Online Appendix to:

"Energy Business Cycles"

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Abstract

This appendix is divided into three parts. In Appendix A, the complete model set up and equilibrium conditions are provided in level forms; in Appendix B, the mathematical derivations for the model's equilibrium conditions are presented in log-linear forms along with the computation of key steady state solutions; and lastly, in Appendix C, we show the data construction and sources.

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Appendix A The Complete Model

This section summarizes the model set up and equilibrium conditions in general forms that appear in the main text. This is a two-sector two-region open economy real business cycle (RBC) model. There are energy and non-energy intensive sectors, and the regions are the domestic country and the rest of the world (RoW). Although an essential feature of the model is that all aspects of the economy must be *powered*, we only include energy on the production side. The main text contains a full description of the model parameters and variables, therefore, here, we focus purely on parameters and variables not well-covered in the body of the paper.

We use the following notations for convenience: UPPER-CASE letters $X' \equiv X_{t+1}$, $X \equiv X_t$, and $X_{\prime} \equiv X_{t-1}$ for dynamic variables for next, current, and lagged periods, respectively; lower-case letters, say x, to denote non-stochastic steady state variables; hatted letters, $\widehat{x}$, to denote variables in their log-linear form; Greek and $\mathsf{sans\ serif}$ letters to denote the exogenous state variables; $j = e$, n, indexes sector-specific variables, and ${}_{\triangle}x$ (${}_{\triangle^2}x$) to denote the first (second) derivative of x. We characterize the model in terms of the domestic country.

Appendix A.1 Firms

Their joint static maximization problem can be formalized as

$$\max_{\{H_e, H_n, U_e K_{e,-1}, U_n K_{n,-1}, E_e, E_n\}} \Pi = P_e Y_e + P_n Y_n - [W H_e + W H_n + R_e U_e K_{e,-1} + R_n U_n K_{n,-1} + \mathsf{Q} E_e + \mathsf{Q} E_n] \quad \text{(A.1)}$$

subject to the sectoral production functions

$$Y_e = \mathsf{A}_e H_e^{1-\alpha_e} \left(\theta_e \left(U_e K_{e,-1} \right)^{-\nu_e} + (1-\theta_e) \left(\mathsf{O}_e E_e \right)^{-\nu_e} \right)^{-\frac{\alpha_e}{\nu_e}} \quad \text{(A.2)}$$

$$Y_n = \mathsf{A}_n H_n^{1-\alpha_n} \left(\theta_n \left(U_n K_{n,-1} \right)^{-\nu_n} + (1-\theta_n) \left(\mathsf{O}_n E_n \right)^{-\nu_n} \right)^{-\frac{\alpha_n}{\nu_n}} \quad \text{(A.3)}$$

where variables A_e, A_n, O_e, O_n, and Q are, respectively, the exogenous stochastic energy intensive sector neutral technology, non-energy intensive sector neutral technology, energy intensive sector energy efficiency, non-energy intensive sector energy efficiency and the energy price shocks. Each of these are assumed to be following AR(1) processes in logarithm, which we write as

$$\ln \mathsf{A}_e = \rho_{\mathsf{a}_e} \ln \mathsf{A}_{e,-1} + \varepsilon^{\mathsf{a}_e} \quad \text{(A.4)}$$

$$\ln \mathsf{A}_n = \rho_{\mathsf{a}_n} \ln \mathsf{A}_{n,-1} + \varepsilon^{\mathsf{a}_n} \tag{A.5}$$

$$\ln \mathsf{O}_e = \rho_{\mathsf{o}_e} \ln \mathsf{O}_{e,-1} + \varepsilon^{\mathsf{o}_e} \tag{A.6}$$

$$\ln \mathsf{O}_n = \rho_{\mathsf{o}_n} \ln \mathsf{O}_{n,-1} + \varepsilon^{\mathsf{o}_n} \tag{A.7}$$

$$\ln \mathsf{Q} = \rho_{\mathsf{q}} \ln \mathsf{Q}_{-1} + \varepsilon^{\mathsf{q}} \tag{A.8}$$

where for $\mathfrak{s}_1 = (\mathsf{A}_e, \mathsf{A}_n, \mathsf{O}_e, \mathsf{O}_n, \mathsf{Q})$, the autoregressive parameters are conditioned by $\rho_{\mathfrak{s}_1} \in [0, 1)$ and $\varepsilon^{\mathfrak{s}_1}$, and is an independently and identically distributed (i.i.d.) normal distribution with zero mean and an innovation standard deviation, $\sigma_{\mathfrak{s}_1}$. The firms' decision variables must satisfy Equations (A.2)-(A.3), and the following first-order necessary conditions

$$(1-\alpha_e)\frac{Y_e}{H_e} = \frac{W}{P_e} \tag{A.9}$$

$$(1-\alpha_n)\frac{Y_n}{H_n} = \frac{W}{P_n} \tag{A.10}$$

$$\alpha_e\theta_e \frac{(U_e K_{e,-1})^{-\nu_e}}{\left(\theta_e (U_e K_{e,-1})^{-\nu_e} + (1-\theta_e)(\mathsf{O}_e E_e)^{-\nu_e}\right)} \frac{Y_e}{U_e K_{e,-1}} = \frac{R_e}{P_e} \tag{A.11}$$

$$\alpha_n\theta_n \frac{(U_n K_{n,-1})^{-\nu_n}}{\left(\theta_n (U_n K_{n,-1})^{-\nu_n} + (1-\theta_n)(\mathsf{O}_n E_n)^{-\nu_n}\right)} \frac{Y_n}{U_n K_{n,-1}} = \frac{R_n}{P_n} \tag{A.12}$$

$$\alpha_e (1-\theta_e) \frac{(\mathsf{O}_e E_e)^{-\nu_e}}{\left(\theta_e (U_e K_{e,-1})^{-\nu_e} + (1-\theta_e)(\mathsf{O}_e E_e)^{-\nu_e}\right)} \frac{Y_e}{E_e} = \frac{\mathsf{Q}}{P_e} \tag{A.13}$$

$$\alpha_n (1-\theta_n) \frac{(\mathsf{O}_n E_n)^{-\nu_n}}{\left(\theta_n (U_n K_{n,-1})^{-\nu_n} + (1-\theta_n)(\mathsf{O}_n E_n)^{-\nu_n}\right)} \frac{Y_n}{E_n} = \frac{\mathsf{Q}}{P_n} \tag{A.14}$$

Appendix A.2 Households

Their dynamic problem can be formalized as

$$\max_{\{C,H,I,I_e,I_n,B',U_e,U_n,K_e,K_n\}} \sum_{0}^{\infty} \beta\tau \left(\frac{(C - \iota C_{-1})^{1-\epsilon}}{1-\epsilon} - \zeta \frac{H^{1+\omega}}{1+\omega} \right) \tag{A.15}$$

where τ and ζ are the exogenous stochastic intertemporal preference and labor supply shocks, respectively. These are both assumed to be following AR(1) processes in logarithm, which we write as

$$\ln \tau = \rho_\tau \ln \tau_{-1} + \varepsilon^{\tau} \tag{A.16}$$

$$\ln \zeta = \rho_\zeta \ln \zeta_{-1} + \varepsilon^{\zeta} \tag{A.17}$$

where for $\mathfrak{s}_2 = (\tau, \zeta)$, the autoregressive parameters are conditioned by $\rho_{\mathfrak{s}_2} \in [0, 1)$ and $\varepsilon^{\mathfrak{s}_2}$, and is an i.i.d. normal distribution with zero mean and an innovation standard deviation, $\sigma_{\mathfrak{s}_2}$.

Denoting λ^C as the marginal utility of consumption and λ^H as the marginal disutility of labor hours, we have that

$$\lambda^C = \tau \left(C - \iota C_{-1}\right)^{-\epsilon} \tag{A.18}$$

$$\lambda^H = \tau \zeta H^{\omega} \tag{A.19}$$

Further, households can accumulate two types of physical capital goods, which are assumed to proceed from the following laws of motion

$$K_e = \left(1 - \delta_{e0} - \frac{\delta_{e1} U_e^{\mu_e}}{\mu_e}\right) K_{e,-1} + \mathsf{Z}_e I_e - \frac{\psi_e}{2}\left(\frac{K_e}{K_{e,-1}} - 1\right)^2 K_{e,-1} \tag{A.20}$$

$$K_n = \left(1 - \delta_{n0} - \frac{\delta_{n1} U_n^{\mu_n}}{\mu_n}\right) K_{n,-1} + \mathsf{Z}_n I_n - \frac{\psi_n}{2}\left(\frac{K_n}{K_{n,-1}} - 1\right)^2 K_{n,-1} \tag{A.21}$$

where Z_e and Z_n are the exogenous stochastic investment-specific technology shocks for the two types of physical capital, and are both assumed to be following AR(1) processes in logarithm, which we write as

$$\ln \mathsf{Z}_e = \rho_{\mathsf{z}_e} \ln \mathsf{Z}_{e,-1} + \varepsilon^{\mathsf{z}_e} \tag{A.22}$$

$$\ln \mathsf{Z}_n = \rho_{\mathsf{z}_n} \ln \mathsf{Z}_{n,-1} + \varepsilon^{\mathsf{z}_n} \tag{A23}$$

where for $\mathfrak{s}_3 = (\mathsf{Z}_e, \mathsf{Z}_n)$, the autoregressive parameters are conditioned by $\rho_{\mathfrak{s}_3} \in [0, 1)$ and $\varepsilon^{\mathfrak{s}_3}$, and is an i.i.d. normal distribution with zero mean and an innovation standard deviation, $\sigma_{\mathfrak{s}_3}$.

The period-by-period budget constraint faced by the representative households is

$$\begin{aligned} &\mathbb{E}R'B' + C + T + \frac{K_e}{\mathsf{Z}_e} + \frac{K_n}{\mathsf{Z}_n} + \frac{\psi_e}{2}\left(\frac{K_e}{K_{e,-1}} - 1\right)^2 \frac{K_{e,-1}}{\mathsf{Z}_e} + \frac{\psi_n}{2}\left(\frac{K_n}{K_{n,-1}} - 1\right)^2 \frac{K_{n,-1}}{\mathsf{Z}_n} = \\ &B + \left(R_e U_e + \frac{1 - \delta_{e0} - \frac{\delta_{e1} U_e^{\mu_e}}{\mu_e}}{\mathsf{Z}_e}\right) K_{e,-1} + WH + \left(R_n U_n + \frac{1 - \delta_{n0} - \frac{\delta_{n1} U_n^{\mu_n}}{\mu_n}}{\mathsf{Z}_n}\right) K_{n,-1} + \Pi \end{aligned} \tag{A.24}$$

and we impose a borrowing constraint on the households to preclude them from engaging in Ponzi-type schemes: $\lim_{\to\infty} \mathbb{E}B^{+z} / \prod_{v=0}^{z} \leq 0$.

Households' decision variables must satisfy (A.20), (A.21), aggregate investment

$$I = I_e + I_n \tag{A.25}$$

and the following first-order necessary conditions

$$\lambda^H = \lambda^C W \tag{A.26}$$

$$\lambda^C = \beta (1+r) \mathbb{E}\lambda'^C \tag{A.27}$$

$$R_e U_e \mathsf{Z}_e = \delta_{e1} U_e^{\mu_e} \tag{A.28}$$

$$R_n U_n \mathsf{Z}_n = \delta_{n1} U_n^{\mu_n} \tag{A.29}$$

$$\left(1+\psi_e\left(\frac{K_e}{K_{e,-1}}-1\right)\right) = \beta\mathbb{E}\frac{\mathsf{Z}_e}{\mathsf{Z}'_e}\frac{\lambda'^C}{\lambda^C}\left(R'_e U'_e \mathsf{Z}'_e - \frac{\psi_e}{2}\left(\frac{K'_e}{K_e}-1\right)^2 + 1 - \delta_{e0} - \frac{\delta_{e1}U'^{\mu_e}_e}{\mu_e} + \psi_e\left(\frac{K'_e}{K_e}-1\right)\frac{K'_e}{K_e}\right) \tag{A.30}$$

$$\left(1+\psi_n\left(\frac{K_n}{K_{n,-1}}-1\right)\right) = \beta\mathbb{E}\frac{\mathsf{Z}_n}{\mathsf{Z}'_n}\frac{\lambda'^C}{\lambda^C}\left(R'_n U'_n \mathsf{Z}'_n - \frac{\psi_n}{2}\left(\frac{K'_n}{K_n}-1\right)^2 + 1 - \delta_{n0} - \frac{\delta_{n1}U'^{\mu_n}_n}{\mu_n} + \psi_n\left(\frac{K'_n}{K_n}-1\right)\frac{K'_n}{K_n}\right) \tag{A.31}$$

Appendix A.3 Traders

In the present model, we are assuming that agents in the domestic country trade both types of produced goods with the RoW. An implication of international trade in goods is that we end up with a four-goods world: domestically produced energy and non-energy intensive goods, and foreign-produced energy and non-energy intensive goods.[1] In order to weight these choices appropriately between types (energy and non-energy intensive goods) and production origins (domestic and foreign goods), we follow a two-cascade Armington (1969) type aggregator function.[2]

More formally, the maximization problem of the domestic traders can be written as

$$\max_{\{D_e, IM, IM_e\}} \left\{ P\underbrace{\left(\sigma^{\frac{1}{\varsigma}}\left(D_e\right)^{\frac{\varsigma-1}{\varsigma}} + (1-\sigma)^{\frac{1}{\varsigma}}\left(D_n\right)^{\frac{\varsigma-1}{\varsigma}}\right)^{\frac{\varsigma}{\varsigma-1}}}_{D \text{ aggregate by type of goods}} - P_e D_e - P_n D_n + P\underbrace{\left(\kappa^{\frac{1}{\phi}}\left(D^d\right)^{\frac{\phi-1}{\phi}} + (1-\kappa)^{\frac{1}{\phi}}\left(IM\right)^{\frac{\phi-1}{\phi}}\right)^{\frac{\phi}{\phi-1}}}_{D \text{ aggregate by location of production}} \right. \tag{A.32}$$

$$\left. -P^d D^d - IM + \underbrace{\left(\chi^{\frac{1}{\eta}}\left(IM_e\right)^{\frac{\eta-1}{\eta}} + (1-\chi)^{\frac{1}{\eta}}\left(IM_n\right)^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}}}_{\text{Split of imports bundle, } IM\text{, by type of goods}} - \mathsf{P}_e^{im} IM_e - \mathsf{P}_n^{im} IM_n \right\}$$

[1] Remember that we are assuming that all four types of goods are being demanded per period both in the domestic country and in the RoW.

[2] Note that our application of the Armington aggregator function here is an extension of the way Backus et al. (1993) employed it. Their model contains one home-produced and one foreign-produced goods in a two-good, two-country environment, whereas we have four goods being traded between the two regions in our model.

where the real exchange rate is defined as a function of the two sectoral relative prices as[3]

$$P = \left(\sigma \left(P_e\right)^{1-\varsigma} + (1-\sigma)\left(P_n\right)^{1-\varsigma}\right)^{\frac{1}{1-\varsigma}} \tag{A.33}$$

with P_e^{im} and P_n^{im} denoting the prices of imported energy and non-energy intensive goods. Note, however, that only P_e^{im} enters the model simulation, where it is treated as an exogenous stochastic process following an AR(1) process in logarithm

$$\ln \mathsf{P}_e^{im} = \rho_{\mathsf{p}_e^{im}} \ln \mathsf{P}_{e,-1}^{im} + \varepsilon^{\mathsf{p}_e^{im}} \tag{A.34}$$

where for $\mathfrak{s}_4 = (\mathsf{P}_e^{im})$, the autoregressive parameter is conditioned by $\rho_{\mathfrak{s}_4} \in [0,1)$ and $\varepsilon^{\mathfrak{s}_4}$ is an i.i.d. normal distribution with zero mean and an innovation standard deviation, $\sigma_{\mathfrak{s}_4}$, and where the first-order necessary conditions are

$$D_e = \sigma \left(\frac{P_e}{P}\right)^{-\varsigma} D \tag{A.35}$$

$$IM = (1-\kappa)\left(\frac{1}{P}\right)^{-\phi} D \tag{A.36}$$

$$IM_e = \chi \left(\mathsf{P}_e^{im}\right)^{-\eta} IM \tag{A.37}$$

The export functions for the domestic country come from the maximization problem of traders in the RoW, given by

$$\max_{\{EX,EX_e\}} \left\{ \underbrace{\left(\kappa_w^{\frac{1}{\phi_w}} \left(D^f\right)^{\frac{\phi_w-1}{\phi_w}} + (1-\kappa_w)^{\frac{1}{\phi_w}} (EX)^{\frac{\phi_w-1}{\phi_w}}\right)^{\frac{\phi_w}{\phi_w-1}}}_{\mathsf{D}^w \text{ aggregate by location of production}} - P^f D^f - PEX \right. \tag{A.38}$$
$$\left. + P \underbrace{\left(\chi_w^{\frac{1}{\eta_w}} (EX_e)^{\frac{\eta_w-1}{\eta_w}} + (1-\chi_w)^{\frac{1}{\eta_w}} (EX_n)^{\frac{\eta_w-1}{\eta_w}}\right)^{\frac{\eta_w}{\eta_w-1}}}_{\text{Split of exports bundle, } EX \text{, by type of goods}} - P_e EX_e - P_n EX_n \right\}$$

where world demand, D^w, is an exogenous stochastic process assumed to follow an AR(1) process in logarithm

$$\ln \mathsf{D}^w = \rho_{\mathsf{d}^w} \ln \mathsf{D}_{-1}^w + \varepsilon^{\mathsf{d}^w} \tag{A.39}$$

and for $\mathfrak{s}_5 = (\mathsf{D}^w)$, the autoregressive parameter is conditioned by $\rho_{\mathfrak{s}_5} \in [0,1)$ and $\varepsilon^{\mathfrak{s}_5}$ is an i.i.d. normal

[3] See Obstfeld and Rogoff (1996, p. 227) for details.

distribution with zero mean and an innovation standard deviation, $\sigma_{\mathfrak{s}_5}$. The first-order necessary conditions are

$$EX = (1 - \kappa_w)\,(P)^{-\phi_w}\,\mathsf{D}^w \tag{A.40}$$

$$EX_e = \chi_w \left(\frac{P_e}{P}\right)^{-\eta_w} EX \tag{A.41}$$

Appendix A.4 Government, Market Clearing and Equilibrium

Government spending, G, is an exogenous stochastic process assumed to follow an AR(1) process in logarithm that takes the form

$$\ln \mathsf{G} = \rho_{\mathsf{g}} \ln \mathsf{G}_{-1} + \varepsilon^{\mathsf{g}} \tag{A.42}$$

where the autoregressive parameter is conditioned by $\rho_{\mathsf{g}} \in [0, 1)$ and ε^{g} is an i.i.d. normal distribution with zero mean and an innovation standard deviation, σ_{g}. Now, together with Equation (A.25), the other aggregate variables and market clearing conditions are

$$Y = Y_e + Y_n \tag{A.43}$$

$$H = H_e + H_n \tag{A.44}$$

$$E = E_e + E_n \tag{A.45}$$

$$Y_e = D_e + EX_e - IM_n \tag{A.46}$$

$$PEX = \mathsf{Q}E + IM \tag{A.47}$$

$$D = C + I + \mathsf{G} \tag{A.48}$$

$$Y = D \tag{A.49}$$

Thus, given the initial conditions $\{C_{-1}, K^e_{-1}, K^n_{-1}, B_0\}$ and the exogenous stochastic processes $\{\mathsf{A}_e$, A_n, D^w, G, ζ, O_e, O_n, P^{im}_e, Q, τ, Z_e, $\mathsf{Z}_n\}$, a competitive equilibrium is a sequence of

1. sectoral goods prices $\{P_e, P_n\}$;
2. wage rate $\{W\}$;
3. interest rates $\{R, R_e, R_n\}$;
4. real exchange rate $\{P\}$;
5. consumption $\{C\}$;

6. investments $\{I, I_e, I_n\}$;
7. labor hours $\{H, H_e, H_n\}$;
8. capital $\{K_e, K_n\}$;
9. capital utilization rates $\{U_e, U_n\}$;
10. primary energy use $\{E, E_e, E_n\}$;
11. output $\{Y, Y_e, Y_n\}$;
12. domestic absorption $\{D, D_e\}$;
13. imports $\{IM, IM_e\}$;
14. exports $\{EX, EX_e\}$;
15. marginal utility of consumption $\{\lambda^C\}$;
16. marginal disutility of labor hours $\{\lambda^H\}$;

such that markets clear for:

1. labor hours;
2. capital;
3. primary energy use;
4. sectoral output; and
5. total output.[4]

Appendix B Mathematical Derivations

Appendix B.1 Log-linearized Model Equations

In this sub-section, we derive the log-linearized version of the model's simulated equations and calculate its steady state solution. Starting with the production side, log-linearized versions of the production technologies, Equations (A.2) and (A.3), are

$$\widehat{Y}_e = \widehat{\mathsf{A}}_e + (1-\alpha_e)\,\widehat{H}_e + \frac{\alpha_e}{1+\frac{1-\theta_e}{\theta_e}\left(\frac{e_e}{k_e}\right)^{-\nu_e}}\left(\widehat{U}_e + \widehat{K}_{e,-1}\right) + \frac{\alpha_e}{1+\frac{\theta_e}{1-\theta_e}\left(\frac{e_e}{k_e}\right)^{\nu_e}}\left(\widehat{\mathsf{O}}_e + \widehat{E}_e\right) \tag{B.1}$$

$$\widehat{Y}_n = \widehat{\mathsf{A}}_n + (1-\alpha_n)\,\widehat{H}_n + \frac{\alpha_n}{1+\frac{1-\theta_n}{\theta_n}\left(\frac{e_n}{k_n}\right)^{-\nu_n}}\left(\widehat{U}_n + \widehat{K}_{n,-1}\right) + \frac{\alpha_n}{1+\frac{\theta_n}{1-\theta_n}\left(\frac{e_n}{k_n}\right)^{\nu_n}}\left(\widehat{\mathsf{O}}_n + \widehat{E}_n\right) \tag{B.2}$$

[4] Note that we have here included four more variables in the definition of a competitive equilibrium than we did in the main text. In the next section, these "extra" equations get substituted out and we end up with 28 equations in 28 unknown endogenous variables and 12 undetermined exogenous variables for which we have assumed AR(1) processes just as in the main text.

respectively. Further, log-linearizing Equations (A.9)-(A.14), noting that we have merged (A.28) with (A.11) and (A.29) with (A.12), gives

$$\widehat{H}_e = \widehat{P}_e + \widehat{Y}_e - \widehat{W} \tag{B.3}$$

$$\widehat{H}_n = \widehat{P}_n + \widehat{Y}_n - \widehat{W} \tag{B.4}$$

$$\widehat{U}_e = \frac{\widehat{P}_e + \widehat{Y}_e + \widehat{\mathsf{Z}}_e + \left(\frac{\nu_e}{1+\frac{1-\theta_e}{\theta_e}\left(\frac{e_e}{k_e}\right)^{-\nu_e}} - \nu_e - 1\right)\widehat{K}_{e,-1} + \frac{\nu_e}{1+\frac{\theta_e}{1-\theta_e}\left(\frac{e_e}{k_e}\right)^{\nu_e}}\left(\widehat{\mathsf{O}}_e + \widehat{E}_e\right)}{\mu_e + \nu_e - \frac{\nu_e}{1+\frac{1-\theta_e}{\theta_e}\left(\frac{e_e}{k_e}\right)^{-\nu_e}}} \tag{B.5}$$

$$\widehat{U}_n = \frac{\widehat{P}_n + \widehat{Y}_n + \widehat{\mathsf{Z}}_n + \left(\frac{\nu_n}{1+\frac{1-\theta_n}{\theta_n}\left(\frac{e_n}{k_n}\right)^{-\nu_n}} - \nu_n - 1\right)\widehat{K}_{n,-1} + \frac{\nu_n}{1+\frac{\theta_n}{1-\theta_n}\left(\frac{e_n}{k_n}\right)^{\nu_n}}\left(\widehat{\mathsf{O}}_n + \widehat{E}_n\right)}{\mu_n + \nu_n - \frac{\nu_n}{1+\frac{1-\theta_n}{\theta_n}\left(\frac{e_n}{k_n}\right)^{-\nu_n}}} \tag{B.6}$$

$$\widehat{E}_e = \frac{\widehat{P}_e + \widehat{Y}_e - \widehat{\mathsf{Q}} + \frac{\nu_e}{1+\frac{1-\theta_e}{\theta_e}\left(\frac{e_e}{k_e}\right)^{-\nu_e}}\left(\widehat{U}_e + \widehat{K}_{e,-1}\right) + \left(\frac{\nu_e}{1+\frac{\theta_e}{1-\theta_e}\left(\frac{e_e}{k_e}\right)^{\nu_e}} - \nu_e\right)\widehat{\mathsf{O}}_e}{\nu_e + 1 - \frac{\nu_e}{1+\frac{\theta_e}{1-\theta_e}\left(\frac{e_e}{k_e}\right)^{\nu_e}}} \tag{B.7}$$

$$\widehat{E}_n = \frac{\widehat{P}_n + \widehat{Y}_n - \widehat{\mathsf{Q}} + \frac{\nu_n}{1+\frac{1-\theta_n}{\theta_n}\left(\frac{e_n}{k_n}\right)^{-\nu_n}}\left(\widehat{U}_n + \widehat{K}_{n,-1}\right) + \left(\frac{\nu_n}{1+\frac{\theta_n}{1-\theta_n}\left(\frac{e_n}{k_n}\right)^{\nu_n}} - \nu_n\right)\widehat{\mathsf{O}}_n}{\nu_n + 1 - \frac{\nu_n}{1+\frac{\theta_n}{1-\theta_n}\left(\frac{e_n}{k_n}\right)^{\nu_n}}} \tag{B.8}$$

On the side of the households, combining Equations (A.18), (A.19), and (A.26) yields the supply of labor hours as

$$\zeta H^{\omega} = \frac{W}{\left(C - \iota C_{-1}\right)^{\epsilon}} \tag{B.9}$$

which when log-linearized becomes

$$\omega \widehat{H} = \widehat{W} - \widehat{\zeta} - \frac{\epsilon}{1-\iota}\widehat{C} + \frac{\iota\epsilon}{1-\iota}\widehat{C}_{-1} \tag{B.10}$$

The consumption Euler equation is

$$\frac{\tau}{\left(C - \iota C_{-1}\right)^{\epsilon}} = \beta R \mathbb{E} \frac{\tau_{t+1}}{\left(C_{t+1} - \iota C_t\right)^{\epsilon}} \tag{B.11}$$

which is obtained by combining Equations (A.18) and (A.27). Log-linearizing yields

$$\widehat{C} = \frac{1}{1+\iota}\widehat{C}' + \frac{\iota}{1+\iota}\widehat{C}_{-1} + \frac{1-\iota}{\epsilon\left(1+\iota\right)}\left(\widehat{\tau} - \widehat{\tau}' - \widehat{R}\right) \tag{B.12}$$

We obtain the investment Euler equations for the two types of investment goods by combining Equations (A.18), (A.28), and (A.30) to get

$$\left(1+\psi_e\left(\frac{K_e}{K_{e,-1}}-1\right)\right) = \beta\mathbb{E}\frac{\tau'}{\tau}\frac{\mathsf{Z}_e}{\mathsf{Z}'_e}\frac{(C'-\iota C)^{-\epsilon}}{(C-\iota C_{-1})^{-\epsilon}} \times\left(1-\frac{\delta_{e1}U_e'^{\mu_e}}{\mu_e}(1-\mu_e)-\delta_{e0}-\frac{\psi_e}{2}\left(\frac{K'_e}{K_e}-1\right)^2+\psi_e\left(\frac{K'_e}{K_e}-1\right)\frac{K'_e}{K_e}\right) \tag{B.13}$$

and by combining Equations (A.18), (A.29), and (A.31) to get

$$\left(1+\psi_n\left(\frac{K_n}{K_{n,-1}}-1\right)\right) = \beta\mathbb{E}\frac{\tau'}{\tau}\frac{\mathsf{Z}_n}{\mathsf{Z}'_n}\frac{(C'-\iota C)^{-\epsilon}}{(C-\iota C_{-1})^{-\epsilon}} \times\left(1-\frac{\delta_{n1}U_n'^{\mu_n}}{\mu_n}(1-\mu_n)-\delta_{n0}-\frac{\psi_n}{2}\left(\frac{K'_n}{K_n}-1\right)^2+\psi_n\left(\frac{K'_n}{K_n}-1\right)\frac{K'_n}{K_n}\right) \tag{B.14}$$

Log-linearizing Equations (B.13) and (B.14) yields the following two expressions

$$\widehat{\tau}-\widehat{\mathsf{Z}}_e-\frac{\epsilon}{1-\iota}\left(\widehat{C}-\iota\widehat{C}_{-1}\right)+\psi_e\left(\widehat{K}_e-\widehat{K}_{e,-1}\right) = \tau'-\mathsf{Z}'_e-\frac{\epsilon}{1-\iota}\left(\widehat{C}'-\iota\widehat{C}\right) +\beta\delta_{e1}u^{\mu_e}(\mu_e-1)\widehat{U}'_e+\beta\psi_e\left(\widehat{K}'_e-\widehat{K}_e\right) \tag{B.15}$$

$$\widehat{\tau}-\widehat{\mathsf{Z}}_n-\frac{\epsilon}{1-\iota}\left(\widehat{C}-\iota\widehat{C}_{-1}\right)+\psi_n\left(\widehat{K}_n-\widehat{K}_{n,-1}\right) = \tau'-\mathsf{Z}'_n-\frac{\epsilon}{1-\iota}\left(\widehat{C}'-\iota\widehat{C}\right) +\beta\delta_{n1}u^{\mu_n}(\mu_n-1)\widehat{U}'_n+\beta\psi_n\left(\widehat{K}'_n-\widehat{K}_n\right) \tag{B.16}$$

Also, log-linearizing the laws of motion for capital accumulation, Equations (A.20) and (A.21), leads to

$$\widehat{K}_e=(1-\delta u_e)\widehat{K}_{e,-1}-\delta_{e1}u^{\mu_e}\widehat{U}_e+\frac{i_e}{k_e}\left(\widehat{\mathsf{Z}}_e+\widehat{I}_e\right) \tag{B.17}$$

$$\widehat{K}_n=(1-\delta u_n)\widehat{K}_{n,-1}-\delta_{n1}u^{\mu_n}\widehat{U}_n+\frac{i_n}{k_n}\left(\widehat{\mathsf{Z}}_n+\widehat{I}_n\right) \tag{B.18}$$

Additionally, log-linearizing Equations (A.35)-(A.37) and (A.40)-(A.41), relating to international trade, yields

$$\widehat{D}_e=\varsigma\left(\widehat{P}-\widehat{P}_e\right)+\widehat{D} \tag{B.19}$$

$$\widehat{IM}=\phi\widehat{P}+\widehat{D} \tag{B.20}$$

$$\widehat{IM}_e = -\eta \widehat{\mathsf{P}}_e^{im} + \widehat{IM} \tag{B.21}$$

$$\widehat{EX} = -\phi_w \widehat{P} + \widehat{\mathsf{D}}^w \tag{B.22}$$

$$\widehat{EX}_e = \eta_w \left(\widehat{P} - \widehat{P}_e\right) + \widehat{EX} \tag{B.23}$$

Log-linearized real exchange rate, Equation (A.33), is

$$\widehat{P} = \sigma \left(\frac{p_e}{p}\right)^{1-\varsigma} \widehat{P}_e + (1-\sigma)\left(\frac{p_n}{p}\right)^{1-\varsigma} \widehat{P}_n \tag{B.24}$$

Finally, the market clearing conditions and the definitions of aggregate variables, when log-linearized, give

$$\widehat{I} = \frac{i_e}{i}\widehat{I}_e + \frac{i_n}{i}\widehat{I}_n \tag{B.25}$$

$$\widehat{Y} = \frac{y_e}{y}\widehat{Y}_e + \frac{y_n}{y}\widehat{Y}_n \tag{B.26}$$

$$\widehat{H} = \frac{h_e}{h}\widehat{H}_e + \frac{h_n}{h}\widehat{H}_n \tag{B.27}$$

$$\widehat{E} = \frac{e_e}{e}\widehat{E}_e + \frac{e_n}{e}\widehat{E}_n \tag{B.28}$$

$$\widehat{Y}_e = \frac{d_e}{y_e}\widehat{D}_e + \frac{ex_e}{y_e}\widehat{EX}_e - \frac{im_e}{y_e}\widehat{IM}_e \tag{B.29}$$

$$\frac{pex}{e}\left(\widehat{P} + \widehat{EX}\right) - \frac{im}{e}\widehat{IM} = \widehat{\mathsf{Q}} + \widehat{E} \tag{B.30}$$

$$\widehat{Y} = \frac{c}{y}\widehat{C} + \frac{i}{y}\widehat{I} + \frac{\mathsf{g}}{y}\widehat{\mathsf{G}} \tag{B.31}$$

$$\widehat{Y} = \widehat{D} \tag{B.32}$$

Appendix B.2 Steady State

The log-linear forms above reveal that we can only solve the model when we have given values to the vector of parameters, $\mathfrak{p}$, and the vector of variables in steady state, $\mathfrak{v}$, summarized as

$$\begin{aligned}
\mathfrak{p} &= \beta \quad \iota \quad \omega \quad \epsilon \quad \alpha_e \quad \alpha_n \quad \nu_e \quad \nu_n \quad \theta_e \quad \theta_n \quad \mu_e \quad \mu_n \quad \psi_e \quad \psi_n \quad \delta_{e1}u^{\mu_e} \quad \delta_{n1}u^{\mu_n} \quad \varsigma \quad \phi \quad \eta \quad \phi_w \quad \eta_w \quad \sigma \\
\mathfrak{v} &= \tfrac{i_e}{k_e} \quad \tfrac{i_n}{k_n} \quad \tfrac{e_e}{k_e} \quad \tfrac{e_n}{k_n} \quad \tfrac{p_e}{p} \quad \tfrac{p_n}{p} \quad \tfrac{i_e}{i} \quad \tfrac{i_n}{i} \quad \tfrac{y_e}{y} \quad \tfrac{y_n}{y} \quad \tfrac{h_e}{h} \quad \tfrac{h_n}{h} \quad \tfrac{e_e}{e} \quad \tfrac{e_n}{e} \quad \tfrac{d_e}{y_e} \quad \tfrac{ex_e}{y_e} \quad \tfrac{im_e}{y_e} \quad \tfrac{pex}{e} \quad \tfrac{im}{e} \quad \tfrac{c}{y} \quad \tfrac{i}{y} \quad \tfrac{\mathsf{g}}{y}
\end{aligned}$$

The main text provides sufficient description of the calibration and estimation of the vector of parameters above. Here, we focus mostly on the computation of the deterministic steady state of the model. That is, the elements of $\mathfrak{v}$. To compute the steady state, we normalize the exogenous variables to unity and employ the normalization that capital utilization rates are also equal to 1 in steady state. As is standard in the literature, we target labor hours worked using microeconomic data on the U.S. time use survey from the Bureau of Labor Statistics (BLS). This gives $H = 0.279$, $H_e = 0.115$, and $H_n = 0.164$, where the aggregate labor hours is in the ballpark of the figures used in the literature (see, for example, McGrattan et al. (1997) who used 0.27, and Dhawan and Jeske (2008) who used 0.3). We also target the data averages of imports to output, im/y, domestic absorption of energy intensive goods to output, d_e/y, government spending to output, g/y, and investment to output, i/y. Then, the steady state representations of the model variables can be obtained recursively, as follows.

The steady state of the relative price of bonds, R, comes from the Euler equation for consumption, Equation (B.11), as

$$R = \frac{1}{\beta} \tag{B.33}$$

By combining the Euler equations for the two capital stocks, Equations (A.30) and (A.31), with the respective equations that determine the equality between the marginal user costs and marginal user benefits of capital, Equations (A.28) and (A.29), and evaluating them in the steady state, we obtain

$$R_e = \frac{1}{\beta} - 1 + \delta u_e \tag{B.34}$$

$$R_n = \frac{1}{\beta} - 1 + \delta u_n \tag{B.35}$$

Combining Equations (A.11) and (A.13) in the steady state and using Equation (B.34) yields an expression for energy use in terms of capital in the energy intensive sector as

$$e_e = \underbrace{\left(\frac{1-\theta_e}{\theta_e}\frac{1-\beta\left(1-\delta u_e\right)}{\beta\mathsf{Q}}\right)^{\frac{1}{1+\nu_e}}}_{\mathfrak{E}_o} k_e = \mathfrak{E}_o k_e \tag{B.36}$$

Applying the same procedure to Equations (A.12), (A.14), and (B.35) yields the energy use in the non-energy intensive sector as

$$e_n = \underbrace{\left(\frac{1-\theta_n}{\theta_n}\frac{1-\beta\left(1-\delta u_n\right)}{\beta\mathsf{Q}}\right)^{\frac{1}{1+\nu_n}}}_{\mathfrak{N}_o} k_n = \mathfrak{N}_o k_n \tag{B.37}$$

Using Equation (A.11), we can write an expression for steady state capital in the energy intensive sector as

$$\begin{aligned} k_e &= \left(\frac{\beta \alpha_e \theta_e p_e}{1-\beta\left(1-\delta u_e\right)} \left(h_e\right)^{1-\alpha_e} \underbrace{\left(\theta_e + \left(1-\theta_e\right) \mathfrak{E}_0^{-\nu_e}\right)^{-\frac{\alpha_e+\nu_e}{\nu_e}}}_{\mathfrak{E}_1} \right)^{\frac{1}{1-\alpha_e}} \\ &= h_e \underbrace{\left(\frac{\beta \alpha_e \theta_e p_e}{1-\beta\left(1-\delta u_e\right)} \mathfrak{E}_1^{-\frac{\alpha_e+\nu_e}{\nu_e}} \right)^{\frac{1}{1-\alpha_e}}}_{\mathfrak{E}_2} = \mathfrak{E}_2 h_e \end{aligned} \tag{B.38}$$

Applying the same procedure to Equation (A.12) yields the expression for steady state capital in the non-energy intensive sector as

$$\begin{aligned} k_n &= \left(\frac{\beta \alpha_n \theta_n p_n}{1-\beta\left(1-\delta u_n\right)} \left(h_n\right)^{1-\alpha_n} \underbrace{\left(\theta_n + \left(1-\theta_n\right) \mathfrak{N}_0^{-\nu_n}\right)^{-\frac{\alpha_n+\nu_n}{\nu_n}}}_{\mathfrak{N}_1} \right)^{\frac{1}{1-\alpha_n}} \\ &= h_n \underbrace{\left(\frac{\beta \alpha_n \theta_n p_n}{1-\beta\left(1-\delta u_n\right)} \mathfrak{E}_1^{-\frac{\alpha_n+\nu_n}{\nu_n}} \right)^{\frac{1}{1-\alpha_n}}}_{\mathfrak{N}_2} = \mathfrak{N}_2 h_n \end{aligned} \tag{B.39}$$

From the laws of motion for the accumulation of the two capital stocks, Equations (A.20) and (A.21), and the solutions for the two capital stocks, the steady state values for the two investments are given by

$$i_e = \delta u_e k_e = \delta u_e \mathfrak{E}_2 h_e \tag{B.40}$$

$$i_n = \delta u_n k_n = \delta u_n \mathfrak{E}_2 h_n \tag{B.41}$$

Substituting Equations (B.36) and (B.38) into the energy intensive sector production function, Equation (A.2), gives

$$y_e = h_e^{1-\alpha_e} \underbrace{\left(k_e\right)}_{\mathfrak{E}_2 h_e}^{\alpha_e} \underbrace{\left(\left(\theta_e + \left(1-\theta_e\right) \mathfrak{E}_0^{-\nu_e}\right)\right)}_{\mathfrak{E}_1}^{-\frac{\alpha_e}{\nu_e}} = h_e \underbrace{\mathfrak{E}_1^{-\frac{\alpha_e}{\nu_e}} \mathfrak{E}_2^{\alpha_e}}_{\mathfrak{E}_3} = \mathfrak{E}_3 h_e \tag{B.42}$$

Likewise, substituting Equations (B.37) and (B.39) into the non-energy intensive sector production function, Equation (A.3), in steady state gives

$$y_n = h_n^{1-\alpha_n} \underbrace{\left(k_n\right)}_{\mathfrak{N}_2 h_n}^{\alpha_n} \underbrace{\left(\left(\theta_n + \left(1-\theta_n\right) \mathfrak{N}_0^{-\nu_n}\right)\right)}_{\mathfrak{N}_1}^{-\frac{\alpha_n}{\nu_n}} = h_n \underbrace{\mathfrak{N}_1^{-\frac{\alpha_n}{\nu_n}} \mathfrak{N}_2^{\alpha_n}}_{\mathfrak{N}_3} = \mathfrak{N}_3 h_n \tag{B.43}$$

Then, the above sector related variables sums to give the aggregate steady state values

$$\begin{aligned} e &= e_e + e_n \\ i &= i_e + i_n \\ y &= y_e + y_n \end{aligned} \tag{B.44}$$

Further, using Equation (A.36), the real exchange rate in the steady state, noting that $y = d$, is

$$p = \left(\frac{1}{1-\kappa}\frac{im}{y}\right)^{\frac{1}{\phi}} \tag{B.45}$$

Then, from Equation (A.35) the price of energy intensive goods in the steady state is

$$p_e = \left(\frac{1}{\sigma}\frac{d_e}{y}\right)^{-\frac{1}{\varsigma}} p \tag{B.46}$$

We obtain the steady state price of non-energy intensive goods as

$$p_n = \left(\frac{p^{1-\varsigma} - \sigma\,(p_e)^{1-\varsigma}}{1-\sigma}\right)^{\frac{1}{1-\varsigma}} \tag{B.47}$$

Using Equations (A.48) and (A.49), consumption is derived as

$$c = y - i - \mathsf{g} \tag{B.48}$$

Combining Equations (A.36) and (A.37), the steady state of imported energy intensive goods can be written as

$$im_e = \chi\,(1-\kappa)\,p^{\phi} y \tag{B.49}$$

Finally, combining Equations (A.40) and (A.41) gives the exports of energy intensive goods in the steady state as

$$ex_e = \chi_w\,(1-\kappa_w)\,(p_e)^{-\eta_w}\,p^{\eta_w-\phi_w} \tag{B.50}$$

It is easy to see from the above derivations that some parameters only works to pin down the steady state of variables and are not used in the dynamic model simulation. Hence, steady state representation is not worked out (by hand) for every single variable.

Appendix C Data and Sources

The Hodrick-Prescott (1997) filter is applied to the series described below, with the smoothing parameter set as equal to 400, as in Kim and Loungani (1992) and Costello (1993), given that our data timing interval is annual.

Output

Model variables: Y, Y_e, Y_n.

Data: Aggregate output, Y, is measured as the sum of the two sectoral outputs, $Y_e + Y_n$. That is, the sum of the gross outputs of the energy and non-energy intensive sectors. Due to a lack of data on gross output by industry dating back to 1949, which is the general starting year of the variables, we instead construct the two sectoral gross domestic products from the value added by industry data (taken from GDP by Industry). More specifically, aggregate output is defined as the total value added of all industries. Consequently, energy intensive sector output, Y_e, is defined as the sum of the value added from agriculture (including forestry, fishing, and hunting), mining, utilities, construction, manufacturing, and transportation (and warehousing due to lack of further disaggregation); the non-energy intensive sector output, Y_n, is defined as the sum of the value added from wholesale and retail trade, information, finance (including insurance, real estate, rental, and leasing), professional and business services, educational services (including health care, and social assistance), arts (including entertainment accommodation, and food services), and other services except government. Lastly, due to a lack of sufficient disaggregation of government output, we split the output of the public sector into two and added half each to Y_e and Y_n.

Consumption

Model variable: C.

Data: This is defined as personal consumption expenditures less durable goods (taken from Table 1.1.5. Gross Domestic Product).

Investment

Model variables: I_t, I_e, I_n.

Data: The measure of gross investment is taken to be the sum of personal consumption expenditure on durable goods, and private non-residential (structures and equipment) and residential fixed investments. This has to be applied to defining the two types of investment variables, noting that aggregate investment is given as $I = I_e + I_n$. For investment series, we combine Table 2.7: Investment in Private Fixed Assets, Equipment, Structures, and Intellectual Property Products by Type and the series for consumer durables from Table 2.4.5: Personal Consumption Expenditures by Type of Product. Beginning with the consumption of durable goods in Table 2.4.5, the following are assigned investments that are non-energy intensive denoted by I_n^{dg}: furnishings and durable household equipment, recreational goods and vehicles, and other durable goods, such that investment in the energy intensive type consumption durable goods is given by I_e^{dg} =Durable goods$-I_n^{dg}$. Further, assignment of Table 2.7 into investment by type is as follows. Investment in the energy intensive goods are deemed to be given by the sum of equipment and structures less residential equipment and improvements. Thus, investment in the non-energy intensive type goods is the sum of residential equipment, improvements, and intellectual property products.

Capital

Model variables: K_e, K_n.

Data: Following the constructions of energy and non-energy intensive investments above, I build the series for capital using Table 8.1. Current-Cost Net Stock of Consumer Durable Goods and Table 2.1. Current-Cost Net Stock of Private Fixed Assets, Equipment, Structures, and Intellectual Property Products by Type. We construct the capital stock of the energy intensive goods as the sum of non-residential equipment and structures and the capital stock of the non-energy intensive goods is calculated as the sum of residential equipment and structures, and intellectual property products. As in the investment series above, non-energy intensive type capital stocks is taken as the sum of furnishings and durable household equipment, recreational goods and vehicles, and other durable goods, such that the capital stock in the energy intensive type consumption durable goods is given by motor vehicles and parts.

Labor hours

Model variables: H, H_e, H_n.

Data: Aggregate labor hours, H, is defined as hours of all persons engaged in production and hours worked per sector, H_e and H_n, which are calculated by following the procedure of Herrendorf et al. (2013). This involves combining the U.S. Bureau of Economic Analysis (BEA) GDP-by-Industry data reported using the North American Industry Classification System (NAICS) classification with BEA's Income-and-Employment-by-Industry data reported with three different classifications over the sample period (SIC72 for pre-1987, SIC87 between 1987-2000, and NAICS since 2001). This is necessary because, although the former data source follows the classification we would prefer, the latter provides us with the kind of detailed industry level information we require for assignment into the two sectors. For this assignment, therefore, we follow the definitions of other sector variables already shown above as closely as feasibly permitted by the level of data disaggregation. Specifically, for each of the series, employment and hours in agriculture, mining, utilities, construction, manufacturing and transportation are classed as related to the energy intensive sector, while employment and hours in wholesale, retail trade, information, finance, professional services, education, arts, and other services are non-energy intensive. Formally, the sectoral labor hours are obtained using:

$$H^j = NAICS_{FT}^H + \frac{NAICS_{FT}^H}{NAICS_{FT}^E} \times NAICS_{SE}$$

with

$$NAICS_{FT}^H = SIC_{FT}^H \times \frac{NAICS_{FT}^E}{SIC_{FT}^E}$$

$$NAICS_{FT}^E = SIC_{FT}^E \times \frac{NAICS_{FTPT}^E}{SIC_{FTPT}^E}$$

$$NAICS_{SE} = SIC_{SE} \times \frac{NAICS_{FTPT}^E}{SIC_{FTPT}^E}$$

where superscripts H and E denote hours employed and number of employees, respectively, and subscripts FT, SE, and $FTPT$ denote full-time, self-employed, and full-time part-time, respectively.

Capital utilization rate

Model variables: U_e, U_n.

Data: Following the assignment of industries into the two sectors, we deem it appropriate to have two definitions of capital utilization rate. Thus, capacity utilization rate for total manufacturing industry and capacity utilization rate for motor vehicles and parts are used as proxies for the measures of capital utilization rates in the energy and non-energy intensive sectors, respectively.

Energy use

Model variables: E, E_e, E_n.

Data: We take the total energy consumption in the economy, E, to be the aggregate consumption of primary energy, or the consumption of fossil fuels comprising of petroleum, coal, and natural gas measured in trillion British thermal units (BTUs), of the private sector excluding the electric power sector.[5] Energy consumption is provided for four end-use sectors, namely the industrial, transportation, residential and commercial sectors. Given a lack of further disaggregation, we use the primary energy consumption in both the industrial and transportation sectors as a proxy for energy use in the energy intensive sector, and primary energy consumption in both the residential and commercial sectors as a proxy for energy use in the non-energy intensive sector. Hence, aggregate energy consumption in this economy is formally given by

[5] We do not include the consumption of renewables (geothermal, solar/ PV, and biomass) and electricity for both theory and data reasons. On the data, if one chooses to use, for instance, total primary energy consumption data, there is no data for biomass consumption until 1981. Also, we excluded the electric power generating sector, which would have been classed as a highly energy intensive sector given that close to 70% of all primary energy is used, or lost, since this sector provides electricity to the final consumers. Note that we have, however, not included the data for primary energy use by the electric power generating sector because we do not have this in our model.

$E = E_e + E_n =$ dollar value of total primary energy use $= q_t \frac{(E_t \times 1\ trillion/\varkappa \times 1\ million)}{1\ billion}$, where $\varkappa = 5.78$ is the conversion factor assumed for relating BTUs to barrels of oil, which is similar to the figure employed by the industry.

Domestic absorption

Model variable: D.

Data: By theoretical construction, $D = Y$, so that we use the same data for aggregate output for domestic absorption.

Domestic demand of energy intensive goods

Model variable: D_e.

Data: This is constructed as $D_e =$ private consumption of energy intensive goods (taken from Table 2.4.5: Personal Consumption Expenditures by Type of Product) + private investment in energy intensive goods (taken from Table 2.4.5: Personal Consumption Expenditures by Type of Product and Table 2.7: Investment in Private Fixed Assets, Equipment, Structures, and Intellectual Property Products by Type) + government consumption of energy intensive goods (taken from Table 3.9.5: Government Consumption Expenditures and Gross Investment) + government investment in energy intensive goods (taken from Tables 7.5A-7.5B: Investment in Government Fixed Assets). Note that, given a lack of disaggregated data on government consumption expenditure, we have assumed that the share of government consumption in energy intensive goods is the same as for government investment in energy intensive goods, and apply this share to the consumption data.

Aggregate import

Model variable: IM.

Data: This is taken to be the aggregate import (taken from Table 1.1.5. Gross Domestic Product).

Import of energy intensive goods

Model variable: IM_e.

Data: This is taken to be the import of goods (taken from Table 1.1.5. Gross Domestic Product).

Aggregate export

Model variable: EX.

Data: This is taken to be the aggregate export (taken from Table 1.1.5. Gross Domestic Product).

Export of energy intensive goods

Model variable: EX_e.

Data: This is taken to be the export of goods (taken from Table 1.1.5. Gross Domestic Product).

Wage rate

Model variable: W.

Data: This is a real index of hourly compensation (Series ID: PRS85006063, Nonfarm Business Sector: Compensation).

Interest rate

Model variable: R.

Data: This is the three-month Treasury bill rate for 1949-1954 (taken from Smets and Wouters, 2007) where we have converted their quarterly data into annual data by averaging; we use the federal funds rate for 1955-2013.

Real exchange rate

Model variable: P.

Data: General price level in the domestic country, which is taken to be the consumer price index (CPI) for all urban consumers, relative to world CPI.

Price of energy intensive goods

Model variable: P_e.

Data: Calculated as the weighted average of the chain-type price indexes for value added from agriculture, mining, utilities, construction, manufacturing, and transportation (taken from GDP by Industry).

Price of non-energy intensive goods

Model variable: P_n.

Data: Calculated as the weighted average of the chain-type price indexes for value added from wholesale and retail trade, information, finance, professional and business services, educational services, arts, and other services (taken from GDP by Industry).

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