

Cardiff Economics Working Papers



Working Paper No. E2015/3

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of the local production structure

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February 2015

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INNOVATION, R&D SPILLOVERS, AND THE VARIETY AND CONCENTRATION OF THE LOCAL PRODUCTION STRUCTURE

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February 20, 2015

Abstract

This paper presents a Cournot oligopoly model with R&D spillovers both within and across industries. The aim is to provide an appropriate theoretical foundation for three different hypotheses regarding the impact of the local production structure on innovation and output, as well as addressing mixed empirical results in this area. Both the effective R&D and total industry output are shown to increase with the variety of industries, which is aligned with Jacobs externalities. With respect to the concentration, the outcome is more ambiguous, where it depends on the variety, both spillover rates, and the R&D efficiency. If the variety is limited, then partial support is given to both Marshall-Arrow-Romer externalities in the case of effective R&D, and to Porter externalities in the case of the total industry output. The use of a relative rather than an absolute measure of variety is also shown to be important.

JEL classification: O33, R11, L13.

Keywords: concentration, innovation, knowledge spillover, regional economy, variety.

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1 Introduction

Since Glaeser et al. (1992), numerous empirical studies have attempted to determine which local production structures are most conducive to innovation, in terms of their variety and concentration, i.e., whether regional diversity or specialisation, and similarly local competition or concentration, encourage innovation and growth. However, the empirical results obtained have been mixed, where the three main hypotheses in this area, i.e., Marshall-Arrow-Romer (MAR), Jacobs, and Porter externalities, have all found some support (Beaudry and Schiffauerova, 2009; De Groot et al., 2009).

A further difficulty when trying to make sense of these mixed findings is that Glaeser et al. (1992) and later studies did not provide any fully developed theories about how the local production structure affects innovation. The theoretical foundations may be identified in industrial economics because there have been many theoretical studies of knowledge spillovers and innovation incentives. However, most of these studies focused on R&D spillovers within a single industry and how R&D cooperation can affect the outcome in this context (see, De Bondt, 1997).

Indeed, it appears that only Steurs (1995) considered the simultaneous occurrence of intra- and inter-industry knowledge spillovers, where he examined the case of a two-industry, two-firm-per-industry model and allowed R&D spillovers to occur both within and between industries. Steurs' theoretical model demonstrates that the two spillover channels have different but interdependent effects on R&D, but he did not determine how the local production structure affects the outcome. As noted by several authors, among the micro-foundations of urban agglomeration, learning and knowledge spillovers are the least understood, and thus there is an urgent need to advance theoretical research into localised knowledge spillovers, which should inform empirical research rather than lagging behind it (Duranton and Puga, 2004; Fujita and Krugman, 2004; Puga, 2010).

The present study builds on previous theoretical research by consider-

ing the case of non-cooperative Cournot oligopolists, which invest in cost-reducing or demand-enhancing technology. Steurs' (1995) model is extended to consider several industries as well as several firms within each of them. Hence, the firms make their R&D investments in the presence of spillovers both within and beyond their respective industries. This study's main theoretical contribution is showing how the level of concentration within local industries and the variety of these industries affects innovation and output. Thus, I attempt to uncover the theoretical circumstances under which the three aforementioned hypotheses can be expected to hold. Therefore, the aim is to bring some clarity to the mixed empirical results and the overall implications for regional economic policy. The results show that variety increases both effective R&D and the industry output, which is aligned with Jacobs externalities, whereas the effect of concentration depends on several factors. In specific circumstances, there is partial support for both MAR and Porter externalities, which also indicates how the model specification can affect the empirical results. Similarly, the outcome is shown to differ when a relative rather than an absolute measure of variety is used.

The remainder of this paper is organised as follows. Section 2 provides a brief review of theoretical and empirical research into inter- and intra-industry knowledge spillovers. Section 3 presents the theoretical model and its equilibrium analysis. The most interesting questions concern the comparative statics, particularly how the effective R&D and total industry output respond to changes in the variety and concentration of local industries. These issues are addressed in section 4, which is followed by the conclusions.

2 Literature Review

To the best of my knowledge, Bernstein (1988) was the first empirical study of both intra- and inter-industry R&D spillovers to consider these as proper externalities. Bernstein found that the firms' R&D investments were either

substitutes for (or complementary to) intra-industry spillovers depending on the size of the firms' R&D propensities. The effect of inter-industry spillovers was found to be small and the same for all industries. Thus, the extent of intra-industry spillovers was the key factor according to Bernstein (1988).

With the exception of Bernstein (1988), Glaeser et al. (1992) marked the true beginning of empirical research into the relative importance of intra- and inter-industry spillovers, as well as the role of the local production structure in this respect. Based on previous theoretical research, they formulated three distinct, but not necessarily mutually exclusive, hypotheses regarding localised knowledge spillovers. The first, MAR externalities, is attributed to the insights of the economists Alfred Marshall, Kenneth Arrow and Paul Romer. According to this hypothesis, regions or cities that are specialised in one or a few types of industry benefit more from localised knowledge spillovers, which are further induced by the concentration of these industries. Porter externalities, which are attributed to the management scholar Michael Porter, also flourish in specialised local production structures but they benefit from increased levels of competition instead. Finally, Jacobs externalities, named after the urban theorist Jane Jacobs, are most prominent in locations with diverse industrial base. Both the variety of industries and the level of competition within them are considered to be conducive to innovation in this hypothesis. Interestingly, Glaeser et al. (1992) did not present a hypothesis, according to which diverse *and* concentrated local production structures would be the best combination.

Glaeser et al. (1992) proceeded to test the hypotheses with U.S. city-level data and found support for Jacobs externalities. Many studies followed and there has been a continuous stream of related empirical research up to the present. For example, a survey by Beaudry and Schiffauerova (2009) analysed the results of 67 studies in this area. Similarly, De Groot et al. (2009) performed a meta-analysis of 31 empirical studies of agglomeration externalities. A major issue identified by these surveys is that the empirical

results are very mixed. Depending on the context (e.g., the data, time period and empirical model), support was found for each of the three hypotheses. In particular, the choice of the dependent variable tended to lead to different results.

Glaeser et al. (1992) formulated their hypotheses loosely based on the ideas proposed in previous studies but to the best of my knowledge, a formal model has never been developed that could explain the underlying mechanisms. This is an interesting issue in itself, but the lack of a formal theory makes it difficult to interpret the mixed empirical findings because we have no clear idea of how particular circumstances affect the spillover mechanisms, or the importance of the specification of the empirical model.

Since the geography of innovation research is mainly empirically orientated, the theoretical basis must be sought elsewhere. In industrial economics, based on the seminal studies of Spence (1984), d'Aspremont and Jacquemin (1988) and Kamien et al. (1992), there has been much theoretical research into (intra-industry) knowledge spillovers and innovation incentives. Two main ways of modelling R&D spillovers exist: spillovers concerning either R&D inputs or outputs. In both cases, spillovers decrease the R&D investments of firms, because there is less to gain from these investments and more from free riding. However, the effective R&D of firms, which comprises the sum of its own R&D propensity and the received spillovers, is also decreasing with the spillover rate in the case of input spillovers. Thus, input spillovers are unlikely to explain geographically concentrated innovation (Leppälä, 2014). However, in the case of output spillovers, the effective R&D is maximised with a spillover rate of exactly one half.

De Bondt et al. (1992) found that effective R&D is increasing in concentration in the case of a single industry. If the number of firms in the industry increases, the firms have less incentive to invest in R&D, which is not compensated for by the increased number of spillover sources. Interestingly, the presence of inter-industry R&D spillovers affects this result, as shown in the

present study.

A related research area considers whether localised knowledge spillovers lead to agglomeration (e.g. Van Long and Soubeyran, 1998). The results are slightly mixed, but Leppälä (2014) showed that in the absence of any opposing factors firms in an industry have a preference to agglomerate in order to maximise the spillovers between them. Since inter-industry spillovers can be expected to only reinforce this incentive, we do not consider the choice of location in the present study and it is assumed that the firms and industries are already agglomerated.

Even in industrial economics, research into inter-industry spillovers is almost non-existent and most extensions concern R&D cooperation in the context of a single industry. Extra-industry sources of R&D were included in the model proposed by Cohen and Levinthal (1989) but they were taken as exogenous. Later, Katsoutacos and Ulph (1998) and Leahy and Neary (1999) compared cases where firms operate either in the same or different industries, but they did not study the case where both inter- and intra-industry spillovers occur simultaneously. Some theoretical studies have also considered spillovers between vertically related firms (Atallah, 2002; Ishii, 2004).

To the best of my knowledge, Steurs (1995) is the only previous study to consider spillovers between completely segmented industries, as well as within them. Steurs' model considers the case of two industries where both are duopolies; otherwise, the firms and industries are completely identical, but the inter- and intra-industry spillover rates are allowed to be different. Steurs showed that inter-industry spillovers always increase the effective R&D, but they also decrease the rate of intra-industry spillovers, which maximises the effective R&D. Thus, inter- and intra-industry spillovers are strategically interdependent.

To analyse the effect of the variety and concentration of the local production structure, the present study extends Steurs' model to consider several

firms as well as industries. R&D cooperation is not of direct interest in this case because it has no role in the three hypotheses and it has little supporting empirical evidence (Brenner, 2007). Thus, instead of addressing R&D cooperation in the present study, I direct the reader to Steurs (1995) for further details.

3 The model

We consider an agglomeration of m identical industries, where each comprises n identical firms that produce a homogeneous output. q_{ij} is the output produced and sold by firm i in industry j , and the total output of industry j is $Q_j = \sum_{i=1}^n q_{ij}$. It is also assumed that the markets are perfectly segmented and that the final outputs are independent. In each market, the firms face a linear inverse demand curve with the same characteristics: $P_j = a - Q_j, \forall j \in m, a > Q_j \geq 0$.

The (initial) unit cost of all firms across the industries is c . X_{ij} is the firm's effective R&D and $a > c > X_{ij} \geq 0$. In this context, R&D output can be considered as a cost reduction or equally well as a demand-enhancing invention (De Bondt and Veugelers, 1991). Effective R&D is given by

$$X_{ij} = x_{ij} + \beta \sum x_{kj} + Z,$$

where x_{ij} is the firm's own R&D output, $\beta \sum x_{kj}, k \neq i$ are the output spillovers from the other firms in the same industry and Z are the spillovers from firms in the other industries. $\beta \in [0, 1]$ is the intra-industry spillover rate. Furthermore, $Z = \sigma \sum \sum x_{il}, l \neq j$, where $\sigma \in [0, 1]$ is the inter-industry spillover rate.

As assumed in previous studies, the cost of the firm's own R&D output

x_{ij} is quadratic and given by

$$C(x_{ij}) = \frac{1}{2}\gamma x_{ij}^2,$$

where $\gamma > 0$ is an inverse measure of the efficiency of R&D. We assume that the values of β and σ are exogenous, where they reflect the extent to which R&D is leaked and useful across firms and different industries. It is further assumed that the m industries are technologically related, such that some beneficial spillovers exist between them (Frenken et al., 2007). As shown by Bernstein and Nadiri (1988), the set of industries bound by spillovers may not be large. Thus, there may be other industries in the same location as well, but they are not related in this sense. It is a highly stylised assumption that the firms, and particularly the industries, are identical in every respect, but it facilitates our analysis with respect to the impact of variety and concentration. However, we briefly discuss how some differences between the industries may affect the outcome.

The firms play a two-stage game. In the first stage, the firms in all industries simultaneously decide their R&D outputs, x_{ij} . In the second stage, the firms engage in Cournot competition and choose their final good outputs, q_{ij} . For expository reasons, we assume that there is no uncertainty with respect to the R&D output, and discounting between the stages is also ignored. We derive the subgame perfect Nash equilibria by backward induction.

3.1 Symmetric equilibria

In the final production stage, firm i in industry j maximises its profit function given by

$$\pi_{ij} = (a - Q_j - c + X_{ij})q_{ij}.$$

The Cournot equilibrium output is

$$q_{ij}^* = \frac{a - c + nX_{ij} - \sum X_{kj}}{n + 1} = \frac{a - c + Z + (n - (n - 1)\beta)x_{ij} + (2\beta - 1) \sum x_{kj}}{n + 1} \quad (1)$$

for all firms $i, k \in n, k \neq i$. Subsequently, the total output in industry j is

$$Q_j = \frac{n(a - c + Z) + (1 + \beta(n - 1)) \sum x_{ij}}{n + 1}. \quad (2)$$

In the first stage, the firms choose their R&D levels. Given the subsequent output levels, firm i chooses x_{ij} in order to maximise

$$\pi_{ij} = (q_{ij}^*)^2 - \frac{1}{2}\gamma x_{ij}^2,$$

where q_{ij}^* is given by Equation (1).

The first order condition gives the best response function

$$x_{ij}(x_{kj}) = \frac{2(a - c + Z + (2\beta - 1) \sum x_{kj})(n - (n - 1)\beta)}{\gamma(n + 1)^2 - 2(n - (n - 1)\beta)^2} \quad (3)$$

for firm i . This shows us that the R&D outputs x_{kj} are strategic substitutes for x_{ij} if $\beta < 1/2$ and complements if the inequality is reversed. However, inter-industry spillovers through Z are always strategic complements.

The second order conditions in the R&D stage require that the numerator in the best response functions is positive. This holds for all $\beta \in [0, 1]$ when $\gamma > 2n^2/(n + 1)^2$. The stability condition requires that the best response functions cross correctly (Henriques, 1990), which holds $\forall \beta \in [0, 1]$ when $\gamma > 2n/(n + 1)$.

First, assuming that the firms in industry j make a symmetric choice, $x_{ij} = x_j, \forall i \in n$, then the best response function (3) gives the following equilibrium R&D output:

$$x_j^* = \frac{2(a - c + Z)(n - (n - 1)\beta)}{\gamma(n + 1)^2 - 2(n - (n - 1)\beta)(1 + (n - 1)\beta)}. \quad (4)$$

We also assume symmetry across the industries, but Equation (4) allows us to consider the implications of any differences between them. For example, a larger initial market size, $a - c$, or higher R&D efficiency, i.e., lower γ , would imply larger equilibrium R&D outputs in industry j . Similarly, through Z , larger R&D outputs in other industries or a higher rate of inter-industry spillovers, σ , have the same effect. Therefore, there is a feedback loop through inter-industry spillovers, which further reinforces any positive or negative effects. Döring and Schnellenbach (2006) noted that inter-industry spillovers tend to be asymmetric or one-directional. Our model does not consider such cases, but we can conjecture that these would diminish the feedback loop.

Since the industries are identical, there is also symmetry across them and thus $x_j^* = x^*$, $\forall j \in m$. Substituting $Z = \sigma(m - 1)nx^*$ into Equation (4) gives the equilibrium R&D output:

$$x^* = \frac{2(a - c)(n - (n - 1)\beta)}{\gamma(n + 1)^2 - 2(n - (n - 1)\beta)(n\sigma(m - 1) + (n - 1)\beta + 1)}. \quad (5)$$

The interior and positive solutions for R&D outputs are guaranteed for $\gamma > 2n(nm - n + 1)/(n + 1)^2$, $\forall \beta, \sigma \in [0, 1]$. We make the following assumption:

Assumption 1 $\gamma > \frac{2n(nm - n + 1)}{(n + 1)^2}$ if $m \geq 2$ and $\gamma > 2n/(n + 1)$ if $m = 1$.

In addition to guaranteeing the interior and positive solutions, this assumption ensures that the stability condition is also met in the case of a single industry. Thus, we need to ensure that the R&D efficiency is not too high with respect to m and n . This assumption also indicates that $\gamma > 1$ at the very least.

Finally, after multiplying Equation (5) by $(n\sigma(m - 1) + (n - 1)\beta + 1)$, it follows that the effective R&D is given by

$$X = \frac{2(a - c)(n - (n - 1)\beta)(n\sigma(m - 1) + (n - 1)\beta + 1)}{\gamma(n + 1)^2 - 2(n - (n - 1)\beta)(n\sigma(m - 1) + (n - 1)\beta + 1)}. \quad (6)$$

The firm level equilibrium output is

$$q^* = \frac{a - c + X}{n + 1},$$

where X is given by Equation (6), and the total industry output is then $Q = nq^*$.

4 Comparative statics

Instead of a complete welfare analysis, this section concentrates on studying the changes in the effective R&D and total industry output, which are the most relevant with regard to the dependent variables used in empirical research. These variables are typically measures of innovation, economic growth or productivity (Beaudry and Schifffauerova, 2009). Empirical researchers also use relative measures of variety and the final subsection shows how this can make a difference to the outcome.

4.1 Effective R&D

The first interesting issue regarding comparative statics concerns the effects of the two spillover rates on effective R&D.

Proposition 1 *Effective R&D always increases in the inter-industry spillover rate, σ , whereas the intra-industry spillover rate that maximises effective R&D is given by $\beta^* = \max\{\frac{1}{2} \frac{n-1-n\sigma(m-1)}{n-1}, 0\}$.*

Proof. Both β and σ exists only in the term

$$A \equiv (n - (n - 1)\beta)(n\sigma(m - 1) + (n - 1)\beta + 1)$$

in the numerator and denominator of Equation (6), which is increasing in A . Since $\frac{\partial A}{\partial \sigma} = (n - (n - 1)\beta)n(m - 1) > 0$, also $\frac{\partial X}{\partial \sigma} = \frac{\partial X}{\partial A} \frac{\partial A}{\partial \sigma} > 0$.

Setting $\frac{\partial A}{\partial \beta} = -(n-1)(n\sigma(m-1)+(n-1)\beta)+1+(n-(n-1)\beta)(n-1)$ equal to zero gives the optimal intra-industry spillover rate $\beta^* = \frac{1}{2} \frac{n-1-n\sigma(m-1)}{n-1}$. Since the spillovers cannot be negative, $\beta^* = 0$ if $n\sigma(m-1) > n-1$. ■

As noted by Steurs (1995), inter-industry spillovers reinforce the disincentive effect of intra-industry spillovers. However, as may be expected, this depends on the number of industries and firms as well as on the inter-industry spillover rate σ . It is easy to verify that without inter-industry spillovers, the optimal intra-industry spillover rate corresponds to previous results, i.e., $\beta = 1/2$. However, the optimal β approaches zero as the inter-industry spillovers increase through σ or m , whereas an increase in the number of firms has the opposite effect as $\frac{\partial \beta^*}{\partial n} = \frac{1}{2} \frac{\sigma(m-1)}{(n-1)^2} \geq 0$.

This may appear to give some support to MAR and Porter externalities because the variety of sectors limits optimal intra-industry spillovers. However, it is still the case that the maximal inter-industry spillover rate, $\sigma = 1$, and no intra-industry spillovers, $\beta = 0$, yield the highest effective R&D. However, it is unlikely that the spillover rates will differ from each other so greatly, in particular that the inter-industry spillover rate is (substantially) higher than the intra-industry rate. That is, we might consider that it is unlikely that firms could benefit more from external R&D derived from a different industry rather than their own. Therefore, we analyse this trade-off between the spillover rates by studying the case where the two rates are the same.

Proposition 2 *If the intra- and inter-industry spillover rates are equal, $\beta = \sigma = \phi$, then the common spillover rate that maximises effective R&D is given by $\phi^* = \frac{1}{2} \frac{n^2 m - 2n + 1}{n^2 m - nm - n + 1} \in [\frac{1}{2}, 1)$, with $\frac{\partial \phi^*}{\partial m} > 0$, $\frac{\partial \phi^*}{\partial n} < 0$.*

Proof. By setting $\beta = \sigma = \phi$, Equation (6) becomes

$$X' = \frac{2(a-c)(n-(n-1)\phi)(nm\phi-\phi+1)}{\gamma(n+1)^2 - 2(n-(n-1)\phi)(nm\phi-\phi+1)}.$$

The first order condition,

$$\frac{\partial X'}{\partial \phi} = \frac{2(a-c)\gamma(n+1)^2(n^2m - 2n^2m\phi + 2nm\phi + 2n\phi - 2n - 2\phi + 1)}{(\gamma(n+1)^2 - 2(n - (n-1)\phi)(nm\phi - \phi + 1))^2} = 0,$$

gives the optimal spillover rate $\phi^* = \frac{1}{2} \frac{n^2m - 2n + 1}{n^2m - nm - n + 1}$. ϕ^* is increasing in m as $\frac{\partial \phi^*}{\partial m} = \frac{1}{2} \frac{n}{(nm-1)^2} > 0$, and decreasing in n as $\frac{\partial \phi^*}{\partial n} = -\frac{1}{2} \frac{(m-1)(n^2m-1)}{(n-1)^2(nm-1)^2} < 0$.

Both inter- and intra-industry spillovers exist only when $m, n \geq 2$. Using L'Hôpital's rule, the lower bound of ϕ^* is given by $\lim_{n \rightarrow \infty} \phi^* = \frac{1}{2} = \phi_-^*$. Using the same rule again, $\lim_{m \rightarrow \infty} \phi^* = \frac{1}{2} \frac{n}{n-1}$ at $n = 2$ gives the upper bound, $\bar{\phi}^* = 1$. ■

It is interesting that the optimal spillover rate in this case is on the higher range of spillovers, which is bounded below by $\frac{1}{2}$. Unsurprisingly, the optimal rate approaches 1 as the variety increases because this makes inter-industry spillovers relatively more important. The number of firms has the opposite effect because rivalry becomes more intense.

Proposition 3 *An increase in the number of industries, m , always leads to an increase in effective R&D.*

Proof. This proof is similar to that of Proposition 1. m exists only in the term A of Equation (6). It is straightforward to see that A is increasing in m and hence the same holds for X . ■

This proposition provides central support for Jacobs externalities because there is no trade-off with respect to variety. This trade-off may arise if space is limited, but it also requires that the number of firms within an industry has a similar, positive effect. This other aspect of Jacobs externalities, i.e., the positive effect of competition, will be studied next.

Proposition 4 *If $3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1 > 0$, which is always the case if $\beta \geq \frac{2}{3}$ or either $\sigma = 0$ or $m = 1$, and $\beta \neq \frac{1}{2}$, then the effective R&D is maximised for $n^* = \frac{\beta\sigma m + 4\beta^2 - \beta\sigma - 4\beta + 1}{3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1}$ firms, where*

$\frac{\partial n^*}{\partial m}, \frac{\partial n^*}{\partial \sigma} > 0$, and $\frac{\partial n^*}{\partial \beta} \geq 0$ if $0 \leq \beta \leq 1 - \frac{1}{2}\sqrt{\sigma m - \sigma + 1}$. Otherwise, effective R&D is always increasing in n .

Proof. Effective R&D is non-decreasing in n when

$$\frac{\partial X}{\partial n} = \frac{2(a-c)\gamma(n+1)^2(Cn-B)}{(\gamma(n+1)^2 - 2(n-(n-1)\phi)(nm\phi - \phi + 1))^2} \geq 0, \quad (7)$$

with

$$B = -\beta\sigma m - 4\beta^2 + \beta\sigma + 4\beta - 1,$$

$$C = -3\beta\sigma m - 4\beta^2 + 3\beta\sigma + 2\sigma m + 4\beta - 2\sigma - 1.$$

Since the other terms in Equation (7) are always positive, its sign depends on the sign of $(Cn - B)$. Hence, effective R&D is increasing in n when $Cn \geq B$. B is non-increasing in m and σ , since $\frac{\partial B}{\partial m} = -\beta\sigma \leq 0$ and $\frac{\partial B}{\partial \sigma} = -\beta m + \beta \leq 0$. When $m = 1$ or $\sigma = 0$, $B = -4\beta^2 + 4\beta - 1 \leq 0$, which holds as an equality when $\beta = \frac{1}{2}$. Therefore, $B \leq 0$.

If $C < 0$, effective R&D is non-decreasing in n when

$$n \leq \frac{B}{C} = \frac{\beta\sigma m + 4\beta^2 - \beta\sigma - 4\beta + 1}{3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1}, \quad (8)$$

and maximised when Equation (8) holds as an equality. n^* is increasing in m and σ , since

$$\frac{\partial n^*}{\partial m} = \frac{2\sigma(1-\beta)(2\beta-1)^2}{(3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1)^2} > 0$$

and

$$\frac{\partial n^*}{\partial \sigma} = \frac{2(m-1)(1-\beta)(2\beta-1)^2}{(3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1)^2} > 0.$$

n^* is also non-decreasing in β when

$$\frac{\partial n^*}{\partial \beta} = \frac{2\sigma(m-1)(4\beta^2 - \sigma m - 8\beta + \sigma + 3)}{(3\beta\sigma m + 4\beta^2 - 3\beta\sigma - 2\sigma m - 4\beta + 2\sigma + 1)^2} \geq 0$$

or

$$4\beta^2 - \sigma m - 8\beta + \sigma + 3 \geq 0. \quad (9)$$

Equation (9) has two roots, $\beta = 1 \pm \frac{1}{2}\sqrt{\sigma m - \sigma + 1}$. Since the larger root is more than 1 and the leading coefficient is positive, then Equation (9) is non-negative if $0 \leq \beta \leq 1 - \frac{1}{2}\sqrt{\sigma m - \sigma + 1}$.

C is decreasing in β when $\frac{\partial C}{\partial \beta} = -3\sigma m - 8\beta + 3\sigma + 4 < 0$ or $\beta > -\frac{3}{8}\sigma m + \frac{3}{8}\sigma + \frac{1}{2} \leq \frac{1}{2}$. When $\beta = \frac{2}{3}$, $C = -\frac{1}{9}$, and thus $C < 0$ always when $\beta \geq \frac{2}{3}$. If $m = 1$ or $\sigma = 0$, then $C = -4\beta^2 + 4\beta - 1 < 0$ provided that $\beta \neq \frac{1}{2}$.

If $C > 0$, effective R&D is non-decreasing in n when $n \geq \frac{B}{C}$, which holds for all n , since then $\frac{B}{C} \leq 0$. If $C = 0$, then Equation (7) is always non-negative. ■

With respect to competition, the results are more mixed. They are aligned with Jacobs externalities in the sense that a wider variety makes the optimal number of firms larger or even infinite for effective R&D. However, if the variety is low or intra-industry spillovers are high, then a more concentrated industry becomes optimal for effective R&D. In this case, the increased competition within an industry dominates the spillovers received from other industries. This issue is studied more deeply in the following corollary.

Corollary 1 *In the absence of inter-industry R&D spillovers a monopoly maximises effective R&D, except when $\beta = \frac{1}{2}$, in which case the number of firms has no effect and $X = \frac{a-c}{2\gamma-1}$.*

Proof. If either $\sigma = 0$ or $m = 1$, then $B = C$. B and C are not equal to zero if $\beta \neq \frac{1}{2}$, and from Equation (8), we find that effective R&D is maximised for $n = 1$. Equation (6) gives $X = \frac{a-c}{2\gamma-1}$ in this case.

If $\beta = 1/2$, then $B = C = 0$ and Equation (7) is zero. Again, from Equation (6), we find that $X = \frac{a-c}{2\gamma-1}$. ■

This result, which is the same as in De Bondt et al. (1992), indicates that effective R&D is typically decreasing with competition within a single

industry. However, an interesting exception, which was not mentioned by De Bondt et al. (1992), is $\beta = \frac{1}{2}$ when the number of firms has no effect. Nevertheless, this result partially supports MAR externalities. Thus, if the variety is low, then concentration increases effective R&D. However, this result shows that effective R&D might not always be the most important performance measure to consider.

4.2 Industry Output

The dependent variable in empirical research has not always been a measure of R&D. Instead, some researchers have studied the impact of variety and concentration on variables such as employment and output. Therefore, it is important to determine how the total industry output is affected because this is more relevant with respect to some empirical studies.

Proposition 5 *Total industry output is increasing in m and σ , as well as in β when $\beta \leq \max\{\frac{1}{2} \frac{n-1-n\sigma(m-1)}{n-1}, 0\}$.*

Proof. The total output of a single industry is

$$Q = n \frac{a - c + X}{n + 1}, \quad (10)$$

where X is given by Equation (6). Since m , σ and β appear only in X and Q is increasing in X , then Q must always be increasing in m and σ , as well as in β when $\beta \leq \max\{\frac{1}{2} \frac{n-1-n\sigma(m-1)}{n-1}, 0\}$, as determined earlier in Propositions 1 and 3. ■

Again, we are given further support for the Jacobs spillover hypothesis because the total industry output is also increasing in the variety of industries as well as their spillovers. Indeed, empirical research also supports Jacobs externalities most often when studying economic growth (Beaudry and Schiffauerova, 2009). As in Proposition 1, the optimal intra-industry spillover rate is shown to be restricted by inter-industry spillovers. The next

step is to determine how the concentration of industries affects the total industry output.

Proposition 6 *The total industry output is increasing in n if $4\beta\sigma m + 6\beta^2 - 4\beta\sigma - 2\sigma m - 6\beta + 2\sigma + 2 - \gamma \leq 0$, which is always the case if $\beta \leq \frac{1}{2}$ and $\gamma \geq 2\sigma - 2\sigma m + 2$; otherwise, the total industry output is maximised for $n^* = \frac{2(2\beta^2 - 2\beta + \gamma)}{4\beta\sigma m + 6\beta^2 - 4\beta\sigma - 2\sigma m - 6\beta + 2\sigma + 2 - \gamma}$ firms, where $\frac{\partial n^*}{\partial m}, \frac{\partial n^*}{\partial \sigma} < 0$, and $\frac{\partial n^*}{\partial \gamma} > 0$ when $\beta > \frac{1}{2}$.*

Proof. An increase in n changes the total industry output by

$$Q(n+1) - Q(n) = \frac{(a-c)(n+1)n\gamma}{\gamma(n+1)^2 - 2(n-(n-1)\beta)(n\sigma(m-1) + (n-1)\beta + 1)}$$

$$- \frac{(a-c)(n+2)(n+1)\gamma}{\gamma(n+2)^2 - 2(n-n\beta+1)(n\sigma(m-1) + n\beta + \sigma m - \sigma + 1)}$$

or

$$Q(n+1) - Q(n) = \frac{(a-c)(n+1)\gamma(Dn - E)}{F}, \quad (11)$$

with

$$D = -4\beta\sigma m - 6\beta^2 + 4\beta\sigma + 2\sigma m + 6\beta - 2\sigma - 2 + \gamma,$$

$$E = -2(2\beta^2 - 2\beta + \gamma),$$

and

$$F = (\gamma(n+1)^2 - 2(n-(n-1)\beta)(n\sigma(m-1) + (n-1)\beta + 1))$$

$$\times (\gamma(n+2)^2 - 2(n-n\beta+1)(n\sigma(m-1) + n\beta + \sigma m - \sigma + 1)).$$

Given the positivity of outputs, $F > 0$. Hence, the sign of Equation (11) depends on the sign of $(Dn - E)$. Given Assumption 1, $E < 0$ as well. Therefore, if $D \geq 0$, the output is increasing in n . D is decreasing in β when $\frac{\partial D}{\partial \beta} = -4\sigma m - 12\beta + 4\sigma + 6 < 0$ or $\beta > -\frac{1}{3}\sigma m + \frac{1}{3}\sigma + \frac{1}{2} \leq \frac{1}{2}$. When $\beta = \frac{1}{2}$, $D = \gamma - \frac{1}{2}$, which is positive given Assumption 1. When $\beta = 0$,

$D = \gamma - 2\sigma + 2\sigma m - 2$, and thus $D \geq 0$ if both $\beta \leq \frac{1}{2}$ and $\gamma \geq 2\sigma - 2\sigma m + 2$.

If $D < 0$, the industry output is maximised for

$$n^* = \frac{E}{D} = \frac{2(2\beta^2 - 2\beta + \gamma)}{4\beta\sigma m + 6\beta^2 - 4\beta\sigma - 2\sigma m - 6\beta + 2\sigma + 2 - \gamma}.$$

If $\beta > \frac{1}{2}$, then

$$\frac{\partial n^*}{\partial m} = -\frac{2(2\beta^2 - 2\beta + \gamma)(4\beta\sigma - 2\sigma)}{(4\beta\sigma m + 6\beta^2 - 4\beta\sigma - 2\sigma m - 6\beta + 2\sigma + 2 - \gamma)^2} < 0,$$

$$\frac{\partial n^*}{\partial \sigma} = -\frac{2(2\beta^2 - 2\beta + \gamma)(4\beta m - 4\beta - 2m + 2)}{(4\beta\sigma m + 6\beta^2 - 4\beta\sigma - 2\sigma m - 6\beta + 2\sigma + 2 - \gamma)^2} < 0,$$

and

$$\frac{\partial n^*}{\partial \gamma} = \frac{4(2\beta - 1)(\sigma m + 2\beta - \sigma - 1)}{(4\beta\sigma m + 6\beta^2 - 4\beta\sigma - 2\sigma m - 6\beta + 2\sigma + 2 - \gamma)^2} > 0,$$

thereby demonstrating how n^* responds to changes in m , σ and γ . ■

Similar to the case of the effective R&D, whether the total industry output is increasing or decreasing with concentration depends on both spillover rates and the variety of industries, but now also on the R&D efficiency. By comparing Propositions 4 and 6, we can see the conditions, under which n always has a positive effect or the optimal, finite n exists, are different. Thus, there may be cases where concentration positively affects the effective R&D but not the output. Previous empirical research shows that when the independent variable is economic growth, MAR externalities appear to have a positive impact far less often and even a negative effect in several cases (Beaudry and Schifffauerova, 2009). This issue is illustrated most clearly by examining the case where no inter-industry spillovers occur (or they are held constant), as follows.

Corollary 2 *In the absence of inter-industry R&D spillovers, the total industry output is always increasing in n when $\gamma \geq 6\beta^2 - 6\beta + 2$ and never*

maximised by monopoly.

Proof. When $m = 1$, $D = -6\beta^2 + 6\beta + \gamma - 2$, which is non-negative when $\gamma \geq 6\beta^2 - 6\beta + 2$. If monopoly maximises the total industry output, then

$$n^* = \frac{2(2\beta^2 - 2\beta + \gamma)}{6\beta^2 - 6\beta - \gamma + 2} < 2$$

or $\gamma < 2\beta^2 - 2\beta + 1 \leq 1$, but this contradicts Assumption 1. ■

Corollary 2 shows that there is a wide range of cases where the total industry output is increasing with the number of firms when studying a single industry. For example, this holds when β is $\frac{1}{2}$ or close to it, or always when $\gamma > 2$. Thus, this model gives partial support to Porter externalities because increased competition with limited variety can improve the outcome in terms of the total industry output. A further contrast with Corollary 1 is that monopoly is never the optimal market structure in this case. Therefore, it is not surprising that even when monopoly leads to the maximal effective R&D, the monopoly output is not the largest possible. This is a further illustration of how the choice of the dependent variable in empirical research can have major consequences.

4.3 Impact of the Relative Variety of Industries

In our model, an increase in the variety of industries leads also to an increase in the size of the local economy. This contributes partly to the outcome that both the effective R&D and industry output are always increasing in variety. Thus, there is a natural link between the size and variety of the local economic base, but it also combines two different effects, which we may wish to analyse separately. Similarly, empirical researchers have studied the effect of relative variety as well as that of absolute variety. A literature survey showed that how variety is measured also affects the empirical findings (De Groot et al., 2009).

To study the impact of variety in a purely relative sense, we make a simple modification to the model by assuming that the inverse demand in all m industries is given by $P_j = a - mQ_j \forall j \in m$, $a > mQ_j \geq 0$. As the slope of the demand curve now depends on the number of industries, the aggregate local demand becomes independent of it: $\sum_{j=1}^m Q_j = a - \bar{P}$, where \bar{P} is the average price across industries.

Given the modified demand function, the equilibrium R&D output becomes

$$x^* = \frac{2(a-c)(n-(n-1)\beta)}{\gamma m(n+1)^2 - 2(n-(n-1)\beta)(n\sigma(m-1) + (n-1)\beta + 1)}. \quad (12)$$

To guarantee the interior and positive solutions for R&D outputs $\forall \beta, \sigma \in [0, 1]$, we now make the following assumption:

Assumption 2 $\gamma > \frac{2n(nm-n+1)}{m(n+1)^2}$ if $m \geq 2$ and $\gamma > 2n/(n+1)$ if $m = 1$.

By multiplying Equation (12) with $(n\sigma(m-1) + (n-1)\beta + 1)$, it follows that the effective R&D is now given by

$$X = \frac{2(a-c)(n-(n-1)\beta)(n\sigma(m-1) + (n-1)\beta + 1)}{\gamma m(n+1)^2 - 2(n-(n-1)\beta)(n\sigma(m-1) + (n-1)\beta + 1)}. \quad (13)$$

Subsequently, the firm-level equilibrium output is

$$q^* = \frac{a-c+X}{(n+1)m}, \quad (14)$$

where X is given by Equation (13). The total industry output is then $Q = nq^*$.

Proposition 7 *Effective R&D is non-decreasing in relative variety if $\sigma \geq \frac{\beta(n-1)+1}{n} \in [\frac{1}{n}, 1]$.*

Proof. Differentiating Equation (13) with respect to m gives

$$\frac{\partial X}{\partial m} = \frac{2(a-c)\gamma(n+1)^2(n-(n-1)\beta)(\sigma n-1-\beta(n-1))}{(\gamma m(n+1)^2-2(n-(n-1)\beta)(n\sigma(m-1)+(n-1)\beta+1))^2}. \quad (15)$$

Clearly, both the denominator of (15) and $2(a-c)\gamma(n+1)^2(n-(n-1)\beta)$ in the numerator are positive. Hence, (15) is non-negative when $\sigma n-1-\beta(n-1) \geq 0$ or $\sigma \geq \frac{\beta(n-1)+1}{n} \in [\frac{1}{n}, 1]$. ■

Proposition 7 shows that, in contrast to the case of absolute variety, the effect of relative variety on effective R&D depends on the spillover rates. In principle, the effective R&D is increasing in relative variety if and only if the inter-industry spillover rate is sufficiently higher than the intra-industry spillover rate. Therefore, the effect of relative variety is similar to that of competition in Proposition 4.

Proposition 8 *The total industry output is non-decreasing in relative variety if $\gamma(n+1)^2 + 2\beta\sigma n^2 - 2\beta\sigma n - 2\sigma n^2 \leq 0$.*

Proof. Substituting Equation (13) into Equation (14) and simplifying the expression gives

$$q^* = \frac{(a-c)\gamma(n+1)}{\gamma m(n+1)^2-2(n-(n-1)\beta)(n\sigma(m-1)+(n-1)\beta+1)}. \quad (16)$$

Since m appears only in the denominator of (16),

$$\begin{aligned} \frac{\partial q^*}{\partial m} \geq 0 &\leftrightarrow \frac{\partial(\gamma m(n+1)^2-2(n-(n-1)\beta)(n\sigma(m-1)+(n-1)\beta+1))}{\partial m} \leq 0 \\ &\leftrightarrow \gamma(n+1)^2 + 2\beta\sigma n^2 - 2\beta\sigma n - 2\sigma n^2 \leq 0. \end{aligned}$$

■

Similar to the case of competition (Proposition 6), for relative variety to have a positive effect on the total industry output, we require relatively high inter-industry and low intra-industry spillover rates. However, this ef-

fect also requires a sufficiently high cost-efficiency of R&D (low γ). Overall, Propositions 7 and 8 demonstrate that excluding the size effect and examining variety purely in relative terms means that its impact is conditional on other factors.

5 Conclusion

This study had two main aims: to provide appropriate theoretical foundations for MAR, Porter and Jacobs externalities; and to address the mixed empirical results with respect to these three hypotheses. The theoretical model supports Jacobs externalities most strongly, which is aligned with the empirical results (Beaudry and Schiffauerova, 2009; De Groot et al., 2009). That is, both effective R&D and the total industry output are increasing with the variety of local industries. However, we considered the number of industries between which there are spillovers, which is often not the same as the total number of local industries. Following Frenken et al. (2007), more recent empirical studies have attempted to consider this “related variety” of local industries.

With respect to concentration, the theoretical model yields more mixed implications because the outcome depends on the variety of industries, intra- and inter-industry spillover rates, and R&D efficiency. However, this may help to clarify the similarly mixed empirical results (De Groot et al., 2009). If the variety is high, effective R&D can be increasing in competition. However, this requires that intra-industry spillovers are not too high because an optimal industry concentration would exist otherwise. The outcome is similar with respect to the total industry output, but a wider variety of cases exists where output is increasing with competition or the optimal number of firms is higher. Nevertheless, the support for Jacobs externalities is only partial in this case. However, why Glaeser et al. (1992) attributed competition to Jacobs externalities is less clear because Jacobs (1969) merely argues

that larger organisations tend to be less innovative (see, also Beaudry and Schiffauerova, 2009). Nevertheless, it is important to acknowledge that the effect of competition depends on other factors and it may vary for different performance measures. If variety is low, then concentration is found to increase effective R&D, which then gives partial support for the MAR externalities. By contrast, when variety is low, competition typically increases total industry output, which then supports Porter externalities. Therefore, the choice of the dependent variable may be critical in empirical research.

How the independent variables are selected is also a critical issue. For example, under the same circumstances, the effective R&D can be shown to be increasing with the average number of firms but decreasing if only the number of firms in that particular industry increases. A related issue is that many empirical studies have used relative measures of variety and concentration, such as the Herfindahl-Hirschman Index. Furthermore, the choice between relative and absolute measures is known to affect the outcome of the analysis substantially (De Groot et al., 2009). We have shown that this is not surprising because the use of a relative measure means that the positive effect of variety on the two performance measures becomes conditional on the other factors.

The proposed model is based on standard spillover models, and thus a logical next step would be considering how well it corresponds with the empirical models and the reality that they aim to explain. An obvious and crude simplification is the assumed symmetry between firms and industries, but a few others should be mentioned in addition. The underlying idea of Porter externalities is that competition fosters innovation because the firms would not survive otherwise. This aspect of competition is missing from our model, so it may not fully consider Porter externalities. However, this shows that further theoretical work regarding Porter externalities is warranted. Another way to extend the model would be to introduce absorptive capacity, e.g., as presented in Martin (2002), because its relevance has been empha-

sised in innovation studies. However, although the absorptive capacity has been found to increase the R&D investments of firms, it is not expected to reverse the trends identified in the present study with respect to variety and concentration.

Another point that should be emphasised is that some authors have considered spillovers not only in terms of imitating existing technologies, but also how they foster subsequent inventions. This issue was particularly relevant to Jacobs (1969). It is not clear how this phenomenon should be formalised, but it may be that variety increases the R&D efficiency of firms instead of providing direct inter-industry spillovers. Again, this would probably support our main finding that inter-industry R&D spillovers strongly facilitate innovation and growth. In summary, the present study demands more theoretical work and it is merely an early step in this area, but it also highlights several critical issues that should be considered when building empirical models to study localised knowledge spillovers and interpreting their results.

References

- Atallah, G. (2002). Vertical R&D spillovers, cooperation, market structure, and innovation. *Economics of Innovation and New Technology*, 11(3):179–209.
- Beaudry, C. and Schiffauerova, A. (2009). Who’s right, Marshall or Jacobs? The localization versus urbanization debate. *Research Policy*, 38(2):318–337.
- Bernstein, J. I. (1988). Costs of production, intra-and interindustry R&D spillovers: Canadian evidence. *Canadian Journal of Economics*, 21(2):324–47.
- Bernstein, J. I. and Nadiri, M. I. (1988). Interindustry R&D spillovers, rates

- of return, and production in high-tech industries. *American Economic Review*, 78(2):429–34.
- Brenner, T. (2007). Local knowledge resources and knowledge flows. *Industry and Innovation*, 14(2):121–128.
- Cohen, W. M. and Levinthal, D. A. (1989). Innovation and learning: The two faces of R&D. *Economic Journal*, 99(397):569–96.
- d’Aspremont, C. and Jacquemin, A. (1988). Cooperative and noncooperative R&D in duopoly with spillovers. *American Economic Review*, 78(5):1133–1137.
- De Bondt, R. (1997). Spillovers and innovative activities. *International Journal of Industrial Organization*, 15(1):1–28.
- De Bondt, R., Slaets, P., and Cassiman, B. (1992). The degree of spillovers and the number of rivals for maximum effective R&D. *International Journal of Industrial Organization*, 10(1):35–54.
- De Bondt, R. and Veugelers, R. (1991). Strategic investment with spillovers. *European Journal of Political Economy*, 7(3):345–366.
- De Groot, H. L., Poot, J., and Smit, M. J. (2009). Agglomeration externalities, innovation and regional growth: theoretical perspectives and meta-analysis. In Capello, R. and Nijkamp, P., editors, *Handbook of Regional Growth and Development Theories*, pages 256–281. Edward Elgar Publishing, Cheltenham.
- Döring, T. and Schnellenbach, J. (2006). What do we know about geographical knowledge spillovers and regional growth?: a survey of the literature. *Regional Studies*, 40(3):375–395.

- Duranton, G. and Puga, D. (2004). Micro-foundations of urban agglomeration economies. In Henderson, J. V. and Thisse, J.-F., editors, *Handbook of Regional and Urban Economics*, volume 4, pages 2063–2117. Elsevier.
- Frenken, K., Van Oort, F., and Verburg, T. (2007). Related variety, unrelated variety and regional economic growth. *Regional Studies*, 41(5):685–697.
- Fujita, M. and Krugman, P. (2004). The new economic geography: Past, present and the future. *Papers in Regional Science*, 83(1):139–164.
- Glaeser, E. L., Kallal, H. D., Scheinkman, J. A., and Shleifer, A. (1992). Growth in cities. *Journal of Political Economy*, 100(6):1126–1152.
- Henriques, I. (1990). Cooperative and noncooperative R&D in duopoly with spillovers: Comment. *American Economic Review*, 80(3):638–40.
- Ishii, A. (2004). Cooperative R&D between vertically related firms with spillovers. *International Journal of Industrial Organization*, 22(8):1213–1235.
- Jacobs, J. (1969). *The Economy of Cities*. Random House.
- Kamien, M. I., Muller, E., and Zang, I. (1992). Research joint ventures and R&D cartels. *American Economic Review*, 82(5):1293–1306.
- Katsoutacos, Y. and Ulph, D. (1998). Endogenous spillovers and the performance of research joint ventures. *The Journal of Industrial Economics*, 46(3):333–357.
- Leahy, D. and Neary, J. P. (1999). R&D spillovers and the case for industrial policy in an open economy. *Oxford Economic Papers*, 51(1):40–59.
- Leppälä, S. (2014). Theoretical perspectives on localised knowledge spillovers and agglomeration. *Cardiff Economics Working Papers*, E2014/10.

- Martin, S. (2002). Spillovers, appropriability, and R&D. *Journal of Economics*, 75(1):1–32.
- Puga, D. (2010). The magnitude and causes of agglomeration economies. *Journal of Regional Science*, 50(1):203–219.
- Spence, A. M. (1984). Cost reduction, competition, and industry performance. *Econometrica*, 52(1):101–21.
- Steurs, G. (1995). Inter-industry R&D spillovers: what difference do they make? *International Journal of Industrial Organization*, 13(2):249–276.
- Van Long, N. and Soubeyran, A. (1998). R&D spillovers and location choice under Cournot rivalry. *Pacific Economic Review*, 3(2):105–119.