# Cardiff Economics Working Papers

Working Paper No. E2011/6

# Optimal Taxation and Redistribution in a Two Sector Two Class Agents' Economy

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March 2011

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# Optimal Taxation and Redistribution in a Two Sector Two Class Agents' Economy

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Abstract:

We examine the optimal taxation problem in a two sector neoclassical economy with workers and capitalists. We show that in a steady state of this economy the optimal policy may involve a capital income tax or subsidy, differential taxation of labour income and redistribution. The level and the direction of the redistribution associated with such an optimal policy depends on the pre tax allocation of capital but not on the social weights attached to the different groups of taxpayers. Excess production of consumption goods creates a difference between the social marginal values of consumption and investment which in turns violates the production efficiency condition. Such a difference can be undone by taxing capital income from the consumption sector, and with this optimal policy the government can implement a redistribution scheme where both workers and capitalists bear the burden of distorting taxes. On the contrary, an optimal policy that involves a capital income subsidy in the production of consumption can implement allocations that minimize the relative price difference between consumption and investment that resulted from the excess production of investment goods.

JEL Codes: C61, E13, E62, H21.

Keywords: Optimal taxation, Ramsey problem, Two Sector Economy, Redistribution.

# 1 Introduction.

In this paper we show that in a steady state of a two sector economy with two classes of agents, the optimal policy that involves a tax/subsidy on capital income can serve the efficiency as well as the redistributive purposes. In a two sector economy the interdependence of labour and capital margins allows the government to choose an optimal policy that taxes/subsidizes capital income from one sector. A version of this result in a two sector model with heterogeneous agents has been discussed in Selim (2010). In this paper we extend Selim's (2010) result by showing that in a similar economy with two classes of agents the long run optimal policy that involves a tax/subsidy on capital income can serve the efficiency as well as the redistributive purpose. This policy is optimal because it restores the production efficiency condition. We show that even in the extreme case where the government cares about the welfare of only one class of agents, the optimal policy with a capital tax/subsidy in the long run can also serve the redistributive purpose.

Typically investment suffers a decline during economic slowdown, and one common fiscal policy response of most governments during the recent financial crisis has been the implementation of measures that fight such declines in investment. Such measures could involve an increase in the accelerated capital depreciation allowances (e.g. the Economic Stimulus Act of 2008 for the US, or the Fiscal Act 2008 for the UK). The welfare as well as the redistributive properties of accelerated capital depreciation allowances has been at the core of tax debates in the eighties, evidence of which can be found in important papers such as Judd (1984), Judd (1985) and later in Judd (1997). Most of these studies argue that if there are pre-existing distortions in the economy (e.g. imperfectly competitive product market) such fiscal measures can promote investment.

The distributional consequences of capital income tax policy in competitive economies are less well known, however. The two most influential results which are relevant to this discussion are the ones in Chamley (1986) and in Judd (1985). Chamley (1986) shows that in a steady state of a one sector economy, the optimal policy is to set the tax rate on capital income equal to zero. Judd (1985) extends this result in a one sector economy with heterogeneous agents. He shows that with zero capital income tax in the scheme, if the government only values the welfare of workers there will not be any redistribution in the limit, and government expenditures will be financed solely by levying wage taxes on labour. In this paper we extend both these results. We present a two sector neoclassical growth model with two classes of utility maximizing agents: workers and capitalists. We consider two production sectors that produce consumption goods (consumption sector, hereafter) and investment goods (investment sector, hereafter), using raw labour and capital, on which government levies distorting flat-rate income taxes. We construct the classic Ramsey (1927)

problem, i.e. the planner's problem of determining the optimal settings over time for two labour income tax rates and two capital income tax rates. Our model is thus possibly the simplest extension of both Chamley (1986) and Judd's (1985) models. In this setting we examine the optimal policy and its distributional consequences in a steady state.

We show that in a steady state of our model, the optimal capital income tax rate in the investment sector is zero but the optimal capital income tax rate in the consumption sector is in general different from zero. In a two sector economy where investment and consumption are produced as two final goods, capital and labour margins are interdependent, and so is the long run optimal policy of taxing the income from these factors. Due to this interdependence, a long run policy that involves a tax/subsidy on capital income from one sector can serve the efficiency purpose. In a steady state of our model any difference in the relative price of investment and consumption is associated with a difference in the social marginal values of investment and consumption. A tax/subsidy on capital income in one sector leaving the other capital income tax at a zero rate and differential taxation of labour income at tandem can undo this difference which in turns restores the production efficiency condition. This result adds to the literature that argues that optimal capital income tax may be nonzero in a variety of growth contexts, such as Kemp et al. (1993), Aiyagari (1995), Lansing (1999), Chamley (2001), Rehme (2009), Selim (2009) and Selim (2010).

We also show that the optimal policy in a steady state that involves a tax/subsidy on capital income can serve the redistributive purpose. Since the optimal policy in a steady state depends crucially on the initial allocation of capital in the two sectors, any difference in the steady state price of investment and consumption allows the government to use three tax instruments. We show that in an economy where initial allocation of capital results in low production of new investment goods, the optimal policy for the long run should encourage the production of investment goods by setting higher labour income tax and a tax on capital income from the consumption sector. This way the government collects revenue from three tax instruments, and both workers and capitalists bear the burden of taxes. On the other hand in an economy where the initial allocation of capital results in an excess supply of new investment goods, the optimal policy for the long run should be one that subsidizes capital income in the consumption sector (and sets lower labour income tax in that sector). If the government in such an economy runs a balanced budget each period, the revenue collected from labour income taxation will be used to finance both the government purchases and the capital income subsidy. This optimal policy therefore involves some redistribution in the form of capital subsidy. The distributional consequences of the long run optimal policy therefore depends crucially on how capitalists allocate capital in the initial period.

## 2 The Two Sector Two Class Agents' Economy.

Time is discrete and runs forever. There are two classes of agents, indexed by $i \in \{1, 2\}$, and each class is of measure 1. Agents of class 1 are workers who consume and work but do not invest in physical capital. Agents of class 2 are capitalists who consume and invest in physical capital but do not work. The two production sectors, the consumption sector and the investment sector, are indexed by $j \in \{C, X\}$. Firms own nothing except the technology, and they competitively hire working time from the workers and physical capital from the capitalists in order to produce the consumption goods (the numeraire) and the investment goods. Agents own the property rights of the firms.

We denote the wage rate and the rental price of capital in sector $j$ by $w_{jt}$ and $r_{jt}$, respectively, and the relative price of investment goods by $p_t$. Firms in the consumption sector produce a perishable consumption good which can be used for private consumption, $c_t^i$, and government consumption, $g_t$, such that $g_t = \overline{g} > 0$. Firms in the investment sector produce investment goods which can be used to augment the capital stock. Both types of agents purchase the consumption good, and only capitalists purchase the investment goods. Both goods are traded in competitive markets.

The representative worker is endowed with one unit of time at each period, and the representative capitalist is endowed with $k_0^2 > 0$ units of capital at period 0. Working time in sector $j$ is denoted by $n_{jt}^1$. Capital is accumulated by capitalists, and the stock of capital used in sector $j$ is denoted by $k_{jt}^2$. The resource constraints are:

$$0 \leq f^c\left(k_{ct}^2, n_{ct}^1\right) - c_t^1 - c_t^2 - g_t \tag{1a}$$

$$0 \leq f^x\left(k_{xt}^2, n_{xt}^1\right) + (1-\delta)\left(k_{ct}^2 + k_{xt}^2\right) - \left(k_{ct+1}^2 + k_{xt+1}^2\right); \ \delta \in (0,1) \tag{1b}$$

where the technologies $f^j(.)$ satisfy standard regularity conditions, including Inada conditions and linear homogeneity. Workers like consumption and leisure streams that give higher values of

$$\sum_{t=0}^{\infty} \beta^t u^1\left(c_t^1, 1 - n_{ct}^1 - n_{xt}^1\right) \tag{2}$$

where $\beta \in (0,1)$. Workers' utility function is separable in consumption and leisure, linear in labour, and marginal disutility from working in the two sectors are same[1]. Capitalists derive utility from consumption, and higher levels of consumption give higher values of

$$\sum_{t=0}^{\infty} \beta^t u^2\left(c_t^2\right) \tag{3}$$

[1] It is straightforward to show that the main results we derive holds for a broader class of utility functions. We use a simple utility function that satisfy standard regularity conditions (including Inada conditions) mainly for tractability.

where $u^2(.)$ satisfies standard regularity conditions (including Inada conditions).

The government has four flat-rate (distorting) tax instruments to raise the required revenue: two labour income tax rates, and two capital income tax rates. The labour income tax rates and the capital income tax rates for sector $j$ are denoted by $\tau_t^j$ and $\theta_t^j$, respectively. The government also makes non-negative class-specific lump sum transfer $TR_t^i \geq 0$. The government's budget constraints are:

$$0 \leq \tau_t^c w_{ct} n_{ct}^1 + \tau_t^x w_{xt} n_{xt}^1 + \theta_t^c r_{ct} k_{ct}^2 + \theta_t^x r_{xt} k_{xt}^2 - g_t - TR_t^1 - TR_t^2 \tag{4}$$

We assume that the government has access to a *commitment technology* that allows it to commit itself once and for all to the sequence of tax rates announced at period 0. It has a social welfare function which is a non-negatively weighted average of individual utilities, with the weight $\alpha^i \geq 0$ on class $i$, $\sum_{i=1}^{2} \alpha^i = 1$.

## 2.1 Competitive Equilibria.

Competitive pricing in the production sectors imply that factor prices are given by $r_{ct} = f_k^c(t), w_{ct} = f_n^c(t), r_{xt} = p_t f_k^x(t)$, and $w_{xt} = p_t f_n^x(t)$. The representative worker chooses allocations $\{c_t^1, n_{ct}^1, n_{xt}^1\}_{t=0}^{\infty}$ in order to maximize expression (2) subject to the budget constraints:

$$0 \leq \widetilde{w}_{ct} n_{ct}^1 + \widetilde{w}_{xt} n_{xt}^1 + TR_t^1 - c_t^1 \tag{5}$$

where $\widetilde{w}_{jt} \equiv \left(1 - \tau_t^j\right) w_{jt}$. The consolidated first order conditions associated to this problem, assuming $u_{nc}^1(t) = u_{nx}^1(t) = u_n^1(t)$, include (5) and:

$$0 = u_c^1(t)\, \widetilde{w}_{jt} + u_n^1(t)\,;\ j \in \{C, X\} \tag{6}$$

Given $k_0^2 > 0$, the representative capitalist chooses $\{c_t^2, k_{ct+1}^2, k_{xt+1}^2\}_{t=0}^{\infty}$ in order to maximize expression (3) subject to the budget constraints:

$$0 \leq \widetilde{r}_{ct} k_{ct}^2 + \widetilde{r}_{xt} k_{xt}^2 + (1-\delta)\left(k_{ct}^2 + k_{xt}^2\right) p_t + TR_t^2 - c_t^2 - \left(k_{ct+1}^2 + k_{xt+1}^2\right) p_t \tag{7}$$

where $\widetilde{r}_{jt} \equiv \left(1 - \theta_t^j\right) r_{jt}$. The consolidated first order conditions associated to this problem include the transversality conditions, (7) and the Euler equations:

$$0 = u_c^2(t) - \frac{\beta}{p_t} u_c^2(t+1)\left[\widetilde{r}_{jt+1} + p_{t+1}(1-\delta)\right];\ j \in \{C, X\} \tag{8}$$

**Definition 2.1.1** *A feasible allocation is a sequence* $\left\{c_t^1, c_t^2, n_{ct}^1, n_{xt}^1, g_t, k_{ct}^2, k_{xt}^2\right\}_{t=0}^{\infty}$ *that satisfies equation (1).*

**Definition 2.1.2** *A price system is a 5-tuple of non-negative bounded sequences* $\{w_{ct}, w_{xt}, r_{ct}, r_{xt}, p_t\}_{t=0}^{\infty}$.

**Definition 2.1.3** *A government policy is a 6-tuple of sequences* $\left\{\tau_t^c, \tau_t^x, \theta_t^c, \theta_t^x, TR_t^1, TR_t^2\right\}_{t=0}^{\infty}$.

**Definition 2.1.4 (Competitive Equilibrium)** *A competitive equilibrium is a feasible allocation, a price system, and a government policy, such that (a) given the price system,* $k_0^2 > 0$ *and the government policy, the allocation solves both sets of the firms' problems and the agents' problems, and (b) given the allocation,* $k_0^2 > 0$ *and the price system, the government policy satisfies the sequence of government budget constraints (4).*

**Proposition 2.1.1** *For given* $k_0^2 > 0$, $g_t = \overline{g} > 0$, *and a government policy sequence* $\left\{\widehat{\tau}_t^c, \widehat{\tau}_t^x, \widehat{\theta}_t^c, \widehat{\theta}_t^x, \widehat{TR}_t^1, \widehat{TR}_t^2\right\}_{t=0}^{\infty}$, *the competitive equilibrium dynamics can be characterized by a system of equations that include the transversality conditions, optimality conditions in the production sectors, (1), (4), (5), (6), (7), and (8) in the set of 11 unknowns* $\left\{c_t^1, c_t^2, n_{ct}^1, n_{xt}^1, k_{ct}^2, k_{xt}^2, p_t, w_{ct}, w_{xt}, r_{ct}, r_{xt}\right\}_{t=0}^{\infty}$.

**Proof.** Equation (1a) and (1b) represent the resource constraints. Equations (4), (5) and (7) are the budget constraints of the government, the workers and the capitalists. For the policy $\left\{\widehat{\tau}_t^c, \widehat{\tau}_t^x, \widehat{\theta}_t^c, \widehat{\theta}_t^x, \widehat{TR}_t^1, \widehat{TR}_t^2\right\}_{t=0}^{\infty}$, $k_0^2 > 0$, $g_t = \overline{g} > 0$, and for the corresponding price sequence $\{\widehat{w}_{ct}, \widehat{w}_{xt}, \widehat{r}_{ct}, \widehat{r}_{xt}, \widehat{p}_t\}_{t=0}^{\infty}$, if an allocation $\left\{c_t^1, c_t^2, n_{ct}^1, n_{xt}^1, k_{ct}^2, k_{xt}^2\right\}_{t=0}^{\infty}$ satisfy (1), (4) and (5), it must also satisfy (7). Feasibility of the competitive equilibrium allocation thus requires that for the price sequence $\{\widehat{w}_{ct}, \widehat{w}_{xt}, \widehat{r}_{ct}, \widehat{r}_{xt}, \widehat{p}_t\}_{t=0}^{\infty}$ the competitive equilibrium allocation (a set of 6 unknowns $\left\{c_t^1, c_t^2, n_{ct}^1, n_{xt}^1, k_{ct}^2, k_{xt}^2\right\}_{t=0}^{\infty}$) satisfy the 6 equations, (1a), (1b), (4), (5), and the two transversality conditions:

$$\lim_{t\to\infty} \frac{k_{jt+1}^2}{\prod_{s=1}^{t} R_s} = 0;\ j \in \{C, X\};\ R_t \equiv \left[\frac{\widetilde{r}_{jt}}{p_t} + 1 - \delta\right] \tag{9}$$

Equation (6) represent the intratemporal optimality condition for the workers, i.e. they show that for intratemporal optimal allocation of consumption and working time the marginal rate of substitution of consumption and work (in sector $j$) must be equal to the ratio of the price of consumption to the after tax wage (in sector $j$). For the policy $\left\{\widehat{\tau}_t^c, \widehat{\tau}_t^x, \widehat{\theta}_t^c, \widehat{\theta}_t^x, \widehat{TR}_t^1, \widehat{TR}_t^2\right\}_{t=0}^{\infty}$, combine the production sector equilibrium conditions with (6) to derive

$$p_t = \frac{\left(1 - \widehat{\tau}_t^c\right) f_n^c}{\left(1 - \widehat{\tau}_t^x\right) f_n^x} \tag{10}$$

Equation (8) represent the intertemporal optimality conditions for the capitalists, which, combined with the production sector equilibrium conditions imply:

$$p_{t+1} = \frac{\left(1 - \widehat{\theta}^{c}_{t+1}\right) f^{c}_{k}(t+1)}{\left(1 - \widehat{\theta}^{x}_{t+1}\right) f^{x}_{k}(t+1)} \tag{11}$$

For the price sequence $\{\widehat{w}_{ct}, \widehat{w}_{xt}, \widehat{r}_{ct}, \widehat{r}_{xt}, \widehat{p}_t\}_{t=0}^{\infty}$, let $\left\{\widehat{c}^1_t, \widehat{c}^2_t, \widehat{n}^1_{ct}, \widehat{n}^1_{xt}, \widehat{k}^2_{ct}, \widehat{k}^2_{xt}\right\}_{t=0}^{\infty}$ represent the competitive equilibrium allocation. For $\left\{\widehat{c}^1_t, \widehat{c}^2_t, \widehat{n}^1_{ct}, \widehat{n}^1_{xt}, \widehat{k}^2_{ct}, \widehat{k}^2_{xt}\right\}_{t=0}^{\infty}$ the competitive equilibrium prices are characterized by the solution to the system comprising 4 production sector equilibrium conditions and either (10) or (11) in the set of 5 unknowns $\{w_{ct}, w_{xt}, r_{ct}, r_{xt}, p_t\}_{t=0}^{\infty}$. It is trivial that for $\left\{\widehat{c}^1_t, \widehat{c}^2_t, \widehat{n}^1_{ct}, \widehat{n}^1_{xt}, \widehat{k}^2_{ct}, \widehat{k}^2_{xt}\right\}_{t=0}^{\infty}$, given $k^2_0, \overline{g} > 0$, and the competitive equilibrium prices, the policy $\left\{\widehat{\tau}^c_t, \widehat{\tau}^x_t, \widehat{\theta}^c_t, \widehat{\theta}^x_t, \widehat{TR}^1_t, \widehat{TR}^2_t\right\}_{t=0}^{\infty}$ satisfies (4). ■

## 2.2 Steady state.

Assume there is a steady state where competitive equilibrium allocations and prices converge to constant levels. Such a steady state has some interesting characteristics. First, in a steady state there is an interdependence of the capital and labour margins in this two sector economy. From (6), it is straightforward to show that in a steady state the relative price of investment goods is determined by $p\left(1 - \tau^x_{ss}\right) f^x_n = \left(1 - \tau^c_{ss}\right) f^c_n$. Furthermore, (8) imply that in a steady state, $p\left(1 - \theta^x_{ss}\right) f^x_k = \left(1 - \theta^c_{ss}\right) f^c_k$. These conditions imply that in the long run the government can only choose optimal policies that generate allocations which together with the optimal taxes satisfy $\left(1 - \tau^c_{ss}\right)\left(1 - \theta^x_{ss}\right) f^x_k f^c_n = \left(1 - \tau^x_{ss}\right)\left(1 - \theta^c_{ss}\right) f^c_k f^x_n$. Thus the capital and the labour income taxes that can implement the competitive equilibrium allocation will depend on each other.

**Proposition 2.2.1** *Given proposition 2.1.1, there is a unique steady state.*

**Proof.** Given proposition 2.1.1, for the steady state policy $\left(\tau_{ss}^{c}, \tau_{ss}^{x}, \theta_{ss}^{c}, \theta_{ss}^{x}, TR_{ss}^{1}, TR_{ss}^{1}\right)$ the steady state version of the competitive equilibrium becomes:

$$\frac{f_k^x f_n^c}{f_k^c f_n^x} = \frac{\left(1-\tau_{ss}^{x}\right)\left(1-\theta_{ss}^{c}\right)}{\left(1-\tau_{ss}^{c}\right)\left(1-\theta_{ss}^{x}\right)} \tag{12a}$$

$$0 = f^c\left(k_c^2, n_c^1\right) - c^1 - c^2 - g \tag{12b}$$

$$0 = f^x\left(k_x^2, n_x^1\right) - \delta\left(k_c^2 + k_x^2\right) \tag{12c}$$

$$0 = \left(1-\tau_{ss}^{c}\right) f_n^c n_c^1 + \left(1-\tau_{ss}^{x}\right) p f_n^x n_x^1 + TR_{ss}^1 - c^1 \tag{12d}$$

$$0 = \left(1-\theta_{ss}^{c}\right) f_k^c k_c^2 + \left(1-\theta_{ss}^{x}\right) p f_k^x k_x^2 - \delta\left(k_c^2 + k_x^2\right) p + TR_{ss}^2 - c^2 \tag{12e}$$

$$1 = \frac{\beta}{p}\left[\left(1-\theta_{ss}^{c}\right) f_k^c + p\left(1-\delta\right)\right] \tag{12f}$$

$$0 = u_c^1\left(1-\tau_{ss}^{c}\right) f_n^c + u_n^1 \tag{12g}$$

which is a system of 7 equations that can be solved for the set of 7 unknowns $\left(c^1, c^2, n_c^1, n_x^1, k_c^2, k_x^2, p\right)$. Given the solution to the steady state levels of allocations and $p$, it is straightforward to derive the steady state levels of factor prices $(r_c, r_x, w_c, w_x)$. The steady state levels of allocations and prices satisfy the steady state version of the government's budget constraint:

$$0 = \tau_{ss}^{c} w_c n_c^1 + \tau_{ss}^{x} w_x n_x^1 + \theta_{ss}^{c} r_c k_c^2 + \theta_{ss}^{x} r_x k_x^2 - \overline{g} - TR_{ss}^1 - TR_{ss}^2 \tag{13}$$

■

**Corollary 2.2.1** *In a steady state as in proposition 2.2.1, the tax rates that implement the production efficiency condition satisfy*

$$\frac{\left(1-\theta_{ss}^{c}\right)}{\left(1-\theta_{ss}^{x}\right)} = \frac{\left(1-\tau_{ss}^{c}\right)}{\left(1-\tau_{ss}^{x}\right)} \tag{14}$$

In proposition 2.2.1 the steady state versions of (6) and (8) are included for only one sector because the combined steady state condition of (6) and (8) is (12a). Given the steady state conditions $p\left(1-\tau_{ss}^{x}\right) f_n^x = \left(1-\tau_{ss}^{c}\right) f_n^c$ and $p\left(1-\theta_{ss}^{x}\right) f_k^x = \left(1-\theta_{ss}^{c}\right) f_k^c$, (12d) and (12e) can be solved to derive $c^1$ in terms of $k_c^2$, $n_x^1$, $n_c^1$ and to derive $c^2$ in terms of $k_c^2, k_x^2, n_c^1$. (12g) can be solved for $c^1$ as a function of $k_c^2$ and $n_c^1$, which combined with the other solution to $c^1$ will give a solution to $n_x^1$ in terms of $k_c^2$ and $n_c^1$. These solutions combined with the three equations (12a), (12b) and (12c) gives the solution to the steady state levels of factor allocations.

The steady state factor allocations can be used to derive the steady state levels of consumption for workers and capitalists. The remaining steady state condition (12f) can be solved for steady state level of $p$. The production efficiency condition states that the

ratio of marginal products of capital and the ratio of marginal products of labour should be equalized across sectors, which in a steady state can be implemented by a tax policy that satisfies (14).

## 3 The Optimal Taxation Problem.

For each set of government policy there exists a competitive equilibrium. With no lump sum tax instrument or its equivalent in the scheme, and because of $\overline{g} > 0$, this multiplicity motivates the optimal taxation problem. We define the optimal taxation problem as the standard Ramsey problem and derive the conditions that characterize the Ramsey allocation. Then we look for the taxes that can implement these second-best wedges. We assume that the government chooses after tax returns to maximize social welfare, such that the chosen after tax returns generate an allocation that is implementable in a competitive equilibrium. Using the linear homogeneity property of the production functions, we rewrite (4) as:

$$0 = f^c\left(k_{ct}^2, n_{ct}^1\right) + p_t f^x\left(k_{xt}^2, n_{xt}^1\right) - \widetilde{r}_{ct}k_{ct}^2 - \widetilde{r}_{xt}k_{xt}^2 - \widetilde{w}_{ct}n_{ct}^1 - \widetilde{w}_{xt}n_{xt}^1 - g_t - TR_t^1 - TR_t^2 \quad (15)$$

In a model with only one class of agents, say, given the preset revenue target and $k_0 > 0$, the Ramsey problem is the government's problem of choosing the after tax returns that maximize welfare and generate allocations and prices that are consistent with the competitive equilibrium behaviour of agents. Since there are two classes of agents, the optimal taxes must generate allocations and prices that satisfy equilibrium conditions for each class of agents.

So here the government's problem is one in which for a given $\overline{g} > 0, k_0^2 > 0$ and a fixed set of $(\theta_0^c, \theta_0^x)$ the government chooses allocations to maximize social welfare subject to (15),

(1), (5), (6), (7) and (8). The Lagrangian corresponding to this problem is:

$$\mathcal{L}=\sum_{t=0}^{\infty}\beta^{t}\left\{\begin{array}{l}\alpha^{1}u^{1}\left(c_{t}^{1},1-n_{ct}^{1}-n_{xt}^{1}\right)+\alpha^{2}u^{2}\left(c_{t}^{2}\right)\\ +\psi_{t}\left[\begin{array}{c}f^{c}\left(k_{ct}^{2},n_{ct}^{1}\right)+p_{t}f^{x}\left(k_{xt}^{2},n_{xt}^{1}\right)\\ -\widetilde{r}_{ct}k_{ct}^{2}-\widetilde{r}_{xt}k_{xt}^{2}-\widetilde{w}_{ct}n_{ct}^{1}-\widetilde{w}_{xt}n_{xt}^{1}-g_{t}-TR_{t}^{1}-TR_{t}^{2}\end{array}\right]\\ +\phi_{ct}\left[f^{c}\left(k_{ct}^{2},n_{ct}^{1}\right)-c_{t}^{1}-c_{t}^{2}-g_{t}\right]\\ +\phi_{xt}\left[f^{x}\left(k_{xt}^{2},n_{xt}^{1}\right)+(1-\delta)\left(k_{ct}^{2}+k_{xt}^{2}\right)-\left(k_{ct+1}^{2}+k_{xt+1}^{2}\right)\right]\\ +\mu_{ct}^{1}\left[u_{n}^{1}(t)+u_{c}^{1}(t)\,\widetilde{w}_{ct}\right]+\mu_{xt}^{1}\left[u_{n}^{1}(t)+u_{c}^{1}(t)\,\widetilde{w}_{xt}\right]\\ +\mu_{ct}^{2}\left[u_{c}^{2}(t)-\frac{\beta}{p_{t}}u_{c}^{2}(t+1)\left\{\widetilde{r}_{ct+1}+p_{t+1}(1-\delta)\right\}\right]\\ +\mu_{xt}^{2}\left[u_{c}^{2}(t)-\frac{\beta}{p_{t}}u_{c}^{2}(t+1)\left\{\widetilde{r}_{xt+1}+p_{t+1}(1-\delta)\right\}\right]\\ +\varepsilon_{t}^{1}\left[\widetilde{w}_{ct}n_{ct}^{1}+\widetilde{w}_{xt}n_{xt}^{1}+TR_{t}^{1}-c_{t}^{1}\right]\\ +\varepsilon_{t}^{2}\left[\widetilde{r}_{ct}k_{ct}^{2}+\widetilde{r}_{xt}k_{xt}^{2}+(1-\delta)\left(k_{ct}^{2}+k_{xt}^{2}\right)p_{t}+TR_{t}^{2}-c_{t}^{2}-\left(k_{ct+1}^{2}+k_{xt+1}^{2}\right)p_{t}\right]\end{array}\right\} \tag{16}$$

where $\beta^{t}\psi_{t},\beta^{t}\phi_{ct},\beta^{t}\phi_{xt},\beta^{t}\mu_{jt}^{1},\beta^{t}\mu_{jt}^{2},\beta^{t}\varepsilon_{t}^{1}$ and $\beta^{t}\varepsilon_{t}^{2}$ are Lagrange multipliers for (15), (1a), (1b), (6), (8), (5), and (7), respectively. More intuitively, $\psi$ denotes the shadow price of government's resources, $\phi_{j}$ denotes the shadow price of resources in sector $j$, $\mu_{j}^{1}$ denote the shadow price of workers' intratemporal optimality condition for working in sector $j$, $\mu_{j}^{2}$ denote the shadow price of capitalists' intertemporal optimality condition for investing in sector $j$, and $\varepsilon^{i}$ denotes the shadow price of class $i's$ resources.

The Ramsey problem's first order conditions with respect to $k_{ct+1}^{2}$, $k_{xt+1}^{2}$, $n_{ct}^{1}$ and $n_{xt}^{1}$ are:

$$\phi_{xt}+\varepsilon_{t}^{2}p_{t} = \beta\left\{\begin{array}{c}\psi_{t+1}\left[f_{k}^{c}(t+1)-\widetilde{r}_{ct+1}\right]+\phi_{ct+1}f_{k}^{c}(t+1)+\phi_{xt+1}(1-\delta)\\ +\varepsilon_{t+1}^{2}\left[\widetilde{r}_{ct+1}+p_{t+1}(1-\delta)\right]\end{array}\right\} \tag{17a}$$

$$\phi_{xt}+\varepsilon_{t}^{2}p_{t} = \beta\left\{\begin{array}{c}\psi_{t+1}\left[p_{t+1}f_{k}^{x}(t+1)-\widetilde{r}_{xt+1}\right]+\phi_{xt+1}\left[f_{k}^{x}(t+1)+1-\delta\right]\\ +\varepsilon_{t+1}^{2}\left[\widetilde{r}_{xt+1}+p_{t+1}(1-\delta)\right]\end{array}\right\} \tag{17b}$$

$$\alpha^{1}u_{n}^{1}(t) = \widetilde{w}_{ct}\left(\psi_{t}-\varepsilon_{t}^{1}\right)-\left(\psi_{t}+\phi_{ct}\right)f_{n}^{c}(t) \tag{17c}$$

$$\alpha^{1}u_{n}^{1}(t) = \widetilde{w}_{xt}\left(\psi_{t}-\varepsilon_{t}^{1}\right)-\left(\psi_{t}p_{t}+\phi_{xt}\right)f_{n}^{x}(t) \tag{17d}$$

### 3.1 Optimal Policy in a Steady State.

Our concentration is on the optimal policy in the long run, which is why we will focus on

a steady state of the Ramsey optimum. The steady state versions of (17a) and (17b) are:

$$\phi_x + p\varepsilon^2 \left[1 - \beta \left\{ \frac{(1-\theta_{ss}^{c*}) f_k^c}{p} + 1 - \delta \right\}\right] = \beta \left[\psi f_k^c \theta_{ss}^{c*} + \phi_c f_k^c + \phi_x (1-\delta)\right] \quad (18a)$$

$$\phi_x + p\varepsilon^2 \left[1 - \beta \left\{(1-\theta_{ss}^{x*}) f_k^x + 1 - \delta\right\}\right] = \beta \left[\psi p f_k^x \theta_{ss}^{x*} + \phi_x (f_k^x + 1 - \delta)\right] \quad (18b)$$

The constraints (5), (6), (7), (8) and the resource constraints (1) in the Ramsey problem ensure that in a steady state the optimal taxes generate a set of allocations (the Ramsey allocations) which are an element in the set of competitive equilibrium allocations. Allocations that are consistent with (18a) and (18b) therefore must be allocations that are consistent with the steady state versions of competitive equilibrium condition (12f) and

$$1 = \beta \left\{(1-\theta_{ss}^x) f_k^x + 1 - \delta\right\} \quad (19)$$

respectively. Similarly, allocations consistent with the steady state versions of (17c) and (17d) must be allocations that are consistent with (12g) and

$$0 = u_c^1 (1-\tau_{ss}^x) p f_n^x + u_n^1 \quad (20)$$

respectively.

**Proposition 3.1.1** *In a steady state the optimal tax rates are given by:*

$$(1-\theta_{ss}^{x*}) = 1; \qquad (1-\theta_{ss}^{c*}) = 1 + \frac{1}{\psi}\left(\phi_c - \phi_x \frac{f_n^x}{f_n^c}\right)$$

$$(1-\tau_{ss}^{x*}) \left[\alpha^1 u_c^1 + \left(\psi - \varepsilon^1\right) \frac{\left\{f_n^c (\psi + \phi_c) - \phi_x f_n^x\right\}}{f_n^c (\psi + \phi_c)}\right] = \psi; \;\; (1-\tau_{ss}^{c*}) \left[\alpha^1 u_c^1 + \left(\psi - \varepsilon^1\right)\right] = \psi + \phi_c$$

**Proof.** Since the optimal taxes generate the allocations that satisfy both (18b) and (19), the optimal taxes and the allocations must satisfy $\phi_x \left[1 - \beta \left(f_k^x + 1 - \delta\right)\right] = \beta\psi f_k^x p \theta_{ss}^{x*}$, and since $\beta \left(\frac{\phi_x}{p} + \psi\right) \neq 0$, together with (19) it implies that $\theta_{ss}^{x*} = 0$. Similar steps using (18a) and (12f) give:

$$\phi_x \left[1 - \beta (1-\delta)\right] = \beta \left[\psi f_k^c \theta_{ss}^{c*} + \phi_c f_k^c\right] \quad (21)$$

which, together with (12f) imply that the optimal capital income tax rate in the consumption sector is given by:

$$(1-\theta_{ss}^{c*}) \left(\frac{\phi_x}{p} + \psi\right) = \psi + \phi_c \quad (22)$$

In a steady state, the optimal tax policy must be consistent with the equilibrium price of investment goods, which is given by $p (1-\tau_{ss}^x) f_n^x = (1-\tau_{ss}^c) f_n^c$. Substituting for the

equilibrium price in (22) we derive

$$(1-\theta_{ss}^{c*}) = \frac{(\psi+\phi_c)(1-\tau_{ss}^c) f_n^c}{\phi_x (1-\tau_{ss}^x) f_n^x + \psi (1-\tau_{ss}^c) f_n^c} \tag{23}$$

which holds for any steady state policy $(\tau_{ss}^c, \tau_{ss}^x)$.

It is straightforward to verify that the steady state version of (17c) and (12g) and the steady state version of (17d) and (20) imply that the optimal labour income tax rates are given by $(1-\tau_{ss}^{c*}) = \frac{\psi+\phi_c}{\alpha^1 u_c^1 + (\psi-\varepsilon^1)}$ and $(1-\tau_{ss}^{x*}) = \frac{\psi}{\alpha^1 u_c^1 + \frac{(\psi-\varepsilon^1)}{f_n^c(\psi+\phi_c)}[f_n^c(\psi+\phi_c)-\phi_x f_n^x]}$. Substitute these in (23) to derive $(1-\theta_{ss}^{c*}) = 1 + \frac{1}{\psi}\left(\phi_c - \phi_x \frac{f_n^x}{f_n^c}\right)$. ■

**Proposition 3.1.2** *In a steady state, $\theta_{ss}^{c*} = 0$ if and only if $\tau_{ss}^{c*} = \tau_{ss}^{x*}$, $\theta_{ss}^{c*} \neq 0$ otherwise.*

**Proof.** From proposition 3.1.1, in a steady state optimal labour income taxes satisfy $\frac{1-\tau_{ss}^{x*}}{1-\tau_{ss}^{c*}} = \frac{\psi}{\psi+\phi_c-\frac{f_n^x}{f_n^c}\phi_x}$, and $\tau_{ss}^{x*} = \tau_{ss}^{c*}$ if and only if $\frac{\phi_c}{\phi_x} = \frac{f_n^x}{f_n^c}$. The optimal capital income tax rate in the consumption sector is given by $(1-\theta_{ss}^{c*}) = 1 + \frac{1}{\psi}\left(\phi_c - \phi_x \frac{f_n^x}{f_n^c}\right)$, and $\theta_{ss}^{c*} = 0$ if and only if $\frac{\phi_c}{\phi_x} = \frac{f_n^x}{f_n^c}$. ■

# 4 The Intuition.

Substituting (19) in (18b) gives the steady state condition that characterizes the optimal and implementable allocation of capital in the investment sector:

$$\phi_x = \beta\left[\psi(r_x - \widetilde{r}_x) + \phi_x (f_k^x + 1 - \delta)\right] \tag{24}$$

Equation (24) states that a marginal increment of capital in the investment sector increases the quantity of capital by the amount $(f_k^x + 1 - \delta)$, which has social marginal value equal to $\phi_x$. In addition, there is an increase in tax revenues (equal to $\theta_{ss}^x r_x$) enabling the government to reduce other taxes by the same amount. Since $\psi$ is the shadow price of the government's resources, the reduction of this excess burden equals $\psi(r_x - \widetilde{r}_x)$. The sum of the two effects is discounted by $\beta$, and the discounted effect is equal to the social marginal value of investment goods, $\phi_x$. In a steady state the optimal policy is to set $\theta_{ss}^x = 0$, and therefore investment in the investment sector is consistent with the condition $1 = \beta(f_k^x + 1 - \delta)$, which characterizes the socially optimal allocation of capital in the investment sector.

In a steady state the optimal capital income tax rate in the consumption sector is therefore nonzero in general, and zero only conditionally. Unlike a one sector model where

the final good is either consumed or invested in capital, in the current setting capital is a good produced in a different sector. This is why capital and labour margins in equilibrium are interdependent. It is therefore the initial allocation of capital across the two sectors that determines the social marginal values of investment (*vis a vis* the equilibrium price of the investment goods) and consumption in a steady state. Due to this interdependence, the equilibrium price of investment goods depend on the optimal policy of taxing labour income and the equilibrium labour margins.

In addition, from (12a) and corollary 2.2.1 it is clear that in a steady state the optimal policy of taxing income from capital and income from labour are also interdependent. Due to this, there exists a unique equilibrium price of investment goods, or more simply a unique condition explaining the social marginal values of consumption and investment (i.e. $\phi_c f_n^c = \phi_x f_n^x$) for which a zero capital income tax rate in the consumption sector is in the set of optimal policies. The zero capital income tax policy is therefore one of many implementable optimal policies, supported by the optimal policy that involves equal labour income tax rates across the two sectors. For any other set of allocations, the government can set a tax/subsidy on capital income from the consumption sector and can use differential labour income taxation to undo the tax distortions.

From proposition 3.1.2 in a steady state $\phi_c = \frac{\phi_x}{p} \Leftrightarrow \theta_{ss}^{c*} = 0$. This implies that a zero capital income tax rate in the consumption sector is optimal if and only if $p = \frac{\phi_x}{\phi_c}$, i.e. $\theta_{ss}^c = 0$ is optimal if and only if the relative price of investment goods is equal to the ratio of the social marginal value of investment to the social marginal value of consumption. Substituting (12f) in (21) and rearranging, we derive:

$$\phi_x = \beta \left[ \psi \left( r_c - \widetilde{r}_c \right) + \phi_c f_k^c + \phi_x \left( 1 - \delta \right) \right] \tag{25}$$

Suppose the initial allocation of capital is such that in a steady state of the Ramsey equilibrium, $p = \frac{\phi_x}{\phi_c}$ and the consequent optimal policy involves $\theta_{ss}^{c*} = 0$. In such a case (25) together with $p = \frac{\phi_x}{\phi_c}$ imply:

$$1 = \beta \left[ \frac{\phi_c}{\phi_x} f_k^c + 1 - \delta \right] \tag{26}$$

The zero capital income tax policy (for the consumption sector) is optimal only if the resulting allocations replicate the socially optimal allocation of capital in the consumption sector, for which $1 = \beta \left( f_k^c + 1 - \delta \right)$ must hold. Together with (26) this implies that in a steady state the zero capital income tax policy generates an allocation that is consistent with $\frac{\phi_c}{\phi_x} = 1$, i.e. an allocation consistent with $p = 1$.

We now explain the converse, i.e. if in a steady state the price of investment goods and

the price of consumption goods are equal, the optimal policy is to set $\theta^c_{ss} = 0$. Say the initial allocation of capital across the two sectors is such that in a steady state $p = 1$. From (22),

$$(1 - \theta^c_{ss})(\phi_x + \psi) = (\psi + \phi_c) \tag{27}$$

This now defines the steady state optimal capital income tax policy for the consumption sector. This policy must satisfy the steady state conditions

$$1 = \beta \left[ \left( \frac{\psi + \phi_c}{\psi + \phi_x} \right) f^c_k + 1 - \delta \right] \tag{28a}$$

$$\phi_x = \beta \left[ \psi f^c_k \left\{ 1 - \left( \frac{\psi + \phi_c}{\psi + \phi_x} \right) \right\} + \phi_c f^c_k + \phi_x (1 - \delta) \right] \tag{28b}$$

which are derived by substituting (27) in (12f) and (25). Equations (28a) and (28b) together imply that the optimal policy in a steady state implements the socially optimal level of capital if it is consistent with the condition $(\psi + \phi_c) = (\psi + \phi_x)$. Together with (27) this implies that the only optimal policy that satisfies this condition is to set $\theta^c_{ss} = 0$.

If initial allocation of capital across sectors is such that in a steady state the price of investment goods and the price of consumption goods are not equal (i.e. $p \neq 1$), the government can implement the optimal policy that taxes/subsidizes capital income in the consumption sector and taxes labour income from the two sectors at different rates. Following corollary 2.2.1 if there is no difference in the relative price of the two goods, the policy that satisfies the production efficiency condition must involve $\theta^x_{ss} = 0$, $\theta^c_{ss} = 0$, $\tau^c_{ss} = \tau^x_{ss}$. This policy is one of many implementable Ramsey policies, and it is the optimal policy only if $p = 1$. For all other cases, the optimal policy involves $\theta^x_{ss} = 0$, $(1 - \theta^c_{ss})(1 - \tau^x_{ss}) = (1 - \tau^c_{ss})$ with $\tau^c_{ss} \neq \tau^x_{ss}$.

### 4.1 The initial allocation of capital and the long run optimal policy.

Say the capitalists allocate the initial stock of capital in a way that the economy reaches a steady state with an inefficiently large production of consumption goods and low production of investment goods, such that investment goods are more expensive than consumption goods (i.e. $p > 1$). If a new policy is designed in this steady state (as the initial period), in the long run the optimal policy should be chosen to encourage more production of investment goods. This can be accomplished by choosing an optimal policy that sets $\theta^x_{ss} = 0$, $\theta^c_{ss} > 0$ and $\tau^c_{ss} > \tau^x_{ss}$. This policy encourages the capitalists to shift more capital and the workers to shift more working hours to the investment sector (because of the advantages of a zero capital income tax and a lower labour income tax in this sector), which in turns increases the production of investment goods. Higher production of investment goods minimizes the

relative price difference. Following corollary 2.2.1 this optimal policy is perfectly consistent with (14), and therefore it restores production efficiency.

Consider another example where the capitalists want to buy investment goods at a cheaper price than consumption good. Suppose they allocate the initial level of capital in a way that the economy reaches a steady state with inefficiently large production of investment goods, and $p < 1$. If that steady state is the initial period when the government designs a new policy, the long run optimal policy is one that sets $\theta_{ss}^{x} = 0$, $\theta_{ss}^{c} < 0$ and $\tau_{ss}^{c} < \tau_{ss}^{x}$, which encourages the capitalists to shift capital and the workers to shift working time from the investment sector to the consumption sector. This long run policy thus results in a new steady state with relatively higher production of consumption goods. It minimizes the relative price difference and because of its validity with (14) it restores production efficiency.

Consider (25), which states that a marginal increment of capital in the consumption sector increases the quantity of consumption goods by the amount $f_k^c$, which has social marginal value equal to $\phi_c$. This increment is adjusted by capital depreciation in the investment sector, which has social marginal value equal to $\phi_x$. The aggregate increment in the quantity of available consumption goods in social marginal value terms is equal to $[\phi_c f_k^c + \phi_x (1 - \delta)]$. The first term is due to an increase in capital in the consumption sector, while the second terms stands for an indirect increase in production of consumption goods through an increase in depreciated capital in the investment sector. This is obvious since with $\theta_{ss}^{x} = 0$ it is best to keep depreciated capital in the investment sector. The increased tax revenue, equal to $(r_c - \widetilde{r}_c)$, enables the government to reduce other taxes by the same amount, and the reduction of this excess burden in terms of government's resources is equal to $\psi (r_c - \widetilde{r}_c)$. The sum of these effects is discounted, and is equal to the social marginal value of the available capital, i.e new investment goods.

It is therefore optimal to set zero tax rate on capital income from the consumption sector when the social marginal value of investment and the social marginal value of consumption are same, implying in turns that their relative prices are same. Any difference in the social marginal value of these two is reflected in a difference in the relative price of investment goods. With a zero tax rate on capital income from the investment sector in the scheme, the only optimal policy that can implement the difference in the social marginal values of consumption and investment (*vis a vis* a relative price difference) involves a tax/subsidy to capital income in the consumption sector and differential labour income tax rates.

### 4.2 Redistribution.

We will focus on redistribution properties of the optimal policy in a limiting steady state, and thus we will not present any discussion about how much redistribution is accomplished along the transition. In deciding the optimal policy, $\alpha^2$ plays no role, and thus we can conduct the analysis from the point of view where the government cares only about the welfare of the workers, i.e. $\alpha^1 > \alpha^2 = 0$.

We first consider the special case that extends the findings of Judd (1985) in our setting. Suppose that the initial allocation of capital across sectors is such that the equilibrium price of investment good and equilibrium price of consumption goods are same, and therefore in a steady state the optimal capital income tax rate is zero in both sectors. With this optimal policy, the government collects all revenue that is required to finance its purchases by levying labour income taxes, and the labour income tax rates across sectors is same. So the entire burden of tax is on the workers. In this case (and even if the government values only the welfare of the workers) there will not be any redistribution in the limit. Judd (1985) finds a similar conclusion using a one sector economy with workers and capitalists.

Now consider the case where the initial allocation of capital across sectors are such that investment goods are more expensive than consumption goods. As we have discussed before, starting at this particular steady state the government's long run objective is design a tax policy that encourages production of investment goods. The long run optimal policy now involves a tax on capital income from the consumption sector, zero tax on capital income from the investment sector and differential labour income taxation with a higher labour income tax in the consumption sector. With this optimal policy, the government collects revenue from three tax instruments, and both the workers and the capitalists bear the burden of taxes. This happens even if the government only values the welfare of workers. Therefore, with this optimal policy there is a redistribution in the limit. Rehme (2009) shows a similar result in a neoclassical framework with incomplete income taxation where he argues that in a steady state capital income taxes and redistribution may be nonzero and this depends on among others the social weight of those who receive redistributive transfers. In the current setting this result is not conditional on the social weight.

If the economy starts at a steady state where investment goods are cheaper than consumption goods, the long run optimal policy involves a zero tax on capital income from the investment sector, a subsidy to capital income from the consumption sector and differential labour income taxation with lower labour income tax in the consumption sector. In this case the government collects revenue from two labour income tax instruments. The revenue collected from labour income taxation will be used to finance both the government

purchases and the capital subsidy. Since there is no lump sum tax or its equivalent, this optimal policy involves some redistribution in the limit in the form of capital subsidy. The capital subsidy is part of the optimal policy only in the case where the economy starts with inefficiently large production of investment goods. This will happen if capitalists allocate large proportion of the initial capital in the production of investment goods. This pushes the private return to capital to a level that is lower than the socially optimal level. This inefficiency in production can be undone by subsidizing income from capital in the consumption sector. This policy will boost the production of consumption goods and reduce the production of investment goods. Since direct subsidies to capital income is potentially associated with negative marginal incentive effect of capital accumulation, during economic slowdown one possible way to implement such a policy would be to increase the accelerated capital depreciation allowance. Our analysis shows that such fiscal measures can be associated with some redistribution.

## 5 Conclusion.

We examine optimal income taxation in a two sector economy with two classes of agents: workers and capitalists. We contribute by showing that in a steady state of this economy the optimal capital income tax rate in the consumption sector is in general different from zero and this policy can serve both the efficiency and the redistributive purposes. Any difference in the social marginal value of investment and the social marginal value of consumption is reflected in the relative price difference between the same, and such a difference can be implemented by the optimal policy that has zero tax on capital income from the investment sector, a tax/subsidy on capital income from the consumption sector, and different rates of labour income taxes across sectors. Such a long run optimal policy can serve both the efficiency and the redistributive purposes. The level and the direction of the redistribution is completely independent of the social weight attached to the particular groups of taxpayers.

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