Rule-of-Thumb Consumers and Labor Tax Cut Policy in the Zero Lower Bound

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June 2012
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June 28, 2012

Abstract

This paper finds that labor tax cut can be an effective policy tool to mitigate the negative effects of a shock that made the zero lower bound on the nominal interest rate binding if the economy features rule-of-thumb households (besides Ricardian ones) and nominal rigidities in prices and wages. Our results are meant to contribute to the discussion initiated by Eggertsson (2010a) who found labor tax cut policy destabilising under zero nominal interest rate in a New Keynesian economy consisting only Ricardian consumers.

JEL classification: E52, E62

Keywords: fiscal policy, zero lower bound, labor tax cut, New Keynesian

1 Introduction

Following the enaction of the American Recovery and Reinvestment package of 2009 in the United States there has been discussion on the sign and magnitude of fiscal multipliers. The $787 billion fiscal package contains payroll tax cuts as well. On one hand some influential papers concluded that an increase in non-productive government spending can be very effective in stimulating the economy under the recent

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1In December 2011 President Obama announced that the payroll tax cut is extended until end of 2012.
zero nominal interest rate environment (see, e.g., Christiano et al. (2011), Eggertsson (2010) and Woodford (2010)). On the other hand it turned out that labor tax cuts can be contractionary when the zero lower bound on the nominal interest rate is binding (see Eggertsson (2010a, 2010b)). Christiano (2010) showed that an increase in the labor tax rate is not effective in stimulating the economy when we have wage-setting frictions—beyond the usual price-setting block—in the new-Keynesian model as well. The latter is true because changes in the labor tax rate operates through the labor supply curve which is largely irrelevant in the new-Keynesian model with wage rigidity.

There is a growing empirical literature which founds labor tax cuts being stimulative. In a well-known study using narrative accounts Romer and Romer (2010) found that tax increases are contractionary. Also, Mertens and Ravn (2011) found using a new narrative account of federal tax liability changes to proxy tax shocks that the short run effects of a tax decrease on output are positive and large. Hall (2009) reviews several empirical studies arguing that households do respond with an increase in their consumption expenditures to a temporary cut in payroll tax. Thus, there is enough empirical evidence in support of the positive effects of a labor tax cut.

Christiano (2010) argued that the labor tax cut can happen to be stimulative in a model featuring standard Keynesian elements like the presence of rule-of-thumb (or rule-of-thumb/non-Ricardian) consumers who are spending their increase in disposable income after an income tax cut. Especially, he concludes that the inclusion of rule-of-thumb consumers might turn the negative sign of the labor tax cut multiplier of Eggertsson (2010) into positive, which possibility deserves to be explored.

This paper investigates whether the incorporation of rule-of-thumb (or non-Ricardian) consumers into the baseline new-Keynesian model can render labor tax cut policy desirable in stabilisation of the economy. We found that the labor tax cut can be an effective policy tool in an economy with wage-setting frictions. The

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2 These papers assume lump-sum taxation. However, Uhlig (2010) used a simple neoclassical model without nominal rigidities and assumed that spending is financed with distortionary taxation (using marginal capital and labor tax rates). He found low multipliers. Also Cogan et al. (2009) found low multipliers in case of a permanent stimulus—in contrast to a transitory stimulus assumed in the previously cited papers. Drautzburg and Uhlig (2011) considered the Smets and Wouters setup (2007) augmented with rule-of-thumb consumers and derived low multipliers under distortionary taxation.

3 Rule-of-thumb households are excluded from the financial market. Hence, they have no consumption-savings tradeoff (lack of Euler equation) and their decision problem is restricted to the optimal choice between consumption and leisure. The inclusion of rule-of-thumb households into DSGE models is a trivial way of generating incomplete asset markets.
most relevant literature on models containing rule-of-thumb consumers are Mankiw (2000), Alvarez et al. (2001), Gali et al. (2004, 2007), Bilbiie (2008). The model used in this paper is closest to Ascari et al. (2011), Furlanetto (2011), Furlanetto and Seneca (2009) who enrich the model of Gali et al. (2007) with wage-setting frictions. Furthermore, we make use of Rossi (2012) who adds a detailed fiscal block to Gali et al. (2007) framework. However, we also include a discount factor (or savings) shock—originally proposed by Eggertsson and Woodford (2003) and recently utilised e.g. by Christiano et al. (2011), Eggertsson (2010a) and Woodford (2010)—to explicitly address the zero-lower-bound (ZLB) problem.

As Bils and Klenow (2008) and Christiano (2010) argued it matters whether we cut the employer’s or the employee’s part of the labor taxes. In the latter case the labor tax cut acts like a traditional stimulus tax cut working through the labor supply, and, thus, wage-setting frictions under which labor supply curve is potentially irrelevant make a difference. However, in the previous case the payroll tax cut directly affects the marginal cost and, as we argue below, acts like a further destabilising factor on the economy besides the original shock. In this paper it is the employee’s part of the labor tax which is reduced.

Before we discuss the intuition of why the labor tax cut can be stimulative in an economy consisting of both optimiser and non-optimiser households it is useful to revise how the economy works in the ZLB. Our discussion rests on Christiano et al. (2011). The ZLB on the nominal interest rate becomes binding due to a rise in savings which is accomplished by a jump in the discount factor. If the shock is small then the fall in the real interest might be enough to restore zero savings in equilibrium before the ZLB on the nominal interest rate becomes binding in an economy without investment. However, for a large enough shock the ZLB may become binding before the real rate can fall by enough to re-establish the zero-savings equilibrium. Hence, the only way through which excess savings ‘disappears’ in equilibrium is a large transitory decline in output. Of particular interest is the staggered price setting channel that magnifies the contraction in output. For a given drop in output there is a fall in marginal cost which leads—through the new-Keynesian Phillips curve—to a reduction in inflation (and also in expected inflation) which raises the real interest rate through the Fisher relationship when nominal interest is zero. The higher is the real interest rate the more Ricardian agents reduce their consumption. As we argue below the introduction of wage-rigidity reduces the extent to which the staggered price-setting channel contributes to the decline in output. In the particular case of constant returns-to-scale technology the marginal cost equal to the real wage which does exhibit only a mild reaction to shocks under the assumption of wage rigidity

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4Originally the introduction of wage rigidity into the new-Keynesian model is motivated by
As shown by Eggertsson (2010a), in an economy with only Ricardian consumers and flexible labour market, the labor tax cut makes people feel inclined to work more and thus, labor supply shifts to the right and the partial equilibrium effect is a drop in the real wage. However, the conventional wisdom which dictates that production increases in equilibrium after a cut in the labor tax is not valid in an environment of zero nominal interest rates any more. In fact, when the ZLB on nominal interest rate is binding the labor demand curve (and also the aggregate demand curve) becomes positively sloped (see Eggertsson (2010a, 2010b)). Hence, a positively-sloped labor demand curve implies that the rightward movement in the labor supply is not only deflationary (the decline in wage rate) but also leads to a fall in output. The deflationary effect emerges from the fact that the wage tax cut decreases the cost of labor for the firm that is willing, as a result, to lower its product-price. However, in the presence of price-setting frictions à la Calvo (1983) only some of the firms are able to reduce their prices and the rest maintains them at their previous level. As a result, most of the firms will be able to cut their prices in future periods. Therefore, the initial drop in the marginal cost has little effect on the current price level but leads to a period of falling prices in the future. Given the fact that firms are forward-looking when maximising their current and discounted future profit, the initial decline in the marginal cost creates deflationary expectations which, in an environment of zero nominal interest rate, leads to an increase in the real interest rate. The rise in the real rate further depresses spending and, hence, the marginal cost that starts another deflationary cycle. In a recent study Ascari et al. (2011) found in a model very similar to ours that the assumption of wage stickiness implies that hours react more than wages to a change in labor demand relative to the case of flexible wages when the reverse is true.

The workings of the labor tax cut is very similar to the effects of the discount factor shock in the sense that both result in a decline of the marginal cost. However, this paper finds that the joint presence of rule-of-thumb consumers and wage rigidity provide rationale for labor tax cut policy. The robustness of this finding needs to be carefully explored. Thus, our strategy is as follows. There are some additional features popular in business cycle modelling like the inclusion of endogenous capital and habit formation in consumption that may alter our policy conclusions. Initially, we take a model featuring price-rigidity (and no wage rigidity) and two types of households—henceforth, referred to as Experiment 1—and show that the tax cut reinforces the conclusions of Eggertsson (2010a) who found labor tax cuts exaggerating the effects of the discount factor shock. Models used in Experiment 1-5 assume that the tax cuts are backed by lump-sum taxes levied on Ricardian households. In desire to reproduce the acyclicity of the real wage that is observed in time series.
Experiment 2 we extend the previous model with wage-rigidity which is then further augmented with habit formation in consumption in Experiment 3. The model in Experiment 3 is enriched with capital featuring investment adjustment costs (see Experiment 4). The section is closed with a relatively rich setup including capital (with changing utilisation), habits and strategic complementarity in price setting induced by Kimball (1995) demand. The last section considers whether the results obtained from Experiments 2-5 survive if the tax cut is financed by government debt (instead of lump-sum taxes paid by Ricardian households) that is retired through uniform distortionary taxation\(^5\). The validity of labor tax cut policy is explored in each of these settings (Experiment 1-8).

Experiment 2 provides various insights. First, assuming that there is no perfect wage-stickiness, the labor tax cut results in a rightward shift of the labor supply curve leading to a decline in the real wage. Second, rule-of-thumb households will spend the increase in disposable income and raise their consumption expenditures. Third, our model economy features price stickiness and those firms that cannot charge a higher price after the rise in demand will produce more and demand more labor. A higher labor demand is associated with lower markups in our setup. Note that in the model containing only one type of household exhibiting life-cycle consumption behaviour there is no shift in the labor demand (which has positive slope in the ZLB). Fourth, in a model without capital the payroll tax cut remains to be deflationary: the wage deflation is higher than the price deflation leading to a decline in the real wage rate in accordance to the findings of Eggertsson (2010b). But the magnitude of the fall in the real wage is severely constrained if we introduce wage stickiness into the model.

The extension of the baseline model with habit formation in consumption (Experiment 3) does not change the picture too much. However, in a model with endogenous capital accumulation and without habit formation in consumption (Experiment 4) we found that that wage tax cut can happen to raise the real wage if the rightward shift in the labor demand is big enough. The latter implies that the real wage falls by less in case of payroll tax cut relative to the case of no policy intervention after a discount factor shock. Hence, the labor tax cut can happen to be inflationary. The intuition for why the labor tax cut can raise the real wage in a model with capital is as follows. In a model with capital there is an additional kick that can stimulate output, namely, investment. It is well-known since at least Baxter and King (1993) that the increase in the labor input shifts out the marginal product schedule of capital and, thus, investment will increase in the short run. Due to the fact that capital is

\(^5\)Uniform means that both types of households have to bear the burden of (distortionary) labor taxes when debt (and the interests on it) is paid back.
pre-determined and not reacting on impact the rental rate plummets in the short run as a unit of capital is operating with more labor. In the long run capital rises such that the steady-state capital:labor ratio and a corresponding rental-rate:wage-rate ratio is maintained. We shortly note that the policy conclusion about the desirability of tax cut policy is still maintained in Experiment 5 which contains various less controversial features\footnote{Maybe one of the most controversial feature of Smets and Wouters (2007) is indexation implying that wages and prices are changed in each period (see Chari, Kehoe and McGrattan (2009)) which is not in line with micro data (see e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008)). Recently, Dixon and Le Bihan (2012) proposed generalised Taylor contracts—calibrated using micro data—which can reproduce the persistent and hump-shaped impulse response of inflation to a monetary policy shock without indexation.} of business cycle modelling.

Finally, we consider a different way of financing the labor tax cut. Utilizing the framework of Rossi (2012) we assume that the labor tax cut is financed by debt which is paid back through increases in distortionary labor tax levied on both types of households (see Experiments 6-8). Building on the insights of Bilbiie et al. (2012) we found that the sooner the government debt is retired the higher the benefits we can associate with the labor tax cut policy.

The rest of the paper is organised as follows. Section 1 describes the agents in the model and their assumed behaviour. Section 2 contains the calibration. In Section 3 we conduct some experiments in various settings to investigate into effects of the labor tax cut in a deterministic environment. Section 4 concludes.

2 The model

2.1 Households

2.1.1 Ricardians

There are two types of households: Ricardians and non-Ricardians. Ricardian households are able to smooth their consumption using state-contingent assets (risk-free bonds) while non-Ricardians cannot. The share of Ricardian and non-Ricardian households in the economy is \( \lambda \) and \( 1 - \lambda \), respectively. The instantaneous utility function of type \( i \in \{ o, r \} \) household which can be Ricardian (optimiser (OPT), \( o \)) or non-Ricardian (or rule-of-thumb (ROT), \( r \)), is given by:

\[
U^i_t = \frac{(C^i_t - h^i_t C^i_{t-1})^{1-\sigma} - 1}{1 - \sigma} - \frac{(N^i_t)^{1+\varphi}}{1 + \varphi}
\]  

\[\footnote{Maybe one of the most controversial feature of Smets and Wouters (2007) is indexation implying that wages and prices are changed in each period (see Chari, Kehoe and McGrattan (2009)) which is not in line with micro data (see e.g. Bils and Klenow (2004) and Nakamura and Steinsson (2008)). Recently, Dixon and Le Bihan (2012) proposed generalised Taylor contracts—calibrated using micro data—which can reproduce the persistent and hump-shaped impulse response of inflation to a monetary policy shock without indexation.}
where $C_i^t$ ($C_i^t$) denotes the time-$t$ consumption (aggregate consumption) of type $i \in \{o, r\}$ household and parameter $h_i$ governs the degree of habit formation in consumption.

First, we discuss the problem of Ricardian households. They maximise their lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta_i U_t^i,$$

where $E_0$ is the expectation operator representing expectations conditional on period-0 information and $\beta$ is the discount factor. This maximisation of the optimiser household is subject to a sequence of budget constraints:

$$P_t C_o^t + R^t E_t \{B^o_{t+1}\} = (1-\tau_t) W_t N_t^o + P_t R_t^o K_t^o + B_o^t + D_t^o - P_t T_t^o - P_t I_t^o - F_t - P_t S^o$$

where $P_t$ is the aggregate price level, $W_t$ is the nominal wage and $N_t^o$ is hours worked by OPT. Thus, $W_t N_t^o$ is the labor income received by the optimiser household. $R_t^k$ is the real rental rate on capital, $K_t^o$, in real terms and $I_t$ is real investment, $T_t^o$ are lump-sum taxes (or transfers, if negative) paid by the household and $\tau_t$ is a distortionary tax rate on labor income. Thus, $R_t^k K_t^o$ is the income earned on capital. $D_t^o$ are the dividends from ownership of firms. Further, $B^o_{t+1}$ is the amount of risk-free bonds and $R_t$ is the nominal interest rate. Following Gali et al. (2007) and Rossi (2012) we assume, without loss of generality, that the steady-state lump sum taxes ($S^o$) are chosen in a way that steady-state consumption of ROT and OPT households are equal in steady-state. Hence, $S^o$ is a steady-state lump-sum tax used to facilitate the equality of the steady-state consumptions of ROT and OPT households. $F_t$ stands for a nominal union membership fee (see later on it below). For an alternative approach when steady-state consumptions are not equal see Natvik (2008). Also the optimiser household takes into consideration the evolution of capital stock

$$K_{t+1}^o = (1-\delta) K_t^o + \left[ 1 - S \left( \frac{I_t^o}{I_{t-1}^o} \right) \right] I_t^o,$$

when choosing the level of capital optimally. In the latter equation $\delta$ stands for the depreciation rate of capital. As standard in the literature (see e.g. Smets and Wouters (2007)) investment is subject to adjustment costs of the form, $S \left( \frac{I_t^o}{I_{t-1}^o} \right)$. In general, $S$ is chosen such that $S'(1) = 0$, $S''(1) = \phi_{inv}$. Using investment-adjustment costs we depart from Gali et al. (2007) who, instead, used capital adjustment costs.

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7For the rest of the paper, a variable without a time subscript denotes steady-state value.
The OPT household first-order conditions (FOCs) with respect to consumption ($C_t$), investment ($Inv_t$), capital ($K_{t+1}^o$) and bonds ($B_{t+1}^o$), are:

$$\frac{\partial U_i^t}{\partial C_i^t} = (C_t - h_i C_{t-1})^{-\sigma} = \lambda_t, \text{ with } i = o,$$

$$\beta_t E_t \mu_{t+1} S' \left( \frac{Inv_{t+1}^o}{Inv_t^o} \right) \left( \frac{Inv_{t+1}^o}{Inv_t^o} \right)^2 = \lambda_t - \mu_t \left( 1 - S \left( \frac{Inv_t^o}{Inv_{t-1}^o} \right) \right) + \mu_t S' \left( \frac{Inv_t^o}{Inv_{t-1}^o} \right) \frac{Inv_t^o}{Inv_{t-1}^o},$$

$$\beta_t E_t \left( \lambda_{t+1} R_{t+1}^K U_{t+1} - \Omega(U_{t+1}) + \mu_{t+1}(1 - \delta) \right) = \mu_t,$$

$$\beta_t E_t \left( \lambda_{t+1} \frac{1 + R_{t+1}}{1 + \pi_{t+1}} \right) = \lambda_t,$$

where $\lambda_t$ and $\mu_t$ are the multipliers associated with the budget constraint (equation (3)) and with the evolution of capital (equation (4)) in the Lagrangean representation of the OPT household’s problem. Also let us define Tobin’s $Q$ as $Q_t = \mu_t / \lambda_t$. Here $\Omega(U_t)$ is the storage cost of the part of capital that is not utilised for production at time $t$. The equations above can be described as follows. Equations (5), (6), (7) and (8) define, respectively, the marginal utility of consumption, the evolution of Tobin’s $Q$, the capital Euler equation and the bond Euler equation. In all the above equations that contain expectations we ignore covariance terms.

The linearised version of equation (8) is the intertemporal Euler equation:

$$c_{t+1} = \frac{h_o}{1 + h_o} c_{t-1} + \frac{1}{1 + h_o} E_t c_{t+1}^o - \frac{1 - h_o}{1 + h_o} \beta \left[ dR_t - E_t \pi_{t+1} - dr_t \right],$$

where $c_t^o \equiv \log(C_t^o / C)$, $\pi_t \equiv \log(P_t / P_{t-1})$ is the time-$t$ rate of inflation, $dR_t \equiv R_t - R$ ($dr_t \equiv R_t^{real} - R^{real}$), i.e. the deviation of nominal (real) interest rate from its steady-state value. $dr_t$ can also be interpreted as the discount factor shock. Notice that $h_o = 0$ delivers the usual Euler equation without habit formation.

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8 The fact that Eggertsson (2010) log-linearise while Christiano (2010) linearise the same model does not affect the main conclusions. Here we follow the latter strategy.

9 Following the appendix of Christiano (2010) the time varying discount factor is made equal to the inverse of the real interest rate ($R_t^{real}$):

$$\beta_t = \frac{1}{1 + R_t^{real}}$$

which can be linearised as:

$$\beta_t \beta_d = \frac{1}{(1 + r)^2} dr_t,$$

where $\beta_d = (\beta_t - \beta) / \beta$ and $dr_t \equiv R_t^{real} - R^{real}$. It follows by using the steady-state condition
The combination of equation (7) and the definition of Tobin’s $Q$ results, after linearisation, in the following expression\(^\text{10}\):

\[ q_t = \beta(1 - \delta)E_t q_{t+1} + (1 - \beta(1 - \delta))E_t r_t^k - \beta(dR_t - dr_t), \tag{10} \]

where $q_t \equiv \log(Q_t/Q)$, $k_t \equiv \log(K_t/K)$ and $r_t^k \equiv \log(R_t^k/R^k)$. It is instructive to observe that the discount factor shock ($dr_t$) appears in the capital-Euler equation (10) as well causing a decline in Tobin’s $q$ when $dr_t < 0$.

Similarly, the substitution of the definition of Tobin’s $Q$ into equation (6) leads to a dynamic relationship between investment and the implicit price of capital (i.e. Tobin’s $Q$) which can be linearised to yield:

\[ inv_t = \frac{1}{1 + \beta} inv_{t-1} + \beta E_t inv_{t+1} + \frac{1}{\phi_{inv}(1 + \beta)} q_t, \tag{11} \]

where $inv_t \equiv \log(Inv_t/Inv)$.

Also the linear version of the evolution of capital in equation (4) can be written as:

\[ k_{t+1} = \delta inv_t - (1 - \delta)k_t. \tag{12} \]

The linearised equilibrium condition describing the relationship between the rental rate and capital utilisation is:

\[ r_t^k = \frac{\Omega''(\mathcal{U})}{\Omega'(\mathcal{U})} u_t = \Omega_u u_t. \]

Following Christiano et al. (2005) we assume that the capital utilisation in steady-state is $\mathcal{U}_t = \mathcal{U} = 1$ and $\Omega(1) = 0$, $\Omega'(1) > 0$ and $\Omega''(1) > 0$. The labor supply of OPT household is determined by the union’s problem (discussed below).

### 2.1.2 Non-Ricardians

Non-Ricardian households cannot invest either into physical capital or into bonds. In other words, they are excluded from financial and capital markets. Hence, this

\[ \beta = 1/(1 + R_{\text{real}}) = 1/(1 + R) \text{ that:} \]

\[ \beta dr_t. \]

\(^\text{10}\)Here we depart from Gali et al. (2007) by assuming investment adjustment costs instead of their capital adjustment costs. Investment adjustment costs are more plausible empirically and widely used in middle-sized DSGE models like the Smets-Wouters (2007) model.
is the case of limited asset market participation. Therefore, ROT do not make consumption-savings decision (i.e. the lack of consumption Euler equation). ROT households' consumption depends on their disposable income—i.e. the labor income after taxation, \((1 - \tau_t)W_tN_t^r\)—which is reflected by their budget constraint:

\[
\int_0^1 P_t(i)C_t^r(i)di = (1 - \tau_t)W_tN_t^r - P_tS^r, \tag{13}
\]

where \(C_t^r(i)\) and \(N_t^r\) are, respectively, the consumption of product \(i\) and hours worked by rule-of-thumb households. The lump-sum tax, \(S^r\), ensures that the steady-state consumption of each types of households coincide. ROT agents exploit relative price differences in the construction of their consumption basket and, in optimum, they obtain:

\[
P_tC_t^r = \int_0^1 P_t(i)C_t^r(i)di.
\]

Thus, a ROT household maximises its utility (equation (2) with \(i = r\)) with respect to its budget constraint (equation (13)).

The budget constraint of ROT households in equation (13) can be expressed in linear form as:

\[
c_t^r = w_t + n_t^r - \chi \hat{r}_t,
\]

where \(\hat{r}_t \equiv \tau_t - \tau, \chi \equiv 1/(1 - \tau).\) Note that in the case of log utility in consumption—which is our baseline calibration, see below—and in the absence of time-varying transfer payments (or lump-sum taxes) in the budget constraint of ROT hours worked for ROT household is constant and \(n_t^r = 0\) (for such a configuration see Ascari et al. (2011))

ROT households delegate their labor supply decision to unions (see next section).

### 2.2 Unions

To introduce wage stickiness into the model one usually assumes that households have monopoly power in determining their wage as in Erceg et al. (2001) who presume that each household can engage in perfect consumption smoothing. However, the presence of ROT households who cannot engage in intertemporal trade precludes the possibility of consumption smoothing. To motive a wage-setting decision we suppose following Gali et al. (2007) and Furlanetto and Seneca (2009) that there are a continuum of unions (on the unit interval), \(z \in [0, 1]\), each representing a continuum of workers of which a fraction \((\lambda)\) are members of rule-of-thumb and the remaining \((1 - \lambda)\) fraction consists of optimising households. Each union employs one
particular type of labor (independently of the type households they originate from) that is different from the type of labor offered by other unions. The labor services supplied by each union is a Dixit-Stiglitz aggregator of the members’ labor services:

\[ N_t(z) = \left( \int_0^1 [N_t(z, i)]^{\varepsilon_w} d\bar{i} \right)^{-\frac{1}{\varepsilon_w - 1}}, \]

where \( \varepsilon_w \) is the elasticity of substitution across different types of households.

Each period the union maximises the weighted current and discounted future utility of its members:

\[ E_t \sum_{T=0}^{\infty} \beta^{t+T} \left[ \lambda U_{t+T}^r + (1 - \lambda) U_{t+T}^o \right] \]

subject to the labor demand function for labor of type \( z \):

\[ N_t(z) = \left( \frac{W_t(z)}{W_t} \right)^{-\varepsilon_w} N_t \]

where \( W_t(z) \) is the nominal wage set by the union \( z \) and \( \varepsilon_w \) is the elasticity of labor demand. We follow Furlanetto (2011) in assuming that wage adjustments are costly and evolve similarly to Rotemberg (1982) who originally applied it to model price adjustment. In particular, there is a wage adjustment cost which is a quadratic function of the change in the nominal wage and proportional to the aggregate wage bill. The presence of this wage adjustment cost is justified by the fact that unions have to negotiate wages each period and this activity consumes real resources. The larger is the increase in nominal wage achieved by the union the higher is the effort associated with it. Each union members incurs an equal share of the wage adjustment cost. Thus, the nominal membership fee, \( F_t(z) \) paid by a generic union member \( z \) at time \( t \) is given by:

\[ F_t(z) = \frac{\phi_w}{2} \left( \frac{W_t(z)}{W_{t-1}(z)} - 1 \right)^2 W_t N_t \]

where \( \phi_w \) governs the size of the adjustment costs. In the special case of \( \phi_w = 0 \) the model coincides with the one in Gali et al. (2007).

The first-order condition associated with the union’s problem is the same as the one in Furlanetto (2011):

\[ 0 = \left( \lambda \frac{\partial U_r^r}{\partial C_t^r} + (1 - \lambda) \frac{\partial U_o^o}{\partial C_t^o} \right) (1 - \tau_t) \tilde{W}_t [\varepsilon_w - 1 + \phi_w (\Pi_t^w - 1) \Pi_t^w] - \varepsilon_w N_t \phi_w^\ast - \beta \left[ \left( \lambda \frac{\partial U_{t+1}^r}{\partial C_{t+1}^r} + (1 - \lambda) \frac{\partial U_{t+1}^o}{\partial C_{t+1}^o} \right) \phi_w (\Pi_{t+1}^w - 1) \Pi_{t+1}^w W_{t+1} N_{t+1} \right] \]

\( \frac{1}{\bar{P}_{t+1}} N_t, \) \( \quad (14) \)
where $\Pi^w_t \equiv W_t / W_{t-1}$ is the wage inflation, $\bar{W}_t \equiv W_t / P_t$ is the real wage and $\partial U_i / \partial C_t^i$ is defined by equation (5) for $i \in \{o, r\}$. The consumption differs between the two types of consumers. When making a decision on labor demand the firm does not distinguish between different workers of type $z$. Thus, in the aggregate, $N^r_t = N^o_t = N_t$ holds i.e. they work the same amount of hours. The linearisation of equation (14) yields what we call the new-Keynesian wage Phillips curve:

$$\pi_{w,t} = \beta E_t \pi_{w,t+1} - \kappa^w [w_t - mrs_t - \chi_{t} \hat{\tau}],$$

(15)

where $\pi_{w,t} \equiv \log(\Pi^w_t / \Pi^w_{t-1})$, $w_t \equiv \log(\bar{W}_t / \bar{W})$, $\hat{\tau}_t \equiv \tau_t - \tau$, $\kappa^w \equiv \frac{\varepsilon_{w-1}}{\phi_w}$111 and the linearised expression for the marginal rate of substitution is12

$$mrs_t = \chi_r(c^r_t - h_r c^r_{t-1}) + \chi_o(c^o_t - h_o c^o_{t-1}) + \varphi n_t,$$

(16)

where

$$\chi_r \equiv \frac{\sigma}{\lambda} \frac{(1 - h_o)^{\sigma}}{1 - h_r \lambda (1 - h_o)^{\sigma} + (1 - \lambda)(1 - h_r)^{\sigma}},$$

$$\chi_o \equiv \frac{\sigma}{\lambda} \frac{(1 - \lambda)^{\sigma}}{1 - h_o \lambda (1 - h_o)^{\sigma} + (1 - \lambda)(1 - h_r)^{\sigma}}.$$

Note that in case of $h_o = h_r = 0$ equation (16) boils down to the case of CRRA utility without habits. Without loss of generality we assume following Furlanetto and Seneca (2011) that $h_r = 0$ and $h_o > 0$. The connection between the wage inflation ($\pi_{w,t}$), price inflation ($\pi_t$) and the real wage ($w_t$) can be expressed, in linear form, as:

$$\pi_{w,t} = w_t - w_{t-1} + \pi_t.$$  

(17)

### 2.3 Firms

The intermediary goods are produced by monopolistically competitive firms of which a randomly selected $1 - \xi^p$ fraction is able to set an optimal price each period as in Calvo (1983) while the remaining $\xi^p$ fraction keep their price fixed. Intermediary good $z$, denoted as $Y(z)$, is produced by a constant returns-to-scale Cobb Douglas technology:

$$Y_t(z) = \left[ U_t K_{t-1}(z) \right]^\alpha \left[ N_t(z) \right]^{1-\alpha},$$

(18)

111In calculating the value of $\kappa^w$ we use $(1-\xi^w)(1-\beta^w) \frac{1}{1+\phi_w} \frac{1}{\varepsilon_{w-1}}$ which results in case of Calvo wage setting and equivalent to $\frac{\varepsilon_{w-1}}{\phi_w}$ that we obtain under Rotemberg wage setting.

12Note that we assume a tax policy that equates steady-state consumptions across household types (i.e., $C^r = C^o$).
where $K_t$ is capital, $N_t$ is hours worked and $U_t$ is the degree of capital utilisation.

There is a competitive firm which bundles intermediate goods into a single final good through the Kimball (1995) aggregator:

$$
\int_0^1 \mathcal{G}(X_t(z))dz = 1,
$$

where $X_t(z) \equiv Y_t(z)/Y_t$ is the relative demand and $\mathcal{G}$ is a function with properties $\mathcal{G}(1) = 1$, $\mathcal{G}' > 0$ and $\mathcal{G}'' < 0$.

The profit maximisation problem of the perfectly competitive goods bundler gives way to the relative demand for the product of firm $z$:

$$
X_t(z) = \tilde{\mathcal{G}}\left(\frac{P_t(z)Y_t}{v_t}\right)
$$

where $\tilde{\mathcal{G}} \equiv \mathcal{G}'^{-1}(\cdot)$ and $v_t$ is multiplier of the constraint in the Lagrangean representation of this maximisation problem.

The price deflator can be implicitly defined by

$$
P_tY_t = \int_0^1 P_t(z)Y_t(z)dz
$$

and

$$
v_t = P_tY_t\left(\int_0^1 \mathcal{G}'(X_t(z))X_t(z)dz\right)^{-1}.
$$

Let us define the price elasticity of demand by

$$
\Xi(X_t(z)) \equiv -\frac{\mathcal{G}'(X_t(z))}{\mathcal{G}''(X_t(z))X_t(z)}.
$$

In the special case when

$$
\mathcal{G}(X_t(z)) = [X_t(z)]^{\frac{\varepsilon_p}{1-\varepsilon_p}},
$$

equation (19) boils down to the usual Dixit-Stiglitz aggregator which implies constant elasticity of substitution: $\Xi(X_t(z)) = \varepsilon_p$ for all $X_t(z)$ (Woodford (2003)). In the standard Dixit-Stiglitz case the demand function can be written as

$$
X_t(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon_p},
$$
where the price index is defined as:

\[ P_t = \left( \int_0^1 [P_t(z)]^{1-\varepsilon_p} \, dz \right)^{1/(1-\varepsilon_p)}. \]

In the general Kimball case the own price elasticity of the elasticity of demand can be defined as

\[ \epsilon(X_t(z)) \equiv \frac{\partial \Xi(X_t(z))}{\partial P_t(z)} \frac{P_t(z)}{\Xi(X_t(z))}, \]

where in steady-state \( \epsilon(1) = \epsilon > 0 \) i.e. the elasticity declines if the firm sells more or, equivalently, elasticity is increasing in the price (Furlanetto and Seneca (2009)).

Intermediary firm \( z \) that last reset its price at time \( T = 0 \) maximises its present and discounted future profits with the probability of not resetting its price:

\[ \max_{p_t^*} \sum_{T=0}^{\infty} (\xi^p)^T \Lambda_{t,t+T} \left[ p_{t+T}(z)Y_{t+T}(z) - TC(Y_{t+T}(z)) \right], \]

where \( p_t^* \) is the optimal reset price at time \( t \), \( \xi^p \) is the probability of not resetting the price, \( TC \) stands for the total cost of production and \( \Lambda_{t,t+T} \) is the stochastic discount factor defined as:

\[ \Lambda_{t,t+T} \equiv \beta \left( \frac{C_t^\alpha - h_0 C_{t+1}^\alpha}{C_{t+T}^\alpha - h_0 C_{t+T-1}^\alpha} \right)^\sigma \frac{P_t}{P_{t+T}}. \]

This firm’s maximisation problem is subject to the production function in equation (18) and to the demand function of good \( z \) in equation (20).

The first order condition with respect to \( p_t^*(z) \) is given by:

\[ \sum_{T=0}^{\infty} (\xi^p)^T E_t \left\{ \Lambda_{t,t+T} Y_{t+T}(z) \left[ p_t^*(1 - \Xi(X_t(z))) - \Xi(X_t(z))P_t^T S_t|t+T(z) \right] \right\}, \tag{21} \]

where \( S_t|t+T(z) \) is the time \( t + T \) real marginal cost of firm \( z \) that last changed its price at time \( t \).

The cost minimisation problem of the intermediary yields the demand for labor, the demand for capital and the marginal cost respectively:

\[ W_t(z) = S_t(z)(1 - \alpha) \frac{Y_t(z)}{N_t(z)}, \tag{22} \]

\[ R_t^k(z) = S_t(z) \alpha \frac{Y_t(z)}{U_t K_{t-1}(z)}, \tag{23} \]
\[ S_t(z) = \alpha^{-\alpha} (1 - \alpha)^{-(1-\alpha)} (B_t^k(z))^\alpha [W(z)]^{1-\alpha}, \]

which, after imposing symmetric equilibrium, can be expressed in their linear form, respectively, as:

\[ w_t = s_t + y_t - n_t, \]
\[ r_t^k = s_t + y_t - k_{t-1} - u_t, \]
\[ s_t = \alpha r_t^k + (1 - \alpha)w_t, \]

where \( s_t \) stands for the average real marginal cost.

The evolution of the aggregate price level in the Calvo model is given by:

\[ P_t \equiv \left[ (1 - \xi^p) (p_t^*)^{1-\varepsilon_p} + \xi^p P_t^{1-\varepsilon_p} \right]^{1/(1-\varepsilon_p)}. \]

The loglinear version of equation (21) is the so-called new-Keynesian price Phillips curve:

\[ \pi_t = E_t \pi_{t+1} + \kappa s_t, \quad (24) \]

where

\[ \kappa \equiv \frac{(1 - \xi^p)(1 - \beta \xi^p)}{\xi^p} \frac{1}{1 + I \frac{\epsilon}{1-\varepsilon_p}}, \quad (25) \]

where \( I \) is an indicator variable that can take on the value of one or zero. When \( I = 1 \) the model contains real rigidity in the form of Kimball (1995) demand. In Experiment one (see below) which utilises the above model without wage stickiness, habit formation and endogenous capital accumulation real rigidity is necessary because it helps to avoid a non-uniqueness problem (for more on this see footnote (21)). Experiment five which contains a model with capital also employs Kimball preferences. In the robustness analysis section we further elaborate on the importance of strategic complementarity\(^\text{13}\).

### 2.4 Fiscal and Monetary Policy

#### 2.4.1 Fiscal policy

Similarly to Christiano (2010) and Christiano et al. (2011) we consider a deterministic experiment: the tax rate is cut with the same amount in each period for the entire duration of the shock. There are at least two ways to finance the cut in the labor tax rate. The first scenario is the simplest one possible: the payroll tax cut is

\(^\text{13}\)See also Kaszab (2011) who found that strategic complementarity can change the magnitude of fiscal multipliers.
financed by lump-sum taxes levied on Ricardian consumers. In the second case the same tax cut is financed by government debt that is paid back through distortionary taxes that are levied on both types of households (uniform taxation).

In the experiments with distortionary taxation (see below) we assume in contrast to Gali et al. (2007) and in accordance to Rossi (2012) that the steady-state level of debt is not zero and public debt is allowed to fluctuate along the business cycle. The government budget constraint which implicitly describes the evolution of debt reads as:

\[ R_t^{-1} B_{t+1} + P_t \tau_t Y_t = B_t + P_t G_t \]

which gives way after linearisation to:

\[ b_{t+1} = \beta dR_t + \frac{1}{\beta} \left( b_t \tau_t - \frac{\tau}{\gamma_b} (y_t + \frac{1}{\tau} \hat{\tau}_t) + \frac{1}{\gamma_b} g_t \right), \] (26)

where \( \gamma_b \) is the ratio of debt to GDP and the debt is defined as: \( b_t \equiv (B_t - B)/B \), \( g_t \equiv (G_t - G)/Y \) and \( B_t \) is in real terms. \( \hat{\tau}_t \) is defined earlier. For the rest of the paper we set \( g_t = 0, \forall t \).

Rossi (2012) proposes the following government revenue rule based on Leeper (1991):

\[ Y_t \tau_t = \delta_0 + \delta_1 \frac{\tau Y}{b} (b_t - b) + \delta_2 \tau (Y_t - Y), \]

where \( \delta_0 > 0 \). As in Leeper (1991) and Rossi (2012) there is no restriction on the values of \( \delta_1 \) and \( \delta_2 \). We refer to \( \delta_2 > 0 \) (\( \delta_2 < 0 \)) as procyclical (countercyclical) fiscal policy. The latter revenue rule can be linearised to yield:

\[ \hat{\tau}_t = \delta_1 b_t + (\delta_2 - 1)y_t. \] (27)

Coefficient \( \delta_1 \) governs the response of taxes to debt. The higher \( \delta_1 \) is the more government relies on deficit financing and the further we are from balanced budget which is a very strict and empirically questionable feature.

Based on Leeper (1991) and Rossi (2012) we note that determinacy is guaranteed by an active (passive) monetary policy, \( \phi_\pi > 1 \) and a passive (active) fiscal policy, \( \delta_1 > \left( \frac{1}{\beta} - 1 \right), \left( \delta_1 < \left( \frac{1}{\beta} - 1 \right) \right) \) in the standard case when a Taylor rule is in operation. The determinacy properties of the equilibrium under a rule like in equation (27) in the ZLB is subject to future research.

### 2.4.2 Monetary Policy

Monetary policy is described by the rule in Christiano et al. (2011):

\[ R_t = \max(Z_t, 0) \] (28)
where

\[ Z_t = (1/\beta)(1 + \pi_t)^{\phi_1(1-\rho_R)}(Y_t/Y)^{\phi_2(1-\rho_R)}[\beta(1 + R_{t-1})]^{\rho_R} - 1, \]  

(29)

where \( Z_t \) is the shadow nominal interest rate which can take on negative values as well. As usual, we assume that \( \phi_1 > 1, \phi_2 \in [0,1) \) and \( 0 < \rho_R < 1 \). \( \phi_1 \) controls how strongly monetary policy reacts to changes in inflation while \( \phi_2 \) governs the strength of the response of nominal interest to changes in output-gap. The main implication of the rule in equation (28) is that whenever the nominal interest rate becomes negative, the monetary policy set it equal to zero, otherwise it is set by the Taylor rule specified in equation (29). The parameter \( \rho_R \) measures how quickly monetary policy reacts to changes in inflation and output-gap. Furthermore, inflation in steady-state is assumed to be zero which implies that steady-state net nominal interest rate is \( 1/\beta - 1 \).

The monetary policy rule above can be written, in linear form, as:

\[
\begin{align*}
   dR_t &= \begin{cases} 
   dZ_t, & dZ_t \geq - \left( \frac{1}{\beta} - 1 \right), \ 'zero\ bound\ not\ binding' \\
   - \left( \frac{1}{\beta} - 1 \right), & \text{otherwise,} \ 'zero\ bound\ binding',
   \end{cases} \\
   dZ_t &= \rho_R dR_{t-1} + (1 - \rho_R) \frac{1}{\beta} [\phi_1 \pi_t + \phi_2 Y_t].
\end{align*}
\]

Hence, the ZLB on the nominal interest binds when \( dR_t = - \left( \frac{1}{\beta} - 1 \right) \). Otherwise, we set \( dR_t = dZ_t \).

### 2.5 Aggregation, Market Clearing and Equilibrium

The aggregate consumption and hours worked is a composite of those of the two types of households:

\[
   C_t = \lambda C^r_t + (1 - \lambda) C^o_t, \\
   N_t = \lambda N^r_t + (1 - \lambda) N^o_t.
\]

The aggregate capital, investment and dividend payments is determined by \( K_t = (1 - \lambda) K^r_t, Inv_t = (1 - \lambda) Inv^r_t \) and \( D_t = (1 - \lambda) D^o_t \).

The aggregation equation with the assumption of consumption and hours of different types of households are equal in steady-state:

\[
   c_t = \lambda c^r_t + (1 - \lambda) c^o_t,
\]

\[^{14}\text{Precisely, the term } Y_t/Y \text{ does not stand for the output gap as the definition of the output gap contains the deviation of the actual GDP from its flexible price level equivalent. Here we simply use the deviation of output from its steady-state value.}\]
\[ n_t = \lambda n_t^r + (1 - \lambda) n_t^o, \]

which we obtained by setting steady-state consumption and hours worked of each type equal in steady-state \((C^r = C^o \text{ and } N^r = N^o)\) using a lump-sum tax (see the previous discussion in the section describing Ricardian households).

In the absence of lump-sum taxes in the budget constraint of the ROT household and assuming log utility in consumption \((\sigma = 1)\) the hours worked by ROT household is constant and, thus, \(n_t^r = 0\).

The goods market clearing is
\[ Y_t = C_t + Inv_t + G_t + \Omega(U_t)K_{t-1}, \]

which can be expressed in linear form as
\[ y_t = \gamma_c c_t + \gamma_i inv_t + g_t + \gamma_k R^k u_t, \]

where for the rest of the paper we set \(g_t = 0\) and \(\gamma_c\) is calculated as described in Appendix B of Gali et al. (2007):
\[ \gamma_c = 1 - \gamma_t - \gamma_g = 1 - \frac{\delta \alpha}{\alpha(Y/K)} - \gamma_g = 1 - \frac{\delta \alpha}{\mu^p(\rho + \delta)} - \gamma_g, \]

where \(\gamma_c \equiv C/Y, \gamma_t \equiv I/Y, \gamma_g \equiv G/Y, \gamma_k \equiv K/Y\) and the last equality made use of the fact that in steady-state \(R^k = \alpha Y/\mu^p K = \alpha/(\gamma_k \mu^p)\) which assumes that the steady-state marginal cost is constant and equal to the inverse of the markup defined as \(\mu^p \equiv \varepsilon_p/(\varepsilon_p - 1)\). The steady-state rental rate is \(R^k = \rho + \delta\) with \(\rho \equiv \beta^{-1} - 1\).

After having outlaid the building blocks we are ready to define equilibrium of this model. The equilibrium is characterised by a sequence of endogenous quantities
\[ \{K_t, N_t^o, N_t^r, N_t, C_t^o, C_t^r, C_t, Inv_t, U_t, Y_t, B_t\}_{t=0}^\infty, \]

price sequences
\[ \{Q_t, \Pi_t, \Pi_t^{\text{wh}}, R_t^k, W_t, S_t, R_t, Z_t\}_{t=0}^\infty, \]

and a given set of exogenous deterministic shocks
\[ \{R_t^{\text{real}}, \tau_t\}_{t=0}^\infty \]

and initial values for the state variables (capital and debt) that satisfy equilibrium conditions of the household, firms, unions, government and monetary authority such that markets clear, the transversality conditions for the endogenous states are imposed and the aggregate resource constraint is also satisfied.
3 Calibration

Households. The discount factor, $\beta$, is equal to 0.99 implying a real annual interest rate of 4%. The elasticity of intertemporal substitution, $\sigma$, is set to one implying log utility which is usual in the literature. Following Christiano (2010 and see the references therein) the parameter governing the disutility of labor, $\varphi$, is chosen to be one (i.e. Frisch elasticity of labor supply, $1/\varphi$, is also one) which is more conservative than the value of 0.2 used by Gali et al. (2007). Also, similarly to Christiano (2010) we use $\varepsilon_p = \varepsilon_w = 6$. The steady-state consumption-income ratio, $\gamma_c$, is 0.75. Campbell and Mankiw (1991) estimate the share of rule-of-thumb consumers ($\lambda$) to be 35% while Fuhrer (2000) finds values of 26% and 29% depending on the econometric method he uses. Averaging the finding of the previous studies we set $\lambda = 0.3$ which we think is more plausible empirically than the 0.5 used by Gali et al. (2007).

Monetary Policy. The inflation coefficient in the Taylor rule, $\phi_1$, is 2. Following Christiano (2010) and Christiano et al. (2011) there is neither interest-rate smooting ($\rho_R = 0$) nor response to output gap in the Taylor rule ($\phi_2 = 0$).

Fiscal Policy and Experiments. The debt-GDP ratio, $\gamma_b$ is 2.4 implying an annual steady-state ratio of public debt to output of 60% (see Rossi (2012) for this choice). In the simulations below $\delta_1$ can be either high (0.9) or low (0.1) depending on what we assume about the time horizon of debt repayment (see more on this below). This paper consider countercyclical fiscal policy and sets $\delta_2 = 0.5$ which ensures determinacy of the equilibrium\textsuperscript{15} and implies a mild tax response of tax rate to changes in output. The steady-state labor tax rate ($\tau$) is chosen to be 30% as in Christiano (2010). Although the size of the discount factor shock, $r_t$, varies somewhat across simulations (see below) it is always close to the mode of -0.0104 estimated by Denes and Eggertsson (2010) using a modell that contains only price rigidity and specific labor market. The duration of the shock is 10 periods\textsuperscript{16} which is in accordance with the modal estimate of Denes and Eggertsson (2009).

Firms. The benchmark value of Christiano, Eichenbaum and Evans (2005) for $u$ is 0.01. The depreciation rate of capital is also a standard choice: 0.025 at a quarterly rate. The mean posterior estimates of Smets and Wouters (2007) for the

\textsuperscript{15}In general, we found that the calibration $0 < \delta_2 < 1$ result in a determinate and unique of the equilibrium. Also see Rossi (2012) for more on the issue of how fiscal rules affect the determinacy properties of the equilibrium in a model with ROT and OPT households.

\textsuperscript{16}Eggertsson (2010) and Denes Eggertsson (2009) consider a stochastic experiment with a persistence estimate of $\mu = 0.9030$ for the shock process. This $\mu$ is easily translated into our deterministic experiment knowing that the average duration of this AR(1) is $1/(1 - \rho)$ which is roughly 10. For a similar argument see Appendix C of Carlstrom, Fuerst and Paustian (2012).
Calvo parameters, $\xi^p = 0.66$ ($\xi^w = 0.7$) imply an average price (wage) stickiness of around two (three) quarters. The reduced form estimates (see for references Furlanetto and Seneca (2009)) on the new-Keynesian price Phillips curve imply $\kappa = 0.03$. Without real rigidity such a value of $\kappa$ would imply a very long-period of price inertia ($\xi^p = 0.85$). In our baseline calibration without real rigidity (i.e. $I = 0$) $\xi^p = 0.66$ implies $\kappa = .1786$. When $I = 1$ the calibration of $\kappa = 0.03$ is achieved by setting an appropriate value for $\epsilon$. The implied value of $\epsilon$ is 24.77 which is in the range of empirical estimates listed in Furlanetto and Seneca (2009).

4 Experiments

Our experiments are in the spirit of Christiano (2010) and Christiano et al. (2011) who assumed that the discount factor shock and the corresponding fiscal policy shock is on for a deterministic period of time. The deterministic simulations are executed using the codes of Christiano (2010) and Christiano et al. (2011). In particular, their codes implement a standard shooting algorithm to handle the ZLB problem. The details of this algorithm are available in the appendix of Christiano (2010). Also, Sargent and Ljunquist (2003) explain in a simple model how the shooting algorithm works. The algorithm is designed such that variables in the model hit their long-run steady-state values at a specific future date. Thus, we are looking for an initial value(s) of the endogenous state(s) which makes it possible for all variables to reach their target values at a particular future date in a way that equilibrium conditions are met each period.

4.1 Tax cut is financed by lump-sum taxes

In the first experiment we assume that there are neither endogenous capital (i.e. equations (10), (11) and (12) are eliminated) nor wage stickiness in the economy i.e. $\xi^w = 0$ in the wage Phillips curve in equation (15). The zero lower bound experiment is very similar to the one conducted in Christiano (2010). A discount

\[\text{Note that section 2 and 3 of Christiano et al. (2011) consider a stochastic experiment similar to those in Eggertsson (2010) and Woodford (2010) while section 4 and 5 consider deterministic experiments that are accomplished by using a standard shooting algorithm. In case of only price rigidity (or only wage rigidity) the system can be re-written using the Eggertsson-Woodford (2003) type of methodology applicable if the system contains no state variable. The latter is not true any more in case of the inclusion of both price and wage stickiness when one of the variables (potentially the real wage) becomes an endogenous state. Hence, we make use of the shooting algorithm of Christiano (2010).}

\[\text{The codes are available from Lawrence Christiano’s website.} \]
factor shock hits the economy in period one. The model is in deterministic steady-state until \( t = 1 \). At \( t = 1 \) the discount rate drops from its steady-state value of 0.01 (per quarter) to \( r = -0.01 \) and remains low for \( T = 10 \) quarters\(^{19} \). From quarter 11 (\( T + 1 \)) on the discount rate is back to its steady-state value. Note that all the deterministic experiments below assume that the discount factor shock is on for ten periods although its size slightly varies across them. Regarding how the tax cut is financed let us consider the simplest possible fiscal arrangement: the wage tax cut is backed by current and future rise in lump-sum taxes paid by Ricardian agents. Thus, in experiment 1-5 the government budget constraint (equation 26) and the tax rule (equation 27) are not included among the equilibrium conditions.

The steady-state level of labor tax is 30 per cent. In the no policy response simulation the labor tax rate is at its steady-state level for the entire simulation. In the alternative simulation (denoted with dashed line) the labor tax rate is decreased (in contrast to Christiano (2010) and Christiano et al. (2011) who considered a rise in the tax rate) to \( \tau = 0.2 \) for the time period in which the zero lower bound on the nominal interest rate is binding. The problem is solved using the code of Christiano et al. (2010) that determines endogenously the date at which the zero lower bound becomes binding and the date at which the zero lower bound ceases to bind.

Figure 1 shows the first experiment featuring a model that includes two types of households and price rigidity. Wage rigidity, capital and habits are excluded from the model of this experiment\(^{21} \). In the absence of tax policy the ZLB ends in period 6 while the presence of tax policy makes the ZLB bind for 9 periods.

The tax cut magnifies the deflationary effects described by Eggertsson (2010a):
the price deflation and the contraction in hours are more severe with the tax cut. Also note that the drop in real wage—which equals to the marginal cost due to constant returns assumption—is considerable. The consumption falls for both types of households. Hence, the wage tax cut does not alleviate the negative consequences of the savings shock (huge deflation and fall in output). In fact, the labor tax cut makes the zero lower bound even more binding. When zero lower bound ceases to bind the Taylor rule becomes operational again and monetary policy reacts to expansionary fiscal policy (i.e. the labor tax cut) by raising the nominal interest rate. Hence, there is a large upward movement in nominal interest following the zero lower bound period.

Figure 2 shows an experiment similar to the first one but this time we introduce wage stickiness into the model (second experiment). The discount rate is set to \(-0.02\) per quarter. The ZLB binds for period 1 to 6 (7) without (with) policy. Wages are set by unions and assumed to remain fixed for about 3 quarters. Again we operate with a simple fiscal scenario: the wage tax cut is backed by current and future rise in lump-sum taxes paid by Ricardian agents. In this particular case OPT internalise the government budget constraint. The wage tax cut increases the disposable income of ROT households who consume it. In the absence of policy change the real wage does not fall dramatically due to the presence of wage stickiness in sharp contrast to the previous experiment. But still the tax cut remains deflationary (as the labor supply shifts to the right) and the real wage in the case of tax policy falls more than without policy. Observing the graphs we can also see that the wage deflation is higher than the price deflation implying a fall in the real wage rate. With perfect wage-stickiness (\(\xi_w\) is close to one)—which is not the case here but serves as a useful abstraction (see e.g. the argument of Christiano (2010))—the labor supply would remain inact. In the next we analyse the indirect reaction of labor demand to the tax cut. Before that, mention must be made of the slope of labor demand that turns from negative to positive in ZLB environment. Following Eggertsson (2010a, 2010b) who, differently from us, considered a stochastic experiment we show that the combination of the new-Keynesian price Phillips curve, the production function and the Euler equation results in a positive relationship between the real wage and hours worked.

The higher consumption demand of non-Ricardian agents induces many of the firms which cannot charge a higher price due to price stickiness to increase their production. To produce more firms demand more labor i.e. the labor demand shifts out. As it is well-known in sticky-price models a rise in aggregate demand—due to the higher consumption expenditures of ROT households—leads to a fall in the markup, which induces an outward shift in the labor demand. Below we argue that the higher labor demand is supported by an increase in labor supply when wages are
Our setup features a negative preference shock that causes deflation and output loss. Without tax policy the labor demand shifts to the left leading to a decrease in the real wage. The wage tax cut can mitigate this effect by boosting aggregate demand (and labor demand) under sticky prices through the increase in consumption of non-Ricardian consumers.

Let us discuss what happens to wages and hours after an increase in labor demand (induced by a fall in the markup) when nominal wages are rigid. Ascarì et al. (2011) present a graph (see the left-hand side graph on figure 3) indicating that the slope of the wage schedule in the sticky wages case is flatter than the one in the flexible wage case. In the flexible wages case the real wage increases a lot in response to a rise in aggregate demand and hours change only to small extent. Therefore, under flexible wages, there is scope for a negative income effect through a decrease in profits to emerge if labor supply is inelastic (i.e. the change in hours is small). This negative income effect translates into a leftward shift in the labor demand depressing the wage for both positive and zero interest rates. Furthermore, Bilbiie et al. (2012) argue that this negative income effect ensures that the labor supply of OPT increases more than the amount by which the labor supply of ROT decreases. Thus, the rise in labor demand is supported by an increase in aggregate labor supply.

But this is not the case anymore for sticky wages when it is hours worked that respond more after a surge in demand and the real wage changes to small extent (see the right-hand side graph on figure 3). In Experiment 2 wage deflation is always more negative than price deflation leading to an increase in the profits of OPT households for given hours. However, this positive income effect is still not enough for OPT households to generate a rise in consumption (see element (1,2) of figure 2 where ROT consumption in case of tax cut policy coincides with those without policy). This can be explained by the behaviour of OPT households who are assumed to bear the whole burden of the tax cut bringing about a negative wealth effect that decrease.

22The wage schedule is derived from the wage Phillips curve assuming, for the sake of a comparative static exercise—the outward shift in the labor demand—that forward-looking terms are constant.

23When the share of ROT agents are high (i.e. λ is big) a unit fall in profit results in a decline of dividends that is bigger than one as OPT households receive 1/(1 − λ) share of the dividends (Bilbiie (2008)).

24As Ascarì et al. (2011), Bilbiie (2008), Bilbiie et al. (2012) and Rossi (2012) argue there is a negative income effect that has to be accounted for in the presence of heterogenous households and flexible labor market. Namely, after a positive demand shock the wage inflation is always higher than the price inflation which leads to a fall in profit income after a positive demand shock on condition that labor supply is inelastic (i.e. the change in hours is small enough).

25The labor supply of ROT falls because of the positive income effect of the tax cut.
their total discounted life-time consumption. Due to this huge negative wealth effect it remains to be true that labor supply of OPT rises by more than the amount by which labor supply of ROT decreases and, thus, the total hours worked can rise too. Thus, in this experiment, the recession is mitigated by means of labor tax policy (see the dashed line on the graph depicting hours that falls by less in case of a labor tax cut policy). Let us explore whether this result stays robust if we add habit formation to the model.

The third experiment shown on Figure 4 makes use of the model in the previous simulation but now it includes external habit formation in consumption as well. Due to the lagged consumption term habit formation injects some endogenous persistence into the model and leads to hump-shaped impulse responses in OPT consumption and hours. Habit formation is a well-known feature of middle-sized DSGE models like the one of Smets and Wouters (2007) and is found useful in matching the empirical VAR evidence. Also habit formation is usually regarded to have some solid psychological foundation. Our baseline calibration assumes that only OPT households feature habit formation in consumption although our results remain still valid in the case when it it also ROT households who care about their past consumption (the graph of this experiment is not reported here). The presence of habits mitigates the negative effects of the shock. This can be explained as follows. As argued above it is the rise in the real interest that makes people delay their consumption expenditure. The introduction of habits reduces the sensitivity of consumption to changes in the real interest (this can be quickly verified by looking at equation (9) where the coefficient on the interest is smaller in case of habits \( \frac{1-\beta}{1+\beta} \) than it is for the standard CRRA case \( \frac{\beta}{\sigma} \)). To generate a fall in variables of magnitude similar to the those in the previous experiments we consider a somewhat bigger drop in the discount factor (-0.03). The ZLB binds from period 1 to 8 (10) without (with) policy. Still output (hours) declines less when labor tax cut policy is applied.

In the fourth experiment on Figure 5 we add endogenous capital accumulation preserving all the other previous properties but abstracting from habit formation. Here we set the size of the discount factor shock to -0.015. The ZLB ceases to bind with (without) policy in periods 1 to 8 (7). Again output, consumption and hours fall less when there is a cut in the labor tax. Now there is one more channel, notably investment, that supports the favorable effect of tax decrease beyond the positive response of ROT consumption. However, the positive effects of the tax cut on investment are less apparent in the short-run because of the investment adjustment costs. As Monacelli and Perotti (2008) argues the investment is inertial in the short run i.e. it exhibits no response at the beginning and builds up only gradually. The most interesting feature of this experiment the wage deflation under the labor tax.
cut smaller than it is in case of no policy. Hence, the real wage (and also the rental rate on capital) can increase to some extent after the labor tax cut (in other words, it falls by less under the wage tax cut policy scenario relative to the case of no policy change). This finding appears to be quite counterintuitive at the first sight and can be explained as follows. As argued previously the labor tax cut results in a rise in hours worked which shifts out the marginal product of capital and leads to higher investment (which is muted by investment adjustment costs). Being predetermined the capital remains fixed at the beginning and builds up gradually so that the equilibrium capital:labor ratio remains to be satisfied in the long-run. This additional surge in investment might be enough to generate a labor demand shift that finally leads to a rise in the real wage.

In the fifth experiment on figure 6 we consider the setup in the fourth experiment with external habit formation, capital utilisation and real rigidity induced by Kimball-demand setting $I = 1$ in expression (25). Several things are interesting in this graph. Note that the real wage cannot increase (or fall by less) in case of the tax cut policy anymore in this setting due to the presence of habit formation that slows down the reaction of consumption. In particular, the consumption of ROT consumers declines to higher extent in case of habit formation and the rightward shift in the labor demand is limited. As a result, the real wage cannot increase and the finding of Experiment 2-3 is reconrmed. In the next we conduct some robustness checks.

Robustness Analysis

It is instructive to ask how robust our results are to a i) change in the coefficient of intertemporal elasticity of substitution (IES) or to a ii) change in the elasticity of the labor supply or to the iii) higher strategic complementarity in price-setting or iv) to the size of the discount factor shock. All sensitivity checks are conducted by using the model in Experiment 5. Corresponding graphs are not reported in the paper but available on request.

Let us first consider that the coefficient of IES rises from $\sigma = 1$ (the case of log-utility) to $\sigma = 1.5$. It is well-known in economics theory that for a utility that is separable in consumption and leisure—like the one here given by equation (1) with $h_i = 0$—the coefficient of $\sigma$ higher than one implies that an increase in the real wage has an income effect which is stronger than the substitution effect (see e.g. Mankiw and Weinzierl (2012) whose calibration is $\sigma > 1$). For $\sigma = 1$ the

26By employing capital utilisation we can mute the negative wealth effect of the labor tax cut on OPT households. As a result consumption and output will fall by less.
income and substitution effects offset each other. Due to the tax cut both types of households receive more money after each hour worked and they are willing to increase their labor supply (substitution effect). At the same time agents realise that their after-tax income is higher and happy to consume more and are also eager to enjoy some more leisure time (income effect). Also with a coefficient of IES different from one the labor supply of the ROT is not constant any more. OPT households consider, however, their total discounted value of their life-time wealth when choosing their consumption path. They interpret the temporary tax cut as future rise in taxes and not do not feel inclined to consume more. When \( \sigma > 1 \) both types of agents feel wealthier and willing to raise their expenditures. Thus, it follows that the consumption of OPT (in case of \( \sigma > 1 \)) falls to a smaller extent while it rises a bit more for ROT. Consequently, the aggregate consumption falls by less. The labor tax cut remains to be an effective tool in alleviating the negative effects of the shock.

Next we investigate into the effects of an increase in labor supply elasticity (i.e. a fall in \( \varphi \)). As Bilbiie (2008) shows hours and consumption of ROT respond more to changes in real wage if \( \varphi \) is smaller. Let us set \( \varphi = 0.2 \) which is the one considered by Gali et al. (2007). In particular, the more elastic labor supply is the more consumption and hours increase in reaction to a change in real wage. Thus, a rise in labor supply elasticity has effects similar to the case of less wage rigidity. As argued previously the more flexible are wages—or the more elastic is the labor supply—the higher is the fall in output and the larger are the wage and price deflation in the ZLB. Hence, the bigger is the fall in the real wage the less ROT consumption increases after the wage tax cut. Despite all these effects the reaction of output remains similar to that with the baseline calibration because investment adjusts in a way that it makes up for the bigger loss in ROT consumption due to higher labor supply elasticity.

As argued by Woodford (2003) the introduction of some real rigidity like specific capital (or labor) market or Kimball demand induces price-setting decisions of firms to be strategic complementaries i.e. firms change prices similarly in reaction to exogenous shocks. Using the model in experiment 5 we investigate into the effects of increasing strategic complementarity in price setting. A lower value of \( \kappa \) implies a weaker response of inflation to changes in marginal cost. As argued in the Intro-

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27 Practically, it means that the linearised labor supply term does not drop out of the budget constraint of the ROT household because the actual labor supply may differ from its steady-state level.

28 Note that we introduced strategic complementarity in price setting in the form of Kimball demand implying that \( I = 1 \) in the denominator of expression leading to a fall in \( \kappa \). Alternatively, we could have considered firm-specific capital as a source of strategic complementarity (see Woodford (2003) Chap 5).
duction, marginal cost plays a major role in the propagation of the savings shock. In fact, a low value of $\kappa$ mitigates the deflationary effects that are magnified by staggered price setting channel. These findings generalise to all other experiments as well. Overall, the introduction of strategic complementarity can severely constrain the negative downward movement in output and inflation and, thus, the quantitative nature of our results may alter to some extent. However, qualitatively, the results remain the same.

It is also of interest to ask how robust the results are to variation in the duration of price and wage stickiness. According to the calibration section we considered conservative values for Calvo parameters. Not surprisingly, our results are heavily dependent on the value of the wage-stickiness parameter. If we radically reduce the duration of wage stickiness, for example, from the baseline calibration of around 3 quarters to 1 quarters the positive effect of the wage tax cut policy virtually disappears. Christiano (2010) showed that an extreme long period of wage stickiness (say longer than a year—a Calvo parameter of at least $\xi^w = 0.8$) disqualifies the positive effect of a wage tax hike multiplier in a model with only Ricardian households. However, in contrast to the surprising result of Christiano (2010), we found in a model with rule-of-thumb households that the higher wage rigidity is the higher is the benefit (the smaller is the output loss) from the wage tax cut policy.

Concerning the size of the discount factor shock we conclude, similarly to Christiano et al. (2011), that the results are not affected as long as the time at which the ZLB becomes binding and the time at which the ZLB ceases to bind is the same for the shocks with different size.

The next section investigates into the robustness of the results obtained under lump-sum taxation by considering a different and possibly more realistic way of financing the labor tax cut. Thus, we explore government debt that is paid back by distortionary labor tax.

4.2 Tax cut is financed by government debt

In the last two experiments we assume, more realistically, that the uniform labor tax cut is financed by government debt. Debt is retired by future increases in the labor tax rate imposed on both types of households (instead of having lump-sum taxes). To facilitate the transparency of the discussion we conduct the experiment using the model without capital and habits (see the model in experiment two).

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29Uniform means that both types of households enjoy the tax cut and also bear the burden of future tax increase. Although, in general, it is mainly the ROT household which can benefit from the tax cut.
Rossi (2012) argues that the government budget constraint cannot be separated from the equilibrium conditions any more in case of heterogeneous households and a fiscal rule. By using a fiscal rule (sometimes called as tax rule) in equation (27) we specify how taxes respond to debt. The strength of this response is captured by coefficient $\delta_1$. When the steady-state debt is not zero (this is our case) the interest payments on debt as well as inflation will appear in the linear version of the tax rule. Also we can include a stabilisation motive for fiscal policy by determining how taxes respond to output. In particular, fiscal policy is assumed to be countercyclical in the model, i.e. $0 < \delta_2 < 1$. Under this financing scheme the stimulative effect of the labor tax cut is less straightforward now and will depend on the parameter governing the response of the tax rate to debt. Since Baxter and King (1993) it is well-known that even the government spending multiplier can turn to negative if it is financed by distortionary taxes period-by-period.

Although fiscal policy is chosen to be countercyclical in the model (see our calibration) it is not driving our main result. When fiscal policy does not change through business cycles i.e. $\delta_2 = 1$ (taxes do not respond to deviations of output from its long-run value) in the linear version of the tax rule in equation (27) we arrive at the same conclusion.

However, the choice of $\delta_1$ turns out to be crucial. In particular, the closer $\delta_1$ is to one the quicker the debt is paid back. In the particular case of $\delta_1 = 1$ the tax cut financed by government debt is totally retired in the next period and, thus, only the first and second period wealth of Ricardian households is affected. When $\delta_1$ is low (or tends to its theoretical lower bound of $\left(\frac{1}{\beta} - 1\right)$) then debt is paid off far in the future and has a wealth effect on Ricardian consumers similar to the case of lump-sum taxes (Bilbiie et al. (2012)). Due to the presence of interest payments there is a gradual reduction in debt in the form of tax increases through the tax rule. Although the countercyclical fiscal behaviour offset the tax hikes (note that $\delta_2 - 1 < 0$ in the fiscal rule as we posit that $\delta_2 < 1$).

First let us explore the case of a high $\delta_1$. This is called experiment 6 and shown on figure 7. The real interest drops to -0.03. In this experiment the ZLB bind for period 1 to 7 (8) without (with) policy. When $\delta_1$ is close to one there is some scope for consumption to rise for both types of households. Several features appear to be important. Contrary to the previous experiment the consumption of ROT does not increase considerably on impact but appear to respond more in the long run. The

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[30] Also it is true that the existence of lump-sum taxes in the budget constraint of ROT households would call for an explicit inclusion of the government budget constraint and associated fiscal rule among the equilibrium conditions. Although in our experiments we abstact from this possibility for reasons noted above.
latter happens because now it is also the ROT who have to bear the burden of the tax cut and therefore the rise in their consumption is muted on impact and builds up only gradually. In contrast to previous experiments with lump-sum taxation, the consumption of OPTs is higher for their whole life-time because of the income effect of the tax cut that boost both consumption and leisure.

When $\delta_1$ is high the debt is paid back shortly after the tax cut and the total life-time wealth of OPT is not affected. Hence, consumption of OPT declines less under tax cut policy. However, the rise in aggregate consumption is not enough for real wage to grow (i.e. to fall by less). Also aggregate consumption follows mainly the pattern of the ROT. Initially the debt declines sharply. After the shock is over the debt gradually returns back to its long-run steady-state value.

Now let us turn to the case of low $\delta_1$. We refer to this as experiment 7 and is depicted on figure 8. In this experiment the ZLB bind for period 1 to 7 (7) without (with) policy. As already argued the case of low $\delta_1$ implies that that debt repayment takes up long period of time, which ROT experience as a reduction in their life-time wealth and, hence, they are not induced to raise their consumption expenditure. Thus, the favorable effects of the labor tax cut completely disappear in the case of low $\delta_1$. Again, our results are only marginally affected by the choice of $\delta_2$. Sensitivity checks are not reported here but available on request.

Government debt is usually not paid back in the short run. If we assume more realistically that government debt settled in the medium run there is some scope for labor tax cut policy although it has quite limited positive effect on the economy. We model medium run debt repayment by setting $\delta_1 = 0.5$. In this experiment the ZLB bind for period 1 to 7 (7) without (with) policy. As figure 9 indicates the labor tax cut has still some positive effects in terms of hours and aggregate consumption that deteriorate less.

In summary, we found that the payroll tax cut policy can still be effective if we consider a different way of financing the tax cuts i.e. issuing government debt which is retired as soon as possible.

5 Conclusion

After augmenting the baseline new-Keynesian model containing price and wage rigidity with rule-of-thumb (or non-Ricardian) households we argued that a labor tax cut can be an effective tool to combat the negative consequences (fall in output and deflation) of a shock that made the zero lower bound on the nominal interest rate binding. Importantly, we assumed that we cut the labor tax that is levied upon

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$^{31}$Sensitivity checks are not reported here but available on request.
the households and not upon the firms. Under such an arrangement the labor tax cut acts like a traditional fiscal stimulus that raises aggregate demand. Our results remain valid after the extension of the model with capital accumulation with variable utilisation, external habits in consumption and strategic complementarity in price setting. Interestingly, there may be scope for a labor tax cut in a situation when tax cuts are financed by government debt (instead of lump-sum taxes) and the repayment of debt is not delayed to the far future.

It would be interesting to explore whether our result remains true in another popular model featuring monopolistic competition, which predicts that consumption rises after an increase in aggregate demand even without sticky prices—this is the deep habit model of Ravn, Schmitt-Grohe and Uribe (2006).

References


Figure 1: This is called Experiment 1 in the text. Experiments 1-5 assume that the labor tax cut is financed by lump-sum taxes levied on Ricardian consumers. Z stands for the shadow nominal interest rate. The + signs indicates the date at which the zero lower bound on the nominal interest rate becomes binding and circles appear on the date at which the zero lower bound ceases to bind. ss means steady-state.
Figure 2: See details of Experiment 2 in the text. This is the model in Experiment 1 extended with wage rigidity.
Figure 3: Comparison of labor markets under positive and zero nominal interest rate. WS-sticky stands for the wage schedule under sticky wages while WS-flexible means the wage schedule under flexible wages. Source: the left-hand-side figure is a reproduction of Ascari et al. (2011 page 12) while the right-hand-side one is based on figure 4 of Eggertsson (2010b page 15)
Figure 4: This is called Experiment 3 in the text. Here we used the model in Experiment 2 extended with external habit formation in consumption.
Figure 5: This is called Experiment 4 in the text. Here we used the model in Experiment 2 extended with endogenous capital accumulation.
Figure 6: This is called Experiment 5 in the text. The model used here is the same as in Experiment 4 augmented with external habit in consumption, capital utilisation and strategic complementarity in the form of Kimball-demand.
Figure 7: This is called Experiment 6 in the text. In contrast to previous experiments the model used here assumes that the wage tax cut is financed by issuing debt that is repaid in the short run i.e. the coefficient on debt in the fiscal rule in equation \((27)\) is set to \(\delta_1 = 0.9\).
Figure 8: This is called Experiment 7 in the text. Here we employ the model in Experiment 6 but with the assumption that the debt is retired in the long run (i.e. $\delta_1 = 0.1$ is set in the tax rule).
Figure 9: This is called Experiment 8 in the text. Here we employ the model in Experiment 6 but with the assumption that the debt is repaid in the medium run (i.e. $\delta_1 = 0.5$ is set in the tax rule).