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Theoretical Perspectives on Localised Knowledge Spillovers and Agglomeration

Samuli Leppälä

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Cardiff Business School
Aberconway Building
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
business.cardiff.ac.uk

Theoretical Perspectives on Localised Knowledge Spillovers and Agglomeration

Samuli Leppälä
Cardiff University

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Abstract

There is substantial empirical evidence that innovation is geographically concentrated. Unlike what is generally assumed, however, it is not clear that localised knowledge spillovers provide a theoretically valid explanation for this. Studying spillovers of cost-reducing technology between Cournot oligopolists we show that 1) localised knowledge spillovers of any level do encourage agglomeration, but 2) whether this leads to higher levels of effective R&D depends on the type and level of knowledge spillovers, the number of firms, and the industry's R&D efficiency.

JEL classification: O33, R32, L13.

Keywords: knowledge spillovers, agglomeration economies, innovation, location.

1 Introduction

Following the seminal work by Glaeser et al. (1992), there has been an extensive number of empirical studies on localised knowledge spillovers.[1] As a result, there is a largely unanimous view that knowledge spillovers are important for innovation and growth and strongly bounded in space (e.g. Döring and Schnellenbach, 2006). Some issues, however, are still unclear such as the role of industrial structure, as empirical support has been found for both inter- and intra-industrial knowledge spillovers (Beaudry and Schiffauerova, 2009). Furthermore, the literature has been criticised of the lack of a firm theoretical background as well as direct evidence of spillovers (Breschi and Lissoni, 2001), which are problematic because there are various potential explanations for agglomeration economies (see Rosenthal and Strange, 2004). Certain stylised facts have emerged, nevertheless. Most importantly, there is definitive evidence for that innovation is geographically concentrated (Asheim and Gertler, 2005). The explanation given

[1] One survey (Beaudry and Schiffauerova, 2009) and another meta-analysis (De Groot et al., 2009), alone, scrutinise 67 and 31 of them, respectively.

to this is that firms agglomerate in order to benefit from knowledge spillovers that are spatially limited and this leads to higher levels of innovation.

In contrast to empirical research, there has not been much theoretical research on localised knowledge spillovers, notwithstanding the vast industrial organisation literature on knowledge spillovers since d'Aspremont and Jacquemin (1988). Moreover, it is not clear on theoretical basis that firms would choose to locate in close proximity in order to maximise spillovers and that agglomeration would lead to higher R&D levels and growth in output without explicit cooperation in R&D. As noted by several authors, among the micro-foundations of urban agglomeration, learning and knowledge spillovers are the least understood and advancing theoretical research on localised knowledge spillovers, which informs empirical research rather than lags behind it, is of urgent necessity (Duranton and Puga, 2004; Fujita and Krugman, 2004; Puga, 2010).

Only a few papers have considered endogenous, location-dependent spillovers in the context of Bertrand or Cournot competition (e.g. Van Long and Soubeyran, 1998; Baranes and Tropeano, 2003; Piga and Poyago-Theotoky, 2005; Hussler et al., 2007). Summarily, they give no uniform theoretical support for urban agglomeration or higher R&D levels as the consequences of localised knowledge spillovers. As such, this paper attempts to disclose the theoretical circumstances that would support the empirical findings. Hence, it is important to go back to the theoretical models to see whether the explanation for geographically concentrated innovation is tenable or needs to be reconsidered. This paper proceeds by reviewing the existing theoretical literature on knowledge spillovers and location choice. After having identified the clues that the existing literature can give us and what remains to be studied, we develop a theoretical model that reveals 1) whether we can expect localised spillovers to promote to agglomeration, and 2) under what conditions does agglomeration lead to a higher effective R&D output. In our model we will focus on output spillovers of cost-saving technology between non-cooperative firms in a Cournot oligopoly. Following the empirical literature, we assume that the extent of spillovers depends on the spatial proximity between the firms. University-industry spillovers or the specific spillover mechanisms are not considered.

The focus of previous theoretical research has been different and there is no clear answer regarding the relationship between localised knowledge spillovers and agglomeration. Furthermore, the differences between the models require careful interpretation. Our model shows that agglomeration is always an equilibrium for any $n \geq 3$ firms, irrespective of the level of agglomeration spillovers. However, agglomeration does not lead to higher effective R&D if the spillovers are high. The number of firms and industry's R&D efficiency affects this threshold though, which suggest that different industries benefit more from agglomeration than others. In addition to the level of agglomeration spillovers, concentration, and R&D efficiency, further empirical research could pay more attention to the type of spillovers and the measure of R&D.

The structure of the paper is as follows. Section 2 will review the basic models of knowledge spillovers and their extensions in the context of location choice. Building up from these insights, Section 3 presents a Cournot oligopoly model that gives an applicable framework for considering the explanation given to the empirical evidence. Finally, Section 4 summarises the results and provides suggestions for subsequent research.

2 Literature Review

The effect of knowledge spillovers on firms' R&D incentives has been addressed most extensively in the industrial organisation literature studying Cournot oligopoly models. The two seminal papers in this respect are d'Aspremont and Jacquemin (1988) and Kamien et al. (1992). The central way how spillovers are modelled in both papers and subsequent research is illustrated by the equation for a firm's effective R&D:

$$X_i = x_i + \beta \sum_{j \neq i} x_j,$$

where $\beta \in [0, 1]$ is the spillover rate between the firms. The central difference is that d'Aspremont and Jacquemin (1988) study spillovers in R&D outputs whereas the spillovers modelled by Kamien et al. (1992) concern R&D inputs. This means that x_i in d'Aspremont and Jacquemin (1988) is firm i's own R&D output, which together with the spillovers from other firms creates its effective R&D output, X_i. In Kamien et al. (1992) these are investments in R&D, self-financed and effective, respectively. In both cases, $\beta = 1$ would imply complete spillovers and $\beta = 0$ a situation where there are none. The R&D output is typically considered to be a cost reduction, but for quality-enhancing R&D the logic is similar.

Despite the similarities, the outcomes of these models differ in some relevant respects and in the literature there has been a discussion regarding their relative merits (see Amir, 2000; Amir et al., 2008). For example, Amir (2000) considers the additive spillovers of the output spillover model to be less realistic in general, but notes that these might be appropriate especially when modelling agglomeration economies, which presume such additive benefits. One way to perceive the difference between the two is whether the firms are developing complementary technologies or jointly refining the same one[2]. In both models, the firm's own level of R&D, x_i, whether input or output, is decreasing in β in the equilibrium. However, the effective R&D, X_i, is also decreasing in β if the spillovers are in inputs, but reaches the maximum when $\beta = 0.5$ in the case of output spillovers. De Bondt et al. (1992) later demonstrated that the latter result holds for any number of firms n. This result also relates to criticism by Breschi and Lissoni (2001). While they are likely right that the concept of tacit knowledge has been used ambiguously, spillovers and tacitness as used in the literature might not be such an odd couple after all. Some degree of imperfection in the spillovers leads to a higher level of innovation.

The above mentioned results have two main implications for empirical research. First, it is should be important whether the used R&D variable measures

[2]Note that by construction both models assume away the risk of duplication.

firm's self-financed or effective R&D. Second, input spillovers are unlikely the explanation for the spatial concentration of innovation as in this case higher spillovers always lead to less R&D. Whether the spillovers are in R&D outputs or inputs may vary between industries and is ultimately an empirical question. However, our purpose is merely to test whether the proposed explanation is logically valid. For this reason, our model follows the approach by d'Aspremont and Jacquemin (1988) as it is the more favourable case of the two. Nevertheless, we expect the level of agglomeration spillovers to play a critical role. Similarly, we adopt a Cournot model, because in a homogeneous good price competition spillovers can only decrease the R&D levels.

There have been numerous subsequent papers on knowledge spillovers (see De Bondt, 1997). Most have concentrated on different forms of R&D cooperation as in the presence of spillovers both the firms and the consumers may benefit from such cooperation between otherwise rival firms. In the case of agglomeration economies, however, the high spillover levels are not due to cooperation but are instead the outcome of non-cooperative location choices[3]. A straightforward way to combine knowledge spillovers with location choice has been to introduce them into the Hotelling (1929) model. There have been a few papers along these lines, and the equilibrium outcomes vary between agglomeration and dispersion depending on the levels of spillovers and transportation costs (Mai and Peng, 1999; Hussler et al., 2007), product differentiation (Piga and Poyago-Theotoky, 2005), and R&D cost and technical risk (Li and Zhang, 2013). To summarise, agglomeration is rarely established as the natural outcome. However, the Hotelling model has two unwanted characteristics considering our purposes. First, a linear city is unlikely the model that represents the empirical findings in general. While some empirical research has been done regarding cluster formations within cities, most point out to differences between peripheries and cities with respect to the level of innovation. Second, in the Hotelling model the location choice also affects the market shares and not only the spillovers levels. While there are certainly centripetal and centrifugal forces other than localised knowledge spillovers (see Fujita and Thisse, 1996; Belleflamme et al., 2000; Fujita and Krugman, 2004), what we want to establish is whether or not it is logically true that in the absence of interfering factors these spillovers provide a reason to agglomerate.

Regarding non-Hotelling approaches, there are two papers studying whether two firms choose to locate in the same region when this would lead to spillovers. In Alsleben (2005), labour poaching, if located in the same region, leads the firms to choose dispersion. A central factor is that knowledge is in this model embodied in the workers. Hence, knowledge is also lost when "spilt over" to the rival. Conversely, in Baranes and Tropeano (2003), choosing the same location creates more competition and makes the firms willing to share their R&D. Knowledge that is fully embodied in human capital or voluntary sharing of R&D, however, are not what is strictly meant by knowledge spillovers as there

[3]Proximity may also facilitate cooperation, but Brenner (2007) notes that formal R&D cooperation has been found rather less important than the other mechanisms of technology diffusion.

are no externalities involved (Breschi and Lissoni, 2001).

The last three papers reviewed here are more closely related to our approach. In these, the firms choose the distance (or the level of technological differentiation) between each other, which then determines the level of spillovers (but not market shares etc.). In Van Long and Soubeyran (1998), three firms choose to agglomerate given any level of R&D investments when the (input) spillover effect is convex in distance. However, this does not tell us how agglomeration affects the R&D levels, which could then also affect the location choices. In Gil Moltó et al. (2005) duopoly model, the R&D levels are endogenous. While the endogenous (output) spillover level depends on technological differentiation in their model, the situation is similar if this is caused by the distance between the firms. They find out that the firms maximise or almost maximise the spillovers depending on the R&D efficiency and the highest attainable level of spillovers. They state that this leads to less own R&D propensity, x_i, but not how the effective R&D, X_i, is affected[4]. Nevertheless, the duopoly model is unable to tell whether there is an agglomeration equilibrium with more than two firms where one firm's decision to deviate does not affect the spillover levels between the other firms. Studying a three-firm oligopoly, Mota and Brandão (2004) take a relevant step in this direction. Making the simplifying assumption that the equilibrium R&D levels are identical, however, does not give us the proper answer as this symmetry cannot be expected to hold if the (output) spillover levels are not identical. Taking this issue into account and extending our model to an n-firm Cournot oligopoly, we are able to study whether agglomeration is an equilibrium and whether it also leads to higher effective R&D.

3 Model

In this section, we consider a three-stage game between $n \geq 3$ Cournot oligopolists that produce a homogeneous output. The inverse demand function is given by $P = a - Q$, where $Q = \sum_{i=1}^{n} q_i$ is the total quantity produced and $a > Q \geq 0$. The unit cost of firm $i, i = 1, ..., n$, is $c_i = c - X_i$, where c is the initial marginal cost, X_i is the effective cost reduction due to R&D, and $a > c > X_i \geq 0$. Hence, we assume that marginal costs are always positive.

A firm's effective R&D output is

$$X_i = x_i + \sum_{j \neq i} \beta(d_{ij}) x_j.$$

As in d'Aspremont and Jacquemin (1988), the cost of the firm's own R&D output x_i is quadratic and given by

$$C(x_i) = \frac{1}{2} \gamma x_i^2,$$

[4]Similarly, the other cited papers have largely concentrated on the issue of agglomeration alone and not on how it would affect effective R&D

where $\gamma > 0$ is an inverse measure of the efficiency of R&D. We do not explicitly consider the exact spillover mechanism, but rather assume as in the empirical literature that the spillovers are decreasing in distance. Hence, the output spillovers from other firms depend on spillover rate $\beta(d_{ij})$, which is a positive and decreasing function of geographical distance d_{ij} between firms i and j $(i \neq j)$, i.e.

$$0 \leq \beta(d_{ij}) \leq \bar{\beta} \leq 1,$$

and $\beta'(d_{ij}) < 0$ and $\beta(0) = \bar{\beta}$. For convenience, we denote it by $\beta_{ij} = \beta(d_{ij})$ and concentrate on changes in β bearing in mind that they result from changes in distance.

Agglomeration spillover rate $\bar{\beta}$ is the upper bound that can be achieved through co-location and it can be limited by other factors, such labour mobility, technological (dis)similarity or intellectual property rights. Likewise, there could be a lower bound to localised knowledge spillovers as well, but that is not our concern as we concentrate on the agglomeration case. We assume that transportation costs and any other costs directly related to the location choice are zero, which allows us to focus on how localised knowledge spillovers affect the location choice. This also implies that the results extend to other cases of endogenous knowledge spillovers (c.f. Gil Moltó et al., 2005).

The timing of the three-stage game is the following:

1. The firms choose their distance from each other and hence the level of spillovers, β_{ij}, between them.
2. The firms choose their own cost reduction levels, x_i.
3. The firms choose their output levels, q_i, through Cournot competition.

In each stage the choices are made simultaneously and discounting between the stages is ignored for expository reasons. We solve the game by backward induction to see whether agglomeration is a Nash equilibrium and if it maximises the firm's effective R&D. As such, we do not consider whether other equilibria exist and need not make any explicit assumptions regarding the location space, except that there is at least one dimension, or the concavity of spillovers in space. Without loss of generality, we assume that all the other firms except i are agglomerated and concentrate on firm i's location choice. That is, if $\beta_{jk} = \bar{\beta}, \forall j, k \in \{n - i\}, j \neq k$, and $\beta_{ij} = \beta_{ik} = \beta$, then whether $\beta = \bar{\beta}$ maximises firm i's effective R&D and profit.

3.1 Production Stage

In the production stage, firm i maximises its profit function given by

$$\pi_i = (a - Q - c_i)q_i.$$

The Cournot equilibrium output is

$$q_i^* = \frac{a - nc_i + \sum_{j \neq i} c_j}{n+1} = \frac{a - c + nX_i - \sum_{j \neq i} X_j}{n+1} \tag{1}$$

for all firms $i \in n$. The total industry output is

$$Q = \frac{n(a-c) + \sum_{i=1}^{n} X_i}{n+1}$$

and the consumer surplus is $CS = \frac{1}{2}Q^2$. As expected, there is a positive effect of R&D on the economic activity and welfare, since $\partial Q/\partial X_i > 0$.

3.2 R&D Investment Stage

In stage 2, the firms choose their R&D levels. Given the subsequent output levels, firm i chooses x_i in order to maximise

$$\pi_i = (q_i^*)^2 - \frac{1}{2}\gamma x_i^2,$$

where q_i^* is given by equation (1). Assuming $\beta_{jk} = \bar{\beta}$ and $\beta_{ij} = \beta_{ik} = \beta$, since we concentrate on the agglomeration equilibrium, the first order condition gives the best-response function

$$x_i(x_j) = \frac{2(a - c + (n\beta - (n-2)\bar{\beta} - 1)\sum_{j \neq i} x_j)(n - (n-1)\beta)}{\gamma(n+1)^2 - 2(n - (n-1)\beta)^2} \tag{2}$$

for firm i. This shows us that the R&D outputs x_j are strategic substitutes to x_i if $n\beta - (n-2)\bar{\beta} - 1 < 0$ and complements if the inequality is reversed.

Similarly, the best response function for the other firms is

$$x_j(x_i, x_k) = \frac{2(a - c + (2\beta - 1)x_i + (3\bar{\beta} - \beta - 1)\sum x_k)(n - \beta - (n-2)\bar{\beta})}{\gamma(n+1)^2 - 2(n - \beta - (n-2)\bar{\beta})^2} \tag{3}$$

$\forall j, k \in \{n-i\}, j \neq k$. Hence, the R&D output x_i is a strategic substitute to x_j if $\beta < 1/2$, and x_j and x_k are strategic substitutes to each other if $3\bar{\beta} - \beta - 1 < 0$.

The second order conditions in the R&D stage require that the numerators in the best response functions are positive. This holds for all $\beta, \bar{\beta} \in [0,1]$ when $\gamma > 2n^2/(n+1)^2$. The stability condition requires that the best response functions cross correctly (Henriques 1990), and this holds $\forall \beta, \bar{\beta} \in [0,1]$ when $\gamma > 2n(2n-1)/(n+1)^2$.

We assume that firms $j \neq i$ make a symmetric choice: x_{-i}. Then using the best response functions (2) and (3) we get the following equilibrium R&D output levels:

$$x_i^* = 2(a-c)(n - (n-1)\beta)\frac{A}{C} \tag{4}$$

and

$$x_{-i}^* = 2(a-c)(n - \beta - (n-2)\bar{\beta})\frac{D}{C} \tag{5}$$

where

$$A = (n+1)\gamma - 2(\bar{\beta}-1)(\beta-\bar{\beta})n^2 + ((8\beta+6)\bar{\beta} - 8\bar{\beta}^2 - 2\beta^2 - 2\beta - 2)n$$
$$+2\beta + 8\bar{\beta}^2 - (8\beta+4)\bar{\beta} + 2\beta^2,$$

$$C = (8n - 4n^2 - 4)\beta^4 + ((16\bar{\beta}-4)n^2 - 20\bar{\beta}n + (4-4\bar{\beta})n^3 + 8\bar{\beta})\beta^3$$
$$+((2n-2n^3)\gamma + 8\bar{\beta} - 4 + (4-4\bar{\beta})n + (4-8\bar{\beta})n^2 + (4\bar{\beta}-4)n^3)\beta^2$$
$$+((12\bar{\beta} - 20\bar{\beta}^2 - 4)n^2 + ((6-2\bar{\beta})n^3 - 12\bar{\beta} + 4 + 8\bar{\beta}n^2 + (2-2\bar{\beta})n)\gamma$$
$$+(4\bar{\beta}^2 - 4\bar{\beta})n^3 - 16\bar{\beta}^2 + 8\bar{\beta} + (32\bar{\beta}^2 - 12\bar{\beta})n)\beta + (n^3 + 3n^2 + 3n + 1)\gamma^2$$
$$+((4\bar{\beta}^2 - 4\bar{\beta} - 2)n^3 + 16\bar{\beta}^2 + (8\bar{\beta} - 12\bar{\beta}^2 - 6)n^2 + (4\bar{\beta}-4)n - 8\bar{\beta})\gamma$$
$$+(16\bar{\beta}^2 - 12\bar{\beta} + 4)n^2 + (4\bar{\beta} - 4\bar{\beta}^2)n^3 + (8\bar{\beta} - 16\bar{\beta}^2)n,$$

and

$$D = (n+1)\gamma - 2n + (2-2n)\beta^2 + (4n-2)\beta.$$

Interior and positive solutions for R&D outputs, in particular that $A > 0$, are guaranteed for $\gamma > (n+1)/2, \forall \beta, \bar{\beta} \in [0,1]$. We make the following assumption:

Assumption 1 $\gamma > (n+1)/2$.

The equilibrium R&D outputs result to effective cost cost reductions $X_i = x_i^* + \beta(n-1)x_{-i}^*$ and $X_{-i} = (1 + \bar{\beta}(n-2))x_{-i}^* + \beta x_i^*$. The marginal effect of spillovers on firm i's effective R&D is given by

$$\frac{\partial X_i}{\partial \beta} = 2(a-c)\Bigg(\frac{n-(n-1)\beta}{C^2}\left(\frac{\partial A}{\partial \beta}C - A\frac{\partial C}{\partial \beta}\right) - (n-1)\frac{A}{C}$$
$$+\frac{\beta(n-1)(n-\beta-(n-2)\bar{\beta})}{C^2}\left(\frac{\partial D}{\partial \beta}C - D\frac{\partial C}{\partial \beta}\right)$$
$$+(n-1)(n-2\beta-(n-2)\bar{\beta})\frac{D}{C}\Bigg),$$

with

$$\frac{\partial A}{\partial \beta} = (2-2\bar{\beta})n^2 + (8\bar{\beta} - 4\beta - 2)n + 4\beta - 8\bar{\beta} + 2,$$

$$\frac{\partial C}{\partial \beta} = (4\bar{\beta}^2 + (8\beta - 12\beta^2 - 2\gamma - 4)\bar{\beta} + 12\beta^2 - (4\gamma+8)\beta + 6\gamma)n^3$$
$$+((48\beta^2 - 16\beta + 8\gamma + 12)\bar{\beta} - 20\bar{\beta}^2 - 16\beta^3 - 12\beta^2 + 8\beta - 4)n^2$$
$$+(32\bar{\beta}^2 - (60\beta^2 + 8\beta + 2\gamma + 12)\bar{\beta} + 32\beta^3 + (4\gamma+8)\beta - 2\gamma)n$$
$$-16\bar{\beta}^2 + (24\beta^2 + 16\beta - 12\gamma + 8)\bar{\beta} - 16\beta^3 - 8\beta + 4\gamma,$$

and

$$\frac{\partial D}{\partial \beta} = (4-4n)\beta + 4n - 2,$$

which brings us to our first proposition.

Proposition 1 *Agglomeration leads to higher effective R&D when the agglomeration spillovers are moderate, i.e.* $\bar{\beta} \leq \hat{\beta}$ *where* $\hat{\beta} \in \left(\frac{n-1}{n+1}, \frac{n-1}{n}\right), \partial\hat{\beta}/\partial\gamma > 0$.

Proof. When agglomerated, the marginal effect of localised knowledge spillovers on firm i's effective R&D is non-negative if

$$\left.\frac{\partial X_i}{\partial \beta}\right|_{\beta\to\bar{\beta}} = \frac{2\gamma(a-c)(n^2-1)f(n,\gamma,\bar{\beta})}{((n+1)\gamma-(2n-2)\bar{\beta}^2+(4n-2)\bar{\beta}-2n)E} \geq 0 \qquad (6)$$

where

$$f(n,\gamma,\bar{\beta}) = (1-\bar{\beta})(2\bar{\beta}^2-2\bar{\beta}+\gamma)n^3+(1-2\bar{\beta})(16\bar{\beta}-10\bar{\beta}^2-6+\gamma)n^2$$
$$+(56\bar{\beta}^2-34\bar{\beta}^3-(\gamma+30)\bar{\beta}+6-\gamma)n+16\bar{\beta}^3-18\bar{\beta}^2+6\bar{\beta}-\gamma$$

and

$$E = ((2\bar{\beta}^2-2\bar{\beta}+\gamma)n^2+(4\bar{\beta}-4\bar{\beta}^2+2\gamma-2)n+2\bar{\beta}^2-2\bar{\beta}+\gamma)^2$$

Clearly, both $E > 0$ and $2\gamma(a-c)(n^2-1) > 0$. Given Assumption 1, also $(n+1)\gamma-(2n-2)\bar{\beta}^2+(4n-2)\bar{\beta}-2n$ is always positive. Hence, the sign of equation (6) depends on the sign of $f(n,\gamma,\bar{\beta})$.

Since $f(n,\gamma,0) = (n-1)(n^2\gamma+2n\gamma-6n+\gamma) > 0$, given Assumption 1, and $f(n,\gamma,(n-1)/n) = -4(2n-1)(n-1)(n-2)^2/n^3 < 0$, there is at least one $\hat{\beta}$ such that $f(n,\gamma,\hat{\beta}) = 0$. Furthermore, $\partial f/\partial\gamma = (n+1)^2(n-1-\bar{\beta}n) \geq 0$ iff $\bar{\beta} \leq (n-1)/n$. As

$$\frac{\partial f}{\partial\bar{\beta}} = -6(n-8)(n-1)^2\bar{\beta}^2+4(n-1)(2n-1)(n-9)\bar{\beta}-n(n+1)^2\gamma$$
$$-2n^3+28n^2-30n+6$$

and $\gamma > (n+1)/2$, the discriminant of this quadratic function is negative for $n > 8$. As the leading coefficient is then also negative, this implies that $\partial f/\partial\bar{\beta} < 0$ if $n > 8$. Also for $n = 8$, $\partial f/\partial\bar{\beta} < -2383-420\bar{\beta} < 0$. For $n \in [3,7]$, the leading coefficient is positive and the roots of the quadratic function are given by

$$\bar{\beta} = \frac{1}{6}\frac{4n^2-38n \pm \sqrt{-3n^5+19n^4+23n^3+133n^2+132n+36}+18}{(n-8)(n-1)}.$$

The larger root is greater than 1 when $n \in [3,7]$ and the smaller root is less than 0 when $n \in [4,7]$. Therefore, $\partial f/\partial\bar{\beta} < 0$ if $n > 3$. For $n = 3$, $\partial f/\partial\bar{\beta} < 0$ if $\bar{\beta} \geq 0.07805$. This implies that for $n = 3$, $\hat{\beta}$ is bounded above at $2/3$. By the implicit function theorem,

$$\frac{\partial\bar{\beta}}{\partial\gamma} = -\frac{\partial f/\partial\gamma}{\partial f/\partial\bar{\beta}} = \frac{8}{3}\frac{3\bar{\beta}-2}{20\bar{\beta}^2-40\bar{\beta}-8\gamma+19} \geq 0$$

if $\bar{\beta} \in [0.07805, 2/3]$. Hence, we conclude that there is always exactly one $\hat{\beta}$. Its higher bound is $(n-1)/n$, and the lower bound is given by

$$f\left(n, \frac{n+1}{2}, \hat{\beta}\right) = -2(n-8)(n-1)^2\hat{\beta}^3 + 2(n-1)(2n-1)(n-9)\hat{\beta}^2$$
$$-\frac{1}{2}(n^4 + 7n^3 - 53n^2 + 61n - 12)\hat{\beta} + \frac{1}{2}(n-1)(n^3 + 3n^2 - 9n + 1) = 0.$$

Since

$$f\left(n, \frac{n+1}{2}, \frac{n-1}{n+1}\right) = \frac{(n-1)(n^5 + 5n^4 - 26n^3 + 178n^2 - 287n + 81)}{2(n+1)^3} > 0,$$

the lower bound is greater than $(n-1)/(n+1)$. ■

(Remark: Several parts of both proofs rely on the positivity or negativity of polynomial functions of n, which is not always explicitly shown for the sake of brevity. One can confirm these cases by using any method for finding the bounds of (real and positive) polynomial roots. Computationally simpler methods, such as Cauchy's, might not always be tight enough such that all $n \geq 3$ can initially be seen to lie outside the bound. In such a case, one can proceed by individually confirming the positivity or negativity of the function for the remaining integers.)

Proposition 1 shows that for low to medium spillovers firm i's effective R&D is higher if it is agglomerated with the other firms. If the agglomeration spillovers are high, however, it could increase its effective R&D by not agglomerating with the other firms. In this case the increase in the R&D propensities would compensate for the lower spillovers. When this effect takes place depends on the number of firms, n, which provides a moving window for the critical spillover rate. The larger the number firms, the higher the agglomeration spillovers that would still increase the firm's effective R&D. Intuitively, n affects both the quantity of spillovers that a firm can enjoy as well as how much strategic effect its agglomeration decision has on the R&D choices of the other firms. Perhaps surprisingly, the critical level of spillovers is higher for $n \geq 3$ firms than in the standard model (c.f. De Bondt et al., 1992). However, the meaning of $\hat{\beta}$ is different. Instead of measuring the common spillover rate that maximises every firms' effective R&D, it provides an important counterfactual condition. That is, after what spillover rate would a firm enjoy a higher effective R&D outside the agglomeration. As such, it still holds that the highest effective R&D for the agglomerated industry as a whole is gained when the agglomeration spillovers are exactly one half.

To some extent, the effect of agglomeration spillovers depends on R&D cost-efficiency, which determines the critical spillover rate within the window. A larger γ brings the critical rate closer to the upper bound, in which case higher agglomeration spillovers are better for effective R&D due to the cost savings. The magnitude of this effect is small, however, and this is partly a consequence of Assumption 1. By approximating the bounds for some values of n:

$$n = 3 \rightarrow \hat{\beta} \in \left(0.6498, \frac{2}{3}\right), n = 5 \rightarrow \hat{\beta} \in \left(0.7796, \frac{4}{5}\right)$$

$$n = 10 \to \hat{\beta} \in \left(0.8930, \frac{9}{10}\right), n = 25 \to \hat{\beta} \in \left(0.9592, \frac{24}{25}\right),$$

we see that they tend to be very close to each other. γ having only a small effect, we can say that in general the critical spillover rate takes place slightly before $(n-1)/n$.

3.3 Location Choice Stage

Even if high agglomeration spillovers imply smaller effective R&D, the last part is to check the range of spillovers for which agglomeration is an equilibrium outcome. That is, for what levels of $\bar{\beta}$ is agglomeration a Nash equilibrium. As earlier, we assume that all the other firms are agglomerated and concentrate on firm i's decision. Given the anticipated outcome of stages 2 and 3, and the equilibrium cost reductions (4) and (5), firm i's profit function in stage 1 is now

$$\pi_i = \frac{(a - c + nX_i - (n-1)X_{-i})^2}{(n+1)^2} - \frac{1}{2}\gamma(x_i^*)^2$$

$$= \frac{(a - c + (n - (n-1)\beta)x_i^* + (n-1)(n\beta - (n-2)\bar{\beta} - 1)x_{-i}^*)^2}{(n+1)^2} - \frac{1}{2}\gamma(x_i^*)^2.$$

The first order condition with respect to β is

$$\frac{\partial \pi_i}{\partial \beta} = \frac{2(a - c + (n - (n-1)\beta)x_i^* + (n-1)(n\beta - (n-2)\bar{\beta} - 1)x_{-i}^*)^2}{(n+1)^2}$$

$$\times\Bigg((n - (n-1)\frac{\partial x_i^*}{\partial \beta} - (n-1)x_i^* + (n-1)(n\beta - (n-2)\bar{\beta} - 1)\frac{\partial x_{-i}^*}{\partial \beta}$$

$$+n(n-1)x_{-i}^*\Bigg) - \gamma x_i^* \frac{\partial x_i^*}{\partial \beta}$$

with

$$\frac{\partial x_i^*}{\partial \beta} = 2(a-c)\left(\frac{n-(n-1)\beta}{C^2}\left(\frac{\partial A}{\partial \beta}C - A\frac{\partial C}{\partial \beta}\right) - (n-1)\frac{A}{C}\right)$$

and

$$\frac{\partial x_{-i}^*}{\partial \beta} = 2(a-c)\left(\frac{n-\beta-(n-2)\bar{\beta})}{C^2}\left(\frac{\partial D}{\partial \beta}C - D\frac{\partial C}{\partial \beta}\right) + \frac{D}{C}\right).$$

This brings us to our final proposition.

Proposition 2 *Agglomeration is an equilibrium outcome for n firms given any level of agglomeration spillovers* $\bar{\beta}$.

Proof. Agglomeration is an equilibrium, when the marginal profit of agglomeration spillovers is non-negative, i.e.

$$\left.\frac{\partial \pi_i}{\partial \beta}\right|_{\beta\to\bar{\beta}} = \frac{4(n-1)\gamma(a-c)^2 h(n,\gamma,\bar{\beta})}{((n+1)\gamma+(2-2n)\bar{\beta}^2+(4n-2)\bar{\beta}-2n)G} \geq 0 \qquad (7)$$

where

$$G = ((2\bar{\beta}^2-2\bar{\beta}+\gamma)n^2-(4\bar{\beta}^2-4\bar{\beta}-2\gamma-2)n+2\bar{\beta}^2-2\bar{\beta}+\gamma)^3$$

and

$$h(n,\gamma,\bar{\beta}) = (1-\bar{\beta})(2\bar{\beta}^2-2\bar{\beta}+\gamma)(4\bar{\beta}-2\bar{\beta}^2+\gamma-2)n^5$$
$$-(28\bar{\beta}^5-100\bar{\beta}^4+136\bar{\beta}^3-(2\gamma+88)\bar{\beta}^2+(\gamma^2+28)\bar{\beta}-2\gamma^2+2\gamma-4)n^4$$
$$+(72\bar{\beta}^5-208\bar{\beta}^4+(20\gamma+216)\bar{\beta}^3-(42\gamma+96)\bar{\beta}^2+(26\gamma+16)\bar{\beta}+\gamma^2-6\gamma)n^3$$
$$-(88\bar{\beta}^5-184\bar{\beta}^4+(20\gamma+120)\bar{\beta}^3-(10\gamma+24)\bar{\beta}^2+(4\gamma^2-10\gamma)\bar{\beta}-\gamma^2+4\gamma)n^2$$
$$+(52\bar{\beta}^5-64\bar{\beta}^4-(20\gamma-16)\bar{\beta}^3+44\bar{\beta}^2\gamma-(7\gamma^2+18\gamma)\bar{\beta}+2\gamma^2+2\gamma)n$$
$$(2\bar{\beta}^3-3\bar{\beta}\gamma+1\gamma)(2\bar{\beta}-6\bar{\beta}^2+\gamma).$$

Clearly, $4(n-1)\gamma(a-c)^2 > 0$. Given Assumption 1, also $(n+1)\gamma+(2-2n)\bar{\beta}^2+(4n-2)\bar{\beta}-2n$ and G are always positive. Hence, the sign of equation (7) depends on the sign of $h(n,\gamma,\bar{\beta})$.

At the end points,

$$h(n,\gamma,0) = (\gamma^2-2\gamma)n^5+(2\gamma^2-2\gamma+4)n^4+(\gamma^2-6\gamma)n^3+(\gamma^2-4\gamma)n^2+(2\gamma^2+2\gamma)n+\gamma^2$$

$$> h(n,\gamma,1) = \gamma^2n^4+(\gamma^2-2\gamma)n^3-(3\gamma^2+4\gamma)n^2-(5\gamma^2-8\gamma-4)n-2\gamma^2+10\gamma-8 > 0,$$

given Assumption 1. $\bar{\beta}=1$ is not the argument that minimises $h(n,\gamma,\bar{\beta})$, only if $h(n,\gamma,\bar{\beta})$ is convex downward.

The second derivative of $h(n,\gamma,\bar{\beta})$ is:

$$\frac{\partial^2 h}{\partial^2\bar{\beta}} = 4(n-1)((20n^4-120n^3+240n^2-200n+60)\bar{\beta}^3$$
$$-(48n^4-252n^3+372n^2+180n-12)\bar{\beta}^2+(36n^4-168n^3+30n^2\gamma+156n^2-24n-30\gamma)\bar{\beta}$$
$$-n^4\gamma-8n^4+36n^3-21n^2\gamma-12n^2-16n\gamma+6\gamma) \equiv h''.$$

Since

$$\frac{\partial h''}{\partial \gamma} = -4(n^2-1)(n^3-n^2-30n\bar{\beta}+22n+30\bar{\beta}-6) < 0,$$

$$h'' < 2(n-1)(40(n-3)(n-1)^3)\bar{\beta}^3-24(4n^2-13n+1)(n-1)^2\bar{\beta}^2$$
$$+6(n-1)(12n^3-39n^2+18n+5)\bar{\beta}-n^5-17n^4+51n^3-61n^2-10n+6 \equiv \bar{h}'',$$

where $\gamma = (n+1)/2$. When $n = 3$, $\bar{h}''$ becomes a quadratic function that is always negative:

$$\bar{h}'' = 768\bar{\beta}^2 + 1536\bar{\beta} - 3264 < 0.$$

$\bar{h}''$ is also negative at both end points:

$$\bar{\beta} = 0 \rightarrow \bar{h}'' = -2(n-1)(n^5 + 17n^4 - 51n^3 + 61n^2 + 10n - 6) < 0,$$

$$\bar{\beta} = 1 \rightarrow \bar{h}'' = -2(n-1)(n^5 + n^4 - 9n^3 - 17n^2 + 128n - 72) < 0.$$

Differentiating $\bar{h}''$ with respect to $\bar{\beta}$ gives a quadratic equation,

$$\frac{\partial \bar{h}''}{\partial \bar{\beta}} = 12(n-1)^2(20(n-3)(n-1)^2\bar{\beta}^2 - 8(4n^2-13n+1)(n-1)\bar{\beta} + 12n^3 - 39n^2 + 18n + 5, \tag{8}$$

with solutions

$$\bar{\beta} = \frac{8\,n^2 - 26\,n \pm \sqrt{\Delta} + 2}{10\,(n-3)\,(n-1)}. \tag{9}$$

For $n \in [4, 8]$, the discriminant,

$$\Delta = 4n^4 - 41n^3 + 33n^2 + 141n + 79,$$

is negative and $\bar{h}''$ has no local maximum. Since the leading coefficient of equation (8) is positive for $n > 3$, the local maximum for $n > 8$ is given by the smaller value of equation (9). At this point $\bar{h}''$ is

$$-\frac{2(n-1)}{25(n-3)^2}(25n^7 - 141n^6 + 96n^5 - (12\sqrt{\Delta} - 406)n^4 + (123\sqrt{\Delta} + 261)n^3$$
$$-(99\sqrt{\Delta} + 981)n^2 - (423\sqrt{\Delta} + 1854)n + \Delta^{\frac{3}{2}} - 237\sqrt{\Delta} - 884).$$

and decreasing in Δ, and therefore less than

$$-\frac{2(n-1)}{25(n-3)^2}(25n^7 - 141n^6 + 96n^5 + 406n^4 + 261n^3 - 981n^2 - 1854n - 884) < 0.$$

Since $h'' < \bar{h}'' < 0$, $h(\gamma, \bar{\beta})$ is concave and always positive as is then the marginal profit of agglomeration spillovers. ■

Proposition 2 means that agglomeration is always a possible outcome, irrespective of the level of spillovers it yields. In the absence of any offsetting factors, this outcome holds for any $n \geq 3$. Due to Proposition 1 this also means that agglomeration, while an equilibrium outcome, cannot always be expected to yield higher effective R&D. While it is intuitive that a firm prefers to agglomerate when this implies higher effective R&D, it is less so when that does not happen. However, a location outside the agglomeration would then imply less spillovers and more investment in R&D and it is therefore always profitable to be agglomerated when the other firms are. As such, the level of agglomeration spillovers, the number of firms, as well as R&D cost-efficiency play an important role in whether localised knowledge spillovers explain the spatial concentration of innovation. Since these are likely to vary between industries, subsequent empirical research could pay more attention to them. Furthermore, these factors may help to explain the observed differences between industries (Döring and Schnellenbach, 2006; De Groot et al., 2009).

4 Conclusion

The explanation given to the found pattern of geographically concentrated innovation has been localised knowledge spillovers. Here we have analysed the theoretical validity of this explanation in the context of Cournot oligopolists. Indeed, it holds that agglomeration is an equilibrium outcome. However, it is not always the case that agglomeration will then imply higher effective R&D for these firms.

Regarding the incentive to agglomerate, it is higher for three or more firms than two (Gil Moltó et al., 2005) as then the firms would choose to agglomerate no matter what the level of agglomeration spillovers is, even if a very high one. This result suggest that localised knowledge spillovers create even stronger centripetal force when the industry size increases. Agglomeration is then more likely even in the presence of other, centrifugal forces. It is important to note that while we considered localised knowledge spillovers in the absence of other factors, we also concentrated on the existence of the agglomeration equilibrium and not its uniqueness. Further theoretical work could consider the existence of other equilibria or sequential entry into the market, which could lead to interesting asymmetries. In addition, our model examined only a single industry, whereas the empirical research has also considered spillovers between different co-located industries. As such, there is further theoretical work to be done regarding inter-industrial spillovers and the effect of the regional industrial structure.

Importantly, the findings highlight issues for empirical research to pay attention to. We know from the previous theoretical research that the type of spillovers and how R&D is measured is likely to be very important. In this paper we have also explored how the R&D costs (or efficiency), the number of firms, and the level of agglomeration spillovers affect the effective R&D that the firms gain. All of these are likely to vary between different industries and technologies and present interesting hypotheses for subsequent empirical research to test.

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