Money, interest rates and the real activity

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This paper examines the effectiveness of monetary aggregates through various nominal interest rates by integrating the financial sector into the Cash-in-Advance (CIA) economy. The model assumes that there are two types of representative agents in the financial sector, which are: productive banks and financial intermediates. The productive banks supply a financial service, which is an exchange technology service to households and financial intermediates receive savings fund from savers and offer loans to borrowers. The monetary expansions are increased banking costs through the rate of inflation. It leads households to use more exchange credit relative to cash at the goods market. Since the number of savings funds is equal to the number of exchange credits used at the goods market, money injections are lower the nominal interest rate on saving as the saving fund increases with exchange credit. By assuming that firms are the only borrowers at the capital market from Fuerst (1992), a lower nominal interest rate on the saving fund reduces the marginal cost of labour and increases labour demand. Meanwhile, the increasing marginal cost of money through the expected inflation effect has a negative effect on labour supply. With labour demand dominating labour supply effects, both output and employment increase with monetary expansion. The paper is able to generate a decreasing nominal interest rate with an increasing money supply with an absence of limited participation monetary shocks from Lucas (1990); and by allowing firms to borrow wage bills payment from financial intermediates, it examines the positive response of aggregate output subject to monetary expansion under flexible price framework.

Key Words: monetary transmission, business cycles, banking sector, interest rates

Subject Classification: E10, E44, E51
1. INTRODUCTION

Decreasing nominal interest rates with monetary expansion is an important monetary transmission mechanism in both traditional Keynesian (Tobin, 1947) and monetarist (Friedman, 1968 and Cagan, 1972) macroeconomics models. The positive correlations among monetary aggregates and real economic activity are a key empirical fact about the macro economy. Flexible price monetary RBC models, such as Cooley and Hansen’s (1989), (1995) and (1998) Cash-in-Advance (CIA) models cannot account for both the nominal interest rates and the real impacts of the money growth rate. Benk, Gillman, and Kejak (2005) extend the standard CIA model to endogenous velocity through the function of productive banks and emphasizes the contribution of the financial shock to business fluctuations. Furthermore, Benk, Gillman, and Kejak’s (2008) monetary model was able to explain the behavior of velocity through exchange credit production functions. However, they cannot explain the lower nominal interest rate and the increase in real economic activity with monetary expansion. The new Keynesian economists employed Calvo’s (1983) type of price stickiness and combined it with a DSGE framework to examine the positive response of output to money injections. However, they failed to include the negative response of the nominal interest rate to monetary expansion.

Lucas (1990) extended standard the CIA model with limited participation monetary shock, which assumes that households make their consumption-saving decision before recognizing monetary innovation. The model indicates that money injections from the monetary authority enter the capital market instead of the goods market. It generates a liquidity effect on nominal interest rate because the size of the capital market cannot increase with monetary expansion.\footnote{This is due to limited participation monetary shock assumption. The size of capital market is pre-determined by the number of the savings fund.} Furthermore, Fuerst (1992) linked the liquidity effect on the nominal interest rate to real activity by assuming that firms’ borrow the wage bill before any goods have been produced, and in so doing explained both the liquidity and real effects of the money growth rate. In other words, the Lucas-Fuerst’s (1992) limited participation CIA models request that households cannot adjust their consumption-saving portfolio with monetary innovations, and that firms have to borrow wage payment before any goods been produced, in order to generate a lower nominal interest rate and raise output with the money growth rate.

This paper formulates, calibrates, and simulates the dynamic stochastic general equilibrium model which incorporates the function of productive banks and financial intermediates to investigate whether the model is able to account for the relationship between monetary aggregates and real activity under a flexible price framework. It extends Benk, Gillman, and Kejak’s (2005) monetary banking model with financial intermediates, and assumes that households make savings to financial intermediates when they collect exchange credit from productive banks. For every unit of exchange credit which has been collected by households, financial intermediates receive an equal number of sav-
ings fund and they lend to borrowers at the capital market with a positive interest rate. Following Fuerst’s (1992) assumption, firms are the only borrowers at the capital market because they have to borrow the wage payments before any goods have been produced. Money injections from the monetary authority increases the marginal cost of money and credit price. With a higher marginal cost of money, households prefer to collect more exchange credit from banks and increase the saving fund of financial intermediates. Increasing savings funds with a fixed number of demands from firms’ wage bill will lower the capital market interest rate. It reduces the marginal cost of labour and leads firms to increase their borrowing from the capital market. The increasing borrowing from the capital market by firms indicates that there is an increase in firms’ labour demand. With a given initial capital stock, increasing labour demand will raise aggregate output through the production function. In a word, the paper explains the monetary aggregates and output relation through cost channel of monetary policy by extending standard cash-in-advance economy with functions of financial sector.

The paper is organized into eight sections, the first of which is this introduction. Section 2 presents the empirical evidence of money shock on real activity and nominal interest rate. Section 3 sets-up the theoretical two exchange technologies DSGE model. Section 4 explains the model’s calibration procedure. Section 5 discusses how the model’s steady state is affected by the money growth rate. Section 6 examines the model’s dynamic and findings. Section 7 will conclude the paper.

2. EMPIRICAL EVIDENCE

Leeper et al. (1996) examined 13 variables VAR model and found that after an expansionary monetary policy shock both consumption and investment rise. They, therefore, argue that any plausible model of the monetary transmission mechanism should generate a rise in output, consumption, and investment. Christiano, Eichenbaum, and Evans (CEE) (1999) employed an identify VAR model and found that aggregate output declines in response to a negative monetary policy shock.

The empirical evidence which comes from five variables recursive VAR (2) model\(^3\) indicates there are positive effects of monetary innovation on real economic activity. Figure 1 indicates that federal fund rate and real activity are affected by the innovations on money equation of VAR model for U.S economy with time period from 1959Q1 to 2004Q2. The impulse responses of variables reflect the effect of changing in money equation error in VAR model. Clearly, money equation innovation has positive effect on real activity, which includes aggregate output, consumption and investment and negative effect on nominal interest rate, which reflects the liquidity effect of money growth rate.

\(^2\)The five variables include real GDP, consumption, investment, federal fund rate and M1. With the exception of the federal fund rate, all variables are in log form.

\(^3\)The number of lags are selected by using Schwarz criterion.
3. THE STRUCTURE OF ECONOMY

This part of paper explains the model economy and displays the problems which are solved by banks, firms and households. It also describes the behavior of financial intermediates and monetary authority. The model includes three sources of uncertainty which are total factor productivity shocks to firms, banks and money growth rate.

Figure 2 reflects the structure of the economy. It includes four representative agents, which are: financial firms, goods sector, household consumers, and the monetary authority. There are two types of representative agents in the financial sector: ‘financial intermediates’ and ‘productive banks’. The function of financial intermediates is to receive savings from households and make loans to firms for wage payment. Productive banks produce an exchange credit service to households through Cobb-Douglas type of production function. Both loan able funds are supplied by financial intermediates, and exchange credit is provided by productive banks, which can be considered as an intra-temporal source of finance. \footnote{The model does not include the inter-temporal finance between agents have been discussed in Bernanke, Gertler and Gilchrist (1999)} Households receive money injections from the monetary authority and labour, capital incomes from firms. The exchange technology held by households for goods market transactions includes real money balance and exchange credit. Households collect exchange credit from banks and make savings with financial intermediates. The model assumes that the number of households’ savings
are equal to the number of exchange credits which been collected from banks. Firms have to borrow wage payments from financial intermediates before any final goods have been produced. The price of exchange credit has to be above the nominal interest rate on savings fund to allow money to be held by households. If the price of exchange credit is equal to the nominal interest rate, then households will only hold exchange credit for goods purchase and cash will be ruled out in the economy.

Figure 2: The structure of model economy

First of all, the positive monetary shock increases the rate of inflation through the money supply equation. Rising inflation will increase the marginal costs of money and exchange credit price. An increasing marginal cost of money and exchange credit price has a negative effect on real activity. It also decreases the money demand and increases the exchange credit of purchase goods. This indicates that the consumption velocity increases with the money growth rate, and has a positive effect on real activities at the goods market. This has been considered as a velocity channel of money growth rate with two exchange technologies framework. An increasing exchange credit purchase goods implies that households have to increase savings as the marginal costs of money increase. This leads to financial intermediates receiving more saving funds from households. The fixed number of demand of saving fund indicates that the nominal interest rate has to fall in order to allow financial intermediates to lend extra savings at the capital market. When firms borrow to pay wage bill, this can generate an increasing output with monetary expansion. This has been called a liquidity channel of money growth rate. In other words, the model generates anticipated and unanticipated increasing money growth rate.
which has negative and positive effects on real activity.

3.1. Banks

The competitive banks in the economy follow Benk, Gillman, and Kejak’s (2005) banking sector specification. Exchange credit $f_t$ is produced by banks using Cobb-Douglas production function with constant returns-to-scale in labour $l_t^f$ and households’ deposit $d_t$. The shares of labour and deposit are $\gamma$ and $1 - \gamma$, respectively.

$$f_t = A_q e^{\gamma t} l_t^f d_t^{1-\gamma}$$  \hspace{1cm} (1)

Assuming that exogenous exchange credit technology are following an AR (1) process with autoregressive parameter $\rho_q$ and structure shock $\varepsilon_t$.

$$q_t = \rho_q q_{t-1} + \varepsilon_t$$ \hspace{1cm} (2)

Each unit of exchange credit is sold by banks at price $p_t^f$. The banks have to pay the wage bill $w_t l_t^f$ and transfer dividends $r_t^d d_t$ to households. The lifetime budget constraint that has been faced by banks is represented by equation (3). The constraint indicates that households collect exchange credit from banks with labour and deposit costs. For each unit of exchange credit which is obtained by households, there is a dividend and labour income deducted from households’ aggregate income. The marginal cost of labour and deposit are indicated by equations (4) and (5), which come from maximizing the exchange credit production function subject to banks’ lifetime budget constraint. They indicate that the shares of wage bill and dividend from banks are equal to the shares of labour and deposits in an exchange credit production function.

$$r_t^d d_t = p_t^f f_t - w_t l_t^f$$ \hspace{1cm} (3)

$$r_t^d = p_t^f (1 - \gamma) \frac{f_t}{d_t}$$ \hspace{1cm} (4)

$$w_t = p_t^f \gamma \frac{f_t}{l_t}$$ \hspace{1cm} (5)

Where $w_t$ represents real wages and $r_t^d$ represents dividend payment per deposit. Following Benk, Gillman and Kejak (2005), the model assumes that the amount of household deposits in banks is equal to the amount of exchange technologies, which include both cash and credit which has been used in the goods market.

$$d_t = \frac{M_{t-1}}{P_t} + f_t + T_t$$ \hspace{1cm} (6)

Where $M_{t-1}$ represents the initial nominal money stock holding, $P_t$ stands for price level, and $T_t$ represents money injections from monetary authority.
3.2. Financial Intermediaries

In contrast to Lucas-Fuerst’s (1992) limited participation CIA models, the model assumes that monetary innovations are happened after households make a consumption-saving decision and money injections are received by households rather than financial intermediates or banks. This indicates that households are able to adjust their consumption-saving portfolio subject to money injections, such as in the standard CIA economy of Lucas (1982) and Svensson (1985). The function of financial intermediates is to receive savings funds from households, which happen after the exchange credit has been collected, and to make loans to firms in the capital market. The amount of savings funds from households has been supposed to be equal to the amount of exchange credit which has been collected by households from banks. This is indicated by equation (7).

\[ f_t = b_t \] (7)

Where \( b_t \) stands for the amount of bank loans to firms in the capital market. By assuming that financial intermediates have zero profit, the income of lending is received by financial intermediates from capital market equal to the savings payout to households. This indicates that the nominal interest rate on the saving fund, which has been denoted as \( R_t \), will be equal to the borrowing rate across the equilibrium. Due to the marginal cost of money and exchange credit price, which has a positive response to monetary shock, an increasing money growth rate raises the share of exchange credit purchase goods. This leads to more exchange credit being collected by households, and increases the saving funds paid to financial intermediates. With a positive interest rate, financial intermediates have an incentive to lend savings funds in the capital market. If the demand for savings funds at the capital market has been determined by the real sector, then financial intermediates have to lower nominal interest rate in order to lend an extra saving fund to the capital market. This leads to a decreasing nominal interest rate with money injections.

3.3. Household Consumers

Representative households maximize their expected log utility function (8) with a discount factor \( \beta \in (0, 1) \) and allocate their time endowment among leisure, labour in goods production sector \( l^g_t \), and labour in banking sector.

\[ U = E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \Psi \ln x_t) \] (8)

\[ 1 = x_t + l^g_t + l^f_t \] (9)

Aggregate output \( y_t \) includes consumption and investment \( i_t \) goods and is produced by firms. The next period’s physical capital stocks \( k_t \) have been accumulated through the law of motion equation (11), with quarterly depreciation rate \( \delta \).
\[ y_t = c_t + i_t \] (10)

\[ i_t = k_t - (1 - \delta)k_{t-1} \] (11)

Goods market exchange technology constraint has been represented by equation (12), which implies that households can either choose cash or exchange credit to purchase consumption goods. The amount of cash and exchange credit which is held by households for the goods market has to depend on the marginal cost of money and exchange credit. Equation (13) represents the amount of cash purchase consumption. Where \( a_t \) denotes the fraction of cash purchase goods.

\[ \frac{M_{t-1} + T_t}{P_t} + f_t = c_t \] (12)

\[ \frac{M_{t-1} + T_t}{P_t} = a_t c_t \] (13)

There are two sources of household income, which are labour income \( w_t l_t^d \) and capital income \( r_t k_t \). The aggregate income of households will be spent on consumption and investment. Equation (14) represents the next period’s money holding for households.

\[ \frac{M_t}{P_t} = \frac{M_{t-1}}{P_{t-1}} + lt d_t + w_t (1 - x_t) + r_t k_{t-1} + R_t f_t - c_t - p_t f_t - k_t + (1 - \delta) k_{t-1} \] (14)

Equilibrium conditions of households is represented by equations (15)-(19), which come from households maximizing the expected log utility function subject to lifetime and goods market CIA constraint.

\[ \frac{\mu_t}{\lambda_t} = p_t^f - R_t \] (15)

\[ 1 + \frac{\mu_t}{\lambda_t} = R_t^c \] (16)

\[ \frac{x_t}{\Psi c_t} = \frac{R_t^c - r_t^d}{w_t} \] (17)

\[ \beta E_t \left( \frac{R_{t+1}^c}{\lambda_{t+1}} \frac{\lambda_{t+1}}{\lambda_t} \right) = 1 \] (18)

\[ \beta E_t \left( \frac{\lambda_{t+1}}{\lambda_t} (r_{t+1} + 1 - \delta) \right) = 1 \] (19)

Where \( \lambda_t \) and \( \mu_t \) represents the shadow prices of lifetime and goods market CIA constraint. Equation (15) indicates that the marginal cost of holding money has to be equal to the marginal cost of exchange credit. The marginal cost of money can be explained by the relative shadow prices between CIA constraint and lifetime budget constraint, which is indicated by equation (16). The marginal cost of credit is the difference between the unit exchange credit price \( p_t^f \), and the return
from saving $R_t$, due to the exchange credit being used. The substitution between marginal utility of consumption and leisure is affected by the marginal cost of money, the deposit rate and the real wage, and is represented by equation (17). Equation (19) represents the standard RBC type of Euler equation. When combined with equation (18), it indicates that the standard Fisher relation, which is the marginal cost of money, is equal to the real interest rate plus the expected rate of inflation.

$$r^d_t = p^f_t (1 - \gamma)(1 - a_t) \quad (20)$$

According to equation (20), the deposit rate is influenced by the fraction of exchange credit goods and credit price. This implies that there is a velocity effect of money growth rate on marginal utility of consumption and leisure substitution. Furthermore, according to firms’ labour costs equation, the real wage is a negative correlation with the nominal interest rate at the capital market. This means that the nominal interest rate at the capital market is able to affect consumption-leisure substitution through the real wage.

### 3.4. Firms

Aggregate output is produced by representative firms using the Cobb-Douglas production function, which includes exogenous technology $e^{\varepsilon_t}$, capital stocks, and labour. The shares of capital stocks and labour are $\alpha$ and $1 - \alpha$, respectively.

$$y_t = e^{\varepsilon_t} (l^g_t)^{1-\alpha} k^\alpha_{t-1} \quad (21)$$

An exogenous Total Factors Productivity (TFP) shock has been assumed to follow AR (1) process with autoregressive parameter $\rho_z$ and structure shock $\varepsilon^z_t$.

$$z_t = \rho_z z_{t-1} + \varepsilon^z_t \quad (22)$$

In order to generate the real impacts of the money growth rate the model follows Fuerst’s (1992) assumptions: firms are the only borrowers at the capital market and have to borrow working capital to pay the wage bill before aggregate goods have been produced. This creates an additional CIA constraint that is faced by firms at the capital market, which is indicated by equation (23). This shows that the amount of exchange credit which would be collected by households is equal to the firms’ cost of labour demand.

$$b_t = w_t l^g_t \quad (23)$$

Firms have sales income from the goods market, and borrow income from the capital market. They need to pay the wage and capital bill to households, and transfer the interest rate payment to financial intermediates in the capital market. The marginal cost of labour and capital comes from the maximized production function, subject to lifetime and capital market constraints.
\[(1 + R_t)w_t l_t^p + r_t k_{t-1} = y_t \quad (24)\]

\[(1 + R_t)w_t = (1 - \alpha) \frac{y_t}{l_t^p} \quad (25)\]

\[r_t = \alpha \frac{y_t}{k_{t-1}} \quad (26)\]

Where \(R_t\) represents the bank loan rate at the capital market, \(r_t\) represents the real interest rate. The model follows Fuerst’s (1992) assumption that firms must borrow to fund their wage bill. Consequently, the appropriate marginal cost of labour to the firm in equation (25) is the real wage times the gross rate of interest on bank loans. This reflects the cost channel of monetary policy. The interest rate decline which is generated by the liquidity effect lowers the marginal cost of labour. At each real wage, the labour demand increases and equilibrium employment and output rise.

### 3.5. Monetary Policy

Monetary policy which has been implemented by central bank through money supply rule is represented by equation (27).

\[M_t = M_{t-1} + T_t \quad (27)\]

Money injections from monetary authorities are represented by equation (28). It indicates that monetary expansion from central bank depends on constant money growth rate \(\Theta^*\), monetary innovation \(\epsilon^{mu}\) and the initial money stock.

\[T_t = (\Theta^* + \epsilon^{mu} - 1)M_{t-1} \quad (28)\]

The deviation of money growth rate is assumed to follow the AR (1) process, with autoregressive parameter \(\rho_m\) and structure shock \(\epsilon^{m}_t\).

\[u_t = \rho_m u_{t-1} + \epsilon^{m}_t \quad (29)\]

### 3.6. Competitive Equilibrium

Competitive equilibrium of this economy consists a set of feasible allocations \(\{y_t, c_t, k_t, M_t, l_t^p, l_t^f, x_t, f_t, d_t\}\), a set of prices \(\{r_t, w_t, r_t^d, R_t, p_t\}\), exogenous shocks \(\{z_t, q_t, u_t\}\) and aggregate outcomes, such that:

- Given \(r_t, w_t, r_t^d, R_t, p_t\) allocation \(c_t, k_t, M_t, x_t, f_t\) solves the households’ problem;
- Given \(w_t, r_t^d, p_t\) allocation \(l_t^f, f_t, d_t\) solves the banks’ problem;
- Given \(r_t, w_t, R_t\) allocation \(l_t^p, k_t, y_t\) solves the firms’ problem;
- The goods, labour, credit and money market is clear;
4. CALIBRATION

The procedure of calibrating deep structure parameters is to map the model economy into observed features of data. It implies that the steady-state value of the model can be indicated by deep structure parameters. With given deep structure parameters, great ratios are predicted by the model’s steady state - which can be directly observed from the data.

Table 1 below summarizes base line deep structure parameters which are implied by U.S post war data. Compare this with the results of Cooley and Hansen (1995) that had a quarterly depreciation rate \(\delta = 0.019\). The data set, which comes from Gomme and Rupert (2007), with duration from 1950 Q1 to 2004 Q2, indicates a quarterly depreciation rate which is equal to 0.024. This also implies that the investment-output ratio is 0.26. With a given depreciation rate and investment output ratio, the steady state capital-output ratio is 10.8. Capital and labour income shares are calibrated by using U.S data from 1950 Q1 to 2004 Q2. The results here are the same as those of Cooley and Hansen (1995), in that it indicates that the share of wage and capital income is 0.6 and 0.4 respectively. With a capital share of 0.4, depreciation rate 0.024, and capital-output ratio 10.8 the steady-state Euler equation implies that \(\beta\) is equal to 0.987; it further indicates that the quarterly real interest rate is equal to 0.013, after depreciation rate. Furthermore, the U.S data indicates that the steady-state working hours from the goods producing sector and leisure are 1/3 and 2/3, respectively. This requires deep structure parameters \(\psi\) which are equal to 1.61.

Table 1 also concludes the behaviors of technology, exchange credit and monetary innovations. The steady-state technology shock has been normalized to one. The autoregressive process and variation of technology shock follow the work of Cooley and Hansen (1995). By assuming a symmetric process between technology and exchange credit shock, the model has the same autoregressive parameter and standard deviation of exchange credit shock with technology innovation. The monetary shock process is estimated by following regression with time duration from 1959 Q4 to 2009 Q4. This indicates that there is a 1.2% money growth rate per quarter at steady-state with persistence 0.64. This result compares with those of Cooley and Hansen (1995), who had steady-state money growth rates of 1.3% with persistence 0.49, and Benk, Gillman and Kejak (2005), who found 1.23% steady state money growth rate with persistence 0.58. The variance of monetary shock from M1 regression is 0.9%, which is close to the results of Cooley and Hansen (1995) and Benk, Gillman and Kejak (2005), which are 0.89% and 1% respectively.

\[
\log M_t = 0.0045 + 0.64 \times \log M_{t-1} \\
(0.0009) (0.0545)
\]

There are three deep structure parameters within banking sector of model: \(A_q, a\) and \(\gamma\). Given one set of calibrated parameter values, the other two can be implied by the model’s steady-state. The model employs the degree of diminishing return in the credit sector which is set \(\gamma\) equal to 0.21, which borrows from Gillman and Otto’s (2002) estimate
for the U.S.

Table 1: Baseline parameters

<table>
<thead>
<tr>
<th>Preferences</th>
<th>Value</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\beta$</td>
<td>0.987</td>
<td>Discount factor</td>
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<tr>
<td>$\psi$</td>
<td>1.6</td>
<td>Leisure weight</td>
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<td>Good production</td>
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<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.4</td>
<td>Capital share in good sector</td>
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<tr>
<td>$\delta$</td>
<td>0.024</td>
<td>Capital stock depreciation rate</td>
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<tr>
<td>$\epsilon^z$</td>
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<td>Good sector productivity parameter</td>
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<tr>
<td>Banking sector</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.21</td>
<td>Labour share in credit production</td>
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<tr>
<td>Monetary authority</td>
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<td></td>
</tr>
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<td>$\Theta^*$</td>
<td>1.2%</td>
<td>Quarterly money growth rate</td>
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<td>Shocks</td>
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<td>Autocorrelation parameters</td>
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<tr>
<td>$\rho_z$</td>
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<td>Good sector productivity</td>
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<td>$\rho_q$</td>
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<td>Banking productivity</td>
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<tr>
<td>$\rho_m$</td>
<td>0.64</td>
<td>Money growth rate</td>
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<td>Standard deviation of shock innovations</td>
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<tr>
<td>$\sigma_z$</td>
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<td>Good sector productivity</td>
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<td>$\sigma_q$</td>
<td>0.7%</td>
<td>Banking productivity</td>
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<tr>
<td>$\sigma_m$</td>
<td>0.9%</td>
<td>Money growth rate</td>
</tr>
</tbody>
</table>

Table 2: target values

| $\pi^{**}$ | 1.2% | Quarterly inflation rate                     |
| $k^{**}/y^{**}$ | 10.62 | Capita-output ratio                          |
| $i^{**}/y^{**}$ | 0.26 | Investment-output ratio                      |
| $x^{**}$    | 2/3  | Leisure                                      |

5. THE TRANSMISSIONS AND REAL IMPACTS OF MONETARY AGGREGATES AT STEADY-STATE

This part of paper discusses the properties of the model’s steady state with different levels of the stationary-state money growth rate. For the nominal side of the economy, the money supply rule implies that the rate of inflation is determined by the money growth rate at a stationary-state. The marginal cost of money can be examined by time preference and inflation rate. Equation (31) indicates that the marginal cost of money is equal to the difference between the exchange credit price and the nominal interest rate at capital market. The Euler relation indicates that the real interest rate is independent to the rate of inflation, and the money growth rate at a stationary-state. It further implies that great ratios are independent to the nominal side of the economy.
\[ R_{ss}^e = \frac{\pi_{ss}}{\beta} \]  
(30)

\[ (R_{ss}^e - 1) = p_{fs}^f - R_{ss}^e \]  
(31)

\[ \alpha \frac{y_{ss}^s}{k_{ss}} = r_{ss}^s = \frac{1}{\beta} - 1 + \delta \]  
(32)

With great ratios are independent to money growth rate, equation (33) implies that the nominal interest rate has a negative relation with the fraction of exchange credit purchase consumption. The increasing fraction of exchange credit used by households at the goods market is decreasing the nominal interest rate at stationary-state. Equation (34) implies that real wages have a negative relation with the nominal interest rate. A lower nominal interest rate has a negative effect on real wages. Therefore, with a given fraction of cash purchase goods at a stationary state, both the nominal interest rate and the real wage are independent to the rate of inflation. In other words, increasing the money growth rate at a stationary-state raises inflation, marginal cost of money and credit price. Real prices, great ratios, and nominal interest rate are independent to the rate of inflation when the fraction of cash purchase consumption is given at stationary-state. Equation (35) indicates that the deposit rate, or marginal cost of exchange technology moves with the exchange credit price. Clearly, with a given fraction of exchange credit consumption, the money growth rate has a positive effect on the deposit rate through the exchange credit price.

\[ R_{ss}^e = \frac{(1 - \alpha)y_{ss}^s}{(1 - a_{ss})c_{ss}} \]  
(33)

\[ R_{ss}^e w_{ss} = (1 - \alpha)(\frac{r_{ss}^s}{\alpha})^{\pi - \gamma} \]  
(34)

\[ r_{ds}^s = p_{fs}^f (1 - \gamma)(1 - a_{ss}) \]  
(35)

The effectiveness of monetary expansion is indicated by equation (36). This reflects the transmission channel between money growth rate and the real economic activity at a stationary state. By assuming independent cash purchase consumption to inflation rate, the deposit rate indicates that there is a negative relation between the money growth rate and leisure-labour in goods sector substitution. In contrast, the marginal cost of money implies that there is a positive effect of money growth rate on leisure-labour in goods sector substitution. With a positive effect from marginal cost of money, which dominates the negative effect from marginal cost of exchange technology, rising money growth rate at steady-state increases leisure-labour substitutions, which indicates a rise in leisure and lower labour at steady-state with money growth rate. Labour supply substitution between sectors is represented by equation (37), which indicates that the substitution between sectors only depends on the exchange credit price at stationary-state. It further implies the positive effect of money growth rate on banking sector
labour supply. With equation (36) and (37), increasing the money growth rate lowers labour in the goods sector and has a negative effect on aggregate output through the production function. Since the real price ratio is independent to money growth rate, equation (38) shows lower capital stock with decreasing labour in goods sector.

\[
x^{ss} = \frac{(R^{ss}_r - r^{ss}_d)\Psi}{(1 - a^{ss})}
\]  
(36)

\[
l^{ss}_g = \frac{\pi^{ss}}{\frac{\beta}{\beta - 1}}
\]  
(37)

\[
R^{ss}w^{ss} = \frac{1 - \alpha k^{ss}}{\alpha l^{ss}_g}
\]  
(38)

In conclusion, with a given fraction of cash purchase consumption, increasing money growth rate at stationary state will lower real activities, such as output, consumption, investment and labour supply through the marginal cost of money and exchange credit price. This indicates that there is negative effect of money growth rate on the economy at a stationary-state.

6. THE MODEL’S DYNAMIC AND FINDINGS

The following section of paper discusses the transmissions and impacts of monetary innovation on real economic activity. It explains the inflation and liquidity effects on nominal interest rate with monetary expansion, and generates a positive responses of output and employment subject to monetary innovations through varies nominal interest rate. It also examines the effect of technology and credit shocks on monetary transmissions and real economic activity.

6.1. Inflation and Liquidity Effects

By integrating financial intermediates with households’ problem, the model introduces an additional CIA constraint which is faced by a representative agent. In contrast to those limited participation models where the representative agent can only choose the number of bonds and is not saving subject to monetary innovation, this model allows households to choose both the number of bonds and exchange credit or saving subject to monetary innovations. Equations (39)-(41) represents those households who maximize their expected log utility subject to lifetime, the goods market, and capital market CIA constraints.

\[
Max E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \Psi \ln x_t) + \lambda_t \left( \frac{m_{t-1}}{\pi_t} + r_t k_{t-1} + w_t (1 - x_t) + R_t b_t + \right.
\]

\[
\left. R_t b_t + r_t^d d_t - m_t - c_t - k_t - p_t^f f_t + (1 - \delta) k_{t-1} + \mu_t \left( \frac{m_{t-1}}{\pi_t} + f_t - c_t \right) + \eta_t (f_t - b_t) \right)
\]

14
Fisher relations for nominal interest rate and cost of exchange credit have been represented by the following equations:

\[ f_t: \lambda_t p^f_t = \eta_t + \mu_t \]  
\[ b_t: \lambda_t R_t = \eta_t \]  
\[ m_t: \beta E_t \left( \frac{\lambda_{t+1} + \mu_{t+1}}{\pi_{t+1}} \right) = \lambda_t \]  

Clearly, equation (43) indicates there is only an inflation effect on price of the exchange credit. It implies that banking costs always increase with monetary shock as it alone has an expected inflation effect. In contrast, the nominal interest rate on savings funds includes both inflation and a liquidity effect, and the liquidity effect will depend on the \( \eta_t - \mu_t \) term. If \( \eta_t - \mu_t < 0 \) there is a liquidity effect on the nominal
interest rate, if $\eta_t - \mu_t > 0$ there is no liquidity effect on the nominal interest rate. Whether the nominal interest rate decreases with monetary expansion has to depend on the size of liquidity and inflation effect on the nominal interest rate. If the inflation effect dominates the liquidity effect then the nominal interest rate has a positive response to positive monetary innovation. In contrast, if the liquidity effect dominates inflation effect then the nominal interest rate has a negative response to a positive monetary shock.

Figure 3 indicates the effect of monetary innovation on the capital market. The number of bonds which have been traded at the capital market can be considered as a firms’ borrowing (corporative bond) for wage payments. Firms are issuing within period claims for wage payments, which determines the supply of bonds. Households which collect exchange credit and make savings can be considered as the demand on the number of bonds. Monetary innovations increase the demand on the number of bonds through raising households’ exchange credit. When the supply of the numbers of bonds is determined by real economic activity, increasing demand for bonds raises the price of bonds and lowers their return, which is the nominal interest rate on saving fund.

6.2. The Transmission Mechanism and Effectiveness of Monetary Policy

Figure 4 represents the responses of the monetary transmission mechanism and real economic activity to positive monetary innovation. It shows that the model is able to generate a negative response of nominal interest rate to monetary shock, and indicates an increase in real economic activity with monetary expansion. First of all, monetary expansions have a positive pressure on the rate of inflation through the money supply function. The increasing rate of inflation with money injections leads the banking costs and the marginal costs of money to increase. When the cost of holding money increases, households substitute from real money balance to exchange credit for goods market transactions. This means that the share of exchange credit purchase goods rises, and the cash purchase consumption falls. With an increasing banking cost, banks have an incentive to supply more exchange credit to households. This leads financial intermediates to receive more savings funds from households as more exchange credits have been used for goods market transaction. The supply of bonds is determined by firms’ wage payment. Demand for bonds increase through rising in banks’ exchange credit, which has a negative effect on the nominal interest rate of the capital market. With an increasing exchange credit holding by households, financial intermediates have to lower nominal interest rate on savings in order to lend extra savings funds to firms. Therefore, monetary expansion increases the banking cost and marginal costs of money through an expected inflation effect, and decreases the nominal interest rate on savings.
Figure 4 also summarizes the responses of output, investment, consumption, and labour supply to monetary shock. Clearly, with a decreasing nominal interest rate and a cost channel of monetary policy assumption, the model is able to generate a positive response of real activity (except for consumption) to monetary expansion without sticky price/wage and a sticky consumption-saving portfolio. A decreasing nominal interest rate in the capital market has a positive effect on the marginal cost of labour and increases firms’ labour demand. Increasing real wages to monetary innovation leads to an income and substitution effect on labour supply. The labour supply in the goods sector will increase, and leisure will decrease, with a positive monetary shock due to the income effect on real wages, which dominates the substitution effect. With a given initial capital stock, rising labour supply increase the aggregate output through the production function. Increasing output with money growth rate has a positive effect on the real interest rate through the marginal cost of capital equation because the initial capital stock has been given. A change in the real interest rate introduces income and substitution effects on capital stock. Figure 4 shows that the capital stock positive response to monetary shock is due to the income effect of real rate, which dominates the substitution effect. Furthermore, through the law of motion equation, investment moves in the same direction with capital stock. The model does not explain the behavior of consumption subject to monetary innovation because the consumption is affected by the marginal cost of money, rather than nominal interest rate at the capital market.

Therefore, by extending the monetary banking model of Benk, Gill-

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5 Leeper et al (1996) found that consumption increases with monetary expansion.
man, and Kejak (2005) with the function of financial intermediates, this model is able to generate a lower nominal interest rate on saving, subject to monetary expansion, without the limited participation monetary shock assumption from Lucas (1990). In this model monetary innovations increase exchange credit, and decrease money demand, through the expected inflation effect of the money growth rate. This encourages households to use more exchange credit in the goods market and decrease their proportion of cash purchase consumption. For every unit of exchange credit which is used by households, the model further assumes that households have to make saving funds at financial intermediates to back-up the exchange credit which has been collected from productive banks. Therefore, savings funds increase with exchange credit, and increase the demand of bond at capital market. With a pre-determined firm’s wage bill, financial intermediates have the incentive to lend savings funds at a lower rate. In other words, the model examines decreasing nominal interest rate with monetary expansion by integrating the functions of productive banks and financial intermediates into a cash-in-advance framework. By assuming that firms have to borrow working capital before any goods have been produced, it explains the positive relation between monetary aggregates and real activity without sticky price/wage and limited participation monetary shock.

6.3. Other shocks

Figure 5: Variables response to a 1% technology shock
output, consumption, investment and labour supply have a positive response to technology innovation. The nominal interest rate on savings and banking costs increases with the technology shock, while the marginal cost of money has a negative response to technology shock. This happens because of the nominal interest rate, which increases more than the banking costs. For the credit shock, monetary transmissions, such as the marginal cost of money, the nominal interest rate and banking costs are a negative response to the credit shock. Real economy activity, except for consumption, has a positive response to banking sector innovation.

Figure 6: Variables response to a 1% credit shock

In conclusion, the main function of financial intermediates is to receive savings and make loans to firms. The number of savings funds from households occurs when the exchange credit has been used for goods market transaction. This determines the demand for bonds in the capital market. Following Fuerst’s (1992) assumption, the supply of bonds in the capital market is determined by the amount of goods producing firms who have to borrow their wage payments. In this model monetary policy has been implemented by the central bank through the money supply rule. The positive monetary shock increases the marginal cost of money through an expected inflation effect on the money growth rate, and has a negative effect on labour supply. With a rising marginal cost of money households will increase the exchange credit which has been used in the goods market, and this creates more savings for financial intermediates. This indicates that velocity has a positive response to monetary shock, and lowers nominal interest rate. Both the positive response of velocity and the negative response of nominal interest rate have positive effects on labour demand. The
velocity and liquidity effects, which dominate the inflation effect on the labour market, indicate that households increase the labour supply and decrease leisure with a positive monetary shock. With a given initial capital stock, an increasing labour demand raises aggregate output through the production function. Therefore, this model is able to explain the negative response of the nominal interest rate to money injections without limited participation monetary shock, and generates positive responses of real activities subject to monetary expansion under a flexible price framework.

6.4. Business Cycle Facts

This section of the paper concludes some of the observed features of monetary business cycle facts which are replicated with model’s simulations. Table 4 describes the cyclical behaviors of the U.S economy, which are obtained from the detrended HP time series data, with duration from 1959 Q1 to 2004 Q2. It also summarizes the simulated economy statistics with technology, credit, and monetary shocks. Although for the real side of the economy the model is able to explain the relative volatilities of consumption and investment, it cannot explain the relative volatility of working hours. For the nominal side of the economy, the model well explains the volatility of the nominal interest rate, but it fails to examine inflation volatility. For nominal real variables interaction, the model is able to generate the pro-cyclical behavior of inflation and nominal interest rate, but it fails to generate the negative correlation between the money growth rate and real economic activity (except for consumption). The important contribution of this model is that it can generate a negative correlation between the M1 growth rate and the nominal interest rate, which has been interpreted by Cooley and Hansen (1995) as a liquidity effect of the money growth rate. Table 4 concludes that the model is able to explain the pro-cyclical behavior of nominal variables, such as inflation and the nominal interest rate. It well explains the behavior of the nominal interest rate from relative volatility, correlation.

Table 4: Simulated monetary economy with technology, credit and monetary shocks

<table>
<thead>
<tr>
<th></th>
<th>Relative SD (%)</th>
<th>Corr with $y_t$</th>
<th>Corr with $u_t$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Data</td>
<td>Model</td>
<td>Data</td>
</tr>
<tr>
<td>Output</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.5163</td>
<td>0.4295</td>
<td>0.79</td>
</tr>
<tr>
<td>Investment</td>
<td>2.7078</td>
<td>2.7852</td>
<td>0.92</td>
</tr>
<tr>
<td>Hours</td>
<td>1.0036</td>
<td>0.4087</td>
<td>0.82</td>
</tr>
<tr>
<td>Inflation</td>
<td>0.2854</td>
<td>1.9809</td>
<td>0.22</td>
</tr>
<tr>
<td>Treasury rate</td>
<td>0.1809</td>
<td>0.2614</td>
<td>0.24</td>
</tr>
<tr>
<td>M1 growth rate</td>
<td>0.6205</td>
<td>0.8302</td>
<td>-0.09</td>
</tr>
</tbody>
</table>
7. CONCLUSION

The flexible monetary RBC models find it difficult to account for the real impacts of monetary aggregates through existing monetary transmissions. Cooley and Hansen (1995) have argued that the nominal interest rate is the only monetary transmission channel in Lucas and Stocky’s (1987) cash-credit goods CIA model. They simulated the model with monetary business cycle facts and concluded that monetary shock does not contribute much to the economic fluctuations in the real variables displayed by a basic neoclassical growth model when money is introduced by requiring a cash-in-advance constraint. Combining a CIA constraint with a simple RBC structure model cannot account for either the observed cyclical behavior of nominal variables or the interaction between real and nominal variables. This suggests that in order to successfully account for the interaction between real and nominal variables in the data we need to introduce more sources of non-neutrality than the inflation tax alone.

Lucas-Fuerst’s (1992) limited participation CIA models with single exchange technology assumes that households cannot adjust their cash-saving portfolio subject to monetary innovations, and that money injections from the central bank are received by financial intermediates to generate lower nominal interest rate with an increasing money growth rate.

This paper extends Benk, Gillman, and Kejak’s (2005) two exchange technologies monetary banking model with the function of the financial sector, and does not request a sticky price/wage or a sticky consumption-saving portfolio to examine the impacts of monetary aggregates with various monetary transmission channels in a Dynamic Stochastic General Equilibrium (DSGE) framework. There are two types of ‘banks’ in this model financial sector, which are: productive banks and financial intermediates. Productive banks provide exchange credit services to households for goods market transactions, and financial intermediates receive savings funds from households and supply loans to goods producing firms. The model assumes that financial intermediates receive savings funds after an exchange credit has been used for a goods market transaction and representative firms, which are the only borrowers in the economy, have to finance their wage payments in advance. Monetary innovation is an increase of the marginal cost of money, and of the proportion of exchange credit purchase goods, through an inflation effect of the money growth rate. With the function of financial intermediates, the model assumes that the number of savings funds received by financial intermediates is equal to the number of exchange credits which have been used by households in the goods market. Raising the exchange credit from productive banks can lead to either an increase in the savings funds or in the demand for bonds. When the supply of bonds is determined by real activity, which is the cost of labour demand, increasing the savings funds within financial intermediates lowers the nominal interest rate. By employing the cost channel of monetary policy, decreasing the nominal interest rate with money injections has a positive effect on labour demand and increases
real economic activity.

In contrast to Lucas-Fuerst (1992) type of limited participation models, money injections from the central bank in this model are received by households instead of financial intermediates. Monetary shock raises both the marginal cost of money and the price of exchange credit through an expected inflation effect. This lowers the real money balance and increases exchange credit. The increasing exchange credit indicates that there is an increasing demand for bonds through financial intermediates, and it lowers the nominal interest rates because the supply of loanable funds (which is a cost of labour) has been determined by real economic activity. With a lower marginal cost of labour demand, firms have an incentive to employ more labour and increase aggregate output. The model is able to explain the real effects of money growth rate through varying the nominal interest rates without sticky price/wage or limited participation monetary shocks under flexible price.

The policy implication of this model is that it introduces the interaction between monetary policy and exchange credit. The monetary authority can increase the money growth rate to allow households to collect more exchange credits from banks and increase the savings funds through raising exchange credit. This leads to more savings from households being made with financial intermediates, which decreases the nominal interest rate of the capital market. A lower nominal interest rate with monetary expansion increases the labour demand from firms and raises aggregate output.
REFERENCES


