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Taxation under Oligopoly in a General Equilibrium Setting

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Abstract

Taxation under oligopoly is analysed in a general equilibrium setting where the firms are large relative to the size of the economy and maximise the utility of their shareholders. It turns out that the model is an aggregative game, which simplifies the comparative statics for the effects of taxation. This novel analysis of taxation leads to a number of counterintuitive results that challenge conventional wisdom in microeconomics. A lump-sum tax may increase the price of the oligopolistic good and decrease welfare whereas a profits tax may decrease the price of the oligopolistic good and increase welfare. An *ad valorem* tax may decrease the price of the oligopolistic good and increase welfare. Furthermore, in line with conventional wisdom, total tax revenue is always higher with an *ad valorem* tax than with a specific tax that leads to the same price for the oligopolistic good.

Keywords: Oligopoly; General Equilibrium; Aggregative Games; *Ad Valorem* Taxes; Specific Taxes; Profits Taxes; Lump-Sum Taxes.

JEL Classification: C72; D21; D43; D51; H22; H25; L13; L21.

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1. Introduction

The analysis of taxation under oligopoly is usually undertaken in a partial equilibrium setting that assumes the oligopolistic firms are small relative to the size of the economy and ignores the income effect on demand. In a general equilibrium setting where firms are large relative to the size of the economy, a number of additional effects of the decisions of oligopolistic firms have to be considered. Firstly, the decisions of an oligopolistic firm will affect the income of consumers through their effect on the profits of the firm and through their effect on the profits of its competitors, which will affect the demand facing the firm. Secondly, the decisions of an oligopolistic firm will affect the prices paid for the firm's output by its shareholders, which will affect the utility of the shareholders. This paper will analyse taxation under oligopoly in a general equilibrium setting where firms maximise the utility of their shareholders rather than profits, and take into account the income effect on demand of their profits. A number of counterintuitive results will be obtained for the effects of lump-sum taxes, profits taxes, and *ad valorem* consumption taxes. Also, the total tax revenue raised by specific and *ad valorem* taxes that both yield the same price for the oligopolistic good will be compared.

The modelling of oligopoly in a general equilibrium setting when oligopolistic firms are large relative to the size of the economy has proved to be problematic for a number of reasons. Firstly, since the demand of consumers depends upon their income, which includes the profits of the oligopolistic firms paid as dividends, the objective of the oligopolistic firms will be a function of their own profits and the profits of their competitors, which complicates the optimisation problem facing the firms. To avoid this complication, the optimisation problem has been solved by assuming that there is no income effect as in Hart (1982) or that profits are taxed at 100% as in Guesnerie and Laffont (1978) or that firms take the profits of competitors as given as in Myles (1989). Secondly, if the oligopolistic firms are assumed to maximise profits then the choice of the numeraire good can have a real effect on the equilibrium outcome as shown by Gabszewicz and Vial (1972). As explained by Dierker and Grodal (1998, 1999), the reason for this result is that profit maximisation is only a valid assumption if the firms are price-takers or if the shareholders of the firm do not consume the firm's product. A view supported by Kreps (2013, pages 200-201):

> *'it is worth noting that the assertion that owners prefer profit maximization is very bound up in the assumption that the firm has no impact on prices. When firms affect prices, and when owners of the firm consume (or are endowed with) the goods whose prices the firm affects, it is no longer clear that the owners either should or do prefer profit-maximizing choices by firms.'*

When shareholders consume the product of the oligopolistic firm, Dierker and Grodal (1998, 1999) argue that the objective of the firm should be the maximisation of the real wealth of the shareholders, and in this case the choice of numeraire good does not matter and there is no price normalisation problem.[1]

The analysis of taxation under oligopoly has generally been undertaken in a partial equilibrium setting. For example, the incidence of taxes under oligopoly has been analysed by Seade (1985), Stern (1987), and Dierickx *et al.* (1988) who showed that consumption taxes may lead to price overshifting (undershifting) when price increases are larger (smaller) than the tax increase, which is a possibility that does not arise under perfect competition. Another difference between perfect competition and imperfect competition is that specific and *ad valorem* taxes are equivalent under perfect competition, but that equivalence breaks down under imperfect competition. Under monopoly, Suits and Musgrave (1953) showed that an *ad valorem* tax is superior to a specific tax that results in the same tax revenue. This result was extended to the case of oligopoly by Delipalla and Keen (1992) while Anderson *et al.* (2001) showed that an *ad valorem* tax would yield higher tax revenue than a specific tax that results

[1] An alternative approach to the modelling of oligopoly in a general equilibrium setting has been to assume that firms are large in their industry, but that the industries are infinitesimally small in the economy as in Neary (2003). Then, the output decisions of an oligopolistic firm have an infinitesimally small effect on prices facing shareholders.

in the same price. Under monopoly, Skeath and Trandel (1994) show that a specific tax can be replaced by Pareto-superior *ad valorem* tax. However, in a general equilibrium setting with a 100% profits tax, Blackorby and Murty (2007) show that the set of Pareto optima with a specific tax is identical to the set of Pareto optima with an *ad valorem* tax. Myles (1996) considers the optimal combination of *ad valorem* and specific taxes. In a general equilibrium setting, where oligopoly is modelled as a strategic market game, Grazzini (2006) claims to show that a specific tax can dominate an *ad valorem* tax, but the result is driven by the effect on income distribution. Only a few authors have analysed taxation under oligopoly in a general equilibrium setting, notably Myles (1989) who derived the Ramsey taxes.

In this paper, taxation under oligopoly will be analysed in a simple general equilibrium setting where oligopolistic firms are large in the economy. The income effect problem will be addressed by assuming that preferences of consumers are either identical and homothetic or identical and quasi-linear. The price normalisation problem will be addressed by assuming that the oligopolistic firms maximise the utility of their shareholders, which is equivalent to the maximisation of the real wealth of shareholders in Dierker and Grodal (1998, 1999).[2] In both cases, it turns out that the model is an aggregative game so the equilibrium condition can be expressed as a function of the aggregate output of the oligopolistic industry. Cobb-Douglas preferences will be used to provide an example of homothetic preferences while a quadratic utility function that yields a linear demand function will be used to provide an example of quasi-linear preferences. The model will be used to analyse the incidence of lump-sum taxes, profits taxes, specific taxes and *ad valorem* taxes, and to compare the revenue raised by specific and *ad valorem* taxes that result in the same price. In contrast to conventional wisdom, lump-

[2] According to Dierker and Dierker (2006, page 438) 'Birgit Grodal was determined to solve the question of how a firm should measure the wealth of its shareholders without reference to utility functions'. However, in order to have a tractable model to analyse taxation, maximisation of shareholder utility will be used and it will be assumed that all shareholders are identical.

sum taxes and profits taxes may affect the equilibrium price under oligopoly. A lump-sum tax will increase the price with homothetic preferences and a profits tax will decrease the price with quasi-linear preferences. Also, with quasi-linear preferences, an *ad valorem* tax may decrease the equilibrium price.

Section two analyses taxation in a model with identical and homothetic preferences then section three explicitly solves the model for the case of Cobb-Douglas preferences. Section four analyses taxation in a model with identical and quasi-linear preferences then section five explicitly solves the model for the case of quadratic utility that yields a linear demand function. The conclusions are presented in section six.

2. The Model with Identical and Homothetic Preferences

Consider an economy with two goods: X and Y, and one factor of production: labour, L. Good X is a homogeneous product produced by an oligopolistic industry and its price is p_X, while good Y is produced by a perfectly competitive industry and its price is p_Y. Labour is supplied by L worker/consumers who each supply one unit of labour (total labour supply is L) and who each receive a wage w. The utility of the lth worker is: $u_{Ll} = u\left(x_{Ll}, y_{Ll}\right)$, for $l = 1,\ldots,L$, where x_{Ll} is consumption of good X and y_{Ll} is consumption of good Y. Utility maximisation yields the indirect utility function of the lth worker: $v_{Ll} = v\left(p_X, p_Y, m_{Ll}\right)$, where m_{Ll} is the income of the lth worker, which is equal to the wage in the absence of any transfers. The X industry is a Cournot oligopoly consisting of J firms (if $J = 1$ then the industry is a monopoly) owned by K shareholder/consumers with each shareholder owning shares in only one of the oligopolistic firms hence each firm has K / J shareholders.[3] The shareholders are assumed to be price takers when making their consumption decisions, and the utility of the k

[3] If shareholders each held shares in all the oligopolistic firms then the maximisation of shareholder utility would lead to a collusive outcome that maximised the joint utility of shareholders.

th shareholder of the j th firm is: $u_{Kkj} = u\left(x_{Kkj}, y_{Kkj}\right)$, for $k = 1,\ldots,K/J$ and $j = 1,\ldots,J$, where x_{Kkj} is the consumption of good X and y_{Kkj} is the consumption of good Y. Utility maximisation yields the indirect utility function of the shareholder/consumer: $v_{Kkj} = v\left(p_X, p_Y, m_{Kkj}\right)$, where m_{Kkj} is the dividend income of the shareholder.

The unit labour input requirement in the X industry is c_X, and the unit labour input requirement in the Y industry is c_Y. The unit labour input requirements are constant in both industries hence there are constant returns to scale in both industries. The labour market is assumed to be perfectly competitive hence there will be full employment of labour, which implies that: $c_X X + c_Y Y = L$, where X is the total output of the X industry and Y is the total output of the Y industry. In the perfectly competitive Y industry, where firms are assumed to be owned by the workers, firms are price takers and maximise profits. Therefore, assuming that both goods are produced in equilibrium, the wage will be equal to the marginal product of labour in the Y industry, $w = p_Y/c_Y$. For simplicity, and without loss of generality as there is no price normalisation problem, the price of good Y will be normalised at unity, $p_Y = 1$, and the relative price of the oligopolistic good will be denoted by $P = p_X/p_Y$.

The marginal cost of producing good X is the labour input requirement multiplied by the wage, $c_X w = c_X/c_Y$, which is the opportunity cost of good X in terms of good Y. The government can use a number of instruments to tax the oligopolistic X industry: a lump-sum tax, T; a profits tax, κ; a specific consumption tax, t; and an *ad valorem* consumption tax, τ. In a general equilibrium setting, the expenditure and/or transfers financed by the tax revenue have to be explicitly modelled therefore, for simplicity, the tax revenue is assumed to be redistributed to the workers as lump-sum transfers. With these various taxes, the profits of an oligopolistic firm in the X industry are:

$$\Pi_j = (1-\kappa)\left[\left(\frac{P}{1+\tau} - c_X w - t\right)X_j - T\right] \qquad j = 1,\ldots,J \tag{1}$$

The demand facing the firm will, in general, depend upon the profits of all the oligopolistic firms which depend upon the outputs of all the oligopolistic firms. Therefore, the payoff function of each firm depends upon the payoff functions of all the oligopolistic firms as well as the outputs of all the oligopolistic firms. To get round this problem, it will be assumed that all consumers (workers and shareholders) have identical and homothetic preferences then demand will only depend upon aggregate profits of the oligopolistic industry. The aggregate profits of the oligopolistic firms is: $\Pi = \sum_{j=1}^{J} \Pi_j$:

$$\Pi = (1-\kappa)\left[\left(\frac{P}{1+\tau} - c_X w - t\right)X - JT\right] \tag{2}$$

The total tax revenue collected by the government from the various taxes is:

$$R = \frac{\kappa}{1-\kappa}\Pi + tX + \frac{\tau}{1+\tau}PX + JT \tag{3}$$

The aggregate income of the shareholders is equal to the aggregate profits of the oligopolistic firms: $M_K = \Pi$, and the aggregate income of the workers is equal to their wage income plus the transfer they receive from the government, which is equal to total tax revenue: $M_L = wL + R$. Therefore, since $\Pi + R = (P - c_X w)X \equiv \bar{\Pi}$, the aggregate income of all consumers is: $M = M_K + M_L = wL + (P - c_X w)X = wL + \bar{\Pi}$, which does not directly depend upon any of the tax rates but only indirectly through their effect on output. With identical and homothetic preferences, the Marshallian demand for good X can be written as a function of prices and the aggregate income of the consumers:

$$X = D(p_X)M = D(P)\left[wL + (P - c_X w)X\right] \tag{4}$$

The slope of the Marshallian demand function, holding income constant, will be assumed to be negative, $\partial X/\partial P = M\,\partial D/\partial P < 0$. Clearly, income is not constant as the output decisions of the oligopolistic firms will affect income through their effect on profits. Taking account of the effect on profits of output, the Marshallian demand function (4) can be inverted to yield an inverse demand function for good X:

$$P = P\left(X; c_X, w, L\right) = P\left(X\right) \tag{5}$$

This inverse demand function $P\left(X\right)$ includes the effects of changes in the output of firms on profits and thereby on aggregate income. Obviously, this is not the same as the Marshallian inverse demand function due to the effect of output on profits. The slope of this new inverse demand function (5) can be obtained by totally differentiating (4), which yields:

$$P' = \frac{\partial P}{\partial X} = \frac{X}{P}\frac{1-\theta_\Pi}{\eta_X^M + \theta_X} \tag{6}$$

where $\eta_X^M = P\,\partial D/\partial p_X/D < 0$ is the usual own price elasticity of the Marshallian demand function obtained by differentiating (4) with respect to P while holding M constant; $\theta_X \equiv PX/M$ is the share of good X in total expenditure; $\theta_\Pi = \overline{\Pi}/M$ is the share of profits from the oligopolistic industry in total income, and it follows that: $0 < \theta_\Pi < \theta_X < 1$. The slope of this new inverse demand function will be negative if $\eta_X^M + \theta_X < 0$, or the share of good X in total expenditure is less than the absolute value of the Marshallian own price elasticity, $\theta_X < \left|\eta_X^M\right|$, which will always be the case if the Marshallian demand function is elastic. From (6), the own-price elasticity of the new inverse demand function is:

$$\eta_X \equiv \frac{P}{XP'} = \frac{\eta_X^M + \theta_X}{1-\theta_\Pi} \tag{7}$$

The new demand function may be more or less elastic than the Marshallian demand function. The larger is the share of good X in total expenditure then the less elastic will be the new demand function, and the larger is the share of profits in total income then the more elastic will be the new demand function. Henceforth, it will be assumed that the inverse demand function is downward sloping, $P'(X) < 0$.

As the oligopolistic firms in the X industry are large relative to the size of the economy, they each independently and simultaneously choose their output to maximise the utility of their shareholders rather than to maximise profits.[4] Since preferences are identical and homothetic, the utility of the shareholders can be modelled using a representative shareholder. The utility of the representative shareholder of the j th firm is:

$$V_j = v(P)\Pi_j \qquad j = 1,\ldots,J \tag{8}$$

Note that since $\partial V_j / \partial P = \Pi_j \, \partial v / \partial P$ and $\partial V_j / \partial M_j = v$, Roy's identity implies that demand for good X from shareholders is $X_{Kj} = -\Pi_j \, \partial v / \partial P / v$. Assuming an interior solution, where all firms produce positive quantities, the first-order conditions for the maximisation of shareholder utility are:

$$\begin{aligned} \frac{\partial V_j}{\partial X_j} &= \Pi_j \frac{\partial v}{\partial p_X} P' + v \frac{\partial \Pi_j}{\partial X_j} \qquad j = 1,\ldots,J \\ &= v\left[(1-\kappa)\left(\frac{P}{1+\tau} + \frac{X_j P'}{1+\tau} - c_X w - t \right) - X_{Kj} P' \right] = 0 \end{aligned} \tag{9}$$

Hence, the maximisation of shareholder utility implies that the expression in square brackets is zero.[5] Since the preferences of all consumers are identical and homothetic, the

[4] An alternative assumption is that firms maximise real profits defined as nominal profits divided by the true cost of living index of the shareholders. This is equivalent to maximising shareholder utility in this case as the income of shareholders comes entirely from dividends paid by the oligopolistic firm.

[5] The existence of a Nash equilibrium when firms maximise the real wealth of shareholders is proved in theorem three of Dierker and Grodal (1999).

fraction of good X consumed by the shareholders is the same as the fraction of total income received by the shareholders. Hence, consumption of good X by all the shareholders is:

$$X_K = \sum_{j=1}^{J} X_{Kj} = \frac{\Pi}{wL + \left(P(X) - c_X w\right)X} X = \frac{\Pi(X)}{wL + \bar{\Pi}(X)} X \tag{10}$$

where $\Pi(X) \equiv (1-\kappa)\left[\left(P(X)/(1+\tau) - c_X w - t\right)X - JT\right]$ and $\bar{\Pi}(X) \equiv \left(P(X) - c_X w\right)X$. Demand for good X from shareholders can be expressed as a fraction of the total output of the oligopolistic industry, and the fraction is a function of total output of the oligopolistic industry. Aggregating the expression in square brackets in (9) over all the oligopolistic firms then using (10) yields the equilibrium condition in terms of aggregate output:

$$\Omega(X) \equiv (1-\kappa)\left[J\left(\frac{P(X)}{1+\tau} - c_X w - t\right) + X\frac{P'(X)}{1+\tau}\right] - \frac{\Pi(X)}{wL + \bar{\Pi}(X)} XP'(X) = 0 \tag{11}$$

Thus, since the equilibrium condition depends upon aggregate output and not the output of the individual firms, the game is aggregative as in Bergstrom and Varian (1985), which will simplify the comparative static analysis.[6] The function $\Omega(X)$ is assumed to be positive when aggregate output is zero if $P(0) > (c_X w + t)(1+\tau)$. Assuming that both goods are produced in equilibrium (incomplete specialisation) then there will be an equilibrium where $\Omega(X^*) = 0$ for some $X^* \in [0, L/c_X]$. A sufficient condition for a unique equilibrium is that $\Omega(X)$ is everywhere strictly decreasing in X, and a necessary condition is that $\Omega(X)$ is strictly decreasing at every equilibrium. Henceforth, it will be assumed that there is a unique equilibrium and that $\Omega(X)$ is strictly decreasing in equilibrium so $\partial\Omega(X^*)/\partial X < 0$.

[6] Recently, Cornes and Hartley (2012) and Acemoglu and Jensen (2013) have generalised the concept of aggregative games, which may allow this model to be extended to the case of differentiated products.

Therefore, it is possible to solve (11) for the unique aggregate output as a function of the various tax rates and the number of firms: $X^*(\kappa, t, \tau, T, J)$.

To help with the intuition and to allow a diagrammatic representation, the equilibrium condition (11) can be rewritten as:

$$M\Pi \equiv (1-\kappa)\left[J\left(\frac{P(X)}{1+\tau} - c_X w - t\right) + X\frac{P'(X)}{1+\tau}\right] = \frac{\Pi(X)}{wL + \bar{\Pi}(X)} XP'(X) \equiv MX < 0 \quad (12)$$

The left-hand side of the equation is the *marginal profit effect*, an increase in the output of the firm affects the utility of the shareholder through its effect on profits, and the second term is the *marginal expenditure effect*, an increase in the output of the firm affects the utility of the shareholder through its effect on the price of good X (there is a decrease in the minimum expenditure required by the shareholders to reach a given level of utility). In equilibrium, the *marginal profit effect* is equal to the *marginal expenditure effect*, and the *marginal expenditure effect* is clearly negative hence the *marginal profit effect* must be negative.[7] Figure one shows marginal profits $M\Pi$, which are obviously decreasing in the output of the oligopolistic industry, and marginal expenditure MX, which is less steep than $M\Pi$ since $\partial\Omega/\partial X = \partial M\Pi/\partial X - \partial MX/\partial X < 0$. The equilibrium, where shareholder utility is maximised, occurs at the intersection of $M\Pi$ and MX where output is X^*, and the profit-maximising equilibrium occurs where $M\Pi = 0$ where output is X^Π. Therefore, it is immediately obvious that the output that maximises shareholder utility is greater than the profit-maximising output, $X^* > X^\Pi$. Taxes will shift the $M\Pi$ curve through their effect on the marginal profitability of the oligopolistic firms whereas they will shift the MX curve through their effect on the income of shareholders and hence on their consumption of the oligopolistic good.

[7] An equivalent condition is obtained by Dierker and Grodal (1998, page 175) when firms maximise the real wealth of shareholders, but price is the strategic variable in their case. Therefore, *marginal profits* and *marginal expenditure* are derivatives with respect to price rather than output, and hence are positive in equilibrium.

Aggregate welfare, ignoring concerns about income distribution, can be obtained by summing the utility of all consumers since preferences are identical and homothetic. Thus, aggregate welfare is $V = v(P)\left[wL + \bar{\Pi}\right]$, hence the effect of a change in a tax or the number of oligopolistic firms is obtained by differentiating welfare with respect to $\sigma = \kappa, \tau, t, T, J$ and using Roy's identity, which yields:

$$\frac{\partial V}{\partial \sigma} = \left(\left[wL + \bar{\Pi}\right]\frac{\partial v}{\partial p}P' + v\left[\left(P - c_X w\right) + XP'\right]\right)\frac{\partial X}{\partial \sigma} = v\left(P - c_X w\right)\frac{\partial X}{\partial \sigma} \tag{13}$$

If the change in tax or the number of firms increases (decreases) the output of the oligopolistic industry then welfare will increase (decrease) since it will decrease (increase) the deadweight loss from oligopoly.

2.1 The Number of Firms in the Oligopolistic Industry

Conventional wisdom from partial equilibrium analysis says that an increase in the number of firms in a monopoly or oligopoly will lead to a reduction in the price, see Seade (1980a). To obtain the comparative static result for the effect of an increase in the number of firms in this general equilibrium setting, totally differentiate (11), which yields:

$$\frac{\partial X^*}{\partial J} = -\frac{\partial \Omega/\partial J}{\partial \Omega/\partial X} = -\frac{(1-\kappa)}{\partial \Omega/\partial X}\left[\left(\frac{P(X)}{1+\tau} - c_X w - t\right) + \frac{XP'}{wL + \bar{\Pi}}T\right] \tag{14}$$

The term in square brackets is ambiguous since the first term is positive, but the second term is negative if the lump-sum tax is positive. If the lump-sum tax is zero (or sufficiently small) then an increase in the number of firms will increase the aggregate output of the oligopolistic industry thereby leading to a decrease in the price, $\partial P/\partial J = P'\,\partial X^*/\partial J > 0$ and an increase in welfare. This leads to the following proposition:

Proposition 1: *In a general equilibrium setting with identical and homothetic preferences, if the lump-sum tax is zero then an increase in the number of firms in a monopoly or oligopoly will result in a decrease in the price and an increase in welfare.*

In figure two, it can be seen that an increase in the number of firms shifts the $M\Pi$ curve upwards and, if there is a lump-sum tax, the MX curve also shifts upwards resulting in a new equilibrium with higher output X^J, provided the shift in MX is not too large. This is in line with conventional wisdom in a partial equilibrium setting as in Seade (1980a) that an increase in competition will lead to a lower price and higher welfare.

2.2 A Lump-Sum Tax on the Oligopolistic Industry

Conventional wisdom from partial equilibrium analysis says that a lump-sum tax on a monopoly or an oligopoly will have no effect on the output or price set by the firms. To obtain the comparative static result for the effect of a lump-sum tax in this general equilibrium setting, totally differentiate (11), which yields:

$$\frac{\partial X^*}{\partial T} = -\frac{\partial\Omega/\partial T}{\partial\Omega/\partial X} = -\frac{1}{\partial\Omega/\partial X}\left[\frac{XP'}{wL+\overline{\Pi}}\frac{\partial\Pi}{\partial T}\right] = -\frac{(1-\kappa)}{\partial\Omega/\partial X}\left[\frac{XP'}{wL+\overline{\Pi}}J\right] < 0 \tag{15}$$

A lump-sum tax leads to a decrease in the aggregate output of the oligopolistic industry thereby leading to an increase in the price of the oligopolistic good and a decrease in welfare. This leads to the following proposition:

Proposition 2: *In a general equilibrium setting with identical and homothetic preferences, a lump-sum tax on a monopoly or a Cournot oligopoly will result in an increase in the price and a decrease in welfare.*

In figure three, it can be seen that an increase in the lump-sum tax does not shift the $M\Pi$ curve, but shifts the MX curve upwards as the lump-sum tax reduces the income of shareholders thereby reducing their consumption of the oligopolistic good. Hence, there will be a decrease in the output of the oligopolistic industry from X^* to X^T. This result is contrary to conventional wisdom in a partial equilibrium setting, and shows the importance of income effects in a general equilibrium setting.

2.3 A Profits Tax on the Oligopolistic Industry

Conventional wisdom from partial equilibrium analysis says that a profits tax on a monopoly or oligopoly will have no effect on the output or price set by the firms. To obtain the comparative static result for the effect of a profits tax in this general equilibrium setting, totally differentiate (11) and evaluate at the equilibrium, which yields:

$$\frac{\partial X^*}{\partial \kappa} = -\frac{\partial \Omega / \partial \kappa}{\partial \Omega / \partial X} = \frac{1}{\partial \Omega / \partial X} \frac{\Omega\left(X^*\right)}{1-\kappa} = 0 \tag{16}$$

This is zero since $\Omega\left(X^*\right) = 0$ in equilibrium, hence a profits tax has no effect on the aggregate output of the oligopolistic industry thereby leading to no effect on the price of the oligopolistic good, $\partial P / \partial \kappa = P' \partial X^* / \partial \kappa = 0$. This leads to the following proposition:

Proposition 3: *In a general equilibrium setting with identical and homothetic preferences, a profits tax on a monopoly or a Cournot oligopoly will result in no effect on the price or welfare.*

In figure four, it can be seen that the $M\Pi$ curve swivels anti-clockwise around the profit-maximising output as marginal profits are multiplied by $\left(1-\kappa\right)$, and the MX curve shifts upwards as the profits received by the shareholders are also multiplied by $\left(1-\kappa\right)$, see (12). Since preferences are identical and homothetic, the MX curve shifts upwards by the same amount as the $M\Pi$ curve and hence there is no change in the equilibrium output of the oligopolistic industry. Obviously, the assumption of identical and homothetic preferences is particularly important for this result.

2.4 Ad Valorem and Specific Consumption Taxes on the Oligopolistic Industry

To obtain the comparative static result for the effect of a specific consumption tax in this general equilibrium setting, totally differentiate (11) and evaluate at the equilibrium, which yields:

$$\frac{\partial X^*}{\partial t} = -\frac{\partial\Omega/\partial t}{\partial\Omega/\partial X} = \frac{(1-\kappa)}{\partial\Omega/\partial X}\left(J - \frac{X^2 P'}{wL+\bar{\bar{\Pi}}}\right) < 0 \tag{17}$$

The term in brackets is unambiguously positive so the overall effect is negative. In figure five, it can be seen that the specific tax shifts the $M\Pi$ curve downwards and shifts the MX curve upwards leading to a decrease in the output of the oligopolistic industry from X^* to X^t. Hence, a specific consumption tax will increase the price of the oligopolistic good and decrease welfare.

To obtain the comparative static result for the effect of an *ad valorem* consumption tax in this general equilibrium setting, totally differentiate (11) and evaluate at the equilibrium, which yields:

$$\begin{aligned}\frac{\partial X^*}{\partial \tau} &= -\frac{\partial\Omega/\partial \tau}{\partial\Omega/\partial X} = \frac{(1-\kappa)}{(\partial\Omega/\partial X)(1+\tau)^2}\left[JP + XP' - \frac{X^2 PP'}{wL+\bar{\bar{\Pi}}}\right] \\ &= \frac{(1-\kappa)}{(\partial\Omega/\partial X)(1+\tau)}\left[J(c_X w + t) - \frac{XP'}{wL+\bar{\bar{\Pi}}}\left((c_X w+t)X + T\right)\right] < 0\end{aligned} \tag{18}$$

As the *marginal profit* effect is negative in equilibrium, marginal revenue may be negative hence the term in square brackets in the first line cannot be signed immediately, but using the equilibrium condition yields the second line where the term in square brackets can be signed assuming that the taxes are positive. In figure six, the $M\Pi$ curve will shift downwards (upwards) as marginal revenue is positive (negative), $P + XP' > (<) 0$, and the MX curve will shift upwards leading to a decrease in the output of the oligopolistic industry from X^* to X^τ.

Hence, an *ad valorem* consumption tax will increase the price of the oligopolistic good and decrease welfare. These results lead to the following proposition:

Proposition 4: *In a general equilibrium setting with identical and homothetic preferences, a specific tax and an ad valorem tax will both increase the price and decrease welfare.*

These results are in line with conventional wisdom under oligopoly in a partial equilibrium setting as in Seade (1985). One question that arises is whether consumption taxes are over-shifted or under-shifted and this will be addressed in the next section for the case of Cobb-Douglas preferences where explicit solutions can be obtained.

Now compare the situation when there is a specific consumption tax t (and a zero *ad valorem* tax) with the situation when there is an *ad valorem* consumption tax τ (and a zero specific tax) that both result in the same market price (and aggregate output) in the oligopolistic industry. Conjecture, as in Anderson *et al.* (2001), that setting the specific tax $t = \tau c_X w$ will yield the same aggregate output X in the two situations. Also, assume that the lump-sum tax is equal to zero, $T = 0$, and that the profits tax κ is the same in both situations. Setting $\tau = 0$ and $T = 0$ yields aggregate profits with the specific tax, denoted Π^t, while setting $t = 0$ and $T = 0$ yields aggregate profits with the *ad valorem* tax, denoted Π^τ, then using the relationship: $t = \tau c_X w$, yields:

$$\begin{aligned} \Pi^t &= (1-\kappa)\big(P(X) - c_X w - t\big)X \\ \Pi^\tau &= \frac{(1-\kappa)}{(1+\tau)}\big(P(X) - c_X w(1+\tau)\big)X = \frac{(1-\kappa)}{(1+\tau)}\big(P(X) - c_X w - t\big)X = \frac{\Pi^t}{1+\tau} \end{aligned} \tag{19}$$

As in Anderson *et al.* (2001), the aggregate profits of the oligopolistic industry with an *ad valorem* tax are lower than with a specific tax. Now consider the equilibrium condition (11) in the situation when there is a specific tax:

$$\Omega^t\left(X^*\right)=(1-\kappa)\left[J\left(P\left(X^*\right)-c_X w-t\right)+X^*P'\left(X^*\right)\right]-\frac{\Pi^t\left(X^*\right)}{wL+\bar{\Pi}\left(X^*\right)}X^*P'\left(X^*\right)=0 \quad (20)$$

To confirm the conjecture that aggregate output is the same in the two situations when $t=\tau c_X w$, one has to show that the same aggregate output X^* solves the equilibrium condition with an *ad valorem* tax. From (11), and using (19), the equilibrium condition in the situation when there is an *ad valorem* tax:

$$\begin{aligned}\Omega^\tau(X)&=\frac{(1-\kappa)}{1+\tau}\left[J\left(P\left(X^*\right)-c_X w(1+\tau)\right)+X^*P'\left(X^*\right)\right]-\frac{\Pi^\tau\left(X^*\right)}{wL+\bar{\Pi}\left(X^*\right)}X^*P'\left(X^*\right)\\&=\frac{(1-\kappa)}{1+\tau}\left[J\left(P\left(X^*\right)-c_X w-t\right)+X^*P'\left(X^*\right)\right]-\frac{1}{1+\tau}\frac{\Pi^t\left(X^*\right)}{wL+\bar{\Pi}\left(X^*\right)}X^*P'\left(X^*\right) \quad (21)\\&=\frac{\Omega^t\left(X^*\right)}{1+\tau}=0\end{aligned}$$

If the aggregate output X^* solves the equilibrium condition when there is a specific tax then it solves the equilibrium condition when there is an *ad valorem* tax so both taxes yield the same market price. Now compare the total tax revenue of the government, including the profits tax, in the two situations. Total tax revenue when there is a specific tax, denoted R^t, and total tax revenue when there is an *ad valorem* tax, denoted R^τ, are:

$$R^t=\frac{\kappa}{1-\kappa}\Pi^t+tX \qquad R^\tau=\frac{\kappa}{1-\kappa}\Pi^\tau+\frac{\tau P}{1+\tau}X \quad (22)$$

Using $t=\tau c_X w$ and (19), the difference in tax revenue with the two taxes can be shown to be:

$$R^\tau-R^t=\frac{\kappa}{1-\kappa}\left(\Pi^\tau-(1+\tau)\Pi^\tau\right)+\tau\left(\frac{P}{1+\tau}-c_X w\right)X=\Pi^t-\Pi^\tau\geq 0 \quad (23)$$

Hence, an *ad valorem* consumption tax yields higher tax revenue than a specific consumption tax that results in the same market price. This leads to the following proposition:

Proposition 5: *In a general equilibrium setting with identical homothetic preferences, and no lump-sum tax, an ad valorem consumption tax yields higher total tax revenue than a specific consumption tax that results in the same price.*

This proposition extends the partial-equilibrium results of Delipalla and Keen (1992) and Anderson *et al.* (2001) to a general equilibrium setting. Also, it extends these results by considering the tax revenue from a profits tax in addition to the tax revenue from the consumption tax. Note that if the profits tax rate is 100%, $\kappa = 1$, as in Blackorby and Murty (2007) then the difference in total tax revenue is zero since $\Pi^{\tau} = 0$.

3. An Example with Cobb-Douglas Preferences

This section will analyse the model for a particular example of a functional form that is homothetic, namely the Cobb-Douglas utility function. It will be shown that the model can be solved explicitly in this case, and this will allow the validity of assumptions to be assessed as well as providing confirmation for the main results. The model is exactly the same as in the previous section except that the preferences of all consumers (workers and shareholders) can be represented by the Cobb-Douglas utility function: $u_{Ll} = x_{Ll}^{1/2} y_{Ll}^{1/2}$ for workers and $u_{Kkj} = x_{Kkj}^{1/2} y_{Kkj}^{1/2}$ for shareholders. Utility maximisation yields the indirect utility functions: $v_{Ll} = P^{-1/2} m_{Ll}$ for workers and $v_{Kkj} = P^{-1/2} m_{Kkj}$ for shareholders. Given the Cobb-Douglas preferences of the individual consumers, the aggregate Marshallian demand for the oligopolistic good is $X = \left(wL + \bar{\Pi}\right)/2P$ where $\bar{\Pi} = \left(P - c_X w\right)X$. Inverting this Marshallian demand function while taking account of the income effect through profits yields:

$$P\left(X\right) = \frac{L - c_X X}{c_Y X} \tag{24}$$

Note that the price is equal to zero when $X = L/c_X$, which is the maximum quantity of the oligopolistic good that can be produced in the economy. The elasticity of demand is $\eta_X = -1 + c_X X/L \in [-1,0]$ so the inverse demand function is inelastic whereas the elasticity of the Marshallian demand function with Cobb-Douglas preferences is equal to (minus) one.

The indirect utility of the representative shareholder of the j th firm is $V_j = P^{-\frac{1}{2}}\Pi_j$ and differentiation yields the first-order conditions as in (9) then summing the first-order conditions yields the equilibrium condition as in (11), which in this case can be shown to be:

$$\Omega(X) = \frac{(1-\kappa)}{(1+\tau)} \frac{AX^2 - BX + C}{2c_Y^{\frac{1}{2}} X^{\frac{1}{2}} (L - c_X X)^{\frac{3}{2}}} = 0 \tag{25}$$

Note that the equilibrium condition will be equal to zero when the quadratic in the numerator is equal to zero, and the coefficients of the quadratic are defined as follows:

$$\begin{aligned} &A = 2c_X^2 J(2 + \theta_t + \tau + \theta_t \tau) > 0 && \text{where } \theta_t \equiv \frac{tc_Y}{p_Y c_X} \\ &B = c_X L\big(\theta_t + \tau + \theta_t \tau + 2J(3 + \theta_t + \tau + \theta_t \tau)\big) > 0 && \\ &C = L^2\big(J(2 - \theta_T - \theta_T \tau) - 1\big) > 0 && \text{where } \theta_T \equiv \frac{Tc_Y}{p_Y L} \end{aligned} \tag{26}$$

To ensure that the equilibrium was unique and to derive the comparative static results in the previous section, it was assumed that $\Omega(X)$ was downward sloping for all X. To check whether this assumption holds for the case of Cobb-Douglas preferences, differentiate (25) and, for simplicity, evaluate when all taxes are equal to zero, which yields:

$$\frac{\partial \Omega(X)}{\partial X} = -L^2 \frac{(2J-1)L - 2(J-2)c_X X}{4c_Y^{\frac{1}{2}} X^{\frac{3}{2}} (L - c_X X)^{\frac{5}{2}}} < 0 \tag{27}$$

Since $X \in [0, L/c_X]$ the numerator and the denominator are both positive so the derivative is negative and there will be a unique equilibrium. The quadratic in (25) has two positive real roots one of which is less than L/c_X and one of which is greater than L/c_X, but this root is not feasible. The function $\Omega(X)$ is plotted in figure seven for different numbers of oligopolistic firms, when $c_X = c_Y = 1$ and $L = 100$, where it can be seen that $\Omega(X)$ is downward sloping everywhere. Solving the equilibrium condition (25) when all taxes are equal to zero yields the aggregate output and the price of the oligopolistic good:

$$X^0 = \frac{L}{c_X}\left(3 - \sqrt{\frac{4+J}{J}}\right) > 0 \qquad P^0 = \frac{c_X}{c_Y}\left(\frac{1+J+\sqrt{J(4+J)}}{2J-1}\right) > \frac{c_X}{c_Y} \tag{28}$$

The price of the oligopolistic good, P^0, exceeds its opportunity cost, c_X/c_Y, which is equivalent to the market price being higher than marginal cost in a partial equilibrium setting. To evaluate how the price of the oligopolistic good depends upon the number of oligopolistic firms differentiate the market price with respect to J yields:

$$\frac{\partial P^0}{\partial J} = -\frac{c_X}{c_Y}\frac{\left(2+5J+3\sqrt{J(4+J)}\right)}{(2J-1)^2\sqrt{J(4+J)}} < 0 \tag{29}$$

This is unambiguously negative so an increase in the number of oligopolistic firms will result in a decrease in the price as expected given proposition one, and as the number of oligopolistic firms goes to infinity then the relative price tends to the opportunity cost. The price-cost margin of the oligopolistic industry is plotted in figure eight as a function of the number of oligopolistic firms.

Now consider the effects of the various taxes on the price of the oligopolistic good starting with the lump-sum tax. To keep the expressions relatively simple, the price of the oligopolistic good with each tax will be derived for the case when all the other taxes are equal to zero. Solving the equilibrium condition (25) for the aggregate output of the oligopolistic industry then substituting this into the inverse demand function (24) yields the price of the oligopolistic good with the lump-sum tax:

$$P^T = \frac{c_X}{c_Y} \frac{\left(1 + J + J\theta_T + \sqrt{J\left(4 + J + 4J\theta_T\right)}\right)}{\left(\left(2 - \theta_T\right)J - 1\right)} \tag{30}$$

To see how the lump-sum tax affects the price of the oligopolistic good, differentiate (30) with respect to the lump-sum tax rate T, which yields:

$$\frac{\partial P^T}{\partial T} = \frac{c_X}{p_Y L} \frac{J^2\left(2 + J\left(5 + 2\theta_T\right) + 3\sqrt{J\left(4 + J + 4J\theta_T\right)}\right)}{\left(J\left(2 - \theta_T\right) - 1\right)^2 \sqrt{J\left(4 + J + 4J\theta_T\right)}} > 0 \tag{31}$$

This is unambiguously positive so a lump-sum tax will increase the price of the oligopolistic good and decrease welfare as expected given proposition two.

For the profits tax, note that the coefficients of the quadratic (26) in the equilibrium condition (25) are independent of the profits tax so it will have no effect on the aggregate output of the oligopolistic good. Hence, the profits tax will have no effect on the price or on welfare as expected given proposition three.

For the specific consumption tax, solving the equilibrium condition (25) for the aggregate output of the oligopolistic industry then substituting this into the inverse demand function (24) yields the price of the oligopolistic good:

$$P^t = \frac{c_X}{c_Y} \frac{\left(2 + \theta_t + 2J\left(1 + \theta_t\right) + F_t\right)}{2\left(2J - 1\right)} \tag{32}$$

where $F_t \equiv \sqrt{\theta_t^2 + 4J^2\left(1 + \theta_t\right)^2 + 4J\left(1 + \theta_t\right)\left(4 + \theta_t\right)} > 0$, which is increasing in θ_t.

The effect of an increase in the specific tax on the price of the oligopolistic good is obtained by differentiating (32) with respect to t, which yields:

$$\frac{\partial P^t}{\partial t} = \frac{\left(\theta_t + F_t + 2J\left(5 + 2\theta_t + 2J\left(1+\theta_t\right) + F_t\right)\right)}{2p_Y\left(2J-1\right)F_t} > 0 \tag{33}$$

A specific tax will increase the price of the oligopolistic good and decrease welfare as expected given proposition four. Under oligopoly, it is possible for a tax to be over-shifted where the price increases by more than the amount of the tax. Hence, using (33), the specific tax will be over-shifted since:

$$\frac{\partial P^t}{\partial t} - 1 = \frac{\theta_t + 3F_T + 2J\left(5 + 2\theta_t + 2J\left(1+\theta_t\right) - F_t\right)}{2\left(2J-1\right)p_Y F_t} > 0 \tag{34}$$

It can be shown that $5 + 2\theta_t + 2J\left(1+\theta_t\right) > F_t$ by squaring both sides, which are positive, then noting that $\left(5 + 2\theta_t + 2J\left(1+\theta_t\right)\right)^2 - F_t^2 = 4J\left(1+\theta_t\right)^2 + \left(5+2\theta_t\right)^2 - \theta_t^2 > 0$. Hence, the numerator and denominator are both positive so the specific tax is over-shifted. It can be shown that as the number of firms tends to infinity then (34) goes to zero so the price increases by the same amount as the specific tax.

For the *ad valorem* consumption tax, solving the equilibrium condition (25) for the aggregate output of the oligopolistic industry then substituting this into the inverse demand function (24) yields the price of the oligopolistic good:

$$P^\tau = \frac{c_X}{c_Y}\frac{\left(2 + \tau + 2J\left(1+\tau\right) + F_\tau\right)}{2\left(2J-1\right)} \tag{35}$$

where $F_\tau \equiv \sqrt{\tau^2 + 4J^2\left(1+\tau\right)^2 + 4J\left(1+\tau\right)\left(4+\tau\right)} > 0$, which is increasing in τ and is identical to F_t if τ is replaced by θ_t. Note the similarity between (32) and (35).

The effect of an increase in the *ad valorem* tax on the price of the oligopolistic good is obtained by differentiating (35) with respect to τ, which yields:

$$\frac{\partial P^{\tau}}{\partial \tau} = \frac{c_X\left(\tau + F_{\tau} + 2J\left(5 + 2\tau + 2J\left(1+\tau\right) + F_{\tau}\right)\right)}{2c_Y\left(2J-1\right)F_{\tau}} > 0 \tag{36}$$

An *ad valorem* tax will increase the price of the oligopolistic good and decrease welfare as expected given proposition five. Using (36), the *ad valorem* tax will be under-shifted since:

$$\frac{\partial P^{\tau}}{\partial \tau} - \frac{P^{\tau}}{1+\tau} = -\frac{c_X\left(6J\left(1+\tau\right) - \tau + F_{\tau}\right)}{2c_Y\left(2J-1\right)\left(1+\tau\right)F_{\tau}} < 0 \tag{37}$$

Hence, with Cobb-Douglas preferences, the *ad valorem* tax is under-shifted but the specific tax is over-shifted.

4. The Model with Quasi-Linear Preferences

This section will analyse the model for the case of quasi-linear preferences, which is the simplest case of non-homothetic preferences. This will allow the sensitivity of the results to the assumption of homothetic preferences used in the previous two sections to be assessed. The assumption of homothetic preferences was clearly very important for many stages of the analysis in the previous two sections. With quasi-linear preferences, there will be no income effect in the demand for the oligopolistic good, but this does not mean that the oligopolistic industry is small relative to the economy. The oligopolistic firms will still take into account the fact that their output choices affect the prices facing their shareholders. All consumers (workers and shareholders) have identical quasi-linear preferences, the utility of the workers will be $u_{Ll} = \phi\left(x_{Ll}\right) + y_{Ll}$ for $l = 1,\ldots,L$, and the utility of the shareholders will be $u_{Kkj} = \phi\left(x_{Kkj}\right) + y_{Kkj}$ for $k = 1,\ldots,K/J$ and $j = 1,\ldots,J$, where $\phi' > 0$ and $\phi'' < 0$. Utility maximisation yields an indirect utility function of the form $v_{Ll} = \upsilon\left(P\right) + m_{Ll}$ for the workers, recalling that the price of good Y is normalised at unity, and an indirect utility function of the form $v_{Kkj} = \upsilon\left(P\right) + m_{Kkj}$ for the shareholders. Roy's identity yields the Marshallian demand function of the consumers

$x_{Ll} = x_{Kkj} = -\partial\upsilon/\partial P = \tilde{D}(P)$, which is independent of income and identical to the Hicksian demand function. Aggregating over all the consumers gives the aggregate demand for the oligopolistic good $X = (K+L)\tilde{D}(P)$, and this can be inverted to give the inverse Marshallian/Hicksian demand function $P = P(X)$, which is downward sloping, $\partial P/\partial X = P' < 0$, as the Hicksian demand function is always downward sloping.

In the oligopolistic industry, the firms independently and simultaneously choose their outputs to maximise the utility of their shareholders. Since shareholders have identical quasi-linear preferences the utility of the shareholders can be modelled using a representative shareholder. Summing the utility of all the shareholder of the j th firm yields the utility of the representative shareholder of the j th firm: $V_j = K\upsilon(P)/J + \Pi_j$. Assuming an interior solution, where all firms produce positive quantities, the first-order conditions for the maximisation of shareholder utility are:

$$\frac{\partial V_j}{\partial X_j} = \frac{K}{J}\frac{\partial\upsilon}{\partial p_X}P' + \frac{\partial\Pi_j}{\partial X_j} = (1-\kappa)\left(\frac{P}{1+\tau} + \frac{X_j P'}{1+\tau} - c_X w - t\right) - X_{Kj}P' = 0 \tag{38}$$

Again, since $X_{Kj}P' < 0$, the marginal effect of a change in the firm's output on its profits will be negative, $\partial\Pi_j/\partial X_j < 0$. Aggregating over all the oligopolistic firms, then noting that $X_{Kj} = K\tilde{D}(p_X)/J$ and hence $X_K = K\tilde{D}(p_X) = KX/(K+L)$, yields the equilibrium condition in terms of aggregate output of the oligopolistic industry:

$$\Omega(X) \equiv (1-\kappa)\left[J\left(\frac{P(X)}{1+\tau} - c_X w - t\right) + X\frac{P'(X)}{1+\tau}\right] - \frac{K}{K+L}XP'(X) = 0 \tag{39}$$

Since the income of the shareholders is the dividend they receive from the firm, profits must be positive otherwise the income of the shareholders will be negative. The price-cost margin, $P/(1+\tau)-c_X w-t>0$ will be positive if $L>(\tau+\kappa)K/(1-\kappa)$, which will be the case if the tax rates are sufficiently low and is undoubtedly the case if tax rates are zero, $\kappa=\tau=0$. Henceforth, it will be assumed that this condition holds, which can also be stated as $(1-\kappa)L-(\kappa+\tau)K>0$. Again, this game is aggregative as the equilibrium condition only depends upon the aggregate output of the oligopolistic industry and not the outputs of the individual firms. Differentiating (39) yields:

$$\frac{\partial\Omega}{\partial X}=\frac{1-\kappa}{1+\tau}\left[(J+1)P'+XP''\right]-\frac{K}{K+L}\left[P'+XP''\right] \tag{40}$$

The first expression in square brackets is negative if the Seade (1980b) condition for the stability of Cournot equilibrium in a partial equilibrium setting is satisfied, which is basically the same as the Kolstad and Mathiesen (1987) condition for uniqueness. The second expression in square brackets is negative if best-response (reaction) functions are everywhere downward sloping , which is the Hahn (1962) stability condition. If $(J+1)P'+XP''<0$ and $P'+XP''>0$ then it is obvious that (40) is unambiguously negative. Alternatively, if $(J+1)P'+XP''<0$ and $P'+XP''<0$ then (40) can be rearranged as:

$$\frac{\partial\Omega}{\partial X}=\frac{J(1-\kappa)(K+L)P'+\left[(1-\kappa)L-(\kappa+\tau)K\right](P'+XP'')}{(K+L)(1+\tau)} \tag{41}$$

This is unambiguously negative if $(1-\kappa)L-(\kappa+\tau)K>0$, as assumed above, which will be the case if the tax rates are sufficiently low and is undoubtedly the case if tax rates are zero, $\kappa=\tau=0$. Therefore, the Seade (1980b) stability condition implies that the equilibrium is unique in this general equilibrium setting, and it will be assumed to hold for the comparative static analysis.

As with homothetic preferences, to help with the intuition and to allow a diagrammatic representation, the equilibrium condition (39) can be rewritten as:

$$M\Pi \equiv (1-\kappa)\left[J\left(\frac{P(X)}{1+\tau}-c_X w-t\right)+X\frac{P'(X)}{1+\tau}\right]=\frac{K}{K+L}XP'(X)\equiv MX<0 \tag{42}$$

Again, the left-hand side of the equation is the *marginal profit effect* and the right-hand side is the *marginal expenditure effect*. The only difference with (12) is with the *marginal expenditure effect* as the shareholders' consumption of the oligopolistic good is a fixed fraction, $K/(K+L)$ of total output, which is unaffected by changes in the tax rates, because there is no income effect on demand. Therefore, changes in tax rates will not shift the *marginal expenditure* curve with quasi-linear preferences.

Aggregate welfare, ignoring concerns about income distribution, can be obtained by summing the utility of all consumers since preferences are identical and quasi-linear. Thus, aggregate welfare is $V=(K+L)\upsilon(P)+wL+\overline{\Pi}$, hence the effect of a change in a tax or the number of oligopolistic firms is obtained by differentiating welfare with respect to $\sigma=\kappa,\tau,t,T,J$ and using Roy's identity, which yields:

$$\frac{\partial V}{\partial \sigma}=\left[(K+L)\frac{\partial \upsilon}{\partial p}P'+(P-c_X w)+XP'\right]\frac{\partial X}{\partial \sigma}=(P-c_X w)\frac{\partial X}{\partial \sigma} \tag{43}$$

As with homothetic preferences, if the change in tax or the number of firms increases (decreases) the output of the oligopolistic industry then welfare will increase (decrease) as it will decrease (increase) the deadweight loss from oligopoly.

4.1 The Number of Firms in the Oligopolistic Industry

Conventional wisdom in a partial equilibrium setting and the analysis in a general equilibrium setting with homothetic preferences says that an increase in the number of firms in a monopoly or oligopoly will lead to a reduction in the price. To obtain the comparative static result for the effect of an increase in the number of firms in this general equilibrium setting with quasi-linear preferences, totally differentiate(39), which yields:

$$\frac{\partial X^*}{\partial J} = -\frac{\partial \Omega / \partial J}{\partial \Omega / \partial X} = -\frac{(1-\kappa)}{\partial \Omega / \partial X}\left[\frac{P(X)}{1+\tau} - c_X w - t\right] > 0 \tag{44}$$

As profits must be positive, the term in square brackets is positive and the overall effect is positive thereefore an increase in the number of firms will increase the aggregate output of the oligopolistic industry. Hence, there will be a decrease in the price of the oligopolistic good and an increase in welfare. This leads to the following proposition:

Proposition 6: *In a general equilibrium setting with identical and quasi-linear preferences, an increase in the number of firms in a monopoly or a Cournot oligopoly will result in a decrease in the monopoly or oligopoly price and an increase in welfare.*

This proposition is in line with conventional wisdom in a partial equilibrium setting and the results in a general equilibrium setting with homothetic preferences so the result seems to be robust.

4.2 A Lump-Sum Tax on the Oligopolistic Industry

Conventional wisdom from partial equilibrium analysis says that a lump-sum tax on a monopoly or an oligopoly will have no effect on the output or price set by the firms, but the result in a general equilibrium setting with homothetic preferences was that there would be a decrease in aggregate output leading to an increase in the price. To obtain the comparative static result for the effect of a lump-sum tax in this general equilibrium setting with quasi-linear preferences, totally differentiate (39), which yields:

$$\frac{\partial X^*}{\partial T} = -\frac{\partial \Omega / \partial T}{\partial \Omega / \partial X} = 0 \tag{45}$$

Since this is zero, the lump-sum tax does not affect the aggregate output of the oligopolistic industry. Hence, there is no effect on price of the oligopolistic good or welfare. This leads to the following proposition:

Proposition 7: *In a general equilibrium setting with identical and quasi-linear preferences, a lump-sum tax on a monopoly or a Cournot oligopoly will result in no effect on the monopoly or oligopoly price and no effect on welfare.*

The lump-sum tax does not shift the $M\Pi$ curve and, because there is no income effect with quasi-linear preferences, does not shift the MX curve, which implies that there will be no change in the equilibrium output of the oligopolistic industry. This result is in line with conventional wisdom, but contrasts with proposition two where the lump-sum tax increased the price of the oligopolistic good. The difference between the results is because of the income effect with homothetic preferences.

4.3 A Profits Tax on the Oligopolistic Industry

Conventional wisdom from partial equilibrium analysis and the analysis in a general equilibrium setting with homothetic preferences says that a profits tax on a monopoly or oligopoly will have no effect on the output or price set by the firms, To obtain the comparative static result for the effect of a profits tax in this general equilibrium setting with quasi-linear preferences, totally differentiate (39) and evaluate at the equilibrium, which yields:

$$\frac{\partial X^*}{\partial \kappa} = -\frac{\partial \Omega / \partial \kappa}{\partial \Omega / \partial X} = \frac{1}{\partial \Omega / \partial X}\left[J\left(\frac{P(X)}{1+\tau} - c_X w - t \right) + X \frac{P'(X)}{1+\tau} \right] > 0 \tag{46}$$

The term in square bracket is negative since it is $\sum_{j=1}^{J} \partial \Pi_j / \partial X_j$ and it was shown above that $\partial \Pi_j / \partial X_j < 0$. Hence, the profits tax will increase the aggregate output of the oligopolistic industry thereby leading to a decrease in the price of the oligopolistic good and an increase in welfare. This leads to the following proposition:

Proposition 8: *In a general equilibrium setting with identical quasi-linear preferences, a profits tax on a monopoly or a Cournot oligopoly will decrease the price and increase welfare.*

In figure nine, it can be seen that the $M\Pi$ curve swivels anti-clockwise around the profit-maximising output as marginal profits are multiplied by $(1-\kappa)$, but the MX curve does not shift as there is no income effect with quasi-linear preferences, which implies that there will be an increase in the aggregate output of the oligopolistic industry. Hence, there will be a decrease in the price of the oligopolistic good and an increase in welfare. It is surprising that the profits tax has an effect on the price of the oligopolistic good, and even more surprising that the effect is to decrease the price.

4.4 Ad Valorem and Specific Consumption Taxes on the Oligopolistic Industry

To obtain the comparative static result for the effect of a specific consumption tax in this general equilibrium setting with quasi-linear preferences, totally differentiate (39), which yields:

$$\frac{\partial X^*}{\partial t} = -\frac{\partial \Omega / \partial t}{\partial \Omega / \partial X} = \frac{(1-\kappa)J}{\partial \Omega / \partial X} < 0 \tag{47}$$

As this is unambiguously negative, a specific consumption tax will decrease the aggregate output of the oligopolistic industry. Hence, there will be an increase in the price of the oligopolistic good and a decrease in welfare. This leads to the following proposition:

Proposition 9: *In a general equilibrium setting with identical quasi-linear preferences, a specific tax will decrease the aggregate output of the oligopolistic industry, increase the price and decrease welfare.*

To obtain the comparative static result for the effect of an *ad valorem* consumption tax in this general equilibrium setting with quasi-linear preferences, totally differentiate (39), which yields:

$$\frac{\partial X^*}{\partial \tau} = -\frac{\partial \Omega / \partial \tau}{\partial \Omega / \partial X} = \frac{(1-\kappa)}{(\partial \Omega / \partial X)(1+\tau)^2}\left[JP + XP'\right] \tag{48}$$

The term in square brackets is the sum of the marginal revenues of the oligopolistic firms, which need not be positive as marginal profits are negative in equilibrium. If the marginal revenue of the oligopolistic firms is positive (negative), $JP + XP' > (<) 0$, then an *ad valorem* tax will lead to a decrease (increase) in the aggregate output of the oligopolistic industry. Hence, there will be an increase (decrease) in the price of the oligopolistic good and a decrease (increase) in welfare. This leads to the following proposition:

Proposition 10*: In a general equilibrium setting with identical quasi-linear preferences, if* $JP + XP' > (<) 0$ *then an ad valorem tax will increase (decrease) the price, and decrease (increase) welfare.*

In figure ten, the $M\Pi$ curve will shift downwards (upwards) if marginal revenue, $P + XP'$ is positive (negative), but the MX curve will not shift as there is no income effect with quasi-linear preferences, which implies that aggregate output of the oligopolistic industry will decrease (increase). Figure ten shows the counterintuitive case when the output increases from X^* to X^τ, and hence price decreases and welfare increases.[8]

Now compare the situation when there is a specific consumption tax t (and a zero *ad valorem* tax) with the situation when there is an *ad valorem* consumption tax τ (and a zero specific tax) that both result in the same price (and aggregate output) in the oligopolistic industry. The method used in section two, as in Anderson *et al.* (2001), does not work in this case so an alternative method must be employed. Assume that the lump-sum profits tax is equal to zero, $T = 0$, and that the proportional profits tax κ is the same in both situations. Comparing

[8] The possibility of an *ad valorem* tax reducing price has been shown by de Meza *et al.* (1995) for the case of a free-entry Cournot oligopoly in a partial equilibrium setting, but this possibility occurs if there are strong economies of scale whereas there are constant returns to scale in this model and a fixed number of firms.

the equilibrium condition (39) for an *ad valorem* tax with that for a specific tax, it can be shown that the specific tax that yields the same market price is:

$$t = \frac{\tau}{1+\tau}\frac{JP + XP'}{J} \tag{49}$$

Since $JP + XP'$ may be negative, the specific tax that is equivalent to a positive *ad valorem* tax may be negative (a subsidy). Comparing profits for the case of an *ad valorem* tax with profits for the case of a specific tax, it can be shown that the difference in profits is:

$$\Pi^t - \Pi^\tau = -(1-\kappa)\frac{\tau}{1+\tau}\frac{X^2 P'}{J} > 0 \tag{50}$$

Profits are higher with the specific tax than with the *ad valorem* tax so there will be more revenue from the profits tax with the specific tax than with the *ad valorem* tax. Comparing the total tax revenue from a specific tax with that from an *ad valorem* tax, and using (49) and (50), it can be shown that the difference in total tax revenue is:

$$R^\tau - R^t = \left(\frac{\tau}{1+\tau}P - t\right)X + \frac{\kappa}{1-\kappa}\left(\Pi^\tau - \Pi^t\right) = -(1-\kappa)\frac{\tau}{1+\tau}\frac{X^2 P'}{J} > 0 \tag{51}$$

Hence, an *ad valorem* consumption tax yields higher total tax revenue than a specific consumption tax that results in the same market price. Note that the difference in total tax revenue is equal to the difference in profits so $R^\tau - R^t = \Pi^t - \Pi^\tau$. This leads to the following proposition:

Proposition 11: *In a general equilibrium setting with identical quasi-linear preferences, and no lump-sum tax, an ad valorem consumption tax yields higher total tax revenue than a specific consumption tax that results in the same price.*

This proposition extends the partial-equilibrium results of Delipalla and Keen (1992) and Anderson *et al.* (2001) to a general equilibrium setting. It also extends their analysis by considering total tax revenue (including revenue from a profits tax) rather than just considering revenue from the consumption taxes. This seems to be a robust result as it holds in a partial equilibrium setting and in a general equilibrium setting with either homothetic or quasi-linear preferences.

5. An Example with Linear Demand

This section will analyse the model for a particular example of a functional form that is quasi-linear, namely the quadratic utility function that results in a linear demand function for the oligopolistic industry. The preferences of the consumers can be represented by the quadratic utility function: form: $u_{Ll} = \alpha x_{Ll} - \beta x_{Ll}^2/2 + y_{Ll}$ for workers and $u_{Kkj} = \alpha x_{Kkj} - \beta x_{Kkj}^2/2 + y_{Kkj}$ for shareholders, where $\alpha > 0$ and $\beta > 0$. Utility maximisation, assuming that both goods are consumed, yields the indirect utility function: $v_{Ll} = (\alpha - P)^2/2\beta + m_{Ll}$ for the workers and $v_{Kkj} = (\alpha - P)^2/2\beta + m_{Kkj}$ for the shareholders. The aggregate Marshallian (Hicksian) demand for the oligopolistic good is $X = (K+L)(\alpha - P)/\beta$. Inverting the Marshallian demand function yields the inverse demand function:

$$P(X) = \alpha - \beta \frac{X}{K+L} \tag{52}$$

The indirect utility function of the representative shareholder of the jth firm is $V_j = K(\alpha - P)^2/2\beta J + \Pi_j$, and differentiation yields the first-order conditions for utility maximisation as in (38). Then, summing the first-order conditions yields the equilibrium condition as in (39), which in this case can be shown to be:

$$\Omega(X) = G - HX = 0 \tag{53}$$

The equilibrium condition is linear in aggregate output of the oligopolistic industry and the coefficients are:

$$\begin{aligned} G &\equiv \frac{(1-\kappa)J}{(1+\tau)c_Y}\left[\alpha c_Y-(1+\tau)(tc_Y+c_X)\right]>0 \\ H &\equiv \frac{\beta}{(1+\tau)(K+L)^2}\left[(1-\kappa)L-(\kappa+\tau)K+(1-\kappa)J(K+L)\right]>0 \end{aligned} \tag{54}$$

To ensure that the aggregate output of the oligopolistic industry is positive it is assumed that $\alpha>(1+\tau)(tc_Y+c_X)/c_Y>0$, which implies that $G>0$. As in the previous section, it will be assumed that $(1-\kappa)L-(\kappa+\tau)K>0$ to ensure that the price-cost margin is positive, which implies that $H>0$. Solving the equilibrium condition when all the taxes are equal to zero yields the aggregate output and price of the oligopolistic good:

$$X^0=\frac{(K+L)^2(c_Y\alpha-c_X)}{c_Y\left(L+J(K+L)\right)\beta}>0 \qquad P^0=\frac{c_X}{c_Y}+\frac{(c_Y\alpha-c_X)L}{c_Y\left(L+J(K+L)\right)}>\frac{c_X}{c_Y} \tag{55}$$

The price of the oligopolistic good exceeds its opportunity cost, c_X/c_Y, and it is decreasing in the number of oligopolistic firms with the limit as the number of firms goes to infinity equal to its opportunity cost.

Now consider the effect of the various taxes on the price of the oligopolistic good starting with the lump-sum tax. Again, to keep the expressions relatively simple, the relative price of the oligopolistic good with each tax will be derived for the case when all the other taxes are equal to zero. For the lump-sum tax, the equilibrium condition is independent of the lump-sum tax so it will have no effect on the aggregate output or price of the oligopolistic good. This is in line with conventional wisdom and proposition seven, but contrasts with proposition two for the case of homothetic preferences.

For the profits tax, solving the equilibrium condition (53) for the aggregate output then substituting this into the inverse demand function (52) yields the price of the oligopolistic good:

$$P^{\kappa} = \frac{c_X J(1-\kappa)(K+L) + c_Y \alpha\left((1-\kappa)L + \kappa K\right)}{c_Y\left((1-\kappa)L - \kappa K + (1-\kappa)J(K+L)\right)} \tag{56}$$

The effect of an increase in the profits tax on the price of the oligopolistic good is obtained by differentiating (56) with respect to κ, which yields:

$$\frac{\partial P^{\kappa}}{\partial \kappa} = -\frac{JK(K+L)(c_Y\alpha - c_X)}{c_Y\left((1-\kappa)L - \kappa K + (1-\kappa)J(K+L)\right)^2} < 0 \tag{57}$$

A profits tax will decrease the relative price of the oligopolistic good as expected given proposition eight, but which is contrary to conventional wisdom and proposition three for the case of homothetic preferences.

For the specific consumption tax, solving the equilibrium condition (53) for the aggregate output then substituting this into the inverse demand function (52) yields the relative price of the oligopolistic good:

$$P^{t} = \frac{J(K+L)(c_X + c_Y t) + c_Y \alpha L}{c_Y\left(L + J(K+L)\right)} \tag{58}$$

The effect of an increase in the specific tax on the market price of the oligopolistic good is obtained by differentiating (58) with respect to t, which yields:

$$\frac{\partial P^{t}}{\partial t} = \frac{J(K+L)}{L + J(K+L)} > 0 \tag{59}$$

This is clearly positive as expected given proposition nine and is in line with conventional wisdom and proposition four for the case of homothetic preferences. The increase in the price is less than one, hence the specific tax is under-shifted in this case.

For the ad valorem tax, solving the equilibrium condition (53) for the aggregate output then substituting this into the inverse demand function (52) yields the price of the oligopolistic good:

$$P^{\tau} = \frac{c_X J\left(K+L\right)\left(1+\tau\right)+c_Y \alpha\left(L-K\tau\right)}{c_Y \Delta} \tag{60}$$

where $\Delta \equiv L - K\tau + J\left(K+L\right) > 0$.

The effect of an increase in the *ad valorem* tax on the price of the oligopolistic good is obtained by differentiating (60) with respect to τ, which yields:

$$\frac{\partial P^{\tau}}{\partial \tau} = \frac{J\left(K+L\right)\left(c_X\left(1+J\right)\left(K+L\right)-c_Y K\alpha\right)}{c_Y \Delta^2} \tag{61}$$

This will be positive (negative) if $\alpha < (>) c_X\left(1+J\right)\left(K+L\right)/c_Y K \equiv \alpha_{\tau}$ in which case the *ad valorem* tax will lead to an increase (decrease) in the price of the oligopolistic good, as expected given proposition ten. Therefore, an *ad valorem* tax may lead to a decrease in price of the oligopolistic good and an increase in welfare. The parameter space where this counterintuitive possibility occurs is shown in figure eleven. In equilibrium, workers and shareholders must consume positive quantities of the numeraire good Y, which will be the case for workers if $\alpha < \alpha_L$ and for shareholders if $\alpha > \alpha_K$. Given the parameters, $c_X = 1$, $c_Y = 5$, $\beta = 10$, $J = 1$, $L = 100$, and $\tau = 0$, there is a region where the *ad valorem* tax leads to a decrease in the price of the oligopolistic good.

Using (61) and (54), it can be shown that the *ad valorem* tax will be under-shifted since:

$$\frac{\partial P^{\tau}}{\partial \tau} - \frac{P^{\tau}}{1+\tau} = -\frac{\left(c_Y\alpha-\left(1+\tau\right)c_X\right)JK\left(K+L\right)\left(1+\tau\right)+c_Y\alpha\left(L-\tau K\right)\Delta}{c_Y\left(1+\tau\right)\Delta^2} < 0 \tag{62}$$

Hence, with a quadratic utility function and linear demand functions, both the specific tax and the *ad valorem* tax are under-shifted, which is the case with linear demand in a partial equilibrium setting.

6. Conclusions

Taxation under Cournot oligopoly has been analysed in a general equilibrium setting where firms are large relative to the economy, and oligopolistic firms maximise the utility of shareholders. This novel analysis of taxation has led to a number of counterintuitive results that challenge conventional wisdom in microeconomics. A lump-sum tax was shown to lead to an increase in the price of the oligopolistic good in the case of homothetic preferences, and to have no effect on the price in the case of quasi-linear preferences. The former result is counterintuitive and contrasts with conventional wisdom. A profits tax was shown to have no effect on the price of the oligopolistic good for the case of homothetic preferences, and to lead to a decrease in the price in the case of quasi-linear preferences. The latter result is counterintuitive and contrasts with conventional wisdom. A specific tax was shown to increase the price of the oligopolistic good with both homothetic and quasi-linear preferences. An *ad valorem* tax was shown to increase the price of the oligopolistic good in the case of homothetic preferences and to have an ambiguous effect with quasi-linear preferences. Hence, with quasi-linear preferences, there is the counterintuitive possibility that an *ad valorem* tax will increase the price Furthermore, it was shown that total tax revenue is always higher with an *ad valorem* tax than with a specific tax that leads to the same price of the oligopolistic good with both homothetic and quasi-linear preferences.

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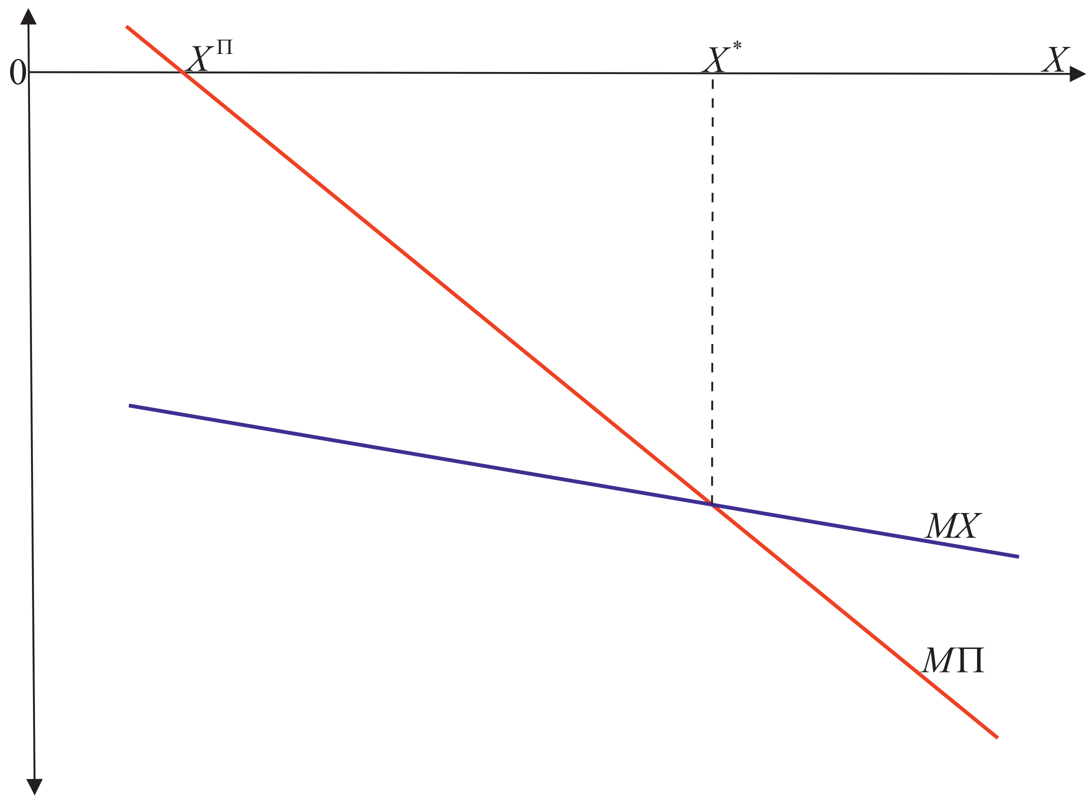

Figure 1: Marginal Profits and Marginal Expenditure

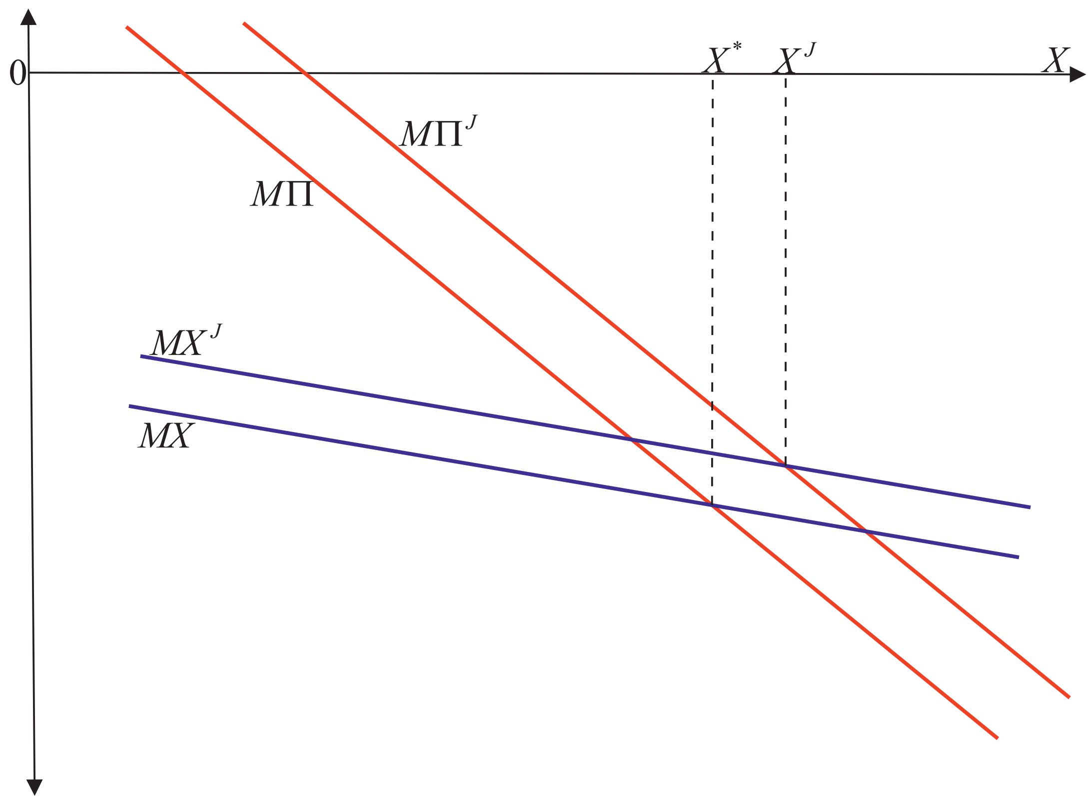

Figure 2: An Increase in the Number of Firms with Homothetic Preferences

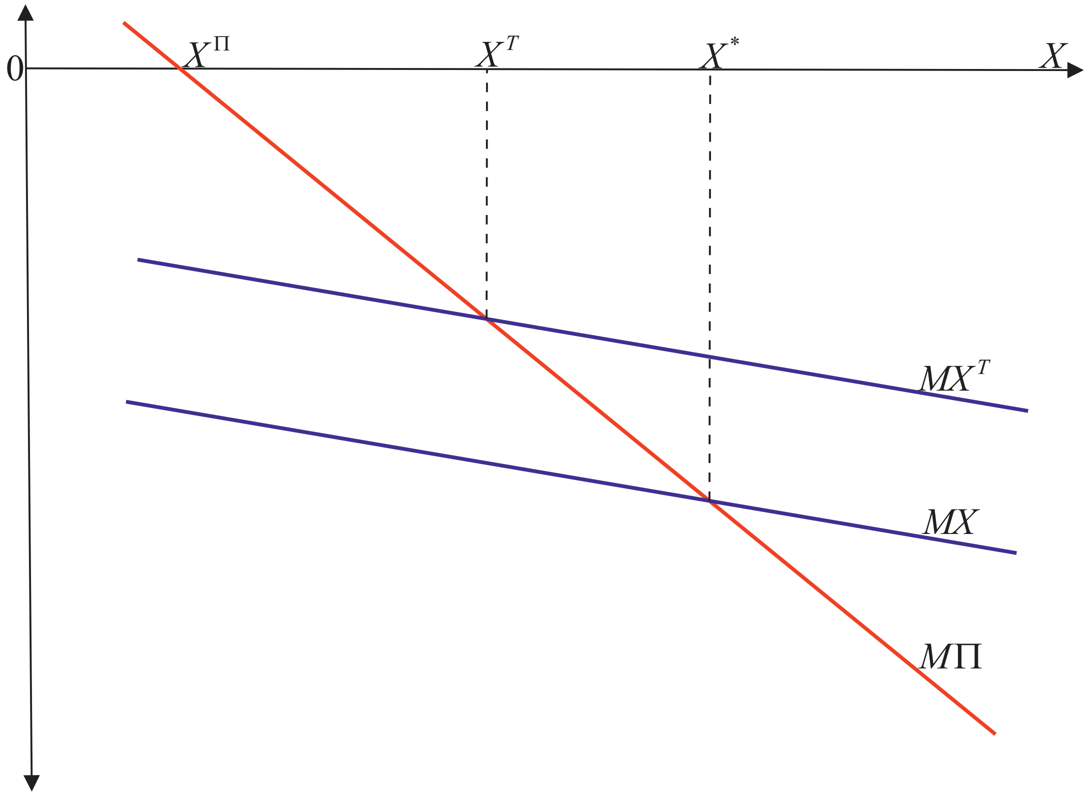

Figure 3: Lump-Sum Tax with Homothetic Preferences

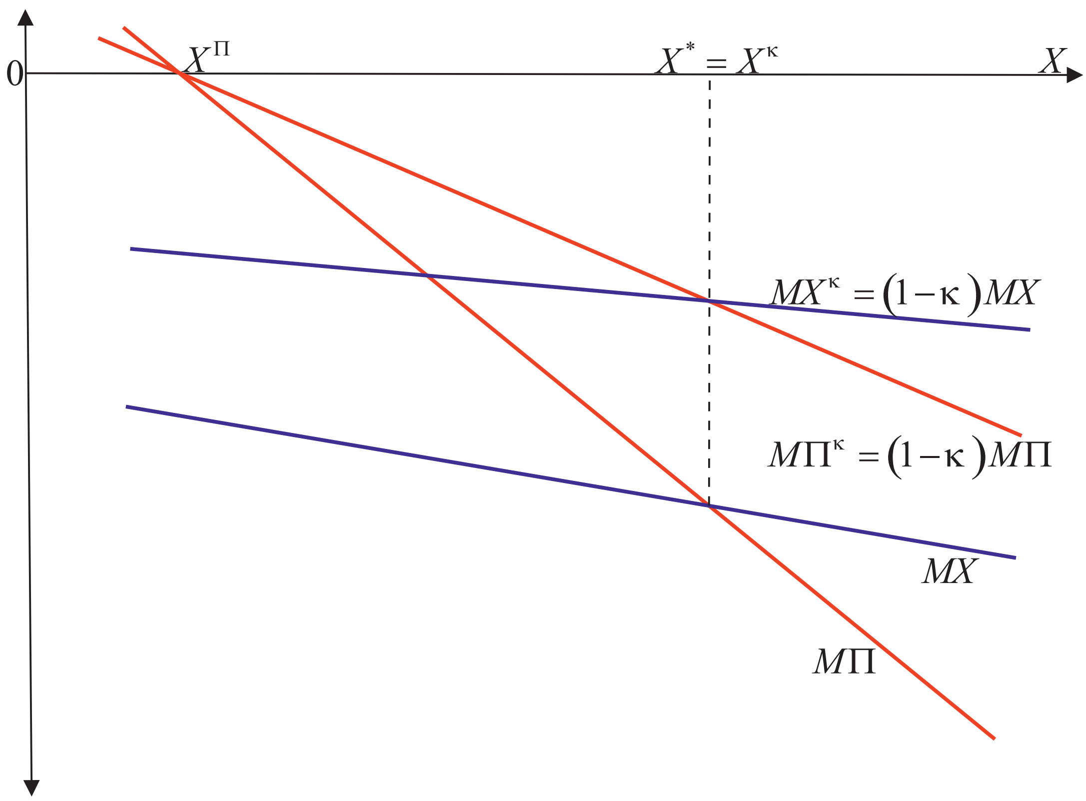

Figure 4: Profits Tax with Homothetic Preferences

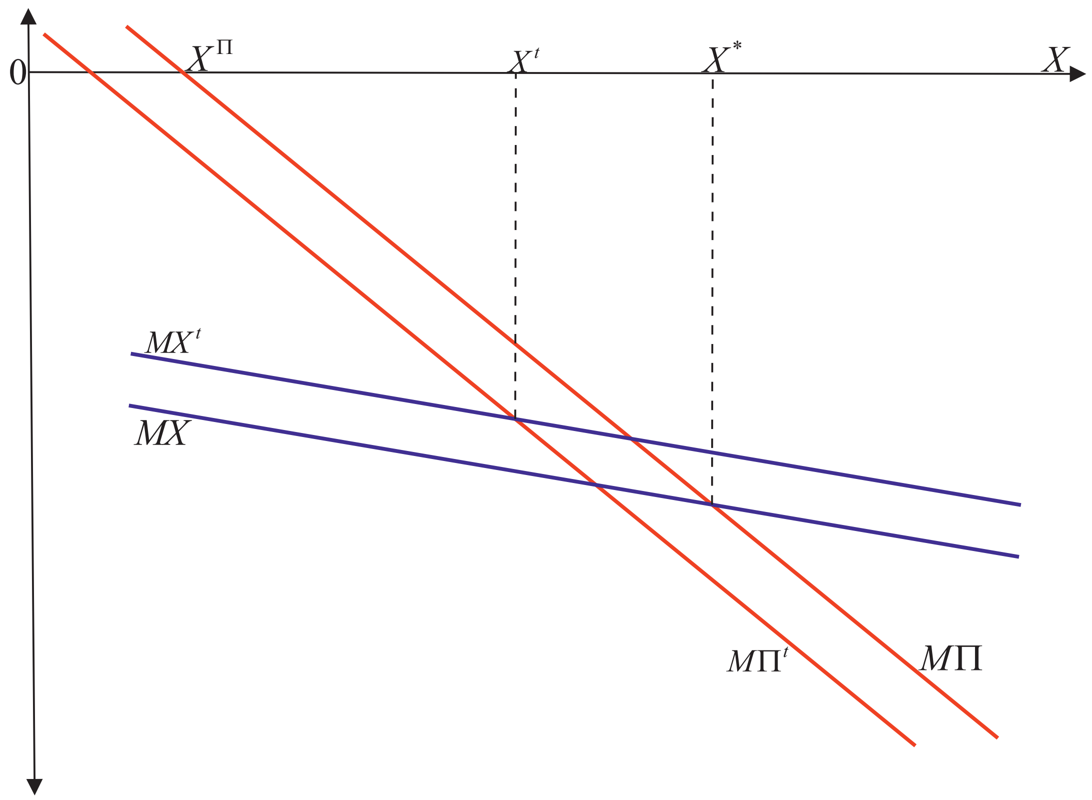

Figure 5: Specific Tax with Homothetic Preferences

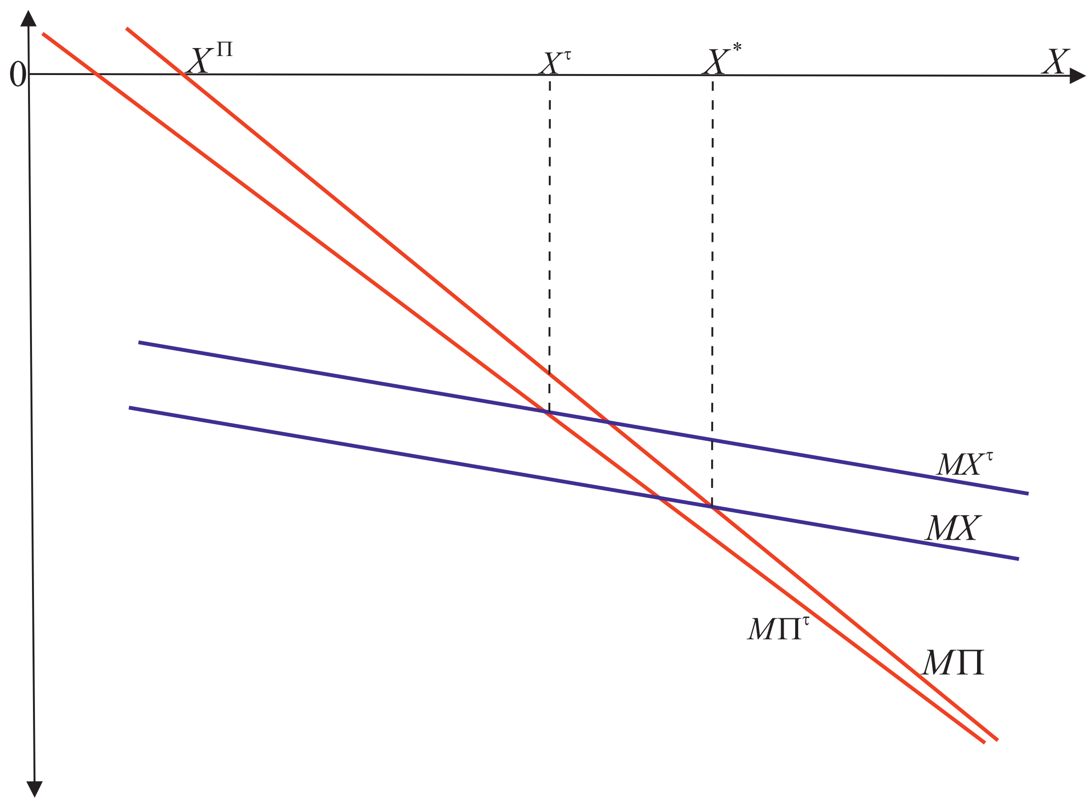

Figure 6: Ad Valorem Tax with homothetic Preferences

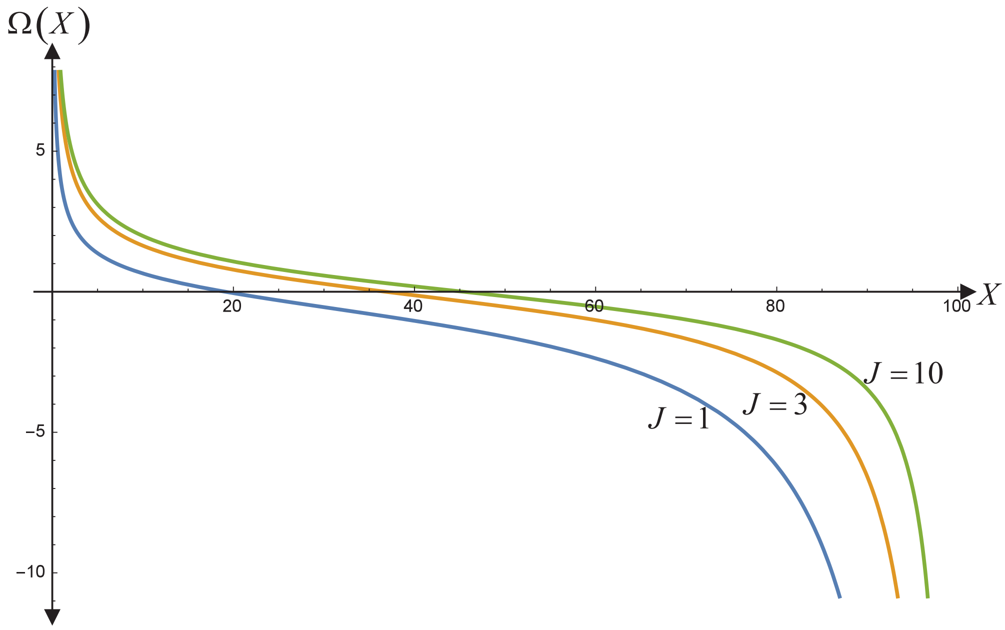

Figure 7: Equilibrium Condition with Cobb-Douglas Preferences

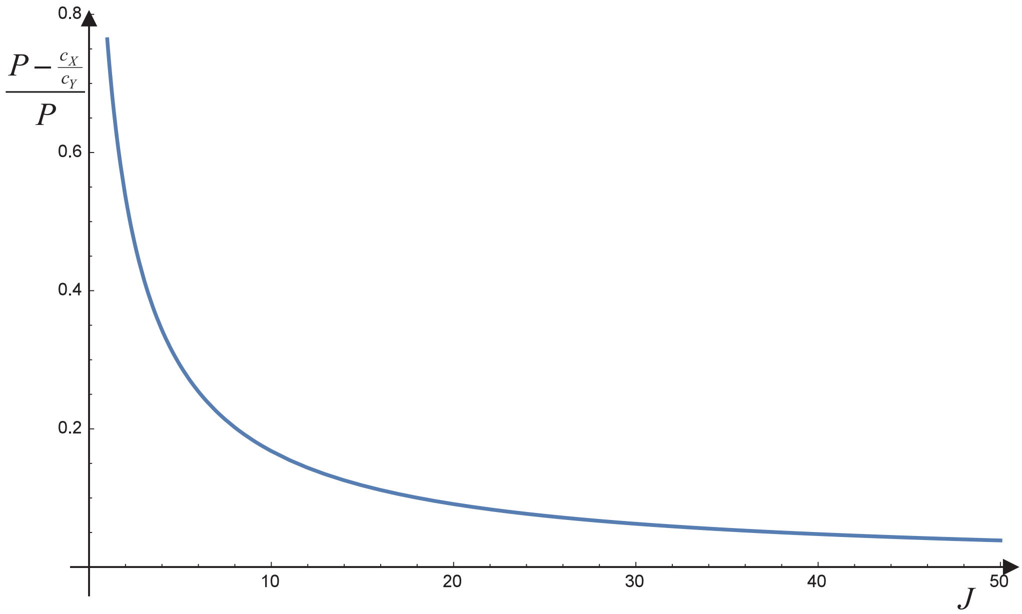

Figure 8: Price-Cost Margin with Cobb-Douglas Preferences

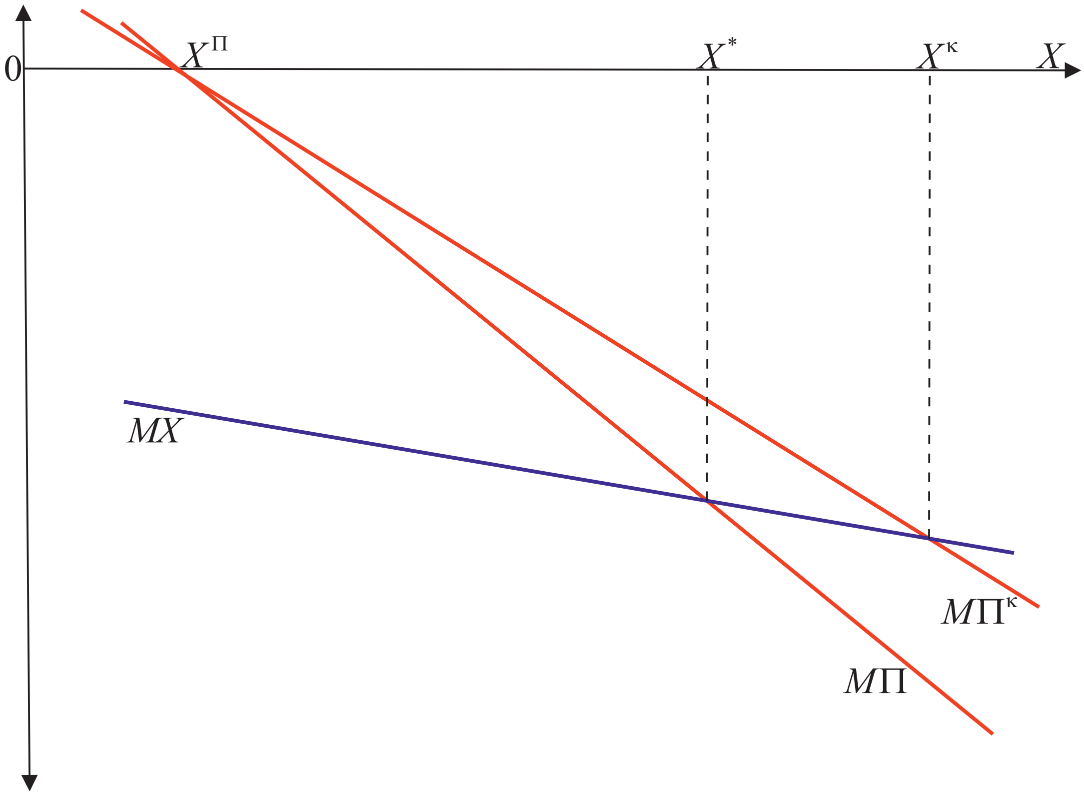

Figure 9: Profits Tax with Quasi-Linear Preferences

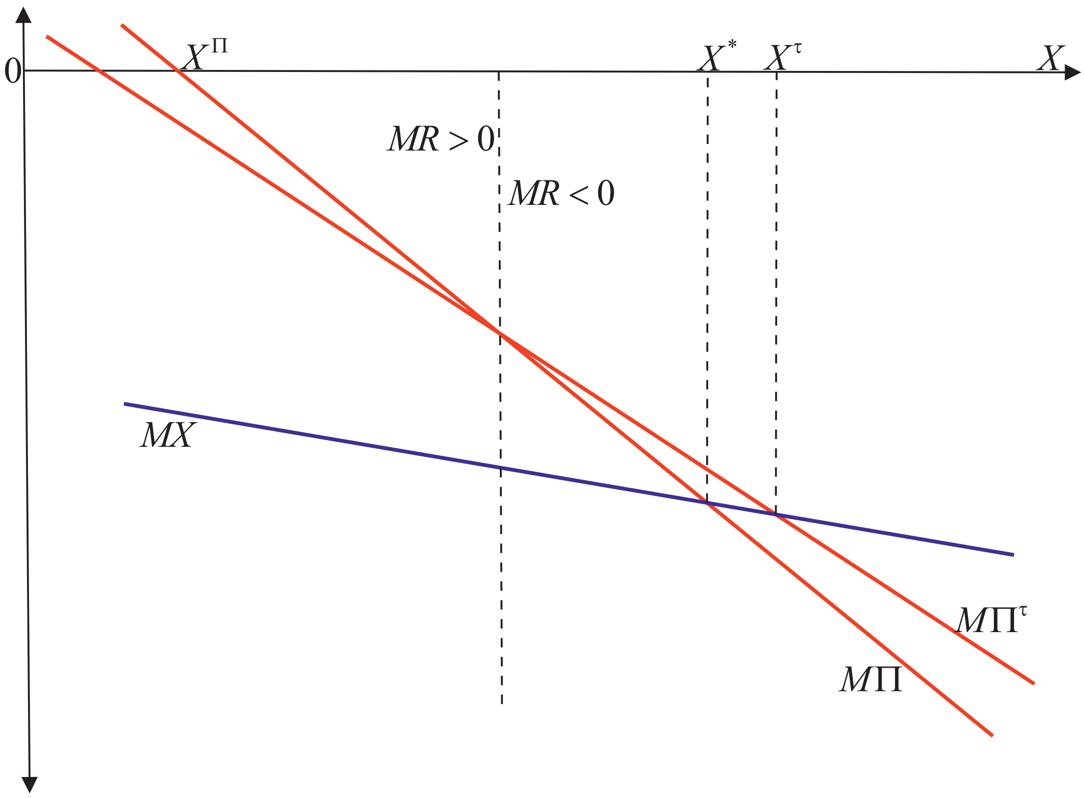

Figure 10: *Ad Valorem* Tax with Quasi-Linear Preferences

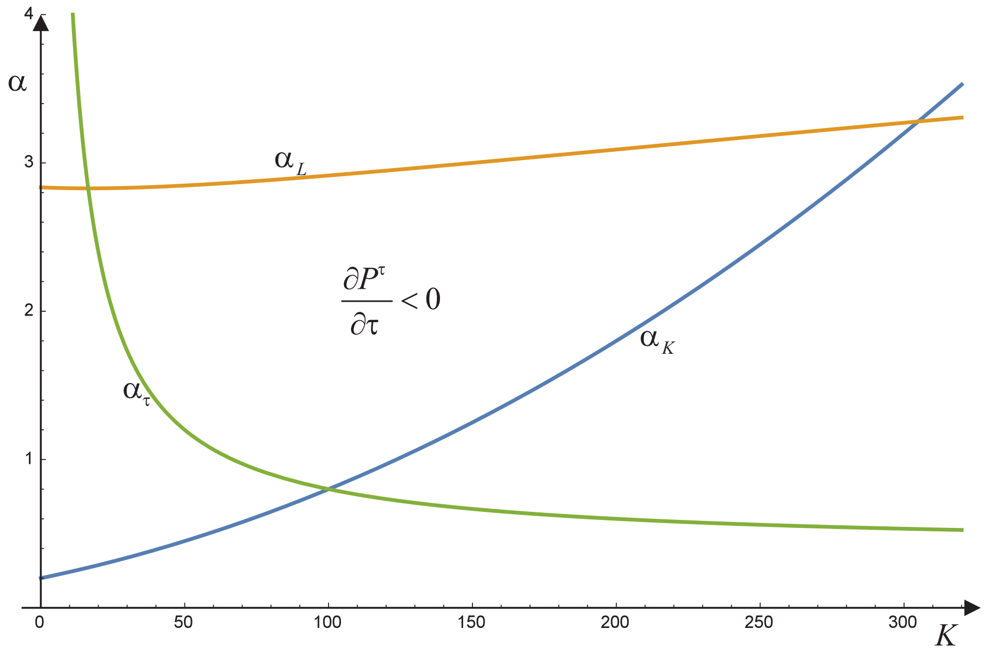

Figure 11: Parameter Space for a Price Decrease with *Ad Valorem* Tax