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Joshy Easaw, Saeed Heravi and Huw David Dixon

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Cardiff Business School
Aberconway Building
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
business.cardiff.ac.uk

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Joshy Easaw[*], Saeed Heravi, Huw Dixon

Cardiff University Business School, Cardiff, UK

[*] Corresponding author: *:EasawJ1@cardiff.ac.uk*

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Abstract

The purpose of the present paper is to investigate perceived inflation gap persistence using actual data of professional forecasts. We derive the unobserved perceived inflation gap persistence and using a state dependent model we estimate the non-linear persistence coefficient of inflation gap. Our main result is that for GDP deflator inflation, the estimates of persistence largely confirm the results obtained indirectly using a linear model. However, when we look at CPI inflation, we find that there is strong evidence for state-dependence and time variation.

Keywords: Perceived Inflation Gaps, Professional's Survey-based Forecasts, State-Dependent Models

JEL classification: E31, E52, E58.

I: Introduction

The inflation gap is defined as the difference between actual inflation and trend inflation. A key question is how this gap behaves in terms of persistence and also how this persistence is influenced by monetary policy and macroeconomic events in general. In a recent paper, Cogley et al (2010) model the inflation gap to argue that in the US its persistence varies over time, mainly reflecting the changing monetary policy regimes. Similarly, Stock and Watson (2007) showed that the predictability of inflation altered with monetary policy. Notably, predictability fell following the Volcker Great Moderation post 1984. Indeed, Benati and Surico (2008) show clearly that there is a structural relationship between these two empirical findings. The purpose of the present paper is to investigate inflation gap persistence using actual data of professional forecasts[1].

The inflation gap, as defined in Cogley et al (2010), is the deviation of actual inflation from a trend component. Cogley et al (2010) estimate or forecast this unobservable component of inflation using both a univariate and multi-variate approach based on a simple New Keynesian model. Their approach rests on the idea that the persistence of the inflation gap is captured by its predictability: "the continuing influence of past shocks can be measured by the proportion of predictable variation in" the inflation gap *j* steps ahead.

We differ from Cogley et al in that we derive the unobserved perceived inflation gap persistence using data on the actual inflation forecasts of professional forecasters. Our data set uses the US based Survey of Professional Forecasters (SPF) includes short-term forecasts for the GDP deflator commencing in 1969 and long-term (10 year) forecasts commencing in 1992 (both series end in 2013). In addition, we have the short-run forecasts for CPI inflation commencing in 1981. Firstly, following Nason and Smith (2013), we derive the inflation gap using the forecasters short-horizon forecast of inflation which is available over a longer

[1] Jain (2013) also uses actual data.

period, for both GDP and CPI inflation. Importantly, the present paper extends the literature by deriving perceived inflation gap with non-linear persistence which is consistent with rational expectations. Secondly, and in addition to the existing literature, we derive perceived inflation gap using the difference between the forecasters' long and short-horizon forecasts for GDP deflator inflation.

We find three main results. First, we find that for the CPI short horizon forecast method over the period 1984-2014 that there is a significant variation in inflation gap persistence. Prior to 1990, inflation gap persistence was much lower than in the moderation period 1990-2008. From 2009 onwards, there has been a drop in persistence, but it is still well above the pre-1990 level. As we would expect, the variation across time is driven by the state-dependence: the inflation gap is more persistent when negative than when positive. Secondly, GDP deflator short horizon forecast over the period 1972-2013 shows little time variation. Persistence is a little lower prior to 1985, but from 1985 onwards is almost constant (in particular there is no change in the "crisis" period 2008-13). There is some state dependence, but mainly in that persistence is higher when the gap is around zero. Thirdly, when we compare the short and the long-horizon method over the period 1994 to 2013, there is little evidence of time variation or state dependence (whether we consider the financial sector or non-financial sector forecasts). The overall lesson form this is that the importance of time varying persistence depends on both the time period covered and the price-index used to measure inflation. In the great moderation, from the late eighties to the mid naughtys, there is no evidence of inflation gap persistence varying over time, for both CPI and GDP deflator inflation. However, if we consider CPI inflation, there is evidence that persistence was lower in the crisis period (post 2008) and much lower in the pre-moderation period (1984-90). On the contrary, there is little evidence for the persistence of the GDP deflator inflation gap to have changed during the post 2008 crisis. There is slightly less persistence prior to 1985, but not much.

How persistent is the inflation gap? When we look at the gap in terms of the GDP deflator interpreted either as the difference between the long and the short horizon forecasts or using the change in short-horizon forecasts, we find that there is an autocorrelation coefficient of around 0.7 which gives a half-life of two quarters (GDP deflator). However, using the short-horizon method for the CPI gives us a lower figure of 0.5 or less for CPI, which of course gives a half-life of only 1 quarter. Even an autocorrelation of 0.7 does not really imply much quantitatively: after 1 year the gap will only be 25% of its initial value. For CPI the inflation gap appears to decay even more rapidly: only 6% is left after one year. Thus when we measure the inflation gap using the data on actual expectations we find it is not very persistent, which confirms the results using statistical methods to measure the gap.

The present paper contributes to the existing literature on inflation persistence in two respects. Firstly, while our approach to investigating inflation persistence focuses on perceived inflation gap persistence, the current analysis considers two different ways to derive professional forecasters perceived inflation gap. Enabling us the compare both forms as well as the robustness of their perceptions. We derive the professional forecaster's perceived inflation gap using both their short- and long-horizon forecast of inflation. Secondly, our non-linear empirical investigation considers the time-varying nature of perceived inflation gap persistence and, thereby, how it relates to monetary policy. We also consider the state-varying nature of perceived inflation gap persistence. We consider how the size and nature of the perceived inflation gap affects persistence. So the papers access whether persistence varies small and big perceived inflation gap as well as whether they are negative or positive.

The paper is outlined as follows. The next section outlines the simple theoretical framework which forms the basis for empirical analysis. Section III introduces and outlines the state-dependent econometric (SDM) model used for our non-linear analysis. Section IV discusses the dataset used and reports the estimation results. Finally, Section V outlines the summary of the key results and draws the concluding remarks.

II: Perceived Inflation Gap Persistence: Theoretical Issues

Since the seminal work of Stock and Watson (2007) the unobserved-components (UC) model supposes that current inflation, π_t , comprise of two components: a stochastic trend, τ_t , and a stationary component, ξ_t :

$$\pi_t = \tau_t + \xi_t \tag{1}$$

The stochastic trend is assumed to follow a random walk process without a drift:

$$\tau_t = \tau_t + \eta_t \tag{2}$$

where η_t is a martingale difference series. Cogley et al (2010) used the UC model to derive the inflation gap which they define as the stationary component ξ_t , the difference between current inflation rate from its trend. The inflation gap, in turn, is assumed to be persistent:

$$\xi_t = \rho_{t-1}\xi_{t-1} + \nu_t \tag{3}$$

where ν_t denotes the inflation gap shock and is a martingale difference series. The covariance of the two innovations τ_t and ξ_t may be non-zero. The AR coefficient ρ_t captures the persistence of the inflation gap. Both Stock and Watson (2007) and Nason and Smith (2013) inflation persistence is assumed to be constant over time, $\rho_t = \rho$.

Cogley et al (2010) estimate the value of a time-invariant ρ to be around 0.5, well below unity. However, we generalize the approach taken in Nason and Smith (2013) to allow for the persistence parameter ρ_t to be time-varying that may be a non-linear function of the inflation gap ξ_t.

We first assume that professional forecasters' have rational expectations and form their long-run inflation forecasts by estimating the stochastic trend $F_t\ (\tau_t)$ based on information at time *t*. It follows since τ_t is a random walk (2):

$$F_t^{'}(\pi_{t+h}) = F_t\ (\tau_t) + (\rho_t\, \rho_{t+1...}\rho_{t+h-1})\xi_t \tag{4}$$

For *h* large and ρ_t bounded away from 1 we have the approximation and limit as the time horizon *h* tends to infinity

$$(\rho_t\, \rho_{t+1...}\rho_{t+h-1})\xi_t \approx 0$$
$$\lim_{h\to\infty}\ (\rho_t\, \rho_{t+1...}\rho_{t+h-1})\xi_t = 0$$

Hence the long-term forecast (*h* large) can be seen as capturing the "trend" element:

$$F_t\ (\pi_{t+h}) \approx F_t\ (\pi_{t+\infty}) = F_t\ (\tau) \tag{5}$$

Forecasters form short-horizon inflation forecasts (*h* small) according to (4). We assume that forecasters make the approximation $\rho_t\, \rho_{t+1...}\rho_{t+h-1} \approx \rho_t^h$ based on information available in period *t*, so that (4) becomes:

$$F_t\ (\pi_{t+h}) = F_t\ (\tau_t) + \rho_t^h F_t\ (\xi_t) \tag{5'}$$

Forecasters, when focusing on short-horizon forecasts, may also wish to form multi-period forecasts. In this case, agents are interested in the momentum of future inflation and the persistence of any transitory shock and, the prospective inflation gap is crucial. The differences of any multi-period forecasts across the horizons are:

$$F_t\ (\pi_{t+h+1}) - F_t\ (\pi_{t+h}) = F_t\ \Delta(\pi_{t+h+1}) = \rho_t^h(\rho_t - 1)F_t(\xi_t) \tag{6}$$

Multiplying equation (5) by $(1-\rho_t L)$, derive the professionals' forecast of *inflation momentum*[2] over time:

$$F_t\,\Delta(\pi_{t+h+1}) = \rho_{t-1}F_{t-1}\Delta(\pi_{t+h}) + \rho_t^h(\rho_t - 1)F_t(v_t) \tag{7}$$

We assume the inflation gap shock v_t is unforecastable white noise, so equation (7) is now:

$$F_t\,\Delta(\pi_{t+h+1}) = \rho_{t-1}F_{t-1}\Delta(\pi_{t+h}) + \varepsilon_t \tag{7'}$$

The advantage of formulation (7') is that we can use the data on changes in short term forecasts (inflation momentum) to estimate the inflation gap persistence parameter ρ_t. This is useful because the data on short term forecasts goes back much further than the data on long-term forecasts. From (7) we can see that even if $F_t(v_t)$ is white noise, there may be some heteroskedasticity due to variations in ρ_t. We will allow for the variance of the error term to vary over time in our estimation procedure. We will also evaluate the accuracy of the approximation $\rho_t\,\rho_{t+1\ldots}\rho_{t+h-1} \approx \rho_t^h$, which we find to be good.

In this paper we consider a general non-linear model in which the parameters ρ_t depend on the past information, and are therefore state-dependent. In particular we take the inflation momentum equation (7') and allow for state-dependence of ρ_t on the previous periods forecast of the change in inflation:

$$F_t\,\Delta(\pi_{t+h+1}) = \rho(x_{t-1})F_{t-1}\Delta(\pi_{t+h}) + \varepsilon_t \tag{8}$$

where $x_{t-1} = F_{t-1}\Delta(\pi_{t+h})$

So far we have considered the derivation of perceived inflation gap using the forecaster's short-horizon inflation forecasts. Professional forecasters' perceived inflation

[2] By *inflation momentum* we mean the persistence of changes in inflation as represented by (7). The key point is that the autoregressive coefficient for inflation momentum is exactly the same as the coefficient for inflation gap persistence.

gap can also be derived using their long-horizon forecast which is available from 1992 in our dataset. Recalling, $F_t\ (\pi_{t+h}) \approx F_t\ (\pi_{t+\infty}) = F_t\ (\tau_t)$, we assume the infinite-horizon forecasts can be proxied by the long-horizon forecasts such as ten-year ahead forecasts[3]:

$$F_t\ (\pi_{t+10}) = F_t\ (\tau_t) \tag{9}$$

On the other hand, a short-horizon inflation forecast such as one-year ahead:

$$F_t\ (\pi_{t+1}) = F_t\ (\tau_t) + \rho_t F_t\ (\xi_t) \tag{10}$$

Subsequently, both short and long-horizon forecasts are related via the stochastic trend component. Their difference is, therefore:

$$F_t\ (\pi_{t+1}) - F_t\ (\pi_{t+10}) = \rho_t F_t\ (\xi) \tag{11}$$

which is simply forecasted one-year ahead inflation gap. As previously we consider a general non-linear model in which the parameter ρ is state-dependent:

$$(F_t\ (\pi_{t+1}) - F_t\ (\pi_{t+10})) = \rho\ (x_{t-1})(F_{t-1}(\pi_{t+1}) - F_{t-1}(\pi_{t+10})) \tag{12}$$

where $x_{t-1} = F_{t-1}(\pi_{t+1}) - F_{t-1}(\pi_{t+10})$.

We have now derived the non-linear version of ρ using two distinct methods and two different professional forecast of inflation. The remainder of the paper focuses on extending the non-linear behavior of ρ using state-dependent models and, subsequent, empirical analyses.

[3] We assume: $F_t\ (\pi_{t+10}) = F_t\ (\tau_t) + \rho_{t|t+10} F_t\ (\xi_t) = F_t\ (\tau_t)$ where $\rho_{t|t+10} = \rho_t^{10} \approx 0$

III: STATE- DEPENDENT MODELS

Priestley (1980) developed a general class of non-linear time series, called "State Dependent Models" SDM, which includes non-linear time series models and linear ARMA as special cases. The principal advantage of SDM is that it allows for a general form of non-linearity and can be fitted without any specific prior assumption about the form of non-linearity. This is not only useful in itself, but may give an indication of specific type of non-linear model which is appropriate to a particular situation, or even, indeed, whether a linear model might prove equally satisfactory . In this section we describe the general structure of the SDM's and the problem involved in identifying SDM's will also be considered. We explain how from the fitted model we may obtain an over view of the non-linear structure of the model, which may lead us toward a more specific non-linear model. A more extensive discussion of these models is given in Preistley (1980), and an extensive study of the application of state-dependent models to real and simulated data is given in Haggan, Heravi and Priestley (1984) and its extension to non-linear dynamical systems is given in Priestley and Heravi (1986).

Consider the following linear AR(*k)* model

$$X_t = \mu + \rho_1 X_{t-1} + ...\rho_k X_{t-k} + \varepsilon_{\mathrm{t}} \qquad (13)$$

Where{ ε_t } is a sequence of independent zero-mean random error terms and $\mu, \rho_1, ...\rho_k$ are constants, then at time $(t-1)$ the future development of the process $\{X_t\}$ is determined by the values $\{X_{t-1}, ..., X_{t-k}\}$, together with future values of { ε_t }. Hence, the vector x_{t-1} $=\{X_{t-1}, ..., X_{t-k}\}'$ may be regarded as the 'state-vector' of the process $\{X_t\}$. That is, the only information in the 'past' of the process relevant to the future development of the process is contained in the state-vector.

The SDM extends the idea of the linear AR time series model by allowing the coefficient of model (13) to become functions of the state-vector x_{t-1}, leading to the general non-linear model.

$$X_t = \mu(x_{t-1}) + \rho_1(x_{t-1})X_{t-1} + ...\rho_k(x_{t-1})X_{t-k} + \varepsilon_t \qquad (14)$$

This model possesses a considerable degree of generality and , in fact, the SDM scheme does include the linear AR model and the main types of specific non-linear time series models if one employs particular forms of the coefficient , μ , $\rho_1, \ldots\ldots\ldots \rho_k$. (see Haggan et al (1984)).

III.i: ESTIMATION FOR SDM's:

In this section we describe the estimation of the SDM's and give a precise formulation of this approach, focusing on how to estimate the parameter ρ in the inflation gap model (12). We also extend the estimation procedure to consider the heteroskedasticity in inflation gap model by allowing the variance of the residuals to change from one point to the next.

As already noted, at time t, the future evolution of the state-dependent model (14) is completely determined by the set quantities $x_t = (X_{t-k+1}, \ldots, X_t)'$ together with the 'innovation', ε_{t+1} , and this set may be regarded as a 'state-vector' at time t. If the state-ector x_t, is augmented with the constant unity to include the mean parameter μ , we can then write the state-vector for model (12) which is an AR of order 1 as:

$$x_t = (1, X_t)'$$

The SDM may be given a formal state-space representation as follows:

$$x_{t+1} = \{\mathbf{F}(x_t)\}\, x_t \; + \varepsilon_{\text{t+1}}$$

$$X_t = \mathbf{H} \,.\, x_t \qquad (15)$$

where the transition matrix **F** is given by $F = \begin{bmatrix} 1 & 0 \\ \mu & \rho \end{bmatrix}$

with $\mathbf{H} = (0 \vdots 1)$ and $\varepsilon_{\text{t}} = \varepsilon_{\text{t}}\,(0 \vdots 1)'$

In fitting the SDM model for this case, we are concerned with the estimation of the parameters μ, ρ However, these coefficients depend on the state vector x_{t-1} , and the estimation problem thus becomes the estimation of the functional form of this dependency. In order to estimate these coefficients, a recursive method similar to that of Harrison and Stevens (1976) is used. The basic difference between the two methods is that in the SDM model the parameters are ‘state-dependent’ which makes the model non-linear, while the Harrison-Stevens approach employs a model with ‘time-dependent’ coefficients.

Priestly (1980) has shown it is possible to base the estimation procedure on the (extended) Kalman Filter algorithm provided some assumptions are made about the parameters. The simplest non-trivial assumption that can be made is that the parameters are linear functions of the state-vector x_t ,

$$\rho\,(x_t) = \rho^{(0)} + x_t' \,.\, \gamma \; = \rho^{(0)} + X_t \,.\, \gamma$$

We may adopt similar model for $\mu\,(x_t)$ as:

$$\mu\,(x_t) = \mu^{(0)} + x_t' \cdot \alpha = \mu^{(0)} + X_t \cdot \alpha$$

where $\mu^{(0)}$, $\rho^{(0)}$ are constants, and α, γ are 'gradient' vectors. Although this assumption clearly cannot represent all types of non-linear model, it seems reasonable to assume that the parameters μ and ρ may be represented *locally* as linear functions of x_t. This assumption is valid, provided μ and ρ are slowly changing functions of x_t. With these assumptions, 'updating' equations for the coefficients μ and ρ can be written as follows:

$$\mu\,(x_{t+1}) = \mu\,(x_t) + \Delta\, x_{t+1}' \cdot \alpha^{(t+1)} = \mu\,(x_t) + (X_{t+1} - X_t) \cdot \alpha^{(t+1)}$$

(16)

$$\rho\,(x_{t+1}) = \rho\,(x_t) + \Delta\, x_{t+1}' \cdot \gamma^{(t+1)} = \rho\,(x_t) + (X_{t+1} - X_t) \cdot \gamma^{(t+1)}$$

where $\Delta\, x_{t+1} = x_{t+1} - x_t = (X_{t+1} - X_t)$. The 'gradient' parameters $\alpha^{(t)}$, $\gamma^{(t)}$ are unknowns, and must be estimated. The basic strategy is to allow these parameters to wander in the form of 'random walks'. The random walk model for the gradient parameters may be written in matrix form as

$$\mathbf{B}_{1+1} = \mathbf{B}_t + \mathbf{V}_{t+1}$$

Where $\mathbf{B}_t = (\alpha^{(t)}, \gamma^{(t)})'$ and $\{\mathbf{V}_t\}$ is a sequence of independent matrix-valued random variables such that $\mathbf{V}_t \sim \mathrm{N}\,(0, \Sigma_v)$.

The estimation procedure then determines, for each *t,* those values of $\boldsymbol{B}_t$, which roughly speaking, minimise the discrepancy between the observed value of X_{t+1} and its predictor, $\hat{X}_{t+1}$ computed from the model fitted at time t. The algorithm is thus sequential in nature and resembles the procedures used in the Kalman filter algorithm (Kalman, 1963).

A reformulation of the SDM model has been given, Priestley (1980), by re-writing the model in a state-space form in which the state-vector is no longer X_t, but is replaced by the state-vector.

$$\theta_t = (\mu^{(t-1)},\ \rho^{(t-1)}, \alpha^{(t)}, \gamma^{(t)})$$

ie, θ_t is the vector of all current parameters of the model. Applying the Kalman algorithm to the reformulated equations yields the recursion

$$\hat{\theta}_{(t)} = \boldsymbol{F}^{*}_{t-1}\ \hat{\theta}_{(t-1)} + \mathbf{K}^{*}_{t}\ \{\mathbf{X}_t - \mathbf{H}^{*}_{t}\ \mathbf{F}^{*}_{t-1}\ \hat{\theta}_{(t-1)}\}$$

where $\boldsymbol{H}^{*}_{t}$ = (1, -X_{t-1}, 0, 0)

$$\boldsymbol{F}^{*}_{t} = \begin{bmatrix} 1 & 0 & (X_{t-1} - X_{t-2}) & 0 \\ 0 & 1 & 0 & (X_{t-1} - X_{t-2}) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $\boldsymbol{K}^{*}_{t}$, the 'Kalman gain' matrix, is given by

$$\boldsymbol{K}^{*}_{t} = \boldsymbol{\Phi}_t\,(\boldsymbol{H}^{*}_{t})'\,\sigma_e^{-2},$$

$\boldsymbol{\Phi}_t$ being the variance-covariance matrix of the one-step prediction error of θ_t, i.e.,

$$\boldsymbol{\Phi}_{t} = E[\{\theta_{t} - \boldsymbol{F}^{*}_{t-1}\ \hat{\theta}_{t-1}\}\{\boldsymbol{\theta}_{t} - \boldsymbol{F}^{*}_{t-1}\ \hat{\theta}_{t-1}\}']$$

and σ_e^2 is the variance of the one-step ahead prediction error of $\boldsymbol{X}_t$, *i.e.*, σ_e^2 is the variance of $e_t = \{\boldsymbol{X}_t - \boldsymbol{H}^{*}_{t}\ \boldsymbol{F}^{*}_{t-1}\ \hat{\theta}_{t-1}\}$. If the variance-covariance matrix of (θ_t - $\hat{\theta}_t$) is denoted by $\boldsymbol{C}_t$, then successive values of $\hat{\theta}_t$ may be estimated by using the standard recursive equations for the Kalman Filter, namely,

$$\boldsymbol{K}^{*}_{t} = \boldsymbol{\Phi}_{t}\,(\boldsymbol{H}^{*}_{t})'\,[\boldsymbol{H}^{*}_{t}\ \boldsymbol{\Phi}_{t}\,(\boldsymbol{H}^{*}_{t})' + \sigma^2_{\varepsilon,t}]^{-1}$$

$$\boldsymbol{\Phi}_{t} = \boldsymbol{F}^{*}_{t-1}\boldsymbol{C}_{t-1}\,(\boldsymbol{F}^{*}_{t-1})' + \sum\nolimits_{W}$$

$$\boldsymbol{C}_t = \boldsymbol{\Phi}_{t} - \mathbf{K}^{*}_{t}\ [(\boldsymbol{H}^{*}_{t})\,\boldsymbol{\Phi}_{t}\,(\boldsymbol{H}^{*}_{t})' + \sigma^2_{\varepsilon,t}]\,\mathbf{K}^{*\prime}_{t}$$

Where $\sum_{W} = \begin{pmatrix} o & o \\ o & \sum_{v} \end{pmatrix}$

In practice, this recursive procedure must be started at some value of $t=t_0$, and hence initial values are required for $\hat{\theta}_{t_0-1}$ and $\hat{C}_{t_0-1}$. Equation (14) represents a 'locally' linear AR model and in finding these initial values, we apply the same procedure as Haggan et. al. (1986) and formulate a practical estimation procedure as follows:

(i) Take an initial stretch of the data, say the first 2m observations, and fit a standard linear AR(1) model. This will provide initial values $\hat{\mu}$, $\hat{\rho}$ and the residual variance of the model, $\hat{\sigma}_{\varepsilon}^{2}$.

(ii) Start the recursion midway along the initial stretch of data at $t_o=m$, and set

$$\hat{\theta}_{t0-1} = (\ \hat{\mu}\ ,\ \hat{\rho}\ , 0, 0)'$$

and $\hat{C}_{t0-1} = \begin{pmatrix} \hat{R}_{\mu,\,\rho} & 0 \\ 0 & 0 \end{pmatrix}$

where $\hat{R}_{\mu,\rho}$ is the estimated variance-covariance matrix of $\hat{\mu}$ and $\hat{\rho}$ obtained from the initial AR(1) model fitting. It also seems reasonable to set all the initial gradients to zero, assuming that the initial values are reasonably accurate at $t_o=m$. We also need to select reasonable values for $\sum_v$, the variance-covariance matrix of $\boldsymbol{V}_t$. The choice of $\sum_v$ depends on the assumed 'smoothness' of the model parameter as functions of x_t . The diagonal elements of $\sum_v$ are set equal to $\hat{\sigma}_e^2$ multiplied by some constant α called the 'smoothing factor', and the off-diagonal elements are set equal to zero. However, if the elements of $\sum_v$ are set too large, the estimated parameters become unstable, but if the elements of $\sum_v$ are made too small, it is difficult to detect the non-linearity present in the data. The best procedure in practice appears to be to reduce the magnitude of the 'smoothing factor' until the parameters show stable behaviour. Haggan et al (1984) suggested to select the smoothing factor in the range of 10^{-2} to

10^{-5}. In addition, the parameters may by smoothed by a multi-dimensional form of the non-parametric function fitting technique (see for example Priestley and Chao (1972)). We also extend the SDM estimation procedure to allow the residual variance to change from one point to the next. In this case, we compute and update the variance of the residuals, $\hat{\sigma}_e^2$ using the current information on the residuals so far obtained. Having done this procedure, it is hoped that the resulting parameters give a clearer idea of the type of non-linearity present in the model.

IV. Data and Empirical Results:

IV.i: Data: Survey of Professional Forecasters (SPF)

In the present paper we focus on professional forecasters in the US, using the Survey of Professional Forecasters, SPF. The two multi-period SPF forecasts of interest are $F_t\ (\pi_{t,t+3})$ and $F_t\ (\pi_{t+1,t+4})$ for both the GDP deflator and CPI inflation rates. They are computed by defining *P = PGDP* or *P = CPI* in the following formulas, written in terms of the SPF labels (for details, see also the SPF survey documentation):

onestepP =100(((1+P2/100)*(1+P3/100)*(1+P4/100)*(1+P5/100))^.25 -1)*

multistepP =100(((1+P3/100)*(1+P4/100)*(1+P5/100)*(1+P6/100))^.25 -1)*

More explicitly, the one-year-ahead forecast of *P* (*onestepP*) is defined as the mean of the SPF forecasts released in quarter *t* for the current and the next three quarters (i.e. *t*, *t+1*, *t+2*, *t+3*), in symbols $F_t\ (\pi_{t,t+3})$; the corresponding multi-step forecast (*multistepP*) is the mean of the SPF forecasts - again released in quarter *t* - for the next four quarters (i.e. *t+1*, *t+2*, *t+3*, *t+4*), in symbols: $F_t\ (\pi_{t+1,t+4})$. Therefore, the one-step- and the multistep-ahead horizons of the SPF forecasts overlap for three quarters. The professionals' forecasts for ten-years ahead is derived in a similar way.

Figure 1 depicts the professional forecasters perceived inflation gap for PGDP and CPI. The data for former covers the period 1970q3-2014q2 and the latter begins in 1981q4. Figure 2 depicts the professional forecasters perceived inflation gap for PGDP using the difference between long and short-horizon forecasts. The data is disaggregated into forecasters belonging to either the financial or non-financial sectors. The ten-year ahead or long-horizon forecasts are available from 1991q4:

Figures 1 and 2[about here]

There is clear serial correlation in the inflation gap in Figure 2. Note that this is perfectly consistent with rational expectations: the serial correlation stems from the underlying error process driving the gap itself.

IV.ii: State-Dependent Model (SDM) Results:

In this section, we apply the state-dependent model fitting technique for data on inflation gap. We estimate our starting values using the first 20 quarters of the data. We start the actual recursive estimation half way through the initial period at 10 quarters. The parameters were smoothed using a non-parametric function fitting technique which employs a rectangular smoothing kernel. The results show the parameters plotted against the state-vector and also, as the algorithm is sequential, we can present the parameters against time scale. As suggested by Haggan et al (1984), we tried applying the smoothing factor in the range of 10^{-2} to 10^{-5}, the results were quite similar and for all the cases the final results are given using smoothing factor of 10^{-4}. It should be emphasized that the SDM algorithm operates purely on the data and has no prior knowledge of the underlying model.

We first report results of perceived inflation gap persistence using the short-horizon forecast, followed by the second method considering the difference between the long and short-horizon forecasts.

IV.iii: Perceived Inflation Gap Persistence: Short-horizon forecasts:

a. Inflation gap persistence (PGDP)

We take as our starting point the simple linear model with a constant ρ estimated on the initial stretch, taking the first 20 periods of the data (1970q3 – 1975q4). The initial estimates over this initial period are:

$\hat{\mu}$ = -0.135, $\hat{\rho}$ = 0.604, $\hat{\sigma}_{\varepsilon}^{2}$ = 0.094

$$\hat{R} = \begin{bmatrix} 0.008 & -0.012 \\ -0.012 & 0.047 \end{bmatrix}$$

We then estimate the SDM model as outlined in the previous section, iteratively starting from half way through the initial stretch, 10 quarters from the beginning of the dataset at 1973q4. For comparison, the dotted line represents the linear estimates taken across the whole period which is depicted by the dotted line in Figure 3 ($\hat{\rho}$ =0.65). The estimates of ρ are shown both against time and against the previous lag of inflation gap in Figures 3 and 4 respectively.

Figures 3 and 4[about here]

The short-horizon forecasts for *PGDP* display little time variation after 1985, being close to the linear estimate over the whole period. For the period from the mid-70's to 1985, when there was more variable and higher inflation, the persistence of the inflation gap is lower than post 1985. There is some state dependence, with a slightly higher value for a positive gap than for a negative gap. The time-variation also seems to add little in the "crisis period" post 2008. For GDP inflation, we can see that there is little to be gained by allowing for a non-linear model as compared to the standard linear estimate. They both predict an auto-correlation coefficient of around 0.65-0.7 for the period 1987-2011. The inflation gap is persistent, but not very much: the half-life is just two quarters. From 1975 to 1985, there was even less persistence with values in the range 0.5-0.6.

b. Inflation gap persistence (SPFCPI)

Once again we take as our starting point the simple linear model with a constant ρ fitted on the first 20 observations (starting from 1984q4) yielding initial estimates,

$$\hat{\mu} = 0.218,\ \hat{\rho} = 0.21,\ \hat{\sigma}_{\varepsilon}^{2} = 0.056$$

$$\hat{R} = \begin{bmatrix} 0.005 & 0.011 \\ 0.011 & 0.061 \end{bmatrix}$$

The estimates of ρ are shown both against time and against the previous lag of inflation gap in Figures 5 and 6 respectively (again, the dotted line represents the simple linear estimate over the whole period, with ρ = 0.34):

Figures 5 and 6[about here]

The non-linear model yields very different results from the linear model for CPI. During the Great Moderation period (1986-2006), the non-linear model predicts a much higher value of ρ (0.5) than the linear model. The value of ρ is significantly lower pre-1990:

the average is about 0.25. Furthermore, there is a clear post 2008 crisis effect: the degree of inflation persistence falls from 0.5 to 0.4.

There is also clear state-dependence: the persistence of inflation is higher for negative inflation gaps than for positive inflation gaps. For CPI inflation, the inflation gap does appear to show significant time variation due to state dependence. The linear estimate of a constant ρ also yields misleading results with an underestimate of ρ for all of the sample period post 1990. However, whilst the estimates for CPI differ, they are all consistent in showing little persistence for the inflation gap: whilst significantly non-zero, the magnitude is low. Even in the Great Moderation period, the autocorrelation coefficient is only 0.5: for other periods it is less. The CPI inflation gap is less persistent than the GDP inflation gap.

IV.iv: Perceived Inflation Gap Persistence: Difference between Long and Short-horizon forecasts:

a. *Inflation Gap Persistence : Financial Sector*

The simple model with a constant ρ estimated on the initial stretch (1991q4-1995q3) yields the starting estimates:

$\hat{\mu}$ = 0.034, $\hat{\rho}$ = 0.658, $\hat{\sigma}_{\varepsilon}^{2}$ = 0.033

$$\hat{R} = \begin{bmatrix} 0.002 & -0.004 \\ -0.004 & 0.032 \end{bmatrix}$$

Starting the recursion from 1994q1, the estimates of ρ are shown both against time and against the previous lag of inflation gap in figures 7 and 8 respectively:

Figures 7 and 8[about here]

For the financial sector, we can see that the non-linear model predicts a consistently lower value of persistence than the linear model for most of the period, being at its largest around 2000. The non-linear estimates are fairly stable, although the degree of persistence rises slightly post crisis at the end of the sample and falls a bit from 1994-2000. Whilst there is inflation persistence, the autocorrelation is mainly between 0.6 and 0.7, indicating a half-life no more than 2 quarters. In terms of state-dependence, there is a small peak when inflation gap is zero, with slightly lower values below zero than above.

If we compare the inflation gap using the long-term forecasts with the gap using only short term forecasts (figures 7 and 4), we find that both methods yield similar magnitudes, so that the method using short term differences is a reasonable approximation.

b. Non-financial forecasts persistence: Non-financial sector

The simple linear model with a constant ρ estimated on the 20 initial stretch of the data gives the following estimates:

$\hat{\mu}$ = 0.04, $\hat{\rho}$ = 0.627, $\hat{\sigma}_{\varepsilon}^{2}$ = 0.017

$$\hat{R} = \begin{bmatrix} 0.002 & -0.005 \\ -0.005 & 0.025 \end{bmatrix}$$

Starting the recursion from 1994Q1, the estimates of ρ is shown both against time and against the previous lag of inflation gap in figures 9 and 10 respectively:

Figures 9 and 10 [about here]

The degree of inflation persistence in the non-financial sector is less than in the financial sector: the non-linear estimates are mostly in the region 0.5-0.6. Furthermore, the linear and non-linear methods show little difference throughout the whole period. This is reflected in a there being little or no state dependence.

IV.v: The Approximation.

In analysing the data on expectations, we made the assumption that agents used the approximation $\rho_t\,\rho_{t+1...}\rho_{t+h-1} \approx \rho_t^h$. Our results will only be valid if this is in general a good approximation. We can use the estimates of ρ_t to generate both $\rho_t\,\rho_{t+1}\rho_{t+2}\rho_{t+3}$ and $\rho^4{}_t$. We can then see how these two are different as a percentage of the average value of $\rho_t\,\rho_{t+1}\rho_{t+2}\rho_{t+3}$ taken over the whole sample (which is 0.0400). Clearly, in the cases where ρ_t does not vary much, the approximation must be good. Therefore we look at the case where ρ_t varies the most: the case of the CPI inflation gap in figure 3. Taken over the whole period, the approximation is good: the average error is -1.5%. If we take the absolute error and do not allow positives to cancel negatives, the average absolute error is 8.60%. If we look more closely, the approximation breaks down in a few brief time periods when there is a big change in inflation gap persistence. The biggest error is in 2008q4 when ρ_t is falling and the approximation overestimates the actual by 130%. Similarly in 1990q1 when ρ_t is increasing rapidly, the approximation underestimates by 93%. However, for over two thirds of the 120 quarters the error is less than 5%. Moreover, given that the average value of ρ_t is about 0.4, the average values of both $\rho_t\,\rho_{t+1}\rho_{t+2}\rho_{t+3}$ and $\rho^4{}_t$ are small relative to the value of the parameter being estimated.

V. Concluding Remarks and Summary:

In this paper we use actual data on the inflation expectations of professional forecasters in order to estimate the inflation gap and its persistence. In this we differ from previous studies which have modelled expectations implicitly from the macroeconomic data itself. Without the actual data on expectations, the indirect modelling process itself may give rise to misleading results. Our method also allows for the degree of inflation persistence to vary over time due to state dependence.

Our main result is that for GDP deflator inflation, the estimates of persistence largely confirm the results obtained indirectly using a linear model. That is, there is little time variation over the period considered and the value of persistence of the inflation gap is similar to earlier estimates derived indirectly without the benefit of data on expectations. We find that the autocorrelation parameter to be in the range 0.6-0.7, which is a little higher than the Cogley at al (2010) estimate of 0.5, but in terms of persistence is still well away from unity.

However, when we look at CPI inflation, we find that there is strong evidence for state-dependence and time variation. There is much less persistence both prior to the great moderation and after it in the post 2008 crisis. However, even during the most persistent period of the great moderation, the autocorrelation coefficient is still only around 0.5. Since CPI inflation is the usual target for monetary policy, it implies that the inflation gap will not provide much information for future inflation.

The persistence of the inflation gap has little to say about the persistence of inflation itself. From the unobserved components perspective, inflation is the sum of two components: one is a unit root process with infinite persistence (the trend), the other is as less persistent inflation gap. The level of trend inflation is what will determine the persistence of inflation. When trend inflation was high (for example in the period prior to the great moderation) then there was a lot of autocorrelation in inflation, despite the fact that we find that there was (for CPI at least) less autocorrelation in the inflation gap. However, during the great moderation and in the crisis period inflation has remained low: trend inflation has been low and less variable. That means a lot of the variation we see in inflation is due to the inflation gap. Since the inflation gap is not persistent, it follows that estimated inflation persistence is low since the great moderation and post-moderation crisis. This is reflected in the long-term forecasts of CPI inflation. They have been remarkably stable since 1998, at around 2.5%, having come down from almost 4% in 1991.

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Figure 1: Inflation gap (PGDP) and Inflation gap (CPI) (Short-Horizon Forecast)

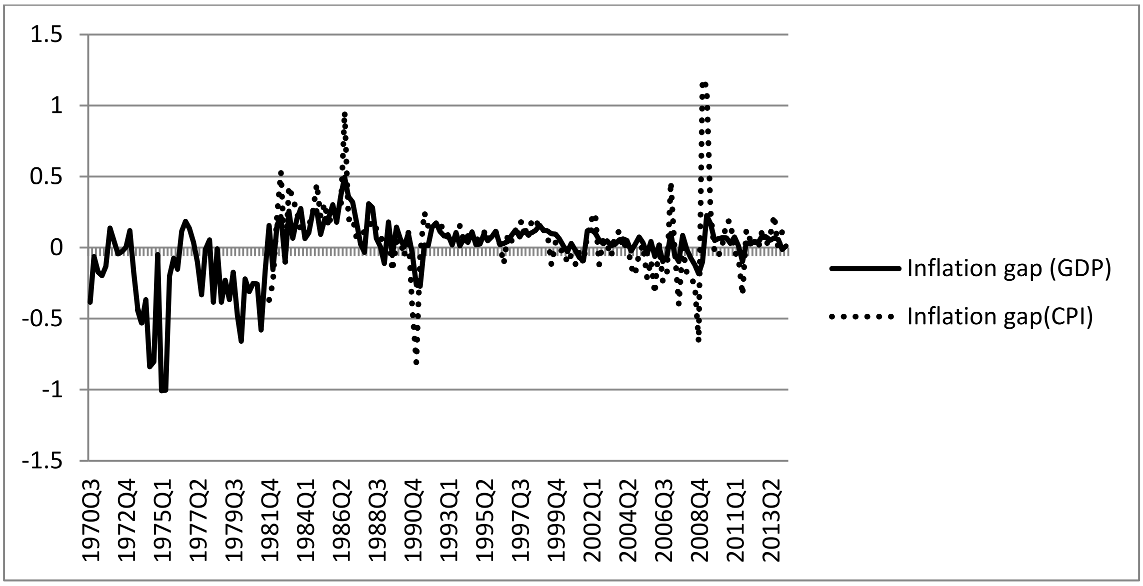

Figure 2: graph of Financial and Non-Financial Data
(Difference between Long and Short Horizon Forecasts)

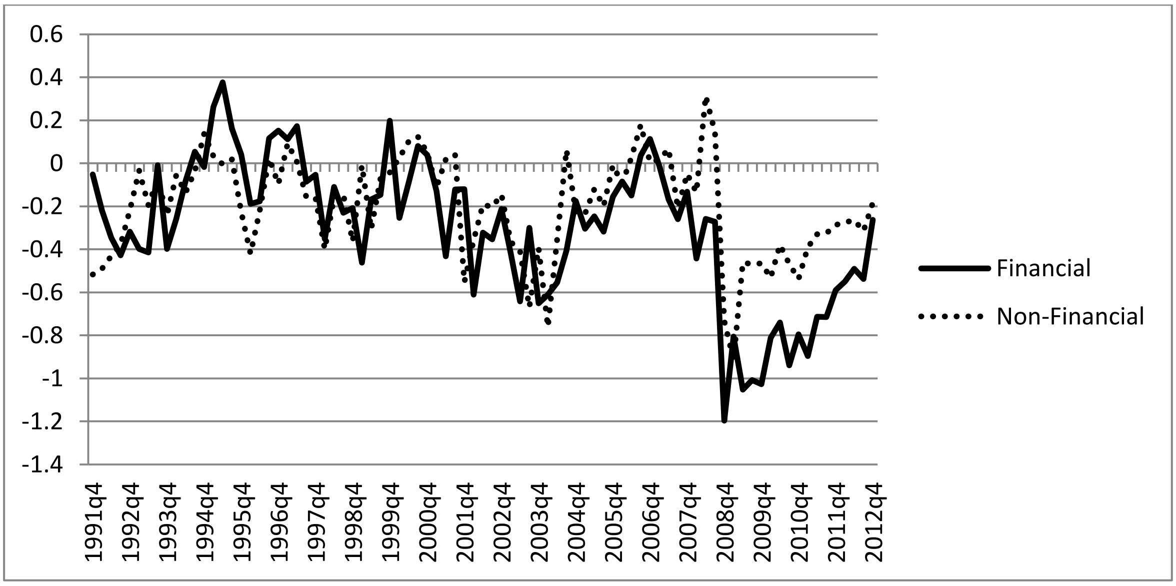

Figure 3: Time-varying Inflation Gap Persistence ($\hat{\rho}$ t): *PGDP* (Short Horizon Forecasts)

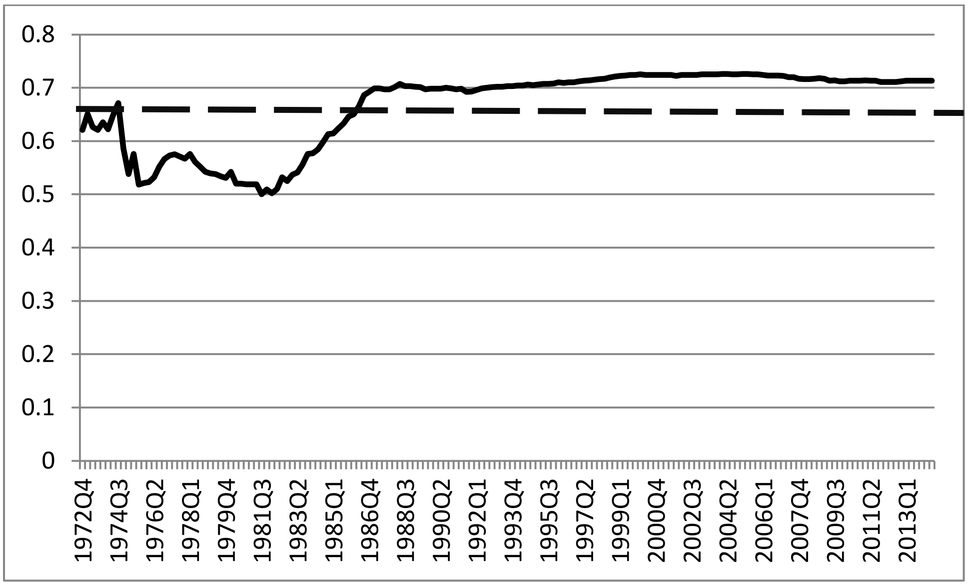

Figure 4: State-varying Inflation Gap Persistence ($\hat{\rho}$): PGDP (Short Horizon Forecasts)

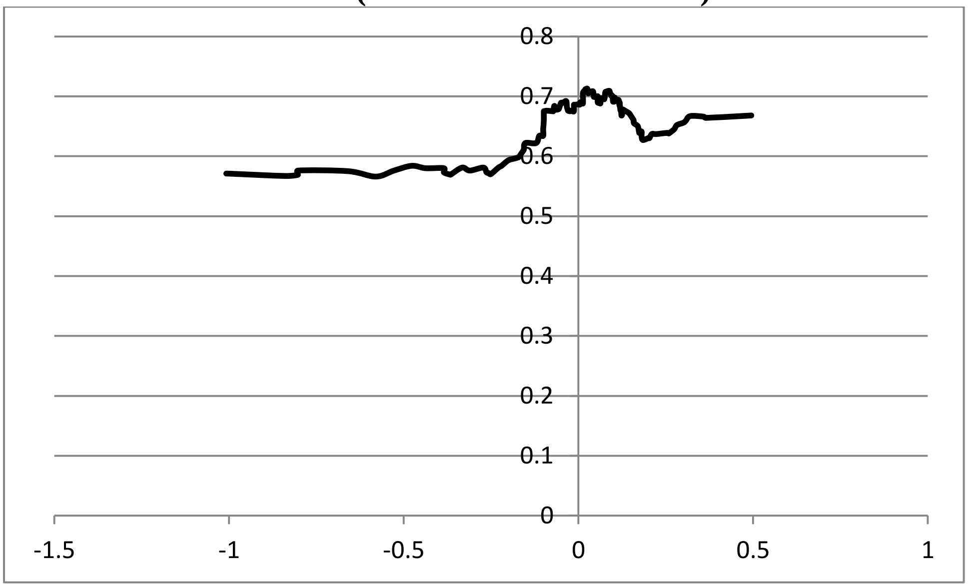

Figure 5: Time-varying Inflation Gap Persistence ($\hat{\rho}_t$): CPI (Short Horizon Forecasts)

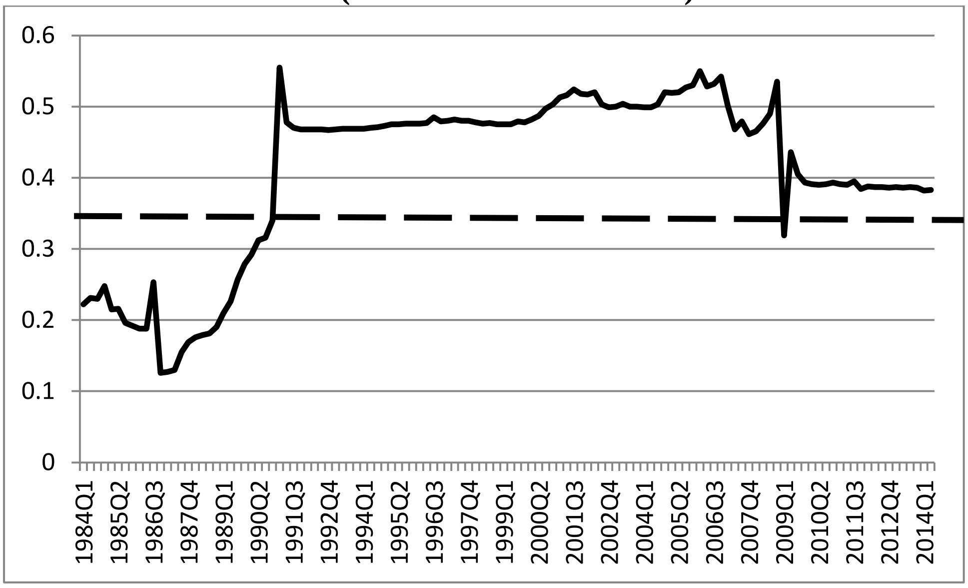

Figure 6: State-varying Inflation Gap Persistence ($\hat{\rho}$): CPI (Short Horizon Forecasts)

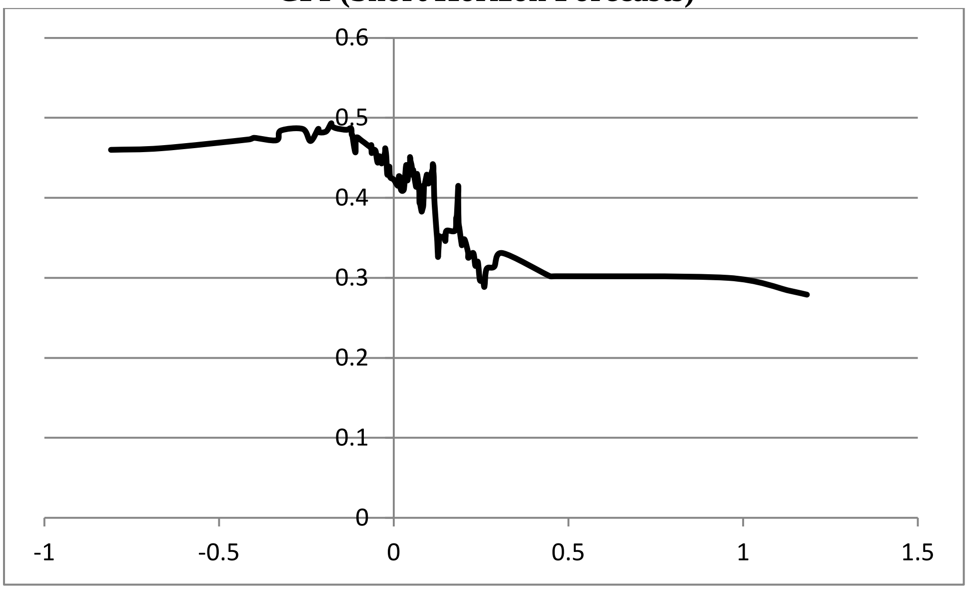

Figure 7: Time-varying Inflation Gap Persistence ($\hat{\rho}$): Financial Sector (Long – Short Horizon Forecasts) *PGDP*.

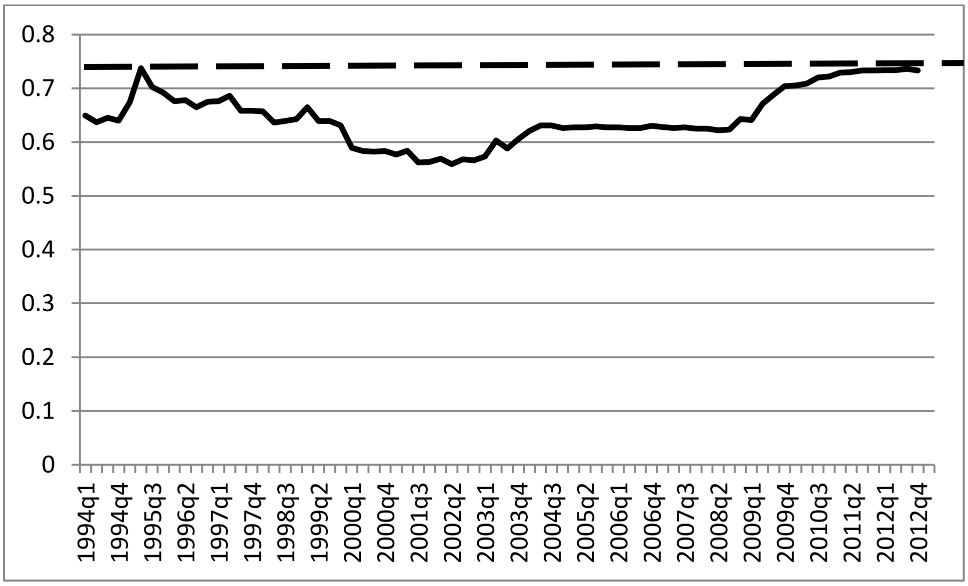

Figure 8: Time-varying Inflation Gap Persistence ($\hat{\rho}$): Financial Sector (Long – Short Horizon Forecasts) *PGDP*.

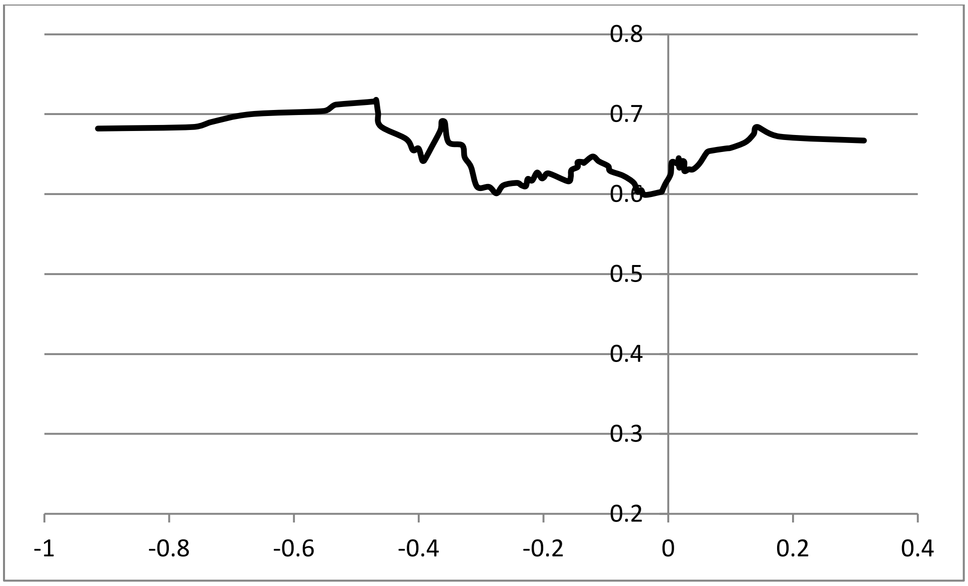

Figure 9: Time-varying Inflation Gap Persistence ($\hat{\rho}$): Non-Financial Sector (Long – Short Horizon Forecasts) *PGDP*.

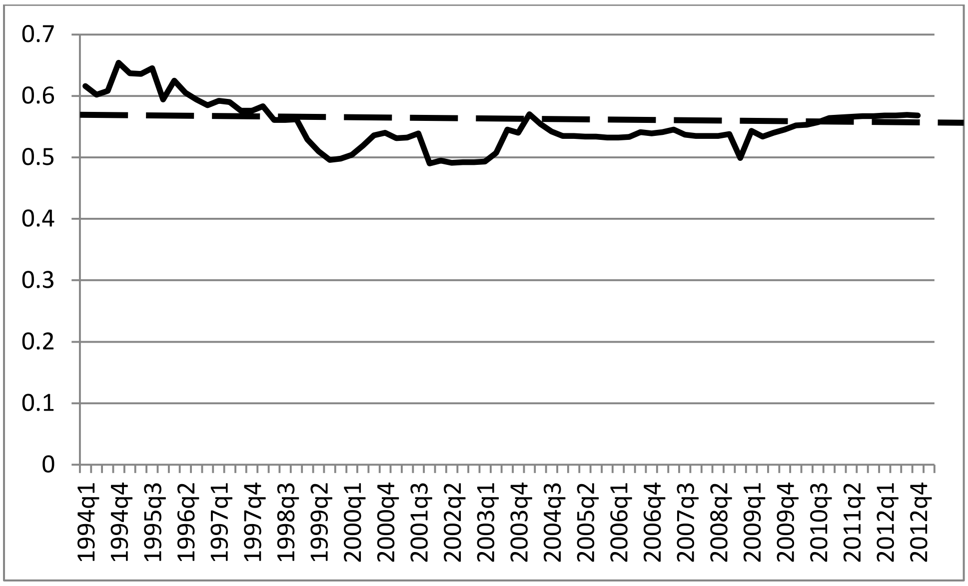

Figure 10: Time-varying Inflation Gap Persistence ($\hat{\rho}$): Non-Financial Sector (Long – Short Horizon Forecasts) *PGDP*.