Monetary aggregates, financial intermediate and the business cycle

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This paper explains and evaluates the transmissions and effectiveness of monetary policy shock in a simple Cash-in-Advance (CIA) economy with financial intermediates. Lucas-Fuerst’s (1992) limited participation CIA models are able to explain decreasing nominal interest rates and increasing real economic activity with monetary expansion through limited participation monetary shock and the cost channel of monetary policy. Calvo’s (1983) sticky price monetary model examines the real effects of money injections through firms’ price setting behaviour, but it fails to generate a negative correlation between nominal interest rates and money growth rate, which has been observed in the data. This paper employs McCandless (2008) financial intermediates CIA model to explain the transmissions and impacts of monetary shocks. The model does not request limited participation monetary shock or Keynesian type of sticky price/wage, to examine the lower nominal interest rate and increasing real economic activity with monetary expansion. By extending the model with Stockman’s (1981) CIA constraint, it is able to account for both positive response of consumption subject to monetary innovations, which has been found in Leeper et al. (1996) and the positive correlation between output and consumption which has been observed in the data.

**Key Words:** Monetary business cycle; financial intermediate; cash-in-advance model

**Subject Classification:** E44; E52
1. INTRODUCTION

The negative correlation between short-term nominal interest rates and money growth rate is an important monetary transmission channel in both traditional Keynesian (Tobin, 1947) and monetarist (Friedman, 1968 and Cagan, 1972) macroeconomics models. The increasing real activity with monetary expansion also has been recognised as stylise fact of the monetary business cycle. The standard CIA models with flexible price from Lucas (1982) and Svensson (1985) indicate positive correlations between money injections and the nominal interest rates. By affecting leisure-consumption substitution, the model generates negative effects on real economic activity\(^1\). In contrast, sticky price models are able to account for positive correlations between monetary aggregates and real activity through Calvo’s (1983) price setting behaviour, but it cannot explain the negative correlation between nominal interest rates and money growth rate. Limited participation CIA models were developed by Lucas (1990), and followed by Fuerst (1992). Christiano and Eichenbaum (1995), and Christiano, Eichenbaum and Evan (1997) are able to explain the negative response of nominal interest rates, and the positive response of real activity, to monetary innovation under a flexible price framework.

Lucas (1990) modified standard cash-in-advance economy by assuming that households cannot adjust their consumption-saving portfolio subject to monetary innovations to explain liquidity effect on nominal interest rate. Since the supply of government bonds has been determined by the amount of savings from households before a monetary shock occurs, the money injections increase the demand for government bonds and lower the return of bonds. This negative response of bond rate subject to monetary expansion has been referred as the liquidity effect of money growth rate. However, Christiano (1991) argued that the introduction of a liquidity effect into the model may not be enough to generate a lower nominal interest rate. He suggests that the liquidity effect must be sufficiently strong to dominate the anticipated inflation effect. This, in turn, depends on the precise relationship among its variables or the values of its parameters.

The real impacts of monetary aggregates through the liquidity effect from Fuerst (1992), who supposed that firms have to borrow from the capital market to pay their wage bills in advanced. With a lower borrowing rate, firms are more willing to increase their borrowing from the capital market and employ more labour. With a given initial stock capital, an increasing labour demand raises the aggregate output through the production function. This allows limited participation CIA models to explain the positive response of real activity to money injections, without sticky price, through the cost channel of monetary policy. However, the weakness of this model is that it fails to examine the increasing consumption with money injections, and it cannot generate the positive correlation between output and consumption which been observed in the data. Christiano and Eichenbaum (1995), and Christiano, Eichenbaum and Evan (1997) modified the CIA constraint, which allows aggregate consumption to be equal to the next period of money demand. This modification generates the behaviour of consumption that is observed in the data.

This paper employs McCandless (2008) financial intermediates CIA model to explain the transmissions and impacts of monetary innovation to real activity. The model includes two nominal interest rates which reflect the price of money at the goods and capital markets, respectively. Although this model is able to account

\(^1\)Cooley and Hansen (1995) referred as the inflation tax effect of monetary aggregates on real activity
for the positive correlation between output and monetary aggregates through two nominal interest rates, it fails to examine the behaviour of consumption subject to monetary expansion, such as the response of consumption to monetary shock and correlation with output. After extending the model with Stockman’s (1981) CIA constraint, which indicates that both consumption and investment have to be purchased by households using real money balance, the model is able to explain the positive response of consumption subject to monetary shock and also the replicate positive correlation between output and consumption in the data. Stockman’s (1981) CIA constraint is crucial to explain the behaviour of consumption subject to monetary innovations.

The paper is organised into seven sections, the first of which is this introduction. Section 2 presents the theoretical dynamic stochastic general equilibrium (DSGE) CIA model with functions of financial intermediate. Section 3 explains the procedure of model calibrations. Section 4 discusses how the model’s steady state is affected by the money growth rate. Section 5 examines the model’s dynamic and findings. Section 6 is a conclusion.

2. THE MODEL ECONOMY

Figure 1: the structure of model economy

Figure 1 describes the structure of the model economy. The economy includes four representative agents, who are: household consumers, firms, financial intermediates, and the monetary authority. It assumes that money injections from the monetary authority are received by financial intermediates instead of households. Financial intermediates also receive savings from households, money injections from the central bank in the form of savings funds, and issue loans to firms in the capital market. Firms have to borrow their wage payments from financial intermediates before any goods have been produced and pay wages, and capital and loan bills.
Households have to divide their real money balance into goods market transaction and savings which are paid to financial intermediates. Therefore, the model includes two nominal interest rates, which are: the interest rate on the savings funds and the interest rate on firms’ borrowing. The interest rates are equal without money injections. Money injections lower the interest rate on firms’ borrowing with excess demand in the capital market, and increase the interest rate on savings funds through an expected inflation effect. The lower nominal interest rate at capital market has positive effect on real economic activity through marginal cost of labour and the increasing saving rate on saving has negative effect through leisure-consumption substitution.

2.1. Household Consumers

Representative households maximise their expected log utility function (1), which includes consumption $c_t$ and leisure $x_t$, with discount factor $\beta \in (0,1)$, and allocate their time endowment between leisure and working hours $h_t$.

$$U = E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \Psi \ln x_t)$$

$1 = x_t + h_t$

Aggregate output $y_t$ includes consumption and investment $i_t$ goods. The next period’s physical capital stocks $k_t$ have to be accumulated through the law of motion equation, with a constant quarterly depreciation rate $\delta$.

$$y_t = c_t + i_t$$

$$i_t = k_t - (1 - \delta)k_{t-1}$$

At the beginning of each period, the initial real money balance is held by households and divided into the amount of savings and goods market transaction. Households have to deposit their saving funds into financial intermediates, and receive a gross interest rate $R^m_t$. Cash is the only exchange technology which can be used for goods market transactions. Equation (5) represents the exchange technology constraint that is faced by households in the goods market. The model assumes that money injections are received by financial intermediates instead of households. This indicates that money injections from the central bank do not enter a CIA constraint at the goods market. Where $n_t$ represents households savings, $P_t$ represents price level, and $M_{t-1}$ represents the initial nominal money stock.

$$\frac{M_{t-1}}{P_t} - n_t = c_t + \Omega i_t$$

When $\Omega = 0$ the exchange constraint implies that the real money balance is used to purchase consumption goods only. When $\Omega = 1$ the exchange technology constraint represents that both consumption and investment goods can be purchased by real money balance, which is indicated by Stockman’s (1981) CIA constraint. The next period’s money holding includes labour $w_t h_t$, capital $r_t k_{t-1}$, and saving
incomes $R_t^m n_t$. And the expenditures are on consumption, investment, and savings. The lifetime budget constraint which is faced by households is represented in equation (6).

$$\frac{M_t}{P_t} = \frac{M_{t-1}}{P_t} + w_t (1 - x_t) + r_t k_{t-1} + R_t^m n_t - n_t - c_t - k_t + (1 - \delta) k_{t-1}$$ \hspace{1cm} (6)

Equilibrium conditions of households are represented by equations (7)-(11), which come from maximising the expected log utility function subject to lifetime and good market CIA constraints.

$$\frac{1}{c_t} = \lambda_t + \mu_t$$ \hspace{1cm} (7)

$$\frac{x_t}{\Psi_c_t} = \frac{R_t^m}{w_t}$$ \hspace{1cm} (8)

$$1 + \frac{\mu_t}{\lambda_t} = R_t^m$$ \hspace{1cm} (9)

$$\beta E_t \left( \frac{P_t c_t}{P_{t+1} c_{t+1}} R_t^m \right) = 1$$ \hspace{1cm} (10)

$$\beta E_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left( r_{t+1} + 1 - \delta \right) + (1 - \delta) \Omega \frac{\mu_{t+1}}{\lambda_{t+1}} \right] = 1 + \frac{\mu_t}{\lambda_t} \Omega$$ \hspace{1cm} (11)

Where $\lambda_t$ and $\mu_t$ represent the shadow price of lifetime and goods market CIA constraint. Equation (9) implies that the marginal cost of money is equal to the saving rate across equilibrium. The Fisher’s relation, which states that the nominal interest rate on the savings fund depends on expected inflation and current-future consumption substitution, has been indicated by equation (10). Clearly, equation (8) indicates the nominal interest rate on the savings fund has a positive effect on leisure-consumption substitution. It reflects that the expected inflation effects of money growth rate have a negative impact on real activity. Equation (7) and (9) show that the marginal utility of consumption depends on the lifetime shadow price and the marginal cost of money. The model’s Euler equation is represented by equation (11). When $\Omega = 0$ the exchange constraint is represented by equation (5), this implies a standard CIA constraint in which consumption and investment are substitution goods. The Euler relationship which is represented by equation (12) indicates that both the current and future consumption substitution is affected by the real interest rate and the movement of the nominal interest rate on the savings fund. Monetary injections raise the nominal interest rate on savings through an expected inflation effect. This has a negative effect on money holding and lowers consumption through the CIA constraint.

$$\beta E_t \left[ \frac{c_t R_t^m}{c_{t+1} R_{t+1}^m} (r_{t+1} + 1 - \delta) \right] = 1$$ \hspace{1cm} (12)

If $\Omega = 1$ then the exchange constraint which is represented by equation (5) indicates that Stockman’s CIA constraint is applied, which indicates that consumption and investment goods are complementary goods. The Euler relationship which is represented by equation (13) indicates that substitution between current and future marginal consumption of utility depends on a real interest rate which is discounted.
by the marginal cost of money. According to equations (7) and (13), the behaviour of consumption subject to money injections depends on the real interest rate and the marginal cost of money. Monetary expansion increases the marginal cost of money through an inflation effect of the money growth rate, and has a negative effect on consumption. Real interest rates rise with money injections as output increases, and have a positive effect on consumption. Therefore, consumption increases with money injections as the real interest rate dominates the expected inflation effect.

\[ \beta E_t[c_t / c_{t+1} \left( \frac{r_{t+1}}{R^m_{t+1}} + 1 - \delta \right)] = 1 \]  

(13)

2.2. Financial Intermediates

The function of financial intermediates is followed Fuerst (1992) and McCandless (2008) that money injections are received by financial intermediates instead of households. The cash flow constraint which has been faced by financial intermediates is represented by equation (14).

\[ n_t + T_t = b_t \]  

(14)

Where \( T_t \) stands for money injections from the central bank, and \( b_t \) denotes the demand of the capital market. By assuming that financial intermediates have zero profit, the gross return of the saving fund which is received by households is equal to the income of financial intermediates from the capital market; this is indicated by equation (15).

\[ R^m_t n_t = R_t b_t \]  

(15)

Where \( R^m_t \) represents the nominal interest rate on savings, and \( R_t \) stands for the nominal interest rate on borrowing at the capital market. The money injections from the central bank which are received by financial intermediates increase the loan able funds for the capital market. They have a negative effect on the borrowing rate in order to increase the loan able money supply of the capital market. This indicates that there is a lower nominal interest rate on the capital market with monetary expansion.

2.3. Firms

Aggregate output is produced by representative firms through the Cobb-Douglas production function; with labour, capital stock, and exogenous technology. The shares of labour and capital are \( 1 - \alpha \) and \( \alpha \), respectively.

\[ y = e^{z_t} h_t^{1-\alpha} k_t^\alpha \]  

(16)

The exogenous Total Factors Productivity (TFP) is assumed to follow an AR (1) process with autoregressive parameter \( \rho_z \), and structure shock \( \varepsilon_t^z \).

\[ z_t = \rho_z z_{t-1} + \varepsilon_t^z \]  

(17)

Following Fuerst’s (1992) assumption, goods producing firms are the only borrowers at the capital market and they have to borrow cash from capital market for wage payment before any goods have been produced. This creates an additional CIA constraint which is faced by firms at capital market, represented by equation
Therefore, a lower nominal interest rate on borrowing with monetary expansion reduces the marginal cost of labour and encourages firms to employ more labour and raise aggregate output. This has been referred to in the literature as the cost channel of monetary policy.

\[ b_t = w_t h_t \]  

(18)

The aggregate incomes of firms include capital market borrowing and goods market sales. The capital market income is used to pay wages. Goods market income is used for renting capital and borrowing payments. Equation (19) indicates that a lifetime budget constraint has been faced by firms. Representative firms maximise their production function subject to their lifetime budget and capital market CIA constraint to obtain the marginal cost of labour and capital, which are indicated by equations (20) and (21).

\[ R_t w_t h_t + r_t k_{t-1} = y_t \]  

(19)

\[ R_t w_t = (1 - \alpha) \frac{y_t}{h_t} \]  

(20)

\[ r_t = \alpha \frac{y_t}{k_{t-1}} \]  

(21)

Where \( w_t \) and \( r_t \) represents the real wage and real interest rate, respectively. Equation (20) indicates that the marginal cost of labour is varied with the borrowing rate at the capital market. This further implies that an increasing borrowing rate has a negative effect on labour demand, while a decreasing borrowing rate has a positive effect on labour demand. In contrast to a standard CIA model, there is a gap between household labour income and firms’ labour costs in this model. The gap, which is given by \((R_t - 1)w_t h_t\), can be considered as a financial friction with money injections. It allows the borrowing rate to be varied with money injections. Monetary expansion affects real economic activity through the labour demand equation as the marginal costs of labour are varied with the nominal interest rate on borrowing.

\[ \text{2.4. Monetary Policy} \]

The monetary policy that has been implemented by the central bank through the money supply rule is represented by equation (22).

\[ M_t = M_{t-1} + T_t \]  

(22)

Monetary expansion has been represented by equation (23). It indicates that money injections from the central bank depend on steady-state money growth rate \( \Theta^* \), monetary shock \( e^{\alpha t} \), and the initial money stock.

\[ T_t = (\Theta^* + e^{\alpha t} - 1)M_{t-1} \]  

(23)

The deviation of the money growth rate was assumed to follow AR (1) process, with autoregressive parameters \( \rho_m \) and structure shock \( \varepsilon_m^t \).

\[ u_t = \rho_m u_{t-1} + \varepsilon_m^t \]  

(24)
2.5. Competitive Equilibrium

Competitive equilibrium of this economy consists of a set of feasible allocations \( \{y_t, c_t, k_t, M_t, h_t, n_t, x_t\} \), a set of prices \( \{r_t, w_t, R_t^m, R_t\} \), exogenous shocks \( \{z_t, u_t\} \), and aggregate outcomes, such that:

- Given \( r_t, w_t, R_t, \) allocation \( c_t, k_t, M_t, x_t, n_t \), solves the households’ problem;
- Given \( r_t, w_t, R_t, \) allocation \( h_t, k_t, y_t \), solves the firms’ problem;
- The goods, labour, and money market are clear;

3. CALIBRATION

The procedure of calibrating deep structure parameters is to map the model economy into the observed features of data. This means that the steady-state value of the variables can be implied by the deep structure parameters. With given deep structure parameters, the model’s steady-state can generate great ratios which can be observed directly from data.

Table 1 concludes the behaviour of technology and monetary shocks. Steady-state technology shock has to be normalised equal to one. Autoregressive process and variation of technology shock follows Cooley and Hansen (1995), which has a persistence parameter of 0.95 and a variance of structure shock of 0.7%. The persistence and variance of money growth rate will be estimated from following the M1 money growth rate regression, with time duration from 1959Q4 to 2009Q4. The results indicate that there is a 1.2% money growth rate per quarter at steady-state, with a persistence of 0.64. This compares with Cooley and Hansen (1995) who had steady-state money growth rates of 1.3%, with a persistence of 0.49. It also compares with Benk, Gillman and Kejak (2005) who found a 1.23% steady-state money growth rate, with a persistence of 0.58. The variance of monetary shock from M1 regression is 0.9%, which is also close to the results of Cooley and Hansen (1995) and Benk, Gillman and Kejak (2005), which are 0.89% and 0.1%, respectively.

\[
\Delta \log M_t = 0.0045 + 0.64 \times \Delta \log M_{t-1} \\
(0.0009) (0.0545)
\]

Table 1 also summarises the base line deep structure parameters, which can be implied from two groups of U.S data. Firstly, the data set from Gomme and Rupert (2007) with duration from 1948 Q1 to 2004 Q2 indicates that the quarterly depreciation rate is 0.024, which compares with the results of Cooley and Hansen (1995) which found that . It also indicates that the investment-output ratio is 0.26. With a given depreciation rate and investment output ratio, the steady-state capital-output ratio is going to be 10.8.

There are two sources of income for households, which are: labour and capital income. Capital and labour income shares are calibrated in the model by using U.S data from 1950 Q1 to 2009Q4. The data implies that the share of wage income is equal to 0.6, and which compares with the results of Cooley and Hansen (1995) which were that . With a capital share of 0.4, a depreciation rate 0.024, and a capital-output ratio of 10.8, the model implies that the real interest rate is equal to 0.037 before depreciation rate. Since the quarterly steady-state inflation is equal to the money growth rate, the steady-state Euler equation implies that the preference parameter is equal to 0.987. U.S data also indicates that the steady-state working hours from the goods producing sector and leisure are 1/3 and 2/3, respectively.
This requires deep structure parameters which are equal to 1.58. With Stockman’s (1981) CIA constraint, the model’s Euler equation and great ratios imply that is equal to 0.988, and that is equal to 1.59.

Table 1: Baseline parameters

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<tr>
<td>$\beta$</td>
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<tr>
<td>$\psi$</td>
<td>1.58/1.59</td>
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<td>Leisure weight</td>
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<tr>
<th>Goods Production</th>
<th>Value</th>
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<tr>
<td>$\delta$</td>
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<table>
<thead>
<tr>
<th>Preferences</th>
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<td>$\theta^*$</td>
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<table>
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<tr>
<th>Monetary authority</th>
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<tbody>
<tr>
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<table>
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<tr>
<th>Goods Production</th>
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<td>$\epsilon$</td>
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<tr>
<td>$i^{ss}/y^{ss}$</td>
</tr>
<tr>
<td>$k^{ss}/y^{ss}$</td>
</tr>
<tr>
<td>$x^{ss}$</td>
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</table>

4. MONETARY TRANSMISSIONS AND REAL ACTIVITIES AT STEADY-STATE

This part of paper discusses monetary properties of model at stationary state with varying money growth rate. First of all, the inflation effect of the money growth rate at a stationary-state is given by equation (25), which indicates a positive correlation between the nominal interest rate on saving and the rate of inflation. By combining the households’ and firms’ steady-state CIA constraint, equation (26) and (27) reflects a negative relationship between borrowing and inflation rates at a stationary-state. In other words, the stationary state money growth rate is positively correlated with the saving rate, and negatively correlated with the borrowing rate at capital market.

\[
R^{m}_{ss} = \frac{\pi^{ss}}{\beta} \tag{25}
\]

\[
R^{s} = \frac{1}{\beta} \left( m^{ss} - \pi^{ss} c^{ss} \right) \tag{26}
\]

\[
R^{s} = \frac{1}{\beta} \left( m^{ss} - \pi^{ss} y^{ss} \right) \tag{27}
\]
With a standard CIA constraint, the model’s stationary state Euler relation, which is represented by equation (28), indicates that the real interest rate is independent of the rate of inflation and leads to great ratios such as investment–output, consumption–output and capital–output ratios, which are independent to the rate of inflation or the monetary growth rate. With Stockman’s (1981) CIA constraint, the model’s stationary state Euler relation, which is implied by equation (29), indicates that the real interest rate varies with inflation or money growth rate, and leads both real price and great ratios to vary with the money growth rate. This reflects the third monetary transmission channel at stationary-state, which has been called a ‘real interest rate’ effect of the money growth rate.

\[
\alpha \frac{y^{ss}}{k^{ss}} = r^{ss} = \frac{1}{\beta} - 1 + \delta
\]  
(28)

\[
\alpha \frac{y^{ss}}{k^{ss}} = r^{ss} = \left( \frac{1}{\beta} - 1 + \delta \right) \frac{\pi^{ss}}{\beta}
\]  
(29)

Since the firms’ wage bill varies with the lending rate, both leisure-consumption and leisure-labour substitutions are affected by the saving rate, and the borrowing and real interest rates. The effects of saving, borrowing, and real interest rates on leisure-consumption and leisure-labour substitutions at a stationary state will be considered as inflation, ‘liquidity’, and real interest rate effects of the money growth rate.

With a standard CIA constraint, the real interest rate effect is zero because real prices are independent of the money growth rate. Furthermore, the negative effect of monetary aggregates on leisure-labour substitution through saving rate is dominated by positive effect of money growth rate through borrowing rate, equation (31) indicates lower leisure-labour ratio with money growth rate, which means increasing labour supply. Equation (32) indicates lower capital-labour ratio with money growth rate due to borrowing rate channel. This is the opposite of the Tobin effect (1965), where increases in the steady-state money growth rate will also increase the capital-labour ratio. And consistent with Stockman (1981) argument, where it needed the Stockman CIA constraint and the model does not. It is increase capital stock through equation (32) and raises the aggregate output through Cobb-Douglas production function; the level of investment with the law of motion equation.

With a Stockman CIA constraint, equation (30)-(32) indicates that the money growth rate has a negative effect on leisure-labour substitution through the borrowing rate; positive effect through saving rate and the real interest rate. If a negative effect from the borrowing rate is dominated by the positive effect from the saving and real interest rate, then the model concludes that there is an increase in leisure and a decrease labour supply with the monetary aggregates. This is lower capital stocks with equation (32) and aggregate output through production function.

\[
\frac{i^{ss}}{y^{ss}} = \frac{\alpha \delta}{r^{ss}}
\]  
(30)

\[
\frac{x^{ss}}{h^{ss}} = \frac{\Psi R^{ss} R^{ss} c^{ss}}{(1 - \alpha)y^{ss}}
\]  
(31)

\[
\frac{k^{ss}}{h^{ss}} = \frac{\alpha R^{ss} w^{ss}}{(1 - \alpha)r^{ss}}
\]  
(32)
In conclusion, the financial intermediate CIA model with standard constraint indicates both a real interest rate and great ratios which are independent to the money growth rate at stationary-state. By assuming that firms borrow funds from the capital market to pay the wage bill in advance, real wages are varied with the borrowing rate and this leads to a leisure-labour substitution which is affected by both the saving rate from Fisher relation and by the bond rate from the wage equation. A rising stationary state money growth rate will increase the goods sector labour demand, the capital stock, and the level of real activities. Therefore, the model generates the positive effect of the money growth rate on real activity at stationary-state.

By extending the model with Stockman’s (1981) CIA constraint, the model indicates that both the real interest rate and the consumption-output ratio are increased with the stationary state money growth rate. This leads leisure-labour substitution to affect the saving rate, bond rate, and consumption-output ratio. Increasing the stationary state money growth rate will lower the level of real activities since the positive effect of liquidity effect is dominated by negative effects of saving rate and real interest rate. Therefore, the model with stockman constraint generates the negative effect of the money growth rate on real activity at stationary-state.

5. THE MODEL’S DYNAMIC AND SIMULATION

This part of the paper analyses the dynamic behaviour of monetary transmissions, discusses the effectiveness of monetary policy shock on real economic activity through various monetary transmission channels, and evaluates the effectiveness of the model economy with business cycle facts.

5.1. The Effects on Nominal Interest Rates

The effectiveness of monetary aggregates on nominal interest rates is investigated through the centralised economy as follows. The representative agents’ problem under a centralised economy can be considered as maximising the expected utility subject to lifetime, goods market CIA constraints, and capital market CIA constraints.

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t (\ln c_t + \Psi \ln x_t) + \lambda_t \left( \frac{m_{t-1}}{\pi_t} + r_t k_{t-1} + w_t (1 - x_t) + (R_t^m - 1) n_t \right)
\]
\[-m_t - c_t - k_t -(R_t - 1)b_t + (1 - \delta)k_{t-1} + \mu_t \left( \frac{m_{t-1}}{\pi_t} - n_t - c_t \right) + \eta_t (b_t - w_t (1 - x_t)) \]

\[n_t : \lambda_t (R_t^m - 1) = \mu_t \quad (33)\]
\[b_t : \lambda_t (R_t - 1) = \eta_t \quad (34)\]
\[m_t : \beta E_t \left( \frac{\lambda_{t+1} + \mu_{t+1}}{\pi_{t+1}} \right) = \lambda_t \quad (35)\]

Fisher relations on nominal interest rates for the borrowing and saving funds have been represented by following equations, where \( \eta_t - \mu_t = (R_t - R_t^m) \lambda_t \):
\[ R_t = \frac{(\eta_t - \mu_t) + (\lambda_t + \mu_t)}{\beta E_t^{\frac{\lambda_{t+1} + \mu_{t+1}}{\pi_{t+1}}}} \]  
(36)

\[ R_t^m = \frac{\lambda_t + \mu_t}{\beta E_t^{\frac{\lambda_{t+1} + \mu_{t+1}}{\pi_{t+1}}}} \]  
(37)

Clearly, there is only an inflation effect on the saving rate and the liquidity effect on the borrowing rate depends on \( \eta_t - \mu_t \). With equation (14) and (15), the borrowing rate is less than the saving rate with positive money injections from the central bank, which implies that the liquidity effect term is negative. This is because the value of cash in the capital market is less than the value of cash in the goods market as money injections are received by financial intermediates and loaned to firms in the capital market.

5.2. Monetary Transmissions and Real Effects

Figure 2: Variables response to a 1% positive technology shock

Figures 2 and 3 reflect the monetary transmissions and real variables response to 1% positive technology and monetary shocks in the model economy, both with and without a Stockman CIA constraint. A negative response of inflation to technology shock reflects the counter-cycle behaviour of the price level. Real economic activities are increased with a technology shock. The monetary transmissions which include savings and borrowing rates have a positive response to a technology shock. The responses of velocity to a technology shock reflect the relationship between the exchange technology constraint and the quantity theory of money. The Stockman type of exchange technology constraint implies that velocity does not move with a technology shock due to the effectiveness of technology shock on output which is
reflected by real money demand. The standard type of exchange technology constraint indicates that only the effectiveness of technology shock on consumption is reflected on real money demand. Because the output increases more than consumption with a technology shock, the quantity theory of money requests a velocity increase with output.

Figure 3: Variables response to a 1% positive monetary shock

For the model with a standard CIA constraint, the positive response of the saving rate to monetary expansion reflects the expected inflation effect of money growth rate. This has been referred to by Cooley and Hansen (1995) as an ‘inflation tax effect’ and it has a negative effect on real economic activity. Since the money injections are received by financial intermediates instead of households, and have to be loaned to firms for wage payments in the capital market, it creates excess demand in the capital market. Since the supply of the capital market is determined by real activity (which is firms’ wage bills) financial intermediates have to lower the borrowing rate in order to allow the extra savings funds to be loaned to firms at the capital market. It leads negative response of the borrowing rate subject to monetary expansion and has a positive effect on real activity through the marginal cost of labour. In other words, there are two monetary transmission channels when the standard CIA model is extended with function of financial intermediates, which are: saving and borrowing rates. The money injections increase leisure, decrease the labour supply, and have a negative effect on real activity through the saving rate channel. They decrease the leisure, increase labour demand, and have a positive effect on output through the borrowing rate channel. Figure 3 concludes that both employment and output are increased with monetary expansion because the positive effect from the borrowing rate dominates the negative effect from the saving rate. The model without Stockman CIA constraint fails to generate the behaviour of consumption subject to monetary shock since the consumption is decreased instead of increased with monetary expansion. This indicates that the positive effect from
the borrowing rate cannot dominate the negative effect from the saving rate on consumption.

Beside the borrowing and saving rates, the model with a Stockman CIA constraint introduces an additional monetary transmission channel through the real interest rate in the Euler equation, and has positive effects on real activity, particularly on consumption. This allows the model to overcome the weakness on consumption behaviour. Figure 3 shows that the consumption does increase with monetary expansion since the positive effect from borrowing rate is able to dominate the negative effect from the saving rate, with a lower substitution rate\(^2\) of consumption due to the Stockman CIA constraint. The velocity has negative response to monetary shock since money demand increases more than aggregate output. Therefore, the CIA model which employs the Stockman CIA constraint, and assumes that money injections are received by financial intermediates instead of households, is able to examine the behaviour of real activity subject to monetary expansion through the cost channel of monetary policy under a flexible price framework, without a limited participation monetary shock.

5.3. Business Cycle Facts

Table 4 describes contemporaneous correlations with aggregate output from the log detrended time series data with duration from 1959Q1 to 2004Q2. It also summarises the simulated economy statistics with both technology and monetary shocks. The statistics of simulation are computed from an artificial time series consists of 182 periods with 50,000 times simulations. In order to compare these results with real data statistics, it has to be taken logged and detrended by a Hodrick-Prescott filter. When a standard CIA constraint is applied (where \(\Omega = 0\)), the monetary innovation is able to generate a positive correlation between the nominal interest rate and output; however, it fails to replicate the positive correlation between aggregate output and consumption. Therefore, it does not imply the consumption behaviours which have been observed in the data. By extending the model with Stockman’s (1981) constraint (where \(\Omega = 1\)), the model is able to overcome the weakness of the model economy (i.e. consumption behaviour cannot explained) and is able to replicate the positive correlation between output and consumption.

Table 4: Contemporaneous correlations with aggregate output

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>(\Omega = 0)</th>
<th>(\Omega = 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>Consumption</td>
<td>0.79</td>
<td>-0.11</td>
<td>0.91</td>
</tr>
<tr>
<td>Investment</td>
<td>0.92</td>
<td>0.89</td>
<td>0.99</td>
</tr>
<tr>
<td>Employment</td>
<td>0.82</td>
<td>0.81</td>
<td>0.80</td>
</tr>
<tr>
<td>Treasury bill rate</td>
<td>0.24</td>
<td>0.47</td>
<td>0.38</td>
</tr>
</tbody>
</table>

6. CONCLUSION

This paper integrates the CIA economy with the function of financial intermediates, and allows money injections to be received by financial intermediates instead

\(^2\)The 'substitution rate' refers to the real interest rate discounted by saving rate, which is indicated by the model's Euler equation.
of households. It is, therefore, able to explain the transmission and real impacts of monetary aggregates without sticky price/wage and limited participation monetary shock.

For the model with a standard CIA constraint, money injections increase the saving rate through an expected inflation effect, and have a negative effect on labour supply. At the same time, it lowers the borrowing rate at the capital market and has a positive effect on labour demand through the marginal cost of labour. Since the positive effect on labour demand dominates the negative effect on labour supply, employment rises with monetary expansion and increases output through the production function. However, the model with a standard CIA constraint is unable to explain the behaviour of consumption because consumption has a negative response to monetary shock, and there is a negative correlation between consumption and output.

By replacing the standard CIA constraint with a Stockman constraint, the model’s Euler equation indicates that the substitution of current and future marginal utility of consumption is affected by the real interest rate discounted by saving rate. This means that the real interest rate is affected by the money growth rate through Euler equation, and introduces a positive effect of monetary expansion on real interest rate with an increasing saving rate. This has been considered as a third monetary effectiveness channel (which is referred to as a ‘real interest rate’ effect in this chapter). An increasing real interest rate with a positive monetary shock has a positive effect on consumption. When combined with a lower borrowing rate at the capital market, they are able to dominate the expected inflation effect on consumption and replicate the consumption behaviour which has been observed in the data. Stockman’s (1981) CIA constraint plays a major rule to obtain a positive response of consumption to monetary expansion, and a positive correlation between output and consumption is observed in the data.

In a word, the paper employs McCandless (2008) working capital CIA model by replacing monetary transactions services with Stockman’s (1981) CIA constraint, which exchanges the technology used for both consumption and investment, to explain the real effects of monetary shock and the monetary transmission mechanism. The model does not request limited participation monetary shock from Lucas (1990), but it does assume that money injections are received by financial intermediates instead of households to generate the real impacts of monetary aggregates. The model with standard CIA constraint which from McCandless (2008) cannot explain the behaviour of consumption and generate negative correlation between consumption and output. The paper employs the Stockman CIA constraint to explain the cyclical behaviour of consumption and velocity. It generates a positive response of consumption and a negative response of velocity to monetary innovation, and a positive correlation between output and consumption. The financial intermediate CIA model with Stockman’s (1981) constraint not only generates the positive response of output, consumption, and investment to monetary expansion but also the negative response of the nominal interest rate to monetary shock. It is able to explain both monetary transmissions and the impacts of monetary shock to real activity without limited participation monetary shock and sticky price/wage setting.
REFERENCES


