The Storm Before the Calm? Adverse Effects of Tackling Organised Crime

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Abstract

Policies targeted at high-crime neighbourhoods may have unintended consequences in the presence of organised crime. Whilst they reduce the incentive to commit crime at the margin, those who still choose to join the criminal organisation are hardened criminals. Large organisations take advantage of this, substituting away from membership size towards increased individual criminal activity. Aggregate crime may rise. However, as more would-be recruits move into the formal labour market, falling revenue causes a reversal of this effect. Thereafter, the policy reduces both size and individual activity simultaneously.

Keywords: Organised crime; crime policy; occupational choice.

JEL Classification: D82; J24; J28; K42; L21.

1 Introduction

Over recent years, numerous policies have been suggested to increase the expected cost of engaging in crime. Under normal circumstances, these discourage participation in illegal activity and cause crime to fall. When applied to neighbourhoods where organised crime is prevalent, however, they may face a stumbling block. Things may get worse before they get better: the storm before the calm.

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Assessments of anti-crime policy have a long history in economics (going back to Becker [1968]). As with any other decision, the rational offender chooses whether to commit crime by comparing the benefits with the costs. Policy can do little to affect the gain the offender enjoys from committing crime. Instead, it influences the cost. For example, increasing the resources available to the police raises the probability of conviction for any given crime. As punishment becomes more likely, the net benefit to the offender declines. Raising the severity of punishment by lengthening prison sentences has a similar effect (Stigler 1970; Polinsky and Shavell 1979; Ehrlich 1981).

More recently, the focus has switched away from traditional law enforcement towards influencing the opportunity cost of committing crime. Improving labour market conditions involve the offender giving up more in order to engage in criminal activity (Ehrlich 1973). Two real-world examples illustrate. The Perry Preschool Project (c.f. Heckman et al. 2010) identified poor black children with low IQs. From an early age, they were given intensive preschool classes. The project better prepared the children for school, leading to higher educational performance, ultimately improving their labour market opportunities. The internal rate of return on this project is estimated to be between seven and ten per-cent. As a by-product, participants’ criminality also fell. This sort of intervention is proving increasingly popular. More recently, the Paper Project has provided financial incentives to students in poor U.S. neighbourhoods to encourage school attendance and better exam performance (Fryer 2011).

All of these approaches have proved successful in reducing the number of individuals involved in crime, in line with the predictions of the rational offender model. However, in the presence of organised crime, there is evidence to suggest that they may backfire. During the 1980s, arrests for heroin and cocaine trafficking in the U.S. rose dramatically (from 4% of all drug arrests in 1980 to almost 20% in 1989) as part of the ‘War on Drugs’ (Lee 1993). Successful conviction of traffickers increased (from 85% in 1985 to 92% in 1989) and they were incarcerated for longer periods of time (up from 61 months to 76 months on average). However, over the same period, surveys suggest that availability of both drugs increased, and drugs prices were stable. Various attempts have been made to explain this outcome based on the unique features of the drugs market. Consumers may have changed their purchasing behaviour, buying larger quantities less often (Lee 1993). The market structure itself might have changed, depending upon whether distributors or retailers are targeted (Poret 2002). Competition could have increased, lowering the price (Mansour et al. 2006). It is certainly true that the drugs market underwent significant changes during this period. If backfiring policies were limited to this case, we might have little more to say. However, other examples exist.
In 1990, the Philippines customs authority clamped down on common forms of duty avoidance (Yang 2008). Evidence suggest that, as a result of the policy, not only had other forms of duty avoidance increased, but that the problem of avoidance may have got worse. Whilst the government raised an additional $24.6 million from additional inspections, it is estimated that displacement towards other forms of avoidance cost an additional $33.3 million in lost duties over a fifteen month period. Not only had criminal groups substituted between different activities, but the overall cost of crime increased. The policy had backfired due to substitution between activities.

In response to the London riots of 2011, the Metropolitan Police developed a strategy of arresting known gang members in several neighbourhoods in London. One year later, a survey asked residents of these neighbourhoods to assess its impact (Centre for Social Justice 2012). They stated that gang violence had increased. Despite the arrests, those who remained at liberty became more violent. A more recent report suggests that gang violence has subsequently declined (Home Office 2013). This indicates that the backfiring effects may be temporary.

In an attempt to explain these phenomena, I assess the impact of law enforcement and labour market policies in a simple model of profit maximisation.

A criminal organisation generates revenue by recruiting members to engage in criminal activity. In exchange for their efforts, it pays them a wage. These members are drawn from a neighbourhood in which the organisation is the sole employer of criminals. Many criminal groups recruit in this way, from mutually exclusive geographical territories, ethnicities or even families (Jankowski 1991; Polo 1997; Paoli 2003). By doing so, it is easier for them to maintain the loyalty of their members, preventing defections to other groups or the police.

Individuals in the neighbourhood have varying criminal ability. Evidence suggests that informational asymmetries abound in organised crime (Gambetta 2009). Signalling of ability, for example through the use of violence, is a pervasive method for gaining membership (Decker 1996; Skaperdas 2001), advancement or respect (Jankowski 1991; Silverman 2004). As such, criminal ability is private information. Members with higher ability incur a lower cost of activity. This could represent the effort involved in committing crime. It could also incorporate the inherent dangers involved in this line of work. For example, members of a drugs gang in Chicago have been found to have a 25% chance of dying in four years (Levitt and Venkatesh 2000). This compares with a 0.4% chance in the same demographic across the U.S. as a whole.

There is a rich history of modelling organised crime as a profit-maximising firm. For recent contributions to this literature, see, for example, Gambetta 1996; Anderson and Bandiera 2000; Garoupa 2000; Chang et al. 2005; Kugler et al. 2005; or Dixit 2007.
Each individual either accepts the contract offered by the organisation, or works in the formal labour market. Since it is assumed that all individuals have the same labour market opportunities, the contract acts as a simple screening device. All members receive the same wage, and engage in the same amount of activity. For those with a high criminal ability, membership provides them with a large surplus. Individuals with low ability find the wage the organisation offers is insufficient to compensate for the activity required of them. They instead join the formal labour market. The contract ensures that recruits are from the upper tail of the criminal ability distribution.

When a policy increases the cost of engaging in crime, the surplus each would-be member enjoys declines. Those with relatively low ability no longer choose to join the organisation. The policy appears to be effective, as the number of criminals falls. However, individual criminal activity may have become more profitable. Those who still join are hardened criminals. The compensation they require for the acts they commit falls, making it less expensive for the organisation to raise individual activity (a cost effect). Yet, with fewer members, increases in individual activity have a smaller impact on the aggregate amount of crime the organisation commits, and hence on the revenue it generates (a revenue effect). Whether individual activity becomes more or less profitable depends upon the relative size of these effects.

Left to its own devices, the organisation maximises profit by employing a large number members. It recruits individuals with very low ability. When a policy causes these would-be members to move to the formal labour market, the organisation need not increase its wage by as much to compensate remaining members for increased activity. The cost effect is large. Conversely, diminishing marginal returns imply that the revenue effect is small. Size and individual activity are profit substitutes. Reductions in membership cause the organisation to increase the activity of its remaining members. For large enough organisations, aggregate criminal activity will increase. This is the storm.

As the policy continues to increase the cost of engaging in crime, and the size of the organisation continues to decline, the revenue effect becomes important. Reductions in size lead to smaller declines in the marginal cost of activity, as those who leave already have relatively low effort costs. Diminishing marginal returns are also less restrictive. Size and activity eventually become complements. Further reductions in size cause the organisation to reduce the activity required of its members. Overall crime falls rapidly. This is the calm.

Two recent theoretical contributions to the crime literature also discuss substitution within the context of a gang (Poutvaara and Priks 2009, 2011). In their analysis, the
gang’s leader enjoys being head of a large, violent group. Following a change in police
tactics (2009) or unemployment (2011), they show that the gang leader may reduce
membership in favour of more violent activity. The relative price of size changes, and
the leader maximises utility by substituting towards violence. The intuition in my
contribution is similar. In contrast, I identify two effects (revenue and cost) which jointly
determine the relationship between members and their individual effort. This enables
me to not only discuss profit complements and substitutes in the same framework, but
also to discern a link between the size of the organisation and how it views the two
inputs. These insights generate a new prediction about the reaction to policy. Whilst
membership always falls, individual (and, potentially, aggregate) criminal activity will
first increase, then stabilise, before rapidly declining.

The model presented is not implicitly one of organised crime - it could equally apply
to a firm in the formal economy facing increasing labour market wages. Here too,
there is evidence of substitution from the literature on work sharing (beginning with
Calmfors [1985] and Booth and Schiantarelli [1987]). These contributions assume that
a firm’s output depends upon total hours worked (individual hours multiplied by the
number of workers). My approach nests this assumption. Empirical estimates for the
elasticity of substitution between size and individual hours range between −0.1 and −1.7
(for a survey, see Freeman [2000]). The upper estimates are consistent with the view that
aggregate activity increases when the number of workers falls. Whilst organised crime
is, of course, different with regard to the constraints it faces, this nevertheless suggests
the possibility that it too may substitute, and that this could result in greater amounts
of crime.

The remainder of the paper proceeds as follows. Section 2 outlines a simple model
of organised crime. Section 3 discusses the equilibrium, highlighting the link between
organisation size and whether size and activity are complements or substitutes. The
following two sections identify the impact of a gradual improvement in formal labour
market conditions or an increase in the expected punishment on the optimal size and
activity. Section 6 concludes. All proofs are provided in the appendices.

2 A Simple Model of Organised Crime

A criminal organisation recruits members from a neighbourhood with a population of
mass $N$. Although its product markets may be competitive, the organisation acts as a
monopsonist employer in the crime sector. The neighbourhood may be a geographical
territory, or may represent an ethnicity or collection of families. It offers an identical
contract to everyone in the neighbourhood, comprising of a wage, $g$ and a level of individual criminal activity $a$.\footnote{A previous working paper considers the case in which the organisation offers different wages in exchange for different activity levels \cite{Long2013}. The effects are similar, although the intuition changes slightly.} The contract is binding on both sides.

Individuals vary in their intrinsic criminal ability, denoted by $\sigma \geq 0$. Ability is exponentially distributed, with parameter $\lambda > 1$. An individual’s ability is not observed by the organisation (there is adverse selection). Their effort cost of criminal activity $a$ is given by $\frac{a}{\sigma}$. The cost is increasing in the level of activity required by the organisation, but more able individuals always suffer less. Given the nature of the contract, those with high ability consequently enjoy a greater surplus from membership of the organisation than those with low ability.

The organisation’s members also face the risk of punishment. With probability $p$ they are caught and suffer punishment $-f$. Following convention, they are still assumed to receive the benefit of their crime - the wage from the organisation - irrespective of whether they are punished \cite{Garoupa1997, Garoupa2000}. For simplicity, both the probability of being caught and the punishment an individual receives is independent of the level of individual activity and size of the organisations. Whilst this is perhaps unrealistic, it is not obvious what alternative assumption would be more appropriate given that all members engage in identical crimes. With limited police resources, an increase in criminal activity reduces the probability that any particular offence will be punished \cite{Sah1991}. However, greater activity can lead to greater resources being made available \cite{Levitt1997}, which may reverse the relationship.

The payoff from accepting the organisation’s contract is thus:

$$g - \frac{a}{\sigma} - pf.$$ \hfill (1)

If, instead, an individual chooses not to become a criminal then they receive a flat wage, $w$. This is equivalent to the expected wage from employment in the formal labour market, and may include the cost of investment in education. The assumption that all individuals have identical formal labour market opportunities reflects the fact that many workers from crime-ridden neighbourhoods perform low-skilled jobs, where variation in wages is small \cite{Levitt2000}.

The organisation recruits $M$ members in order to generate revenue $r(M, a)$.\footnote{The black box nature of revenue (as opposed to production) is purely for notational ease. One can think about it as an indirect revenue function: the one resulting from the optimal allocation of inputs across the wide range of activities the gang engages in. \cite{Kugler2005} consider a more structured approach, decomposing revenue into the number of crimes committed, and the booty collected from.} Revenue
is subject to diminishing marginal returns to either input and, for simplicity, is assumed to have constant returns to scale. Size and activity are revenue complements: the marginal revenue product of activity, $MRP_a$, is not only increasing in $M$, but also diminishes more slowly as $M$ increases. The extent of revenue complementarity proves important in the analysis to follow. One measure of this is the cross elasticity of the marginal revenue product of activity with respect to size:

$$\eta(M, a) = \frac{M}{MRP_a} \frac{\partial MRP_a}{\partial M}.$$  \hspace{1cm} (2)

$\eta$ states the percentage increase in the marginal revenue from activity following a one-percent increase in organisation size ($\eta > 0$). Since marginal revenue is one half of a firm’s profit-maximisation decision, $\eta$ intuitively tells us how the organisation’s incentive to increase activity varies when its size increases. When $\eta$ is large, size and activity are strong revenue complements. An increase in size creates a strong incentive to increase activity, as it will bring in much more revenue.

The organisation chooses its contract to maximise profit, given by its revenue minus its total wage bill:

$$\pi(M, a) = r(M, a) - gM.$$ \hspace{1cm} (3)

Summarising, the timing is as follows. The policy environment is first announced, and becomes common knowledge. The criminal organisation then chooses its contract. Next, individuals choose whether to join the organisation or to work in the formal sector. Crime then takes place and wages are paid. Finally, members of the organisation may be arrested and punished.

### 3 Equilibrium

Solving this simple framework yields a (semi-separating) perfect Bayesian equilibrium. Proceeding by backwards induction, first consider the choice of the neighbourhood’s individuals. Given the contract on offer, an individual will work for the organisation if and only if:

$$g - \frac{a}{\sigma} - pf \geq w \iff \sigma \geq \hat{\sigma}(g, a, w + pf) \equiv \frac{a}{g - (w + pf)}.$$
The payoff from joining the organisation is increasing in ability. All members receive the same wage, engage in the same amount of criminal activity, and face the same expected punishment. However, those with higher ability have a lower cost of committing crime. The formal labour market, on the other hand, does not respect criminal ability. Everyone receives $w$. So there exists a unique marginal individual, with ability $\hat{\sigma}(g,a,w + pf)$ defined above, such that only those with ability exceeding that of the marginal individual join. Whilst the marginal individual is indifferent between either form of employment, all other members of the organisation receive a positive surplus. Individuals with ability below $\hat{\sigma}$ strictly prefer working in the formal labour market. Since crime is always prohibitively costly for those with ability close to zero, $\hat{\sigma}(g,a,w + pf) > 0$. The contract acts as a very simple screening device. The required level of individual activity provides a hurdle which only those with sufficiently high ability are willing to overcome.

Note that all policy tools in this framework, $w$, $p$ and $f$, have the same impact on the ability of the marginal individual. By raising the expected cost of engaging in crime, each policy reduces the surplus that members receive from joining the organisation. Fewer individuals accept any given contract. The ability of the new marginal individual is higher. As we are only interested in the composite effect of policy, we denote by $\phi \equiv w + pf$, the expected cost of engaging in crime. In the following sections, we will ask how changes in $\phi$ affect the organisation.

We now turn to the choice of optimal contract. It proves helpful to rephrase the organisation’s profit maximisation problem slightly. For a given wage, $g$, individual activity, $a$, and cost of engaging in crime, $\phi$, the expected size of the organisation is:

$$M(g,a,\phi) = Ne^{-\lambda \hat{\sigma}(g,a,\phi)}.$$  \hspace{1cm} (4)

Size has a one-to-one relationship with $g$. Rather than choosing the contract, (4) suggests a different approach: the organisation chooses its size, $M$, and individual activity level, $a$. Knowing how individuals respond in equilibrium, it then rearranges (4) to identify the ability it needs to make indifferent:

$$\hat{\sigma}(M) = \frac{\ln N - \ln M}{\lambda}.$$  \hspace{1cm} (4)

The organisation then computes the wage needed to achieve this indifference, given its
chosen individual activity level and the expected cost of engaging in crime. It needs:

\[
g(M, a, \phi) - \frac{a}{\sigma(M)} - pf = w
\]

\[
\iff g(M, a, \phi) = \phi + \frac{a}{\sigma(M)}.
\]  

(5)

The wage it offers just compensates the marginal individual for both the expected cost of engaging in crime (\(\phi\)) and the effort cost of the activity the organisation requires of them. Infra-marginal members receive a positive surplus from joining, given by:

\[
g(M, a, \phi) - \frac{a}{\sigma} - pf - w = \frac{a}{\sigma(M)} - \frac{a}{\sigma} \geq 0.
\]

Members with higher ability always receive a greater surplus from the contract on offer than members with lower ability.

The organisation’s profit maximisation problem can thus be rewritten as:

\[
\max_{M,a} \{r(M, a) - g(M, a, \phi)M\},
\]

(6)

where \(g(M, a, \phi)\) is defined by (5). Any unconstrained solution must satisfy the following:

**Proposition 1 (First-order Conditions)** Suppose that \(\eta > \frac{1}{2}\). Then the profit maximisation problem (6) has a solution, \((M^*, a^*) > 0\), given by:

\[
MRP_M(M^*, a^*) = g(M^*, a^*, \phi) + \frac{a^*}{\lambda \sigma(M^*)^2},
\]

(7)

\[
MRP_a(M^*, a^*) = \frac{M^*}{\sigma(M^*)}.
\]

(8)

**Proof.** See Appendix A.

The assumption that \(\eta > \frac{1}{2}\) is sufficient to ensure that all solutions to this system locally maximise profit. Equation (7) describes the first-order condition for size. Given individual activity, \(a^*\), new members increase the organisation’s revenue by \(MRP_M(M^*, a^*)\). However, they must be paid \(g(M^*, a^*, \phi)\). Moreover, attracting new members involves recruiting those with lower ability than the current marginal individual. All individuals with higher ability already receive a positive surplus from membership, and hence have already chosen to join. In order to compensate for the new recruits’ higher cost of activity, the organisation must increase its wage (in (5) the ability of the new marginal individual has lower ability). This involves offering higher compensation to the infra-
marginal individuals too. The marginal cost of members exceeds \( g(M^*, a^*, \phi) \).

Equation (8) gives the first-order condition for activity. Increasing the level of individual activity enables the organisation to generate more revenue. Each member commits more crime, and total revenue increases by \( M R P_a(M^*, a^*) \). However, in order to ensure that no member chooses to switch towards the formal labour market, the organisation must compensate them for the higher effort cost that they incur. The member requiring the greatest payment is the marginal individual. From (5) the organisation must raise its wage by \( \frac{M^*}{\hat{\sigma}(M^*)} \). However, all \( M^* \) members receive this pay rise. The marginal cost of activity is thus \( \frac{M^*}{\sigma(M^*)} \). Note that (8) is independent of \( \phi \). Whilst increasing the expected cost of engaging in crime affects an individual’s decision to join the organisation, it does not impact upon their effort cost of criminal activity. In particular, controlling for organisation size, the ability, and hence the effort cost, of the marginal individual is unaffected. Increasing activity requires the same increase in wages and results in the same overall increase in the organisation’s wage bill.

The profit-maximising level of individual activity and organisation size can be described as the point of intersection between two restricted demand curves, \( \tilde{M}(a) \) and \( \tilde{a}(M) \). Each curve gives the optimal choice of one input, for any given quantity of the other. They are implicitly defined directly from the first-order conditions, as follows:

\[
M R P_M \left[ \tilde{M}(a), a \right] = g \left[ \tilde{M}(a), a, \phi \right] + \frac{a}{\lambda \tilde{\sigma} \left[ \tilde{M}(a) \right]^2}, \tag{9}
\]

\[
M R P_a \left[ M, \tilde{a}(M) \right] = \frac{M}{\hat{\sigma}(M)}. \tag{10}
\]

For each \( a \), the solution to equation (9) states the organisation’s profit-maximising size. Equation (10) has a similar interpretation for activity. Profits are maximised when both equations are satisfied, as size maximises profit given individual activity and activity maximises profit given the organisation’s size. The restricted demand curves provide a very intuitive way to assess the endogenous effects of changes in the policy environment on the organisation’s optimal choice of inputs. Also, by substituting \( \tilde{a}(M) \) for \( a \) in (9), we can express the organisation’s profit-maximisation problem in terms of a single input, \( M \). Understanding the shape of these curves in more detail is hence our next task.

Consider how an increase in size impacts upon the marginal profitability of activity, given by (8), at the profit-maximising combination of inputs:

\[
\frac{\partial}{\partial M} \left( \frac{\partial \pi}{\partial a} \right) = \frac{1}{\hat{\sigma}(M)} \left\{ \eta(M, a) - \left[ 1 + \frac{1}{\lambda \hat{\sigma}(M)} \right] \right\}. \tag{11}
\]
Whether individual activity becomes more profitable depends upon the sign of the term in curly brackets. The first element, \( \eta(M, a) \), states the percentage increase in the marginal revenue product of activity following a one per-cent rise in size. It represents a revenue effect. With more members, a small rise in each member’s individual activity leads to larger growth in aggregate crime and hence in the organisation’s revenue. Size and activity are revenue complements. The second term is the percentage increase in the marginal cost of activity. It represents a cost effect. When membership expands, the new marginal individual has a lower ability. They require a greater increase in wages to compensate them for higher individual activity. All infra-marginal members receive the raise too, exacerbating the problem. The marginal cost of individual activity also increases in size.

Activity only becomes more profitable following an increase in size if the revenue effect dominates the cost effect. As alluded to in the introduction, this has important implications for how the organisation responds to a change in the policy environment. Fortunately, we can easily distinguish between the two cases:

**Proposition 2 (Complements vs. Substitutes)**  
There exists a unique \( M \geq 0 \) such that the revenue effect dominates the cost effect if and only if \( M < \bar{M} \).

**Proof.** See Appendix B.

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Figure 1 illustrates. From (11), the cost effect only depends upon the size of the organisation. As size increases, the ability of the marginal individual declines. The cost effect is increasing in size. In contrast, when we incorporate endogenous changes in

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\(^4\) These effects are, of course, entirely symmetric. (11) also describes \( \frac{\partial \bar{a}}{\partial a} \).
activity using (10), the revenue effect is declining. The figure has two regions. Small organisations only recruit very high ability individuals. Any increase in activity only requires a small rise in wages. The cost effect is small, and is dominated by the revenue effect. Size and activity are profit **complements**. Larger organisations, on the other hand, are forced to recruit lower ability individuals. Any increase in activity necessitates a much larger rise in wages in order to maintain the indifference of the marginal individual. The cost effect is very high. Moreover, the revenue effect is small. If size is large enough, it is dominated by the cost effect. In this region, size and activity are profit **substitutes**. Where the two curves intersect, the revenue and cost effects exactly cancel each other out. An increase in activity has no effect on the optimal size of the organisation and vice-versa.

The restricted demand functions clearly behave differently depending upon whether size and activity are complements or substitutes. Their respective slopes are:

\[
\frac{\partial \tilde{M}}{\partial a} = -\frac{\partial^2 \pi}{\partial M \partial a},
\]
\[
\frac{\partial \tilde{a}}{\partial M} = -\frac{\partial^2 \pi}{\partial M \partial a}.
\]

If size and activity are complements (that is to say that \(\frac{\partial^2 \pi}{\partial M \partial a} > 0\)), an increase in activity causes an endogenous increase in size. As each member is more active, additional members will bring in more revenue. The marginal revenue product of size is higher (the revenue effect). However, greater activity requires greater compensation for the marginal individual. Increasing size worsens the problem by attracting individuals who are more sensitive to crime. The marginal cost of size increases too (the cost effect). Since size and activity are complements, the revenue effect dominates, and size optimally increases. The restricted demand for size, \(\tilde{M}(a)\) is upward-sloping.

The effects are symmetric. An increase in size causes an endogenous increase in activity. As size grows, the organisation is better placed to generate revenue through activity. The marginal revenue product of activity is higher (the revenue effect). Concurrently, it is forced to recruit less able members, which increases its wage bill (the cost effect). Since size and effort are complements, the revenue effect dominates, increasing the marginal profitability of activity. The organisation asks its members to work harder. The restricted demand for activity, \(\tilde{a}(M)\) is also upward-sloping.

If, on the other hand, size and activity are substitutes, the cost effect dominates the revenue effect. An increase in activity reduces the marginal profitability of size. \(\tilde{M}(a)\)
is downward-sloping. Similarly, an increase in size reduces the marginal profitability of activity. $\tilde{a}(M)$ is also downward-sloping.

Finally, if $M = \overline{M}$, then size and activity do not affect one another. Both curves are stationary.

Figure 2: Profit-Maximising Size and Activity Level.

Figure 2 displays the restricted demand functions. The diagram can be split into two sections. Considering the region to the left of $\overline{M}$ in isolation, size and activity are complements. The revenue effect dominates. Both restricted demand curves are upward sloping. As the organisation’s size increases, the inputs’ complementarity weakens ($\frac{\partial^2 \pi}{\partial M \partial a}$ approaches zero). From Figure 1, the difference between the revenue and cost effects declines. An increase in size causes a smaller endogenous increase in activity ($\tilde{a}(M)$ becomes less steep). An increase in activity causes a smaller endogenous increase in size ($\tilde{M}(a)$ becomes steeper). When $M = \overline{M}$, $\frac{\partial^2 \pi}{\partial M \partial a} = 0$, so the restricted demand for activity achieves a maximum, whereas the the restricted demand for size asymptotes towards infinity.

In the complements region, profit is maximised at point C. At C, size maximises profit given the level of individual activity (we are on the $\tilde{M}(a)$-curve) and activity maximises profit given size (we are also on the $\tilde{a}(M)$-curve). Since the restricted demand for activity becomes less steep as size increases, whereas the restricted demand for size becomes steeper, the curves can only intersect once. C is unique.

Turning attention to the region to the right of $\overline{M}$, size and activity are substitutes. The cost effect dominates, and the restricted demand curves are downward sloping. As the size of the organisation increases, the difference between the cost and revenue effects

\footnote{The uniqueness is guaranteed by the assumption that $\eta > \frac{1}{2}$. From the second-order conditions for profit maximisation, at any point of intersection between the restricted demand functions we have that,}
gets larger (see Figure 1). The inputs become stronger substitutes. An increase in size causes a larger endogenous decline in individual activity ($\tilde{a}(M)$ becomes steeper), and vice versa ($\tilde{M}(a)$ becomes less steep).

In the substitutes region, there is also a profit-maximising point, $S$, where the restricted demand curves intersect. Once again, $S$ is unique, due to how changing size affects the slopes of both curves.

Considering the whole range of inputs on offer to the organisation, we have two candidates for the profit-maximising combination of inputs, $C$ and $S$. Fortunately, one offers strictly higher profits than the other:

**Proposition 3 (Profit Maximisation)** Suppose that the profit-maximising size of the organisation at $S$ is strictly greater than $\bar{M}$. Then $S$ is the unique profit-maximising combination of inputs for the organisation.

**Proof.** See Appendix C. ■

At either $C$ or $S$, profit is positive. Making use of Euler’s Theorem, profit at any point where the restricted demands for both inputs intercept can be calculated as:

$$\pi(M^*, a^*) = \frac{M^* a^*}{\sigma(M^*)} \left[ 1 + \frac{1}{\lambda \sigma(M^*)} \right]$$

Using (10) to substitute for $a^*$, this can be shown to be increasing in the size of the organisation. Since size at $S$ is larger than at $C$, and the restricted demands intercept at both points, profit must be higher at $S$. Left to its own devices, the profit-maximising

if $\frac{\partial^2 \pi}{\partial M \partial a} > 0$:

$$\frac{\partial^2 \pi}{\partial M^2} \frac{\partial^2 \pi}{\partial a^2} \left( \frac{\partial^2 \pi}{\partial M \partial a} \right)^2 > 0,$$

$$\iff \left| \frac{\partial^2 \pi}{\partial M^2} \right| > \frac{\partial^2 \pi}{\partial M \partial a},$$

$$\iff \left| \frac{\partial^2 \pi}{\partial M \partial a} \right| < \frac{\partial^2 \pi}{\partial a^2},$$

$$\iff \frac{1}{\partial M} \frac{\partial a}{\partial M} > 0.$$  

Noting that the slope of the $\tilde{M}(a)$ curve in Figure 2 is $\frac{1}{\partial M}$, the restricted demand curve for size is steeper than the restricted demand curve for activity whenever they intersect. They can therefore only intersect once.

$^6$The assumption that $\eta > \frac{1}{2}$ is sufficient for uniqueness. The proof is identical to the case with complements, except that $\frac{\partial^2 \pi}{\partial M \partial a} < 0$. Thus:

$$\frac{1}{\partial M} \frac{\partial a}{\partial M} < 0.$$
criminal organisation will operate in a region where size and individual activity are substitutes.

4 The Storm

We now begin to consider the impact of a steady increase in either the wage that individuals would earn in the formal labour market, \( w \), or the expected punishment from crime, \( pf \). The organisation is operating at the point where the restricted demand curves intersect. The effect of increasing the expected cost of engaging in crime can be seen by asking what happens to these restricted demands.

Consider first the effect of an increase in \( \phi = w + pf \) on the profitability of size, holding individual activity constant. From (9):

\[
\frac{\partial \tilde{M}}{\partial \phi} = \frac{1}{\frac{\partial^2 \pi}{\partial M^2}} < 0.
\]

When \( \phi \) increases, the surplus each member enjoys from the organisation declines. The marginal individual, who was indifferent between employment in either sector, now prefers the formal labour market. If the organisation wishes to maintain its size, it must increase the wage it offers in order to restore this indifference. This increases the marginal cost of size. New recruits must be paid more. \( MRP_M(M^*, a^*) \), on the other hand, is unaffected. The increase in \( \phi \) makes size less profitable for any level of individual activity. The restricted demand for size shifts left.

Turning to the profitability of individual activity for any given size, from (10), we have:

\[
\frac{\partial \tilde{a}}{\partial \phi} = 0.
\]

The increase in \( \phi \) has no impact upon the marginal cost of activity. For a given organisation size, the marginal individual has the same ability, and hence faces the same effort cost. Varying activity whilst maintaining this individual’s indifference thus necessitates the same change in wages. Similarly, \( MRP_a(M^*, a^*) \) has not changed. For any given organisation size, the level of individual activity maximises profit is unchanged. The restricted demand for size does not shift.

From Proposition 3, the organisation initially operates in the substitutes region. Figure 3 shows the impact of the policy change: the \( \tilde{M}(a) \) curve shifts left. Whilst it is clear that the policy is effective at reducing membership, it has had an unintended consequence.
Figure 3: Activity Increases with Substitutes.

For every level of activity, the organisation’s optimal size has fallen. Due to revenue complementarity between inputs, the marginal revenue product of activity declines. In (10), this provides the organisation with an incentive to reduce activity. However, as it now recruits fewer members, the ability of the new marginal individual is higher. The remaining recruits require less compensation for greater individual activity. The marginal cost of activity has also declined. Since size and activity are substitutes, the cost effect dominates, and the organisation chooses to increase its activity.

The rise in individual activity generates further endogenous effects. The surplus each member receives from being part of the organisation declines. For the marginal individual, this is sufficient to cause them to prefer the formal labour market. In order to maintain its size, the organisation would have to increase the wage it offers. The marginal cost of size has gone up. At the same time, greater individual activity means that new members generate more revenue for the organisation. However, in the substitutes region, this increase in $MRP_M$ is not sufficient to maintain the profitability of size. The organisation reduces its membership further.

The secondary reduction in membership leads to further endogenous increases in activity, which further impacts upon membership. Eventually, in Figure 3, the profit-maximising combination of inputs moves from $S_0$ to $S_1$. Whilst the organisation has fewer members, each member is more prolific. This is the storm.

**Proposition 4 (The Storm)** An increase in the expected cost of engaging in crime, $\phi$, initially reduces size, but increases individual activity.

**Proof.** See Appendix D.
How bad can things get? On the one hand, the organisation is smaller. This reduces aggregate crime. On the other, each member is more active, increasing it. If activity increases enough, the aggregate amount of crime could increase.

**Corollary 1 (Aggregate Crime)** There exists $\hat{M} > \bar{M}$ such that, if $M > \hat{M}$ either an increase in the expected cost of engaging in crime will raise aggregate crime, $M^* a^*$.

**Proof.** See Appendix E

For large organisation, the cost effect is huge (it approaches infinity as $M$ approaches $N$, see Figure 1). The marginal individual has very low ability. Small increases in activity require a very large increase in compensation, both due to the large number of members and the sensitivity of marginal individual. When policy reduces the size of the organisation, the marginal cost of activity declines very rapidly relative to the marginal revenue product. The organisation optimally increases individual activity more than proportionally. In effect, the restricted demand for activity is elastic with respect to size. This is consistent with the higher estimates of the elasticity of hours with respect to workers reported by Freeman 2000. As size declines, total criminal activity increases.

As the policy reduces size further, the revenue effect becomes important (in Figure 1 the gap between the revenue and cost effects decline). Whilst individual activity continues to increase, it becomes less responsive. Eventually, the restricted demand for activity becomes inelastic. At this point, further improvements in the formal labour market or increases in expected punishment lower the total amount of criminal activity the organisation commits (consistent with the lower estimates in Freeman 2000). Nevertheless, individual criminal activity increases. This could represent a greater number of the same type of crime being committed, or a movement towards more serious crime.

As the policy environment continues to increase the cost of engaging in crime, the marginal cost of size grows. Size declines and activity increases. Both of these changes raise the marginal revenue product of size. However, this cannot continue indefinitely. In the substitutes region, size is bounded below by $\bar{M}$ and effort is bounded above by $\tilde{a} (\bar{M})$. So:

$$M R P_M (M^*, a^*) \leq M R P_M [\bar{M}, \tilde{a} (\bar{M})].$$

The marginal cost of size faces no such restriction. In fact, it is bounded below by $\phi$. As $\phi$ increases, the compensation the organisation must provide its members grows indefinitely. Eventually, the marginal cost of size increases above the marginal revenue product. At this point, the organisation reaches a corner solution with $M = \bar{M}$, and the policy appears to become ineffective:
Proposition 5 (Impasse) There exists $\phi^S > 0$ such that, when $\phi$ increases above $\phi^S$, size and individual activity become unresponsive to the changing policy environment.

Proof. See Appendix F.

$\phi^S$ is defined by:

$$M_{RP} [\bar{\lambda} (\bar{M})] = g [\bar{\lambda} (\bar{M}), \phi^S] + \frac{\bar{\alpha} (\bar{M})}{\lambda \sigma (\bar{M})^2},$$

$$\iff \phi^S \equiv M_{RP} [\bar{\lambda} (\bar{M})] - \frac{\bar{\alpha} (\bar{M})}{\sigma (\bar{M})} \left[ 1 + \frac{1}{\lambda \sigma (\bar{M})} \right], \quad (12)$$

the expected cost of engaging in crime at which $\bar{M}$ becomes the optimal organisation size in the substitutes region. Since, when $\phi = \phi^S$, $\bar{M}$ is (unconstrained) optimal size in the substitutes region, by Proposition 3 the profit it generates still strictly exceeds that of any combination of inputs in the complements region. It is therefore the unique profit-maximising organisation size. Profit-maximising individual activity is given by $\hat{a} (\bar{M})$.

As $\phi$ rises above $\phi^S$, although profit declines rapidly, $\bar{M}$ still maximises profits. It is optimal in the substitutes region, and offers greater profits than any combination of inputs in the complements region. Changing the policy environment appears to have run out of steam. Size and activity are unresponsive. Of course, this is not sustainable...

5 The Calm

As the expected cost of engaging in crime continues to increase, the profits the organisation continue to decline. Stuck in a corner solution, the organisation is unwilling to adjust its size and activity, but must nevertheless pay out higher wages to its members. Whilst profits in the complements region also decline in the face of this increasing cost, the greater flexibility afforded by an interior solution curtails the rate at which they fall. The benefits from producing in the substitutes region is reduced. Eventually, the organisation finds it profitable to switch:

Proposition 6 (Calm) There exists $\phi^C > \phi^S$ such that as the expected cost of engaging in crime increases above $\phi^C$, size and activity both decline.

Proof. See Appendix G.

Having moved production into the complements region, the impact of further increases in the $\phi$ are shown in Figure 4.
As before, an increase in $\phi$ exogenously increases the marginal cost of size. Without a corresponding exogenous increase in its marginal revenue product, the organisation recruits fewer members. In Figure 4, the $\tilde{M}$ curve shifts in to $\tilde{M}_3$. This has two effects. Firstly, the marginal revenue product of activity falls (the revenue effect). Secondly, those who are still recruited have relatively high ability. They consequently need little compensation for the criminal activity they engage in. The marginal cost of activity also falls (the cost effect). Since the organisation is operating in the complements region, the revenue effect dominates, and the marginal profitability of activity falls. The decline in size causes an endogenous decrease in activity.

The endogenous effects that were so troublesome in the substitutes region now reinforce the impact of the policy. The decline in activity reduces the marginal revenue product of size. Since each member engages in less crime, new recruits generate smaller increases in aggregate crime, and hence in the organisation’s revenue. However, lower wages are needed to attract low ability individuals to the organisation. In the complements region, the revenue effect dominates, and lower activity makes members less profitable. The organisation further reduces its size.

The secondary fall in size leads to further declines in individual activity. Eventually, the organisation’s profit-maximising input combination moves from $C_2$ to $C_3$, consisting of both fewer members and lower activity. Now, increases in the cost of engaging in crime lead to rapid declines in both the organisation size, and the extent of its criminal activities.
6 Conclusions

Recent years have seen numerous innovative policies put forward to tackle high-crime neighbourhoods. These approaches tend to be based upon a rational offender argument. Increasing the expected punishment a criminal suffers or improving formal labour market conditions increase the expected cost of engaging in crime. The offender weighs up these higher costs against the benefits they enjoy from successfully committing crime. The crime rate falls.

The presence of organised crime may confound this argument, at least initially. Whilst those on the margin do indeed move away from a life of crime, the criminal organisation reacts by adjusting its recruitment policy. Those who still opt for a career in the organisation are hardened criminals. They require relatively little compensation for engaging in criminal acts. With this in mind, the organisation substitutes away from a large, inactive membership towards a small, prolific one. This may help to explain evidence suggesting that policy can backfire in the presence of organised crime.

All is not lost, however. As the size of the organisation continues to fall, the endogenous effects that hampered the policy now reinforce it. With so few recruits, increasing each member’s individual activity does little to increase the organisation’s revenue. Conversely, its costs continue to grow, as members must be compensated for their efforts. This counteracts the incentive to substitute. Eventually, the organisation switches to a strategy whereby increasing the expected cost of engaging in crime reduces both its size and the amount of crime each of its members commits.

The model presented herein is simple. As such, it generates the stark results necessary to illustrate the underlying intuition. In particular, it is assumed that changes in policy do not impact upon the marginal cost of individual activity. In many cases, this is accurate. Increases in a minimum wage or the number of vacancies in a neighbourhood simply raise the opportunity cost of engaging in crime. They do not affect the marginal cost of one more criminal act within the organisation. Similarly, more police on the streets or tougher sentences across the board (even if different crimes warrant different punishments) will simply raise the expected cost of punishment - they act as a fixed cost associated with each crime. However, if more serious crimes saw their expected punishment rise more sharply than less serious crime, then criminal activity would become more costly at the margin. This would generate an exogenous decline in individual criminal activity. Such policies may help mitigate the storm. Stretching the metaphor a bit far, perhaps, they are all-weather policies.
Appendices

Throughout the appendices, I will use subscript to denote derivative. For example, $r_M = \frac{\partial r}{\partial M}$ and $r_{Ma} = \frac{\partial^2 r}{\partial M \partial a}$.

A Proof of Proposition [1]

Proof. The first-order conditions for the profit maximisation problem are:

$$
\pi_M = r_M(M^*, a^*) - g(M^*, a^*, w) - \frac{a^*}{\lambda \hat{\sigma}(M^*)^2} \equiv 0,
$$
$$
\pi_a = r_a(M^*, a^*) - \frac{M^*}{\hat{\sigma}(M^*)} \equiv 0.
$$

The associated second derivatives are:

$$
\pi_{MM} = r_{MM}(M^*, a^*) - \frac{a^*}{\lambda M^* \hat{\sigma}(M^*)^2} - \frac{2a^*}{\lambda^2 M^* \hat{\sigma}(M^*)^3},
$$
$$
\pi_{Ma} = r_{Ma}(M^*, a^*) - \frac{1}{\hat{\sigma}(M^*)} - \frac{1}{\lambda \hat{\sigma}(M^*)^2},
$$
$$
\pi_{aa} = r_{aa}(M^*, a^*).
$$

Diminishing marginal returns guarantee that $\pi_{MM} < 0$ and $\pi_{aa} < 0$. For $(M^*, a^*)$ to constitute a maximum, we therefore require that:

$$
\pi_{MM} \pi_{aa} - \pi_{Ma}^2 > 0.
$$

Using the fact that $r(M, a)$ has constant returns to scale and Euler’s theorem, this condition is satisfied if and only if:

$$
\eta(M^*, a^*) > \frac{\lambda^2 \hat{\sigma}(M^*)^2 - 2\lambda \hat{\sigma}(M^*) + 1}{2\lambda^2 \hat{\sigma}(M^*)^2 + \lambda(2 + a^*) \hat{\sigma}(M^*) + 2\lambda}.
$$

A sufficient condition is that $\eta(M, a) > \frac{1}{2}$.

Finally, it is necessary to show that the organisation does not wish to shut down.
Making use of constant returns to scale:

\[
\pi^* = M^*r_M(M^*, a^*) + a^*r_a(M^*, a^*) - g(M^*, a^*, \phi)M^*
\]

\[
= M^* \left[ g(M^*, a^*, \phi) + \frac{a^*}{\lambda \hat{\sigma}(M^*)} \right] + a^*r_a(M^*, a^*) - g(M^*, a^*, \phi)M^*
\]

\[
= \frac{a^*M^*}{\lambda \hat{\sigma}(M^*)^2} + a^*r_a(M^*, a^*) > 0,
\]

where the second line comes from substituting \( r_M(M^*, a^*) \) for \( r_M(M^*, a^*) \). The organisation makes positive profits. This completes the proof.

**B Proof of Proposition 2**

**Proof.** Firstly, consider the first-order condition for \( a \) given by (8). For each \( M \), this gives the \( a \) that maximises profit. Let us denote this restricted demand for \( a \) by \( \tilde{a}(M) \), defined implicitly by:

\[
\tilde{a}(M) = \frac{M}{\hat{\sigma}(M)} \equiv 0.
\]

We can now consider whether size and effort are complements in equilibrium purely as a function of \( M \). Consider (11). For any \( M \) size and effort are complements if and only if:

\[
\eta[M, \tilde{a}(M)] \geq 1 + \frac{1}{\lambda \tilde{\sigma}(M)}.
\]

(13)

As \( M \) increases, \( \tilde{\sigma}(M) \) falls, and so the right hand side increases. Moreover, \( 1 + \frac{1}{\lambda \tilde{\sigma}(M)} \to \infty \) as \( M \to N \).

Taking derivative of the left hand side with respect to \( M \):

\[
\frac{d\eta[M, \tilde{a}(M)]}{dM} = \frac{\partial \eta}{\partial M} + \frac{\partial \eta}{\partial a} \frac{d\tilde{a}}{dM}
\]

\[
= (r_{Ma} + M r_{Ma}) \frac{r_a}{r_a} - M r_{Ma}^2 r_a - M r_{Ma} r_{aa} \frac{1}{\sigma} + \frac{1}{\lambda \tilde{\sigma}^2} \frac{1}{r_{aa}},
\]

where the simplification makes use of Euler’s Theorem once again. If \( M r_{Ma} r_{aa} - M r_{Ma} r_{aa} > 0 \) then \( \eta \) declines as \( M \) increases. Since \( r_{Ma} > 0 \) by assumption, the left hand side of (13) is declining in \( M \). So there exists a \( M \geq \tilde{M} \) such that, when \( M \geq \tilde{M} \) the cost effect comes to dominate the revenue effect. This completes the proof.
C Proof of Proposition 3

Proof. From the proof of Proposition 1, the profit generated by at any point of intersection between the restricted demand curves is:

\[ \pi^*(M, a) = \frac{aM}{\lambda \sigma(M)^2} + ar_a(M, a) \]
\[ = \frac{aM}{\sigma(M)} \left[ 1 + \frac{1}{\lambda \sigma(M)} \right], \]

where the second line comes from substituting for \( r_a(M, a) \) using (8). We can assess what happens to profit as we move along the \( \tilde{a}(M) \) curve, increasing \( M \):

\[ \pi^*[M, \tilde{a}(M)] = \tilde{a}(M)M \left[ 1 + \frac{1}{\lambda \sigma(M)} \right]. \]

So:

\[ \frac{d\pi^*[M, \tilde{a}(M)]}{dM} = \frac{\partial \pi^*}{\partial M} + \frac{\partial \pi^*}{\partial a} d\tilde{a} \]
\[ = \frac{\tilde{a}}{\sigma} \left( 1 + \frac{1}{\lambda \sigma} \right) + \frac{\tilde{a}}{\lambda \sigma^2} \left( 1 + \frac{2}{\lambda \sigma} \right) + \frac{M}{\sigma} \left( 1 + \frac{1}{\lambda \sigma} \right) \frac{r_Ma - \frac{1}{\sigma} - \frac{1}{\lambda \sigma^2}}{ra_a} \]
\[ = \frac{\tilde{a}}{\lambda \sigma^2} \left( 1 + \frac{2}{\lambda \sigma} \right) - \frac{M}{ra_a \sigma^2} \left( 1 + \frac{1}{\lambda \sigma} \right)^2 > 0. \]

Now, it must be the case that the organisation size is greater at \( S \) than at \( C \). So the organisation makes more profit at \( S \) than \( C \). This completes the proof.

D Proof of Proposition 4

Proof. Applying the Implicit Function Theorem to (7) and (8), we have that:

\[ M_\phi^* = \frac{\pi_{aa}}{\pi_{MM} \pi_{aa} - \pi_{Ma}^2} < 0, \]
\[ a_\phi^* = -\frac{\pi_{Ma}}{\pi_{MM} \pi_{aa} - \pi_{Ma}^2}. \]

Size unambiguously falls. Since size and activity are substitutes, \( \pi_{Ma} < 0 \). So \( a_\phi^* > 0 \). This completes the proof.
E  Proof of Corollary 1

Proof. Aggregate criminal activity is given by $M^*a^*$. Differentiating with respect to $\phi$:

$$\frac{\partial M^*a^*}{\partial \phi} = M^*_\phi a^* + a^*_\phi M^* = \frac{\pi_{aa} a^* - \pi_{Ma} M^*}{\pi_{MM} \pi_{aa} - \pi_{Ma}^2}.$$ 

So $\frac{\partial M^*a^*}{\partial \phi} > 0$ if and only if:

$$\pi_{Ma} M^* - \pi_{aa} a^* < 0 \iff M^* r_M(M^*, a^*) - \frac{M^*}{\sigma(M^*)} \left[ 1 + \frac{1}{\lambda \hat{\sigma}(M^*)} \right] - a^* r_{aa}(M^*, a^*) < 0.$$ 

Making using of Euler’s Theorem (noting that $r_a(M^*, a^*)$ is HD0), we require:

$$\eta(M, a) - \frac{1}{2} \left[ 1 + \frac{1}{\lambda \hat{\sigma}(M)} \right] < 0.$$ 

The first term is the revenue effect. The second term is half the cost effect. Since the revenue effect declines in $M$ and the cost effect increases towards infinity as $M$ approaches $N$, for large enough $M$ this must be negative. Moreover, if $M = \hat{M}$ the revenue and cost effect are equal, so the term above is still positive. There therefore exists a unique $\hat{M} \in (\hat{M}, N)$ such that for $M > \hat{M}$ aggregate crime increases. This completes the proof. ■

F  Proof of Proposition 5

Proof. In the substitutes region:

$$r_M(M^*, a^*) = r_{MM} M^*_\phi + r_{Ma} a^*_\phi > 0.$$ 

From the previous proof $M^*_\phi < 0$ and $a^*_\phi > 0$. As $\phi$ increases, the marginal revenue product of size increases. Moreover, conditional on $M \geq \hat{M}$:

$$r_M(M^*, a^*) \leq r_M \left[ \hat{M}, \hat{\sigma}(M) \right].$$
So, if:
\[
\phi \geq \phi^S \equiv r_M \left[ M, \hat{a} (M) \right] - \frac{\hat{a} (M)}{\hat{\sigma} (M)} \left[ 1 + \frac{1}{\lambda \hat{\sigma} (M)} \right],
\]
then \( M^* = \bar{M} \) and \( a^* = \hat{a} (\bar{M}) \). The organisation has reached a corner solution. When \( \phi = \phi^S \), \( \bar{M} \) is the unconstrained profit-maximising organisation size. By Proposition 3, the organisation makes strictly more profit than it would it were to switch into the complements region. As the wage continues to increase, the organisation thus prefers to remain at \((\bar{M}, \hat{a} (\bar{M}))\). This completes the proof. ■

G Proof of Proposition 6

**Proof.** The proof is presented in two stages. We first compare the profit from the optimal input combination under complements to that when size is \( \bar{M} \), summarising the result in the following lemma:

**Lemma 1** There exists \( \phi^C \) such that, for all \( \phi \geq \phi^C \) the organisation makes more profit using an input combination in the complements region than it does using an input combination in the substitutes region.

**Proof.** The proof consists of two steps. Firstly, I show that, for high enough \( \phi \), the profit from optimal input combination under complements exceeds that when size is \( \bar{M} \). Secondly, I show that, as \( \phi \) increases, profit when size is \( \bar{M} \) decline more rapidly than when size and activity are complements.

The profit when size is \( \bar{M} \) is given by:
\[
\pi [\bar{M}, \hat{a} (\bar{M})] = r [\bar{M}, \hat{a} (\bar{M})] - \bar{M} \phi - \frac{\bar{M} \hat{a} (\bar{M})}{\hat{\sigma} (\bar{M})}.
\]

So, for:
\[
\phi > r \frac{[M, \hat{a} (M)]}{M} - \frac{\hat{a} (M)}{\hat{\sigma} (M)},
\]
we have that \( \pi [\bar{M}, \hat{a} (\bar{M})] < 0 \). Now, with complements, we always have an interior solution. So:
\[
\pi^*(M, a) = \frac{a M}{\hat{\sigma}(M)} \left[ 1 + \frac{1}{\lambda \hat{\sigma}(M)} \right] \geq 0.
\]
For high enough $\phi$ the profit under complements exceeds that under substitutes. Moreover:

\[
\frac{d\pi[M, \hat{a}(M)]}{d\phi} = -M, \\
\frac{d\pi(M, \hat{a}(M))}{d\phi} \bigg|_{M<\bar{M}} = -M.
\]

Since, under complements, $M < \bar{M}$ we have that $-\frac{d\pi[M, \hat{a}(M)]}{d\phi} > -\frac{d\pi}{d\phi}|_{M<\bar{M}}$. And $\pi[M, \hat{a}(M)]|_{M<\bar{M}}$ can only intersect once as $\phi$ increases. Call the expected cost of engaging in crime which equates the two profit functions $\phi^C$. For all $\phi$ above $\phi^C$, $\pi[M, \hat{a}(M)]|_{M<\bar{M}} > \pi[M, \hat{a}(M)]$. ■

Following on from the proof of Proposition 4, size unambiguously falls. However, in the complements region, $\pi_M > 0$, and so $a^*_\phi < 0$. Activity also declines. This completes the proof. ■

References


