceivable that for some parameter values of L and H (such as, $L = 0.2$, $H = 0.9$), the ranges of p where $S_{separating}$ and $S_{pooling}$ respectively exist, do separately overlap with the interval $[\frac{L}{1+L}, \frac{H}{1+H}]$. Hence, there exist possible values of p for which both $S_{separating}$ and the Bayesian Nash equilibrium achieve type-coordination (such as, $p = 0.2$ for the parameter values $L = 0.2$, $H = 0.9$) and similarly, values of p for which both $S_{pooling}$ and the Bayesian Nash equilibrium achieve type-coordination (such as, $p = 0.4$ for the parameter values $L = 0.2$, $H = 0.9$).

However, when $\frac{L}{1+L} \leq p \leq \frac{H}{1+H}$, the structure of the Bayesian Nash equilibrium is such that the players fully miscoordinate when their true type profile is (H, H) or (L, L) . This contrasts with $S_{separating}$ where the players achieve some degree of coordination in both the states (H, H) and (L, L) , and with $S_{pooling}$ where the players manage to coordinate with positive probability when the state is (H, H) although there is complete miscoordination when the true type profile is (L, L) .

Hence, both $EU_{separating}$ and $EU_{pooling}$ are strictly greater than that of the Bayesian Nash equilibrium, confirming that cheap talk is strictly beneficial to both players.

3.5 Comparison with Mediated Equilibria (Banks and Calvert 1992)

Banks and Calvert (1992) characterised the (ex-ante) efficient symmetric incentive compatible direct mechanism for a similar game so that the players are truthful and obedient to the mechanism (mediator). Following Banks and Calvert (1992), one may analyze (as in Ganguly and Ray 2009) a (direct) *symmetric mediated equilibrium* that provides the players with incentives (i) to truthfully reveal their types to the mediator and (ii) to follow the mediator's recommendations following their type-announcements. Clearly, our cheap talk equilibria can be achieved as outcomes of such mediated equilibria using incentive compatible mechanisms. Formally, one can easily prove that (i) the distribution over the outcomes generated by $S_{separating}$ can be achieved as a symmetric mediated equilibrium if $\frac{L^2+2L^2H+L^2H^2}{1+L+H+LH^2+L^2+L^2H+L^2H^2+H^2} \leq p \leq \frac{H-L+LH^2+L^2H+L^2H^2+H^2}{1+L+H+LH^2+L^2+L^2H+L^2H^2+H^2}$ and (ii) the distribution over the outcomes generated by $S_{pooling}$ can be achieved as a symmetric mediated equilibrium if $\frac{L+LH+LH^2}{1+L+LH^2+H^2} \leq p \leq \frac{H+H^2+H^3}{1+H+H^2+H^3}$. Not surprisingly, these ranges of p strictly contain the corresponding ranges for the cheap talk equilibria implying a larger range of p for which the corresponding mechanism

is in equilibrium. Rather intuitively, this indicates that there are priors for which an outcome can be obtained as an equilibrium via a direct mechanism but not using the unmediated one-round cheap talk that only allows direct communication between players of different types.

3.6 One-sided Talk

One-sided cheap talk with two-sided private information has also been studied in the literature (see, for example, Seidmann (1990) and more recently, Moreno de Barreda (2012)). One thus may be interested to know whether the feature of the two-sided cheap talk equilibria analysed in our paper, namely, the type-coordination property, can be achieved with one-sided cheap talk.

Proposition 3 *Type-coordination is not possible in any equilibrium within our cheap talk game when only one player (say, player 1) is allowed to talk.*

The proof of this proposition is in the Appendix. Proposition 3 suggests that two-sided cheap talk achieves more type-coordination than one-sided cheap talk in our set-up, in contrast with the well-known experimental results for cheap talk in the complete information BoS (Cooper *et al* 1989).

4 CONCLUSION

In this paper, we have analysed a simple game, namely BoS with incomplete information and studied the possibility of information revelation and desirable coordination using one round of direct cheap talk. The main takeaway of our paper is that full revelation is possible and it is unique whenever such an equilibrium exists; moreover, full revelation achieves the desirable type-coordination. Such a coordination may also be achievable with partial revelation when fully revealing cheap talk equilibrium does not exist.

We here have characterised the unique fully revealing symmetric cheap talk equilibrium in the BoS with private information. There are of course many fully revealing but asymmetric cheap talk equilibria of this game. Clearly, babbling equilibria exist in which the players ignore the communication and just play one of the Nash equilibria of the complete information BoS for all type-profiles. There are other asymmetric equilibria as well, as Ganguly and Ray (2009) have already shown.

We are aware of many interesting open questions that come out of our analysis. For example, one may ask whether non-babbling cheap talk equilibrium always exist in our game for any given p or not. We also do not characterise the general case where neither type reveals truthfully in the cheap talk phase. Finally, following Banks and Calvert (1992), one may also be interested in characterising the ex ante efficient cheap talk equilibrium in our set up. We postpone all these issues for future research.

5 APPENDIX

We collect the proofs of our results in this section.

Proof of Theorem 1. Using Definitions 2 and 3, we first observe the following fact.

In a fully revealing symmetric cheap talk equilibrium $((a_1, \sigma_1), (a_2, \sigma_2))$, the players' strategies in the action phase must constitute a (pure or mixed) Nash equilibrium of the corresponding complete information BoS, that is, $(\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))$ is a (pure or mixed) Nash equilibrium of the BoS with values t_1 and t_2 , $\forall t_1, t_2 \in \{H, L\}$. Thus, in a fully revealing symmetric cheap talk equilibrium $((a_1, \sigma_1), (a_2, \sigma_2))$, conditional on the announcement profile (H, H) or (L, L) , the strategy profile in the action phase must be the mixed strategy Nash equilibrium of the corresponding complete information BoS, that is, whenever $t_1 = t_2$, $(\sigma_1(t_1, t_2), \sigma_2(t_1, t_2))$ is the mixed Nash equilibrium of the BoS with values $t_1 = t_2$.

Based on the above fact, one can easily identify all the candidate equilibrium strategy profiles of the extended game that are fully revealing and symmetric. It implies that these profiles are differentiated only by the actions played when $t_1 \neq t_2$, that is, when the players' types are (H, L) and (L, H) .

As the strategies are symmetric, it is sufficient to characterise these candidate profiles only by $\sigma_1[A|H, L]$. There are only three possible candidates for $\sigma_1[A|H, L]$ as the complete information BoS with values H and L has three (two pure and one mixed) Nash equilibria. These profiles are (i) $\sigma_1[A|H, L] = \sigma^1(HL)$ where $\sigma^1(HL)$ is the probability of playing A in the mixed Nash equilibrium strategy of player 1 of the complete information BoS with values $t_1 = H$ and $t_2 = L$, that we call S_m ; (ii) $\sigma_1[A|H, L] = 1$, that we call S_{ineff} and (iii) $\sigma_1[A|H, L] = 0$, which indeed is $S_{separating}$.

We first show that S_m is not an equilibrium. Under S_m , H -type will announce his type truthfully only if $p(\frac{H}{1+H}) + (1-p)(\frac{H}{1+H}) \geq p(\frac{H}{1+L}) + (1-p)(\frac{H}{1+L})$, where the LHS is the expected payoff from truthfully announcing H and the RHS is the expected payoff from announcing L and choosing the corresponding optimal action strategy. This inequality implies $\frac{1}{1+H} \geq \frac{1}{1+L}$ which can never be satisfied as $H > L$.

The second candidate strategy profile, S_{ineff} is an equilibrium only when $\frac{1+H}{1+L+HL^2+L^2} \leq p$ and $p \leq \frac{1+L+HL-H^2}{1+L+HL+H^2L}$. To see this, note that under S_{ineff} , H -type will announce his type truthfully only if $p(\frac{H}{1+H}) + (1-p) \geq pH + (1-p)(\frac{H}{1+L})$ which implies $p \leq \frac{1+L+HL-H^2}{1+L+HL+H^2L}$. Similarly, L -type will announce his type truthfully only if $pL + (1-p)(\frac{L}{1+L}) \geq p(\frac{H}{1+H}) + (1-p)$ which implies $\frac{1+H}{1+L+HL^2+L^2} \leq p$. However, it can be shown that $\frac{1+H}{1+L+HL^2+L^2} > \frac{1+L+HL-H^2}{1+L+HL+H^2L}$. Hence, S_{ineff} cannot be an equilibrium.

Finally, we prove that $S_{separating}$ is an equilibrium only when $\frac{HL^2+L^2}{1+L+HL^2+L^2} \leq p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$. Under $S_{separating}$, H -type will announce his type truthfully only if $p(\frac{H}{1+H}) + (1-p)H \geq p + (1-p)(\frac{H}{1+L})$ which implies $p \leq \frac{HL+H^2L}{1+L+HL+H^2L}$. Similarly, L -type will announce his type truthfully only if $p + (1-p)(\frac{L}{1+L}) \geq p(\frac{H}{1+H}) + (1-p)L$ which implies $\frac{HL^2+L^2}{1+L+HL^2+L^2} \leq p$. ■

Proof of Theorem 2. For this profile to be an equilibrium, we first note that in the cheap talk phase, player 1(H -type) needs to be indifferent between announcing H and L . With the given values of $q_0 = \frac{1}{1+H}$, $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$, the expected payoff from announcing H is $H(1-p) + p\left(H(1-r) + H\frac{r}{H+1}\right)$ while the expected payoff from announcing L is $pr + H(1-p)$ which will be equal when $r = \frac{H+H^2}{1+H+H^2}$.

We now check that $q_1 = 0$, $q_2 = 1$, $q_3 = 0$, $q_4 = 1$ and $q_5 = 1$ indeed form an equilibrium using the following equilibrium conditions.

If player 1(H -type) receives the message profile (H, L) , then the expected payoff from playing A ($= 0$) is less than the expected payoff from playing B ($= H$), implying $q_1 = 0$.

If player 1(H -type) receives the message profile (L, H) , then the expected payoff from playing A ($= 1$) is greater than the expected payoff from playing B ($= 0$), implying $q_2 = 1$.

If player 1(H -type) receives the message profile (L, L) , then the expected payoff from playing A ($= \frac{p-rp}{1-rp}$) is less than the expected payoff from playing B ($= (1 - \frac{p-rp}{1-rp})H$), implying $q_3 = 0$, only when $p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$.

If player 1(L -type) receives the message profile (L, H) , then the expected payoff from playing A ($= 1$) is greater than the expected payoff from playing B ($= 0$), implying $q_4 = 1$.

If player 1(L -type) receives the message profile (L, L) , then the expected payoff from playing A ($= \frac{p-rp}{1-rp}$) is greater than the expected payoff from playing B ($= (1 - \frac{p-rp}{1-rp})L$), implying $q_5 = 1$, only when $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p$.

Finally, in the cheap talk phase, it should be incentive compatible for player 1(L -type) to announce L which requires $p \geq \underset{x}{Max} (1-p)((1-x)L) + p(r\frac{H}{1+H} + (1-r)((1-x)L))$, where x is the optimal probability of playing A in the action phase if player 1(L -type) deviates and announces H and receives the message profile (H, L) . The derivative of the RHS of this inequality with respect to x is $L(p-1) + Lp(\frac{H+H^2}{1+H+H^2} - 1) < 0$, which implies $x = 0$ and in turn shows that this condition is satisfied (LHS $= p \geq$ RHS $= \frac{(L+LH+LH^2+H^2p-LHp-LH^2p)}{1+H+H^2}$) only when $p \geq \frac{L+LH+LH^2}{1+H+LH+LH^2}$. Since $\frac{H+H^2+H^3}{1+H+H^2+H^3} - \frac{L+LH+LH^2}{1+L+LH+LH^2} > 0$, the above gives us a meaningful range for p .

Hence, the profile constitutes an equilibrium if $\frac{L+LH+LH^2}{1+L+LH+LH^2} < p < \frac{H+H^2+H^3}{1+H+H^2+H^3}$. ■

Proof of Proposition 3. To prove the result, we first note that the type-coordination property cannot be achieved as a fully revealing equilibrium with one-sided cheap talk maintaining our set-up. This is because, if player 1 reveals his type truthfully, in order to achieve type-coordination, player 1 has to play B (A) after announcing H (L). Player 2's best response would then be to play B (A) after receiving H (L) irrespective of her own type. This is not an equilibrium because player 1(H -type) would then deviate and announce L .

Similarly, the type-coordination property cannot be achieved as a partially revealing equilibrium

with one-sided cheap talk where player 1(H -type) announces H with probability r and L with probability $(1 - r)$ while player 1(L -type) announces L with probability 1 followed by an action strategy (of player 1) of playing B by the H -type and A by the L -type. In order to achieve type-coordination, we must have that player 1(H -type) plays B after announcing either H or L and player 1(L -type) plays A . This implies that, in an equilibrium, after receiving the message H , player 2(H -type) and player 2(L -type) should play B . To preserve type-coordination, after receiving the message L , player 2(H -type) needs to play A because the announcement could have been made by player 1(L -type) and player 2(L -type) needs to play B because the announcement could have been made by player 1(H -type). Given these strategies in the action stage, player 1(H -type) cannot be indifferent between announcing H and L in the cheap talk phase because the expected payoff from announcing H is equal to H whereas the expected payoff from announcing L is equal to $(1 - p)H$. This contradicts the fact that player 1(H -type) uses a mixed strategy in the cheap talk stage. Hence, there is no one-sided cheap talk equilibrium with the type-coordination property. ■

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