Supporting Annex to 'Revisiting the Great Moderation: policy or luck?'*

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1 Full Derivation of the Baseline Model and the Optimal Timeless Rule

1.1 Derivation of the 'IS' curve:

Let the representative household consume a composite of differentiated goods produced by monopolistically competitive firms that make up of a continuum of measure 1. The composite consumption entering the utility function in each period is:

$$C_t = [\int_0^1 c_{jt}^{\frac{\theta-1}{\theta}} dj]^{\frac{\theta-1}{\theta}} \tag{A.1}$$

where θ (>1) is the price elasticity of demand for good j. The cost minimization process implies the demand for good j is:

$$c_{jt} = (\frac{p_{jt}}{P_t})^{-\theta} C_t \tag{A.2}$$

where p_{jt} is the price of good j and P_t is the general price level.

For simplicity, assume the representative household cares only about leisure and consumption and define his life-time utility function as:

$$U_t = E_t \sum_{i=0}^{\infty} \beta^i [\frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \chi \frac{N_{t+i}^{1+\eta}}{1+\eta}] \tag{A.3}$$

where σ is the inverse of intertemporal elasticity of substitution of consumption and η is the inverse of elasticity of labour.

Now, suppose the household owns and also works for the firms. The real budget constraint he faces is:

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$$C_t + \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} = \frac{W_t}{P_t}N_t + \frac{M_t}{P_t} + (1+i_t)\frac{B_t}{P_t} + \Pi_t \tag{A.4}$$

where M_t and B_t are the initial stocks of money and nominal bond respectively, W_t is the nominal wage, Π_t is the firms' profit and i_t is the nominal interest rate. We introduce the bond market here to give interest rate a role; we also assume labour is the only productive factor so besides Π_t the household' disposal income is all from wages.

Given the Cash-in-advance constraint, the household's maximization problem can be described as follows:

$$\begin{aligned} \underset{C_t,N_t,M_{t+1},B_{t+1}}{Max} L_0 &= E_0 \sum_{t=0}^{\infty} \beta^t \{ [\frac{C_t^{1-\sigma}}{1-\sigma} - \chi \frac{N_t^{1+\eta}}{1+\eta}] \\ &\quad -\lambda_t [C_t + \frac{M_{t+1}}{P_t} + \frac{B_{t+1}}{P_t} - \frac{W_t}{P_t}N_t - \frac{M_t}{P_t} - (1+i_t)\frac{B_t}{P_t} - \Pi_t] \\ &\quad -\mu_t [C_t - \frac{M_t}{P_t}]\} \end{aligned} \tag{A.5}$$

The first order conditions are:

$$C_t: \quad C_t^{-\sigma} = (\lambda_t + \mu_t)$$

$$N_t: \quad \chi N_t^{\eta} = \lambda_t \frac{W_t}{P_t}$$

$$M_{t+1}: \quad \lambda_t (1+\pi_{t+1}) = \beta(\lambda_{t+1} + \mu_{t+1})$$

$$B_{t+1}: \quad \lambda_t (1+\pi_{t+1}) = \lambda_{t+1}\beta(1+i_{t+1})$$

These then imply:

$$C_t^{-\sigma} = \beta(1+i_t)E_t \frac{B_t}{P_{t+1}} C_{t+1}^{-\sigma} \tag{A.6}$$

$$\frac{\chi N_t^{\eta}}{C_t^{-\sigma}}(1+i_t) = \frac{W_t}{P_t} \tag{A.7}$$

that is, the 'Euler's equation' (A.6) and the intratemporal substitution between labour and consumption (A.7).

Log-linearization of (A.6) around zero-inflation steady state implies:

$$\tilde{c}_t = E_t \tilde{c}_{t+1} - \frac{1}{\sigma}(\tilde{\imath}_t - E_t \pi_{t+1}) \tag{A.6'}$$

where '$\sim$' denotes 'percentage deviation from the steady state'.

Since the model has no physical capital (and so no investment), log-linearising the market clearing condition $Y_t = C_t + G_t$ yields:

$$\tilde{c}_t = \tilde{y}_t + \ln(1 - \frac{G_t}{Y_t}) - \ln\frac{C}{Y} \tag{A.8}$$

Combining (A.6') and (A.8) then gives the 'IS' curve:

$$x_t = E_t x_{t+1} - (\frac{1}{\sigma})(\tilde{\imath}_t - E_t \pi_{t+1}) + v_t \tag{A.9}$$

where $x_t \equiv \tilde{y}_t - \tilde{y}_t^f$, $v_t = (E_t \tilde{y}_{t+1}^f - \tilde{y}_t^f) + (E_t \hat{g}_{t+1} - \hat{g}_t)$, *and* $\hat{g}_t \equiv \ln(1 - \frac{G_t}{Y_t})$.

Note the output gap x_t here is defined as the log difference between the actual output and the flexible-price output. However, as the main context explained we follow the common practice of measuring this by taking the log difference between the actual output and its HP trend as a proxy. The 'demand shock' v_t here is an aggregate of shocks to both technology and government expenditure.

1.2 Derivation of the Phillips curve:

Under monopolistically competitive environment each firm has individual production function:

$$y_{jt} = A_t N_{jt} \tag{A.10}$$

where 'j' denotes the jth firm, A_t is the technology level that follows $\log A_t = \xi \log A_{t-1} + z$, where z_t is an i.i.d. error.

By assuming the Calvo (1983) contracts, let the fraction of firms being able to reset their price for any given period be $1 - \omega$. Since equation (A.2) implies the demand curve faced by each firm is:

$$y_{jt} = (\frac{p_{jt}}{P_t})^{-\theta} Y_t \tag{A.11}$$

firms producing differentiated goods then process identical pricing strategy; they would set individual prices to p_{jt}, subject to (A.10), the Calvo probability $1 - \omega$ and (A.11), in order to maximize the discounted real profit.

Let φ be the real marginal cost. The firms' cost-minimization problem requires:

$$\varphi_t = \frac{W_t/P_t}{A_t} \tag{A.12}$$

This then leads to the profit maximization problem:

$$\underset{p_{jt}}{Max} E_t \sum_{i=0}^{\infty} \omega^i \beta^i V_{i,t+i}[(\frac{p_{jt}}{P_{t+i}}) y_{j,t+i} - \varphi_{t+i} y_{j,t+i}] \tag{A.13}$$

where $V_{i,t+i}$ is a discount factor, indicating the ratio of marginal utilities of intertemporal consumptions.

Using (A.11) to substitute away $y_{j,t+i}$ in (A.13), it yields:

$$\underset{p_{jt}}{Max} E_t \sum_{i=0}^{\infty} \omega^i \beta^i V_{i,t+i}[(\frac{p_{jt}}{P_{t+i}})^{1-\theta} - \varphi_{t+i}(\frac{p_{jt}}{P_{t+i}})^{-\theta}] Y_{t+i} \tag{A.13'}$$

The first order condition with respect to p_{jt} is:

$$E_t \sum_{i=0}^{\infty} \omega^i \beta^i V_{i,t+i} Y_{i,t+i} [(1-\theta)(\frac{p_{jt}}{P_{t+i}}) + \theta \varphi_{t+i}] \frac{1}{p_{jt}} (\frac{p_{jt}}{P_{t+i}})^{-\theta} = 0 \tag{A.14}$$

Log-linearizing (A.14) around zero-inflation steady state then yields the optimal reset price for each firm:

$$\tilde{p}^*_{jt} = (1-\omega\beta) \sum_{i=0}^{\infty} \omega^i \beta^i (E_t \tilde{\varphi}_{t+i} + E_t \tilde{P}_{t+i}) \tag{A.15}$$

The general price level in each period under the Calvo contract is then:

$$P_t = (1-\omega) p^*_{jt} + \omega P_{t-1} \tag{A.16}$$

- the weighted average of the reset prices and the unchanged prices[1].

Log-linearizing (A.16) implies:

$$\pi_t = (1-\omega) \tilde{p}^*_{jt} + (\omega - 1) \tilde{P}_{t-1} \tag{A.17}$$

Combining (A.15) and (A.17) gives:

$$\pi_t = \beta E_t \pi_{t+1} + \frac{(1-\omega)(1-\omega\beta)}{\omega} \tilde{\varphi}_t \tag{A.18}$$

[2]or the standard forward-looking New Keynesian Phillips curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa \tilde{\varphi}_t \tag{A.18'}$$

where $\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$.

We can then transform (A.18') to relate inflation to the output gap by log-linearising (A.12) and (A.7) and combining the results. After some algebra it can be shown that:

$$\tilde{\varphi}_t = (\eta + \sigma \frac{Y}{C})(\tilde{y}_t - \tilde{y}^f_t) = (\eta + \sigma \frac{Y}{C}) x_t \tag{A.19}$$

[3]

Following Clarida, Gali and Gertler (2002) to suppose the labour market is not perfectly competitive such that the wage mark-up over intratemporal substitution between consumption and labour is subject to stochastic errors, we can introduce a wage mark-up error to (A.19), so that:

$$\tilde{\varphi}_t = (\eta + \sigma \frac{Y}{C}) x_t + u^w_t \tag{A.19'}$$

where u^w_t is the bias caused to the wage mark-up. We can then rewrite the Phillips curve as:

[1]Note the individual firms would have the same pricing strategy so all p^*_{jt} (and hence all $\tilde{p}^*_{jt}$) are the same.

[2]Note (A.15) can be written as $\tilde{p}^*_{jt} = \frac{(1-\omega\beta)}{(1-\omega\beta B^{-1})}(\tilde{\varphi}_t + \tilde{P}_t)$ under rational expectations

[3]This result is obtained by assuming $Y_t = C_t + G_t$. Walsh (2003) used $Y_t = C_t$ and showed $\tilde{\varphi}_t = (\eta + \sigma) x_t$.

$$\pi_t = \beta E_t \pi_{t+1} + \gamma x_t + u_t \tag{A.20}$$

where $\gamma = \kappa(\eta + \sigma\frac{Y}{C})$, $u_t = \kappa u_t^w$, and $\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$.

1.3 Derivation of the Optimal Timeless Rule:

Most New Keynesian authors would close the above model by adding to it an exogenously-specified monetary policy rule, such as Taylor Rules. While this is straightforward, we show here how the Optimal Timeless Rule can be derived based on the current settings:

Rotemberg and Woodford (1998)—also Nistico (2007)—defines the 'social welfare loss' as 'the loss in units of consumption as a percentage of steady-state output':

$$SWL_t = \frac{U - U_t}{MU_c \cdot Y}$$

They showed that given the utility function (A.3) and under the Calvo contracts, the social welfare loss function can be expressed approximately as:

$$SWL_t = \frac{\psi}{2}[\alpha x_t^2 + \pi_t^2] \tag{A.21}$$

[4]where ψ is some measure of stickiness, α is the relative weight the central bank puts on loss from output variation against inflation variation.

This leads to the minimization problem of the central bank:

$$\underset{\pi_{t+i},x_{t+i}}{Min}\ L_t = E_t \sum_{i=0}^{\infty} \beta^i [\frac{\psi}{2}(\alpha x_{t+i}^2 + \pi_{t+i}^2) + \lambda_{t+i}(\gamma x_{t+i} + \beta\pi_{t+i+1} + u_{t+i} - \pi_{t+i})] \tag{A.22}$$

- See also McCallum and Nelson (2004).

Let the problem starts from period '1', the first order conditions with respect to π_t's and x_t's are:

$$\pi_1 : \quad \psi\pi_1 - \lambda_1 = 0 \qquad \text{(the initial period)} \tag{A.23}$$

$$\pi_t : \quad E_1(\psi\pi_t + \lambda_{t-1} - \lambda_t) = 0 \qquad \text{t=2,3,......} \tag{A.24}$$

$$x_t : \ E_1(\psi\alpha x_t + \gamma\lambda_t) = 0 \qquad \text{t=1,2,3......} \tag{A.25}$$

Under timeless perspective that ignores the initial conditions at the regime's inception (Woodford, 1999; McCallum and Nelson, 2004 for examples), the optimal response can be derived by combining (A.24) and (A.25) and dropping (A.23). This then gives:

$$\pi_t = -\frac{\alpha}{\gamma}(x_t - x_{t-1}) \tag{A.26}$$

[4] This is derived in Nistico (2007) under the Rotemberg (1982) pricing mechanism. In addition to this, Nistico showed the relative weight α is equal to the ratio of the slope of the Phillips curve to the price elasticity of demand, so $\alpha = \gamma/\theta$.

2 Listing of Model Parameters

Definitions of model parameters	
β:	time discount factor
σ:	inverse of intertemporal consumption elasticity
η:	inverse of labour elasticity
ω:	Calvo price non-adjusting probability
$\frac{G}{Y}$:	Steady-state government spending to GDP ratio
$\frac{Y}{C}$:	Steady-state GDP to consumption ratio
κ:	$\kappa \equiv \frac{(1-\omega)(1-\omega\beta)}{\omega}$
γ:	$\gamma \equiv \kappa(\eta + \sigma\frac{Y}{C})$
α:	relative weight on loss from output variations to inflation variations
$\frac{\alpha}{\gamma}\equiv\frac{1}{\theta}$:	rate of optimal trade-off
θ:	price elasticity of demand
γ_π:	interest-rate response to inflation
$\gamma_x^{'}$:	interest-rate response to output gap
ρ:	interest rate smoothness
ρ_v:	persistence of demand shock
ρ_{u^w}:	persistence of supply shock
ρ_ξ:	persistence of policy shock

3 Tests of Structural Break and Stationarity of the Time Series

3.1 Qu-Perron Test for Structural Break

Table A.1: Qu-Perron Test Result

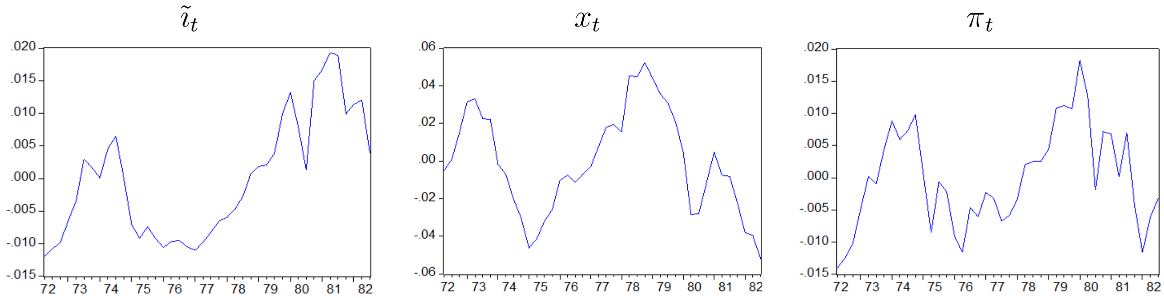
Estimated break date	95% confidence interval lower	upper	supLR test statistic for fixed number of breaks	5% critical value
1984Q3	1980Q1	1984Q4	164.84	31.85

Note: a. Time series model: VAR(1) (without constant). b. H_0: no structural break; H1: one structural break.
c. Observation sample (adjusted): 1972Q2—2007Q4.

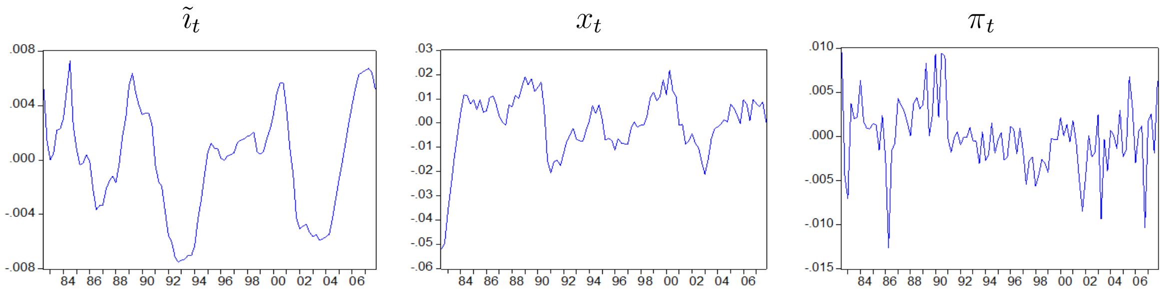
3.2 Plots of Subepisode Time Series

Figure A.1: Demeaned, Detrended Time Series

Panel A: Pre-1982 sample (1972Q2-1982Q3)



Panel B: Post-1982 sample (1982Q3-2007Q4)



Note: $\tilde{i}_t \equiv$ deviation of quarterly Fed rate from steady-state value; $x_t \equiv$ log difference of quarterly real GDP from HP trend; $\pi_t \equiv$ quarterly CPI inflation

3.3 Unit Root Test for Stationarity

Table A.2: Unit Root Test Result

Panel A: pre-break sample (1972Q3—1982Q3)

Time series	5% critical value	10% critical value	ADF test statistics	p-values*
$\tilde{i}_t$	-1.95	-1.61	-1.71	0.0818
x_t	-1.95	-1.61	-1.67	0.0901
π_t	-1.95	-1.61	-2.86	0.0053

Panel B: post-break sample (1982Q4—2007Q4)

Time series	5% critical value	10% critical value	ADF test statistics	p-values
$\tilde{i}_t$	-1.95	-1.61	-2.91	0.0040
x_t	-1.95	-1.61	-4.42	0.0000
π_t	-1.95	-1.61	-3.34	0.0010

Note: '*' denotes the Mackinnon (1996) one-sided p-values.

4 Robustness Issues on the Models' Ranking

The main text of our paper suggests that in both the post-war episodes the Optimal Timeless Rule model fits the data better compared to the Taylor Rule. Such evaluation was first built on partial model comparison based on calibrated parameters, then followed by full model comparison based on estimated parameters. It used a VAR(1) to produce data descriptors against which the models were evaluated indirectly. The cut-off of data between the sub-episodes was chosen in 1982 and was justified by a compromise between economic and econometric facts.

In this section we go one step further to check how robust the model ranking is to the choice of these. We first try higher orders of VAR. We then set the cut-off in 1984—the best breakpoint suggested by the data. While these can in principle be done both to the calibrated model and to the estimated model, we limit our discussion to the estimated models, hence their best numerical versions.

4.1 The choice of auxiliary model

Our main text has used a VAR(1) to produce parsimonious descriptors of the data behaviour. However, depending on the extent to which one requires the model to fit, higher orders of VAR can also be used. Setting a VAR of higher orders as the auxiliary model would increase the test power of Indirect Inference, as more detailed/precise data features are to be met. Hence practically this is also a way of further discriminating between models whose performances are hardly distinguishable under parsimonious auxiliaries.

Table A.3 summarises the models' performance when a VAR(2) or VAR(3) is used. The reported Walds show that increasing the VAR's order will cause strong rejection of both models in most cases due to the surged burden injected. This is the extra information related to the data dynamics from the extra lags. Nevertheless, the normalized t statistics (in parentheses) indicate that—although being rejected—the Timeless Rule model is consistently less rejected and hence more preferable compared to the Taylor Rule model, regardless of the chosen order of the VAR. Our model ranking found in the main text is therefore robust to this choice.

4.2 The choice of cut-off between the Great Acceleration and the Great Moderation

Another factor by which our model ranking might have been affected is the choice of breakpoint in the data. We have chosen 1982 in the main text to combine economic and econometric facts. Such a choice is supported by the Qu-Perron (2007) test that suggests the break in 1984(Q3), with a 95% interval going back to until 1980. Here we use the pure data information to re-estimate and re-evaluate the models by setting the break in 1984. The estimated models are revealed in table A.4 and table A.5. Table A.6 reports their performance.

We can see the Simulated Annealing (SA) estimates of both the Timeless Rule model and the Taylor Rule model are both not very different from what were estimated (as parenthesized) when 1982 was chosen. While in this case the Taylor Rule model does outperform the Timeless Rule model in fitting the data volatility pre-1984 and the data dynamics post-1984 (table A.6), the reported Full Walds have confirmed superiority of the Timeless Rule model overall (92.6 vs 93.9 in the Great Acceleration and 92.3 vs 96.9 in the Great Moderation). This can be made even clearer when we ask the models to explain a VAR(2) or VAR(3)—this last being verified by the normalized t in table A.7 that indicate much less rejection of the Timeless Rule model under most criteria. Hence,

Table A.3: Performance of Models under Differing Auxiliaries

Panel A: pre-1982

Tests for chosen features	VAR(2)		VAR(3)	
	Timeless Rule	Taylor Rule	Timeless Rule	Taylor Rule
Directed Wald for dynamics (Normalized t-stat)	99.7 (3.92)	100 (14.0)	100 (4.86)	100 (15.8)
Directed Wald for volatilities (Normalized t-stat)	66.5 (-0.12)	84.8 (0.44)	86.2 (0.48)	81.2 (0.25)
Full Wald for dyn. & vol. (Normalized t-stat)	99.9 (4.19)	100 (13.0)	100 (4.78)	100 (14.6)

Panel B: post-1982

Tests for chosen features	VAR(2)		VAR(3)	
	Timeless Rule	Taylor Rule	Timeless Rule	Taylor Rule
Directed Wald for dynamics (Normalized t-stat)	99.9 (4.33)	100 (9.38)	100 (10.1)	100 (13.7)
Directed Wald for volatilities (Normalized t-stat)	93.7 (1.41)	99.9 (6.59)	90.7 (1.06)	100 (6.37)
Full Wald for dyn. & vol. (Normalized t-stat)	100 (4.87)	100 (12.1)	100 (9.80)	100 (15.0)

although compared to the benchmark result splitting the data in 1984 is less in favour of the Timeless Rule model, the exercise here shows our earlier model ranking has failed to be overturned.

5 The Difference between the Optimal Timeless Rule Model and Taylor Rule Model: comparing the impulse responses

Our paper has argued that the US post-war monetary policy was better understood as the Optimal Timeless Rule rather than a Taylor Rule. Indeed, if a Taylor Rule was operating it would have generated quite different economic dynamics. We use the post-1982 subsample and the estimated Timeless Rule and Taylor Rule models to illustrate.

Table A.4: SA Estimates of Models when Cut-off at 1984: Timeless Rule model

Parameters	Definitions	SA estimates			
		Pre-1984 (-82)		Post-1984 (-82)	
β	time discount factor	——fixed at 0.99——			
σ	inverse of intertemporal consumption elasticity	1.01	(1.01)	2.67	(1.46)
η	inverse of labour elasticity	1.54	(2.04)	2.53	(3.23)
ω	Calvo contract price non-adjusting probability	0.79	(0.79)	0.48	(0.54)
$\frac{G}{Y}$	steady-state gov. expenditure to output ratio	——fixed at 0.23——			
$\frac{Y}{C}$	steady-state output to consumption ratio	——fixed at 1/0.77——			
κ	$\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$	0.06	(0.06)	0.57	(0.40)
γ	$\gamma = \kappa(\eta + \sigma\frac{Y}{C})$	0.17	(0.19)	3.41	(2.06)
α	relative weight of loss assigned to output variations (against inflation)	0.29	(0.20)	0.58	(0.58)
$\frac{\alpha}{\gamma} \equiv \frac{1}{\theta}$	optimal trade-off on the Timeless Rule	$\frac{1}{0.85}$	$(\frac{1}{0.95})$	$\frac{1}{5.9}$	$(\frac{1}{3.6})$
θ	price elasticity of demand	0.85	(0.95)	5.9	(3.6)
ρ_v	demand shock persistence	0.89	(0.92)	0.94	(0.94)
ρ_{u^w}	supply shock persistence	0.87	(0.86)	0.84	(0.79)
ρ_ξ	policy shock persistence	0.18	(0.14)	0.36	(0.42)

Figure A.2 to A.4 in what follows show the impulse responses of both models to a (one-standard-error) unit shock caused by the demand side, the supply side and the monetary policy. The responses of the Timeless Rule model are shown with solid lines; the dashed lines indicate those of the Taylor Rule model.

Figure A.2 shows an increase in aggregate demand raises nominal interest rates both in the Timeless Rule model and in the Taylor Rule model to similar levels. Under the optimal timeless policy where output and inflation are determined solely by the Phillips curve and the policy (the recursiveness feature, and yet can be seen from the unresponsiveness of these in panels B and C) the expected future inflation remains zero so that the real interest rate overlap the nominal rate. This is different in the Taylor Rule model where the initial rise in real interest rate is largely weakened by the surge in expected inflation due to rise in current inflation and persistence of the shock[5]; the real interest rate under this circumstance picks up the nominal rate slowly as the shock dies out. Thus under the Optimal Timeless Rule any shock to demand will be fully offset by the adjustment of nominal/real interest rate, leaving the rest of the system intact, whereas when a Taylor-type policy is substituted for the shock spreads out as a result of inadequate movements of the real rate of interest. In both cases the impulse responses suggest the shock has quite long-lasting effect. But this according to the Timeless Rule model is purely determined by the shock's persistence while in the Taylor Rule model also that the interest rates are deliberately smoothed.

A shock to aggregate supply shifts the Phillips curve upwards, worsening the trade-off between inflation and the output gap. In either model this raises inflation and interest rates (both nominal and real) and causes an output recession as figure A.3 illustrates.

[5] Note output also rises according to the Phillips curve, the dashed line in panel B.

Table A.5: SA Estimates of Models when Cut-off at 1984: Taylor Rule model

Parameters	Definitions	SA estimates			
		Pre-1984 (-82)		Post-1984 (-82)	
β	time discount factor	——fixed at 0.99——			
σ	inverse of intertemporal consumption elasticity	1.00	(1.15)	2.83	(1.16)
η	inverse of labour elasticity	2.42	(2.66)	3.40	(3.85)
ω	Calvo contract price non-adjusting probability	0.79	(0.79)	0.64	(0.61)
$\frac{G}{Y}$	steady-state gov. expenditure to output ratio	——fixed at 0.23——			
$\frac{Y}{C}$	steady-state output to consumption ratio	——fixed at 1/0.77—-			
κ	$\kappa = \frac{(1-\omega)(1-\omega\beta)}{\omega}$	0.06	(0.06)	0.21	(0.25)
γ	$\gamma = \kappa(\eta + \sigma\frac{Y}{C})$	0.22	(0.23)	1.46	(1.33)
γ_π	interest rate response to inflation	2.10	(2.03)	1.34	(2.06)
γ_x	interest rate response to output gap	0.006	(0.001)	0.09	(0.06)
ρ	interest rate smoothness	0.63	(0.42)	0.83	(0.89)
ρ_v	demand shock persistence	0.89	(0.91)	0.94	(0.95)
ρ_{u^w}	supply shock persistence	0.88	(0.87)	0.80	(0.77)
ρ_ξ	policy shock persistence	0.60	(0.58)	0.51	(0.40)

Table A.6: Performance of Models when Cut-off at 1984

Tests for	Pre-1984 under		Post-1984 under	
chosen features	Timeless Rule	Taylor Rule	Timeless Rule	Taylor Rule
Directed Wald (for dynamics)	92.4	95.8	96.4	88.2
Directed Wald (for volatilities)	82.8	27.1	2.3	99.3
Full Wald (for dynamics & volatilities)	92.6	93.9	92.3	96.9

Yet with differing policies the shock exhibits clear distributional difference according to the magnitude of the impulse responses: under the Optimal Timeless Rule it requires keeping inflation equal to a fixed fraction of (the first difference of) the output gap. This constitutes another 'optimal trade-off' between inflation and output (growth) so that when a supply shock occurs the Phillips curve moves along the policy equation to determine the equilibrium inflation and output; the increase in inflation (panel C) is punished by an output recession (panel B) made by raising the real interest rate (panel A); the latter being initiated by the rise in nominal rate but then deepened as expected future inflation goes negative. The supply shock under this circumstance goes mostly to the output as the impulse responses demonstrate, partly because of the model estimates but more importantly that inflation is bound by the optimal plan. When this is replaced by the Taylor Rule, the shock spreads out more evenly, as except being suppressed primly by the real interest rate set by such rule, inflation commits to nothing but determined

Table A.7: Performance of Models under Differing Auxiliaries (II)

Panel A: pre-1984

Tests for chosen features	VAR(2) Timeless Rule	VAR(2) Taylor Rule	VAR(3) Timeless Rule	VAR(3) Taylor Rule
Directed Wald for dynamics (Normalized t-stat)	99.9 (6.03)	100 (14.3)	100 (5.56)	100 (16.4)
Directed Wald for volatilities (Normalized t-stat)	73.6 (0.05)	88.8 (0.86)	90.8 (1.02)	89.5 (0.90)
Full Wald for dyn. & vol. (Normalized t-stat)	99.9 (6.01)	100 (15.0)	100 (5.61)	100 (15.1)

Panel B: post-1984

Tests for chosen features	VAR(2) Timeless Rule	VAR(2) Taylor Rule	VAR(3) Timeless Rule	VAR(3) Taylor Rule
Directed Wald for dynamics (Normalized t-stat)	100 (5.12)	100 (14.8)	100 (11.2)	100 (33.3)
Directed Wald for volatilities (Normalized t-stat)	42 (-0.56)	99.9 (5.04)	58.9 (-0.16)	99.7 (4.16)
Full Wald for dyn. & vol. (Normalized t-stat)	100 (5.08)	100 (14.7)	100 (11.1)	100 (31.0)

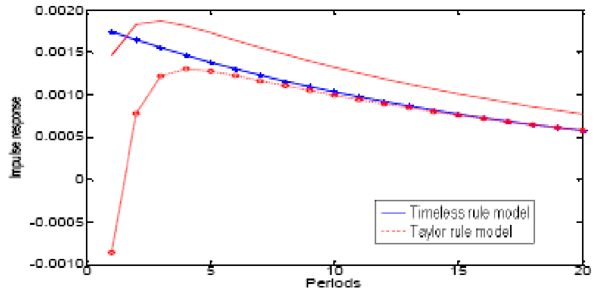
solely by the Phillips curve. The impulse responses suggest when inflation is tolerated in this way a supply shock would cause higher inflation as real interest rates response less; its effect on output is similar, though, to what would be seen under the timeless optimum. In either case, again, the shock's persistence generates the models' persistence as the figure shows, but unlike in the Taylor Rule model where this is partially caused by interest rate smoothing, under the Timeless Rule it is a joint result with the optimal trade-off[6].

Figure A.4 shows finally the models' impulse responses to a tightening monetary policy shock. In the paper we have interpreted this as a 'trembling hand' error made by the policy maker in execution of the monetary policy. But we have also emphasized that given

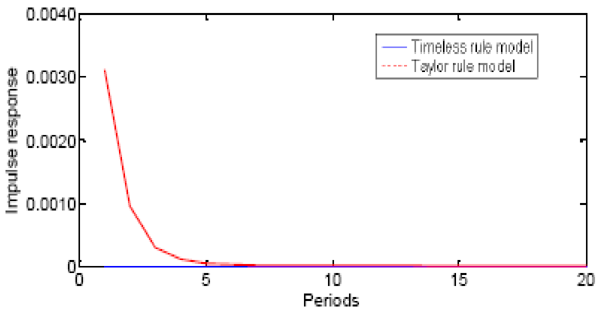
[6]This last does not hold if the optimal trade-off were between inflation and the level of output rather than its growth, i.e., the optimality condition under discretion where the lag of output is not involved-Walsh (2003) provides a neat discussion on this.

Figure A.2: Impulse Responses to a Unit Shock to Demand

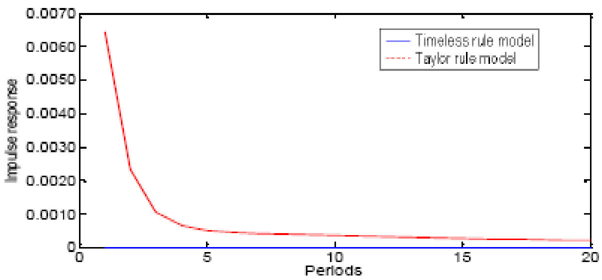
Panel A: Interest rates (nominal, unmarked; real, marked)



Panel B: Output gap



Panel C: Inflation

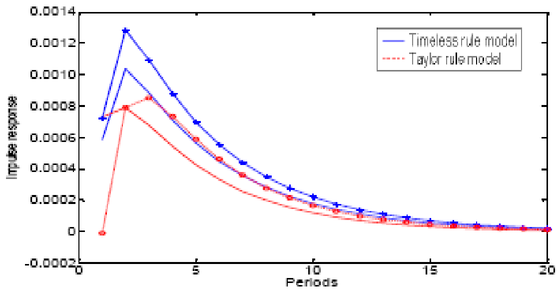


the different natures of the Timeless Rule and the Taylor Rule, its connotation in the two models is different. A tightening monetary shock to the Timeless Rule model deepens the trade-off between inflation and the output gap/growth, sending a signal of harsher punishment on the latter against the former and causing a fall in inflationary expectations. This shifts both the policy equation and the Phillips curve downwards and results in lowered equilibrium inflation (panel C). The equilibrium output gap is also determined by this and is lowered as policy tightens, but part of the contractionary pressure is cancelled out by the fall of expected future inflation that encourages current production so the actual fall of it is small (panel B). Panel A shows to support this equilibrium the real interest rate must rise. But according to the fall of inflationary expectations this is made not by raising but slightly lowering the nominal interest rate[7]. The impulse responses in

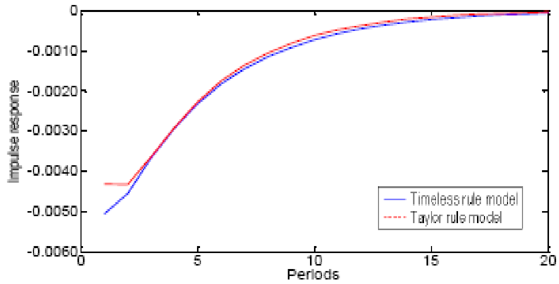
[7] Thus accompanied by an extensive downward movement of the 'IS' curve caused by the fall of

Figure A.3: Impulse Responses to a Unit Shock to Supply

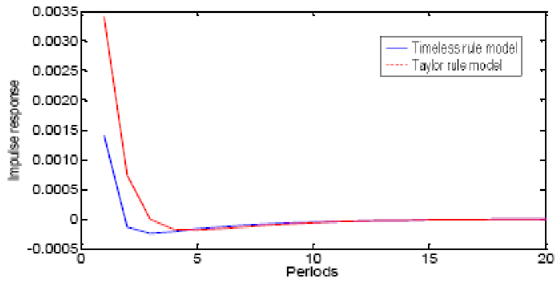
Panel A: Interest rates (nominal, unmarked; real, marked)



Panel B: Output gap



Panel C: Inflation

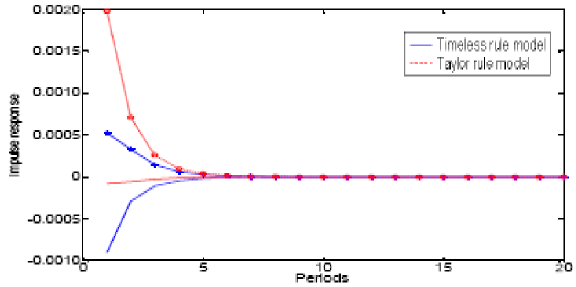


this case thus suggest the policy shock goes mostly to inflation. This would not happen to the Taylor Rule model, however, as a tightening shock to Taylor Rule raises the nominal interest rate instantly, and for given expected inflation causes a temporary rise in the real interest rate. The contractionary signal in the Taylor Rule model is sent from this, reflecting tightened monetary environment but not deepened trade-off between inflation and the output gap, the policy goals under the optimal rule. This then lowers expected future inflation (here because interest rates are smoothed and the shock is persistent) and further raises the real interest rate (panel A), causing a strong reduction in equilibrium output and correspondingly a strong reduction in equilibrium inflation (panels B and C)—these tend to be 'balanced' unless the Phillips curve is extremely steep or flat. The contraction in this particular case also causes a fall in the nominal rate that dominates

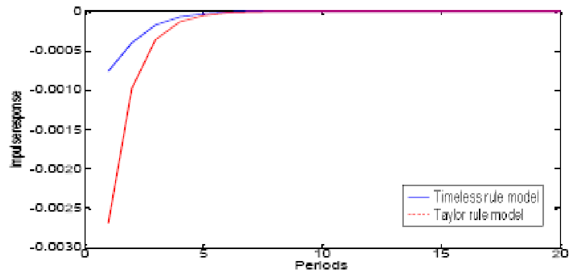
expected inflation and output gap.

Figure A.4: Impulse Responses to a (tightening) Unit Shock to Policy

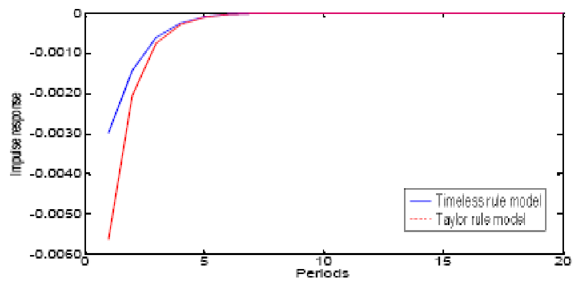
Panel A: Interest rates (nominal, unmarked; real, marked)



Panel B: Output gap



Panel C: Inflation



its initial rise; so in equilibrium it falls a little[8]. However, with either policy the shock's persistence is still the main source of the model's persistence. But the fast die-out of policy shock in either case has determined that it would not have long-lasting impact.

To sum up, implementation of the Optimal Timeless Rule has helped directing different shocks into different sectors of the economy, facilitating the monetary authority in stabilization in that the causes of instability are easier to be identified and eliminated. Compared to a Taylor Rule that specifies systematic interest-rate response, the Timeless Rule advocates active adjustment of these to ensure the policy outcome is at the least cost. Effectively it is trading the volatility of policy instrument, the nominal interest rate here, with that of the policy objectives, i.e., output gap and inflation, that would otherwise be less stabilized as the impulse responses illustrate. Our empirical assessments

[8]This is largely determined by the extent to which inflationary expectations fall in response to the shock.

have suggested for both post-war episodes the Fed's behaviour was closest to the Optimal Timeless Rule. Thus from the point of view of the history, we could argue that by committing to the timeless optimum the Fed had successfully circumvented some costs of monetary management and that the US economy would have suffered greater instability if a Taylor Rule were used.

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