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Taylor Rule or Optimal Timeless Policy? Reconsidering the Fed's behavior since 1982*

Patrick Minford† (Cardiff University and CEPR) Zhirong Ou†† (Cardiff University)

September 2009
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Abstract

We calibrate a standard New Keynesian model with three alternative representations of monetary policy- an optimal timeless rule, a Taylor rule and another with interest rate smoothing- with the aim of testing which if any can match the data according to the method of indirect inference. We find that the only model version that fails to be strongly rejected is the optimal timeless rule. Furthermore this version can also account for the widespread finding of apparent ‘Taylor rules’ and ‘interest rate smoothing’ in the data, even though neither represents the true monetary policy.

Key words: Monetary policy; New Keynesian model; optimal timeless policy; Taylor rules; Bootstrap simulation; VAR; Indirect inference; Wald statistic

JEL Classification: E42, E52, E58

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1. Introduction

In this paper our aim is to uncover the principles according to which the Board of Governors of the US Federal Reserve System (the Fed) conducted monetary policy since the early 1980s. We do so in a novel way by asking which such principles can, when combined with a widely-accepted macro model, replicate the dynamic behaviour of the US economy during the sample period. By ‘principles’ we mean either an explicit rule the Fed follows (such as an interest-rate setting rule) or some other economic relationship that it aims to ensure occurs (such as a fixed exchange rate or as here an optimality condition).

The main context for this work is the influential paper by Taylor (1993), who-building on earlier work by Henderson and McKibbin (1993a, 1993b) which argued for the efficacy of interest rate rules- suggested that the Fed actually had been for some time systematically pursuing a particular interest rate rule, reacting directly to two ‘gaps’, one between inflation and its target rate, the other between output and its natural rate. Such a ‘Taylor rule’ was subsequently adopted widely in New Keynesian models to represent the behaviour of monetary policy (e.g., Rotemberg and Woodford (1997, 1998), Clarida, Gali and Gertler (1999, 2000), Rudebusch (2002), English, Nelson and Sack (2002)).

However, Minford, Perugini and Srinivasan (2001, 2002) and Cochrane (2007) have shown that a Taylor rule is not identified. Estimates of such a ‘rule’ may emerge from the data when the Fed is following quite other monetary policies; this is because a variety of relationships within the economy can imply a relationship between interest
rate, inflation and output (gap) which mimics a Taylor rule. In the presence of such an identification problem, direct estimation of Taylor rules on the data does not establish whether the Fed was actually pursuing them or not. Some other way of testing hypotheses about monetary policy must be found. The one proposed here is to set up competing structural models which differ solely according to the monetary policies being followed, and to distinguish between these models according to the ability to replicate the dynamic behaviour of the data. Thus for example if one were to accept just one of these models and reject the rest, it would be reasonable to argue that this model succeeds because in it not only the rest of the economy but also monetary policy is well-specified. Of course other less decisive empirical outcomes of the tests are entirely possible.

The rest of this paper is organised as follows: section 2 reviews the work estimating monetary policy rules and the critique of it in terms of identification; section 3 outlines the simple micro-founded New Keynesian model with the hypothetical rules to be tested; section 4 explains the test methodology and reveals the results; section 5 discusses how the ‘true’ policy/model can explain the apparent existence of Taylor rules in the data; section 6 concludes.

2. Taylor Rules, Estimation and Identification

Taylor (1993) suggested that, at least for the post-1982 periods during which Alan Greenspan was chairman of the Fed, the Federal funds rate could be well described by the simple equation (with quarterly data) as:
\[ i_t^A = \pi_t^A + 0.5x_t + 0.5(\pi_t^A - 0.02) + 0.02 + \xi_t \]  

where \( x_t \) is for the percentage deviation of real GDP from trend, \( \pi_t^A \) is the annual averaged rate of inflation over four quarters, with both the target of inflation and growth rate of the real GDP (with trend) set at 2 percents.

Equation [2.1] is the original ‘Taylor rule’. However, a number of variants have also been proposed; for example, a Taylor rule where policy inertia is assumed could take the form as in Clarida, Gali and Gertler (1999) as:

\[ i_t^A = (1 - \rho)(\alpha + \gamma_\pi(\pi_t^A - \pi^*) + \gamma_x x_t) + \rho \pi_{t-1}^A + \xi_t \]  

with \( \rho \) showing the degree of ‘interest rate smoothing’. Others have involved lagging or leading the inflation and output gap terms - Rotemberg and Woodford (1997, 1998), Clarida, Gali and Gertler (1999, 2000), Rudebusch (2002), English, Nelson and Sack (2002).

Rules of these types are generally found to fit the actual data well in regression analysis, either via single-equation regression by GLS as in Rotemberg and Woodford (1997, 1998), Clarida, Gali and Gertler (1999, 2000) and Giannoni and Woodford (2005), or via full-model estimation by Maximum Likelihood as in Rotemberg and Woodford (1997, 1998), Smets and Wouters (2003), as well as Ireland (2007). However, besides the usual difficulties encountered in applied work (e.g., Carare and Tchaidze (2005) and Castelnuovo (2003)), these estimates face an identification problem pointed out in Minford, Perugini and Srinivasan (2001, 2002) and Cochrane (2007)- see also Minford (2008) which we use in what follows.
Lack of identification occurs when an equation could be confused with a linear combination of other equations in the model. Thus DSGE models give rise to the same correlations between interest rate and inflation as the Taylor rule, even if the Fed is doing something quite different, such as targeting the money supply. For example, Minford, Perugini and Srinivasan (2001, 2002) show this in a DSGE model with Fischer wage contracts (see also Gillman, Le and Minford (2007)).

In effect, unless the econometrician knows from other sources of information that the central bank is pursuing a Taylor rule, he cannot be sure he is estimating a Taylor rule when he specifies a Taylor rule equation because under other possible monetary policy rules a similar relationship to the Taylor rule is implied. Of course by specifying a Taylor rule he will successfully retrieve the coefficients of the ‘rule’; but he cannot know that these describe the true rule the central bank is following.

To illustrate the point, we may consider a popular DSGE model but with a money supply rule instead of a Taylor rule:

(IS curve): \( y_t = \gamma_1 E_{t-1} y_{t+1} - \phi_r + \nu_t \)

(Phillips curve): \( \pi_t = \zeta (y_t - y^*) + v E_{t-1} \pi_{t+1} + (1 - v) \pi_{t-1} + u_t \)

(Money supply target): \( \Delta m_t = m + \mu_t \)

(Money demand): \( m_t - p_t = \psi_1 E_{t-1} y_{t+1} - \psi_2 R_t + \varepsilon_t \)

(Fisher identity): \( R_t = r_t + E_{t-1} \pi_{t+1} \)

This model implies a Taylor-type relationship that looks like:

\[
R_t = r^* + \pi^* + \gamma \chi^{-1} (\pi_t - \pi^*) + \psi \chi^{-1} (y_t - y^*) + w_t ,
\]
where \( \chi = \psi_2 \gamma - \psi_1 \phi \), and the error term, \( w_t \), is both correlated with inflation and output and autocorrelated; it contains the current money supply/demand and aggregate demand shocks and also various lagged values (the change in lagged expected future inflation, interest rate, the output gap, the money demand shock, and the aggregate demand shock). This particular Taylor-type relation was created with a combination of equations—the solution of the money demand and supply curves for interest rate, the Fisher identity, and the IS curve for expected future output. But other Taylor-type relationships could be created with combinations of other equations, including the solution equations, generated by the model. They will all exhibit autocorrelation and contemporaneous correlation with output and inflation, clearly of different sorts depending on the combinations used.

All the above applies to identifying a single equation being estimated; thus one cannot distinguish a Taylor rule equation from the equations implied by the model and alternative rules when one just estimates that equation. One could attempt to apply further restrictions—e.g., on the error process— but such are hard to justify—e.g., the error in a Taylor rule (‘monetary judgement’ based on extraneous factors) can be autocorrelated (because those factors may be persistent).

---

1 From the money demand and money supply equation, 
\[
\psi_2 \Delta R_t = \pi_t - m + \psi_1 \Delta E_t + \Delta \varepsilon_t - \mu_t. 
\]
Substitute for \( E_t \) from the IS curve and then inside that for real interest rates from the Fisher identity giving 
\[
(\psi_2 - \psi_1')(\Delta R_t - \Delta R') = (\pi_t - m) - \psi_1' \Delta E_t + \psi_1' \Delta \varepsilon_t - \mu_t \]
where the constants \( R' \) and \( y' \) have been subtracted from \( R \) and \( y_t \) respectively, exploiting the fact that when differenced they disappear. Finally, 
\[
R_t = r' + \pi' + \Delta R_t + \{R_t - R'\} - \psi_1' \Delta E_t + \psi_1' \Delta \varepsilon_t - \mu_t, 
\]
where we have used the steady state property that \( R = r' + \pi' \) and \( m = \pi' \).
However, when a ‘monetary rule’ is chosen for inclusion in a complete DSGE model, then the model imposes over-identifying restrictions through the rational expectations terms which involve in principle all the model’s parameters. Thus a model with a particular rule is in general over-identified so that estimation by full information methods of that particular model as specified is possible (as in Rotemberg and Woodford (1997, 1998), Smets and Wouters (2003), Ontaski and Williams (2004) and Ireland (2007)). One way of putting this is that there are more structural parameters than reduced form parameters. Another is to note that the reduced form will change if the structural description of monetary policy changes- a point first made by Lucas (1976) in his ‘critique’ of conventional optimal policy optimization at that time, and some illustrations of how reduced forms will change for a model like the one in this paper (see Meenagh et al. (2009)). So if the econometrician posits a Taylor rule then he will retrieve its coefficients and those of the rest of the model under the assumption that it is the true structural monetary rule. He could then compare the coefficients for a model where he assumes some other rule. He can distinguish between the two models via their different reduced forms and hence their different fits to the data. Thus it is possible to identify the different rules of monetary policy behavior via full information estimation.

However, the identification problem does not go away, even when a model is over-identified in this way. The problem is that the decision to include the Taylor rule in such a model is justified by the fact that it fits the data in single equation estimation; but as we have seen such a choice could be the victim of identification failure as the rule could be mimicking the joint behaviour of the rest of the model and some other (true) monetary rule. If so, including it in the model will produce a mis-specified
model whose behaviour will not fit the data as well as the properly-specified model with the true monetary policy equation. To detect this and also to find the true model we need not only to test this model but also to test possible well-specified alternatives. Thus we need to check whether there is a better model which with its over-identifying restrictions may fit the data more precisely.

This points the way forward. One may specify a complete DSGE model with alternative monetary rules and use the over-identifying restrictions to determine their differing behaviour. One may then test which of them is accepted by the data. This is the approach taken here.

3. A Simple New Keynesian Model for Inflation, Output Gap and Nominal Interest Rate Determinations

We follow a common practice among New Keynesian authors of setting up a full DSGE model with representative agents and reduce it to a three-equation framework consisting of an ‘IS’ curve, a Phillips curve and a monetary policy rule (Clarida, Gali and Gertler (1999, 2000), Rotemberg and Woodford (1997, 1998), Walsh (2000)).

Under rational expectations the ‘IS’ curve derived from the household’s optimization problem and the Phillips curve derived from the firm’s optimal price-setting behaviour given Calvo (1983) contract can be shown as:

\[ x_t = E_t x_{t+1} - \left( \frac{1}{\sigma} \right) (\tilde{\pi}_t - E_t \pi_{t+1}) + v_t \]  \[ 3.1 \]

\[ \pi_t = \beta E_t \pi_{t+1} + \gamma x_t + \kappa u_t^w \]  \[ 3.2 \]
where \( x_t \) is the output gap, \( \tilde{i}_t \) is the deviation of interest rate from its steady-state value, \( \pi_t \) is the price inflation, and \( v_t \) and \( u_t \) are interpreted as ‘demand’ disturbance and ‘supply disturbance’, respectively\(^2\).

We consider three popular regime versions usually suggested for the US economy. These are the optimal timeless policy when the Fed commits to minimize a typical social welfare loss function, the original Taylor rule [2.1], and its ‘interest rate smoothed’ version [2.2].

In particular, the optimal timeless policy is derived following the idea of Woodford (1999) by ignoring the initial conditions confronting the Fed at the regime’s inception. It requires keeping inflation equal to a fixed fraction of the first difference of output gap in each period such that

\[
\pi_t = -\frac{\alpha}{\gamma} (x_t - x_{t-1})
\]

with \( \alpha \) indicating the relative weight the Fed puts on loss from output variations against inflation variations\(^3\). We assume implementing such a rule is subject to ‘policy disturbance’—which would arise as well when alternatives are being pursued—due to ‘trembling hand’; so the stochastic version of if will read:

\(^2\) Note that \( \gamma \) and \( \kappa \) are functions of other structural parameters and some steady-state relationships (See table 4.2 for calibrations in next section). Full derivations of equation [3.1] and [3.2] are shown in the Supporting Annex available on the Cardiff Business School working paper webpage at:


\(^3\) See also Clarida, Gali and Gertler (1999, pp.1681) and McCallum and Nelson (2004, pp.45). Note this implication is based on defining social welfare loss as ‘the loss in units of consumption as a percentage of steady-state output’ as in Rotemberg and Woodford (1998) and Nistico (2007); it is also conditional on assuming a particular utility function and zero-inflation steady state—See Supporting Annex at: [http://www.cf.ac.uk/carbs/faculty/minfordp/E2009_19Annex.pdf](http://www.cf.ac.uk/carbs/faculty/minfordp/E2009_19Annex.pdf) for more details.
\[ \pi_t = -\frac{\alpha}{\gamma} (x_t - x_{t-1}) + \xi_t \]  

[3.3]

with the ‘disturbance’ being captured by \( \xi_t \) (as those in the Taylor rules).

We can now construct three pseudo economies where different policies are pursued.

These are summarised in Table 3.1 as follows:

**Table 3.1: Models to be tested**

<table>
<thead>
<tr>
<th>Model one</th>
<th>(‘IS’+PP+ optimal timeless rule )</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘IS’ curve</td>
<td>( x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\tilde{\eta} - E_t \pi_{t+1}) + v_t ) \hspace{1cm} ( v_t = \rho_v v_{t-1} + e^v_t )</td>
</tr>
<tr>
<td>PP curve</td>
<td>( \pi_t = \beta E_t \pi_{t+1} + \gamma x_t + \kappa u^w_t ) \hspace{1cm} ( u^w_t = \rho_u u^w_{t-1} + e^{u^w}_{t} )</td>
</tr>
<tr>
<td>Policy rule</td>
<td>( \pi_t = -\frac{\alpha}{\gamma} (x_t - x_{t-1}) + \xi_t ) \hspace{1cm} ( \xi_t = \rho_\xi \xi_{t-1} + e^\xi_t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model two</th>
<th>(‘IS’+PP+ the original Taylor rule)</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘IS’ curve</td>
<td>( x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\tilde{\eta} - E_t \pi_{t+1}) + v_t ) \hspace{1cm} ( v_t = \rho_v v_{t-1} + e^v_t )</td>
</tr>
<tr>
<td>PP curve</td>
<td>( \pi_t = \beta E_t \pi_{t+1} + \gamma x_t + \kappa u^w_t ) \hspace{1cm} ( u^w_t = \rho_u u^w_{t-1} + e^{u^w}_{t} )</td>
</tr>
<tr>
<td>Policy rule</td>
<td>( i_t^A = \pi_t^A + 0.5 x_t + 0.5(\pi_t^A - 0.02) + 0.02 + \xi_t )</td>
</tr>
<tr>
<td>The transformed policy rule</td>
<td>( \tilde{i}<em>t = 1.5 \pi_t + 0.125 x_t + \xi_t ) \hspace{1cm} ( \xi_t = \rho</em>\xi \xi_{t-1} + e^\xi_t )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Model three</th>
<th>(‘IS’+PP+ Taylor rule with ‘interest rate smoothing’)</th>
</tr>
</thead>
<tbody>
<tr>
<td>‘IS’ curve</td>
<td>( x_t = E_t x_{t+1} - \left(\frac{1}{\sigma}\right)(\tilde{\eta} - E_t \pi_{t+1}) + v_t ) \hspace{1cm} ( v_t = \rho_v v_{t-1} + e^v_t )</td>
</tr>
</tbody>
</table>

\[4\] Note we have assumed an AR(1) process for all disturbances to the structural equations to capture possible omitted variables. We also transform the Taylor rules to quarterly versions so that the frequency of interest rate and inflation is consistent with other variables in the model—note we have dropped the constants as we will use demeaned, detrended data (See section 4.2 in what follows) and we have assumed \( \tilde{i}_t = i_t - i_t - \left(\frac{1}{\beta} - 1\right) \) in zero-inflation steady state.
Note that these models differ only in their policies being implemented. Hence by comparing their capacity to fit the real data, one should be able to tell which rule, when included in a simple New Keynesian model, provides the best explanation for the ‘reality’ and therefore the most appropriate description of the underlying policy.

We perform this in section 4 in what follows.

4. Identification of Monetary Policy Rules with Tests

4.1. Methodology—testing the models using the method of indirect inference

We evaluate the models’ performance in fitting the real data using the method of indirect inference proposed in Minford, Theodoridis and Meenagh (2009)\(^5\). Such an approach employs an auxiliary model that is completely independent of the theoretical one to produce descriptors of the data against which the performance of the theory is evaluated indirectly. Such descriptors can be either the estimated parameters of the auxiliary model or functions of these. While these are treated as the ‘reality’, the theoretical model being evaluated is simulated to find its implied values for them.

\(^5\) See Meenagh, Minford and Wickens (2009) and Le, et al. (2009, 2010) for more applications of this approach.
Indirect inference has been widely used in the estimation of structural models (e.g., Smith (1993), Gregory and Smith (1991, 1993), Gourieroux et al. (1993), Gourieroux and Monfort (1996) and Canova (2005)). Here we make a different use of indirect inference as our aim is to evaluate an already estimated or calibrated structural model. The common element is the use of an auxiliary time series model. In estimation the parameters of the structural model are chosen such that when this model is simulated it generates estimates of the auxiliary model similar to those obtained from the actual data. The optimal choices of parameters for the structural model are those that minimise the distance between a given function of the two sets of estimated coefficients of the auxiliary model. Common choices of this function are the actual coefficients, the scores or the impulse response functions. In model evaluation the parameters of the structural model are taken as given. The aim is to compare the performance of the auxiliary model estimated on simulated data derived from the given estimates of a structural model - which is taken as a true model of the economy, the null hypothesis - with the performance of the auxiliary model when estimated from the actual data. If the structural model is correct then its predictions about the impulse responses, moments and time series properties of the data should statistically match those based on the actual data. The comparison is based on the distributions of the two sets of parameter estimates of the auxiliary model, or of functions of these estimates.

In other words, the testing procedure involves first constructing the errors derived from the previously estimated structural model and the actual data. These errors are then bootstrapped and used to generate for each bootstrap new data based on the
structural model. An auxiliary time series model is then fitted to each set of data and the sampling distribution of the coefficients of the auxiliary time series model is obtained from these estimates of the auxiliary model. A Wald statistic is computed to determine whether functions of the parameters of the time series model estimated on the actual data lie in some confidence interval implied by this sampling distribution.

Following Minford, Theodoridis and Meenagh (2009), this paper takes a VAR(1) for the three macro variables (interest rate, output gap and inflation) as the appropriate auxiliary model and treats as the descriptors of the data the VAR coefficients and the variances of the three variables. The Wald statistic is computed from these. This tests whether the observed dynamics and volatility of the chosen variables are explained by the simulated joint distribution of the corresponding parameters at a given confidence level. The Wald statistic is given by:

\[
(\Phi - \Phi^*) \Sigma_{\Phi}^{-1} (\Phi - \Phi^*) \tag{4.1}
\]

where \(\Phi\) is the vector of VAR estimates of the concerned parameters yielded in each simulation, with \(\Phi^*\) and \(\Sigma_{\Phi}\) representing the corresponding sample means and variance-covariance matrix of these estimates calculated across simulations, respectively. The whole test procedure can be illustrated diagrammatically in Figure 4.1 as follows:

---

Note that the VAR impulse response functions, the co-variances, as well as the auto/cross correlations of the left-hand-side variables will all be implicitly examined when the VAR coefficient matrix is considered, since the formers are functions of the latter.
While the first panel in Figure 4.1 summarises the main steps of the methodology described in the past two paragraphs, the ‘mountain-shaped’ diagram replicated from Meenagh, Minford and Wickens (2009) in panel B gives an example of how the ‘reality’ is compared to model predictions using the Wald test when only two parameters of the auxiliary model are concerned: let either of the spots in panel B
indicate the real-data-based estimates of the two concerned parameters and the ‘mountain’ represent their corresponding joint distribution generated from model simulations; when the real-data-based estimates are given at point A, the theoretical model in hand will fail to provide a sensible explanation for the real world, since what the model predicts is too ‘far away’ from what the ‘reality’ suggests; by contrast, if the real-data-based estimates are given at point B, which, according to the diagram, means the ‘reality’ is captured by the model-implied joint distribution of the corresponding parameters, the hypothesis that ‘the real data are generated by the model under discussion’ will be completely possible, although how likely that will be the case is dependent on what is reported for the Wald-statistic7.

The simulated joint distribution of the VAR parameters mentioned above is a bootstrapped distribution. This is generated from bootstrapping the innovations implied by the data and the theoretical model and is therefore an estimate of the small sample distribution8. Such a distribution is generally more accurate for small samples than the asymptotic distribution and is shown to be consistent by Le, et al. (2010) given that the Wald statistic is ‘asymptotically pivotal’; it also has quite good accuracy in small sample Montecarlo experiments according to Le, et al. (2010)9.

7 Note that in this particular example, only two parameters are considered and they are both assumed to be normally-distributed. Yet, the principle of the Wald test would not be changed for more general cases, where more parameters, which may follow various kinds of distribution, are concerned.

8 Note that by bootstrapping the innovations to the Taylor rules, we mean those from the transformed equations. Also, the bootstraps in our tests are all drawn as time vectors so that any contemporaneous correlation between the innovations will be preserved.

9 Specifically, they found that the bias due to bootstrapping was just over 2% at the 95% confidence level and 0.6% at the 99% level. They suggested possible further refinements in the bootstrapping procedure which could increase the accuracy further; however, we do not feel it necessary to pursue these here.
4.2. Data and calibration

Data

For testing the prevailing monetary policy in the US, this paper employs the quarterly data published by the Federal Reserve Bank of St. Louis from 1982Q2 to 2007Q4, when most of the period are covered by the ‘Greenspan era’, during which the US economy is thought to have been governed by one identical monetary regime and most discussions about the Fed’s behaviour are concerned\textsuperscript{10}.

Regarding the three endogenous variables involved, $\tilde{\tau}$ is measured as the deviation of current Fed rate from the steady-state value, output gap $\tau_x$ is approximated by the percentage deviation of real GDP from its HP trend, whereas $\pi_t$ indicates the quarterly inflation rate defined as the log difference between the current CPI and the one captured in the last quarter\textsuperscript{11}. For simplicity, the tests use data that are in deviations from means\textsuperscript{12}. In particular, a linear trend is taken out of the interest rate series such that stationarity is ensured. Figure 4.2 to figure 4.4 below plot each of these series in deviation forms; the relevant unit root test results are also presented in table 4.1.

\textsuperscript{10} Data base of Federal Bank of St. Louis: \url{http://research.stlouisfed.org/fred2/}
\textsuperscript{11} Notice that the annual Fed rates proposed by the Fred are purposely adjusted into quarterly rates such that the frequencies of all the time series are kept consistently on quarterly basis.
\textsuperscript{12} Nevertheless, the time series of output gap used is in level, as its sample mean is not significantly different from zero.
Table 4.1 Unit Root Tests for Stationarity

<table>
<thead>
<tr>
<th>Time series</th>
<th>5% critical value</th>
<th>ADF test statistics</th>
<th>p-values*</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{t}_t$</td>
<td>-1.94</td>
<td>-2.81</td>
<td>0.0053</td>
</tr>
<tr>
<td>$x_t$</td>
<td>-1.94</td>
<td>-2.95</td>
<td>0.0035</td>
</tr>
<tr>
<td>$\pi_t$</td>
<td>-1.94</td>
<td>-3.60</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

Note: ‘*’ denotes the Mackinnon (1996) one-sided p-values at 5% level of significance; $H_0$: the time series has a unit root.

Note that since all the data used are in deviation from mean, a VAR(1) representation of them would contain no constant but only nine parameters in the autoregressive coefficient matrix. Also, the use of such data requires dropping the constants in any equation of the models as well. This explains why the two transformed Taylor rules involved in model two and three have no constant at all.

Calibration

The values of parameters chosen for the tests are those commonly calibrated and accepted for the US economy in literature. These parameters and their values are listed in table 4.2 as follows:
Table 4.2 Calibration of Parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Definitions</th>
<th>Calibrated values</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>time discount factor</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>inverse of elasticity of intertemporal consumption</td>
<td>2</td>
</tr>
<tr>
<td>$\eta$</td>
<td>inverse of elasticity of labour</td>
<td>3</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Calvo contract price non-adjusting probability</td>
<td>0.53</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>steady-state government expenditure to output ratio</td>
<td>0.23</td>
</tr>
<tr>
<td>$Y/C$</td>
<td>steady-state output to consumption ratio</td>
<td>1/0.77 (implied)</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$\kappa = \frac{(1 - \omega)(1 - \omega \beta)}{\omega}$</td>
<td>0.42 (implied)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>$\gamma = \kappa(\eta + \sigma \frac{Y}{C})$</td>
<td>2.36 (implied)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>price elasticity of demand</td>
<td>6</td>
</tr>
<tr>
<td>$\alpha/\gamma = \theta^{-1}$</td>
<td>parameter driving the optimal timeless policy$^{13}$</td>
<td>1/6 (implied)</td>
</tr>
<tr>
<td>$\rho$</td>
<td>degree of interest rate smoothness</td>
<td>0.76</td>
</tr>
<tr>
<td>$\gamma_x$</td>
<td>interest rate response to inflation</td>
<td>1.44</td>
</tr>
<tr>
<td>$\gamma_x$</td>
<td>interest rate response to output gap</td>
<td>0.14</td>
</tr>
<tr>
<td>$\rho_v$</td>
<td>autoregressive coefficient of demand disturbance</td>
<td>0.91 (sample estimate)</td>
</tr>
<tr>
<td>$\rho_u$</td>
<td>autoregressive coefficient of supply disturbance</td>
<td>0.82 (sample estimate)</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>autoregressive coefficient of policy disturbance: model one</td>
<td>0.35 (sample estimate)</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>autoregressive coefficient of policy disturbance: model two</td>
<td>0.37 (sample estimate)</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>autoregressive coefficient of policy disturbance: model three</td>
<td>0.31 (sample estimate)</td>
</tr>
</tbody>
</table>

As table 4.2 shows, the quarterly time discount rate is calibrated as 0.99, implying an approximately 1% quarterly (or equivalently 4% annual) rate of interest in steady state. $\sigma$ and $\eta$ are set to as high as 2 and 3 respectively as in Carlstrom and Fuerst (2008),

$^{13}$ Nistico (2007) showed the relative weight $\alpha$ is equal to the ratio of the slope of the Phillips curve to the price elasticity of demand, namely, $\alpha = \gamma / \theta$. 

17
who emphasized on the values’ consistency with the inelasticity of both intertemporal consumption decision and labour supply shown by the US data. The Calvo price stickiness of 0.53 and the price elasticity of demand of 6 are both taken from Kuester, Muller and Stolting (2009). Note that these values accordingly imply a contract length of more than three quarters\textsuperscript{14}, while the constant mark-up of price to nominal marginal cost is 1.2. The implied steady-state output-consumption ratio of 1/0.77 is calculated based on the steady-state ratio of government expenditure over output of 0.23 calibrated in Foley and Taylor (2004). Regarding the Taylor rule tested in model three, again, calibration follows those in Carlstrom and Fuerst (2008), where the interest rate’s response to a unit change in inflation and output gap are 1.44 and 0.14 respectively, with the degree of ‘smoothness’ of 0.76. The last five lines in table 4.2 also report the autoregressive coefficients of the structural disturbances implied by the models, which are all sample estimates based on the real data\textsuperscript{15}. Notice that both of the demand and supply shocks are shown to be highly persistent, in contrast to the policy shocks reflected in all the three models.

\textbf{4.3. Evaluating the models’ performance—the test results}

The test results and the corresponding evaluations for the three models proposed are presented in turn in this subsection, where the simulated 95% lower bounds and upper bounds for the concerned parameters, their real-data-based counterparts, as well as the relevant Wald statistics, are considered\textsuperscript{16}. Since there are three endogenous variables,

\textsuperscript{14} To be accurate, $2\omega^{-1} - 1 \approx 3.26$.

\textsuperscript{15} These estimates are all significant at 5\% significance level.

\textsuperscript{16} We have also produced diagrams for VAR impulse response functions and cross-correlations between variables with lower and upper bounds plotted in the Supporting Annex at: http://www.cf.ac.uk/carbs/faculty/minfordp/E2009_19Annex.pdf .
namely, interest rate, output gap, and the rate of inflation, in the VAR(1) representation, twelve components are involved in calculation of the Wald statistics; these are the nine VAR coefficients and the three variances of the L.H.S. variables.\(^{17}\)

The detailed results for each model are as follows:

**Model one (‘IS’+PP+ optimal timeless policy)**

Table 4.3 below summarises the test results regarding the dynamic properties of model one:

<table>
<thead>
<tr>
<th></th>
<th>VAR(1) Coefficients</th>
<th>95% lower bound</th>
<th>95% upper bound</th>
<th>Values estimated with real data</th>
<th>In/Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{11} )</td>
<td>0.6454</td>
<td>0.9420</td>
<td>0.8017</td>
<td>In</td>
<td></td>
</tr>
<tr>
<td>( \beta_{12} )</td>
<td>-0.0844</td>
<td>0.0439</td>
<td>0.0834</td>
<td>Out</td>
<td></td>
</tr>
<tr>
<td>( \beta_{13} )</td>
<td>-0.1774</td>
<td>0.0991</td>
<td>0.0112</td>
<td>In</td>
<td></td>
</tr>
<tr>
<td>( \beta_{21} )</td>
<td>-0.2589</td>
<td>0.2578</td>
<td>-0.2711</td>
<td>Out</td>
<td></td>
</tr>
<tr>
<td>( \beta_{22} )</td>
<td>0.6685</td>
<td>0.9105</td>
<td>0.9009</td>
<td>In</td>
<td></td>
</tr>
<tr>
<td>( \beta_{23} )</td>
<td>-0.4037</td>
<td>0.1871</td>
<td>-0.1090</td>
<td>In</td>
<td></td>
</tr>
<tr>
<td>( \beta_{31} )</td>
<td>-0.1821</td>
<td>0.1595</td>
<td>-0.0187</td>
<td>In</td>
<td></td>
</tr>
<tr>
<td>( \beta_{32} )</td>
<td>-0.0434</td>
<td>0.1361</td>
<td>0.1428</td>
<td>Out</td>
<td></td>
</tr>
<tr>
<td>( \beta_{33} )</td>
<td>0.1010</td>
<td>0.4976</td>
<td>0.2552</td>
<td>In</td>
<td></td>
</tr>
</tbody>
</table>

‘Directed’ Wald statistic (for dynamics) 98.2%

\(^{17}\) Note that the VAR(1) representation is assumed to take the form:

\[
\begin{bmatrix}
\tilde{r}_t \\
\tilde{x}_t \\
\tilde{\pi}_t
\end{bmatrix} = \begin{bmatrix}
\beta_{11} & \beta_{12} & \beta_{13} \\
\beta_{21} & \beta_{22} & \beta_{23} \\
\beta_{31} & \beta_{32} & \beta_{33}
\end{bmatrix} \begin{bmatrix}
\tilde{r}_{t-1} \\
\tilde{x}_{t-1} \\
\tilde{\pi}_{t-1}
\end{bmatrix} + \Sigma_t
\]
According to table 4.3, three out of the nine real-data-based estimates of the VAR coefficients that reflect the actual dynamics are found to lie outside their corresponding 95% bounds implied by the theoretical model. Specifically, the response of interest rate to the lagged output gap and the response of output gap to the lagged interest rate, as well as the response of inflation to the lagged output gap, are all shown to be more aggressive than what the theoretical model would predict. In particular, the interest rate’s response to the lagged output gap in reality is more than twice as great as what could be generated from model simulations. Overall, the ‘directed’ Wald statistic is reported as 98.2%; this indicates the model’s success in capturing the actual dynamics at the 99% confidence level, although it clearly fails at the more conventional 95% level. Clearly, all the DSGE models here have problems fitting the data closely; our main purpose is to rank them and to see if one of them stands out as relatively acceptable.

Turing to the other aspect of the concerned ‘stylized facts’, table 4.4 below shows the extent to which the observed volatilities of real data are explained by the theoretical model:

<table>
<thead>
<tr>
<th>Volatilities of the endogenous variables</th>
<th>95% lower bound</th>
<th>95% upper bound</th>
<th>Values calculated with real data</th>
<th>In/Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>var((\bar{r}))</td>
<td>0.0102</td>
<td>0.0450</td>
<td>0.0171</td>
<td>In</td>
</tr>
<tr>
<td>var(x)</td>
<td>0.0411</td>
<td>0.1601</td>
<td>0.0951</td>
<td>In</td>
</tr>
<tr>
<td>var((\pi))</td>
<td>0.0094</td>
<td>0.0206</td>
<td>0.0153</td>
<td>In</td>
</tr>
<tr>
<td>‘Directed’ Wald statistic (for volatilities)</td>
<td></td>
<td></td>
<td>10.4%</td>
<td></td>
</tr>
</tbody>
</table>

Note: Values reported in table 4.4 are magnified by 1000 times as their original values.
As table 4.4 shows, while the variances of the three considered endogenous variables calculated with the real data are all within the model-implied 95% bounds, the ‘directed’ Wald statistic is reported as 10.4%. That is, at the confidence level of 95%, the observed volatilities are not only individually, but also jointly explained by the theoretical model- with such a low Wald statistic, they are very close to the joint means of the variances.

Note that, by using the ‘directed’ Wald, we have been examining the theoretical model’s partial capacities in explaining either the dynamics or the volatilities of the actual data. To evaluate the model’s overall fitness to the real world, we consider both the dynamics and the volatilities simultaneously, for which we use the ‘full’ Wald statistic. This is reported in table 4.5 as 96.5%; hence the null hypothesis that the theoretical model explains both the actual dynamics and volatilities is easily accepted at the 99% confidence level and only marginally rejected at 95%.

<table>
<thead>
<tr>
<th>The concerned model properties</th>
<th>‘Full’ Wald statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamics + Volatilities</td>
<td>96.5%</td>
</tr>
</tbody>
</table>

To summarise, model one does not only provide a rough explanation for the actual dynamics, but also precisely captures the volatilities shown by the real data; its overall fitness in explaining the data is fairly good, as DSGE models go and we may consider as a reasonable approximation to the real-world economy.

*Model two (‘IS’+PP+the original Taylor rule)*
Leaving the economic environment (i.e., the ‘IS’ curve and the Phillips curve) unchanged, model two replaces the optimal timeless rule assumed in model one with the original Taylor rule, widely regarded as a good description of the Fed’s monetary policy from 1982 to at least the early 1990s. The test results for the dynamic behaviour of the model are reported in table 4.6 as follow:

<table>
<thead>
<tr>
<th>VAR(1) Coefficients</th>
<th>95% lower bound</th>
<th>95% upper bound</th>
<th>Values estimated with real data</th>
<th>In/Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_{11}$</td>
<td>0.6139</td>
<td>1.1165</td>
<td>0.8017</td>
<td>In</td>
</tr>
<tr>
<td>$\beta_{12}$</td>
<td>-0.0743</td>
<td>0.2385</td>
<td>0.0834</td>
<td>In</td>
</tr>
<tr>
<td>$\beta_{13}$</td>
<td>-0.3098</td>
<td>0.2977</td>
<td>0.0112</td>
<td>In</td>
</tr>
<tr>
<td>$\beta_{21}$</td>
<td>-0.1571</td>
<td>0.3175</td>
<td>-0.2711</td>
<td>Out</td>
</tr>
<tr>
<td>$\beta_{22}$</td>
<td>0.6112</td>
<td>0.8960</td>
<td>0.9009</td>
<td>Out</td>
</tr>
<tr>
<td>$\beta_{23}$</td>
<td>-0.4316</td>
<td>0.1654</td>
<td>-0.1090</td>
<td>In</td>
</tr>
<tr>
<td>$\beta_{31}$</td>
<td>-0.1055</td>
<td>0.6202</td>
<td>-0.0187</td>
<td>In</td>
</tr>
<tr>
<td>$\beta_{32}$</td>
<td>-0.1457</td>
<td>0.1983</td>
<td>0.1428</td>
<td>In</td>
</tr>
<tr>
<td>$\beta_{33}$</td>
<td>-0.0043</td>
<td>0.6596</td>
<td>0.2552</td>
<td>In</td>
</tr>
</tbody>
</table>

‘Directed’ Wald statistic (for dynamics) 100%

As revealed in table 4.6, while most of the real-data-based estimates of the VAR coefficients are individually captured by the 95% bounds implied by model simulations, the output gap’s responses to the lagged interest rate and to its own lagged value are found to exceed their corresponding lower bound and upper bound, respectively. Overall, the ‘directed’ Wald statistic is reported as 100%, which means there is no hope at all for the theoretical model to generate a joint distribution of the VAR coefficients that simultaneously explains the ones observed in reality.
theoretical model thus is totally rejected by the Wald test for the dynamics.

Yet the model can still explain most of the data volatilities, as shown in table 4.7. It generates excessive interest rate variance, but reasonably matches series the variances of the output gap and inflation. The ‘directed’ Wald statistic for the variances is 91.5%, comfortably accepted therefore at 95%.

<table>
<thead>
<tr>
<th>Volatilities of the endogenous variables</th>
<th>95% lower bound</th>
<th>95% upper bound</th>
<th>Values calculated with real data</th>
<th>In/Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \text{var}(\bar{r}) )</td>
<td>0.0604</td>
<td>0.2790</td>
<td>0.0171</td>
<td>Out</td>
</tr>
<tr>
<td>( \text{var}(x) )</td>
<td>0.0400</td>
<td>0.1527</td>
<td>0.0951</td>
<td>In</td>
</tr>
<tr>
<td>( \text{var}(\pi) )</td>
<td>0.0475</td>
<td>0.1672</td>
<td>0.0153</td>
<td>In</td>
</tr>
<tr>
<td>‘Directed’ Wald statistic (for volatilities)</td>
<td></td>
<td></td>
<td>91.5%</td>
<td></td>
</tr>
</tbody>
</table>

Note: Values reported in table 4.7 are magnified by 1000 times as their original values.

Lastly, table 4.8 shows the ‘full’ Wald statistic as 100%. This is hardly surprising since it fails so badly to capture the dynamics in the data.

<table>
<thead>
<tr>
<th>The concerned model properties</th>
<th>‘Full’ Wald statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamics + Volatilities</td>
<td>100%</td>
</tr>
</tbody>
</table>

So far, it is clear that model two, where the original Taylor rule is set as the fundamental monetary policy, has only partially captured the characteristics shown by the actual data; unless the discussions are focused exclusively on the ‘size’ of the economy’s fluctuations, such a model is not to be taken as a realistic description of the prevailing economic reality.
Model three (‘IS’+PP+Taylor rule with ‘interest rate smoothing’)

In this last model, a calibrated Taylor type rule whose specification reflects the feature of ‘interest rate smoothing’ is assumed to be the underlying policy reaction function. In effect, the rate of interest implied by such a rule is a weighted average of what was set in the last period and what would be required had the original Taylor rule been put in place, with the weights being the degree of ‘policy inertia’ and its complement, respectively. While Taylor-type rules in which interest rates are ‘smoothed’ are commonly claimed to be supported by empirical evidence as the prevailing monetary policies (e.g., Clarida, Gali and Gertler (1999, 2000), Rotemberg and Woodford (1997, 1998)), the test results regarding model three’s performance are revealed as follows:

Table 4.9: Individual VAR Coefficients and the ‘Directed’ Wald Statistic

<table>
<thead>
<tr>
<th>VAR(1) Coefficients</th>
<th>95% lower bound</th>
<th>95% upper bound</th>
<th>Values estimated with real data</th>
<th>In/Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \beta_{11} )</td>
<td>0.7228</td>
<td>0.9470</td>
<td>0.8017</td>
<td>In</td>
</tr>
<tr>
<td>( \beta_{12} )</td>
<td>-0.0168</td>
<td>0.1287</td>
<td>0.0834</td>
<td>In</td>
</tr>
<tr>
<td>( \beta_{13} )</td>
<td>-0.0029</td>
<td>0.1553</td>
<td>0.0112</td>
<td>In</td>
</tr>
<tr>
<td>( \beta_{21} )</td>
<td>-0.1424</td>
<td>0.2095</td>
<td>-0.2711</td>
<td>Out</td>
</tr>
<tr>
<td>( \beta_{22} )</td>
<td>0.6551</td>
<td>0.8971</td>
<td>0.9009</td>
<td>Out</td>
</tr>
<tr>
<td>( \beta_{23} )</td>
<td>-0.2840</td>
<td>-0.0046</td>
<td>-0.1090</td>
<td>In</td>
</tr>
<tr>
<td>( \beta_{31} )</td>
<td>-0.1668</td>
<td>0.4706</td>
<td>-0.0187</td>
<td>In</td>
</tr>
<tr>
<td>( \beta_{32} )</td>
<td>-0.1260</td>
<td>0.2655</td>
<td>0.1428</td>
<td>In</td>
</tr>
<tr>
<td>( \beta_{33} )</td>
<td>0.0830</td>
<td>0.5427</td>
<td>0.2552</td>
<td>In</td>
</tr>
</tbody>
</table>

‘Directed’ Wald statistic (for dynamics) 99.9%
Table 4.9 above summarises how the actual dynamics are explained by the theoretical model. Again, except for the output gap’s responses to the lagged interest rate and to its own lagged value, all dynamic relationships shown by the real data are individually captured by the simulated 95% bounds. Yet, the ‘directed’ Wald statistic reported is as high as 99.9%, indicating the theoretical model can hardly be used for explaining the observed dynamics, as the set of real-data-based estimates of the VAR coefficients is not captured by the corresponding joint distribution generated from model simulations, even at a 99% confidence level\textsuperscript{18}.

Turning to the volatilities of the endogenous variables, table 4.10 shows the theoretical model has merely correctly mimicked the performance of the output gap, but evoked too much variance for both the interest rate and inflation; the ‘directed’ Wald statistic is reported as 99.4%, which implies the model in hand is not a proper explanation for the observed volatilities, either.

<table>
<thead>
<tr>
<th>Volatilities of the endogenous variables</th>
<th>95% lower bound</th>
<th>95% upper bound</th>
<th>Values calculated with real data</th>
<th>In/Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{var}(\bar{\bar{i}}))</td>
<td>0.0229</td>
<td>0.1174</td>
<td>0.0171</td>
<td>Out</td>
</tr>
<tr>
<td>(\text{var}(x))</td>
<td>0.0380</td>
<td>0.1430</td>
<td>0.0951</td>
<td>In</td>
</tr>
<tr>
<td>(\text{var}(\pi))</td>
<td>0.0532</td>
<td>0.1158</td>
<td>0.0153</td>
<td>Out</td>
</tr>
</tbody>
</table>

“Directed” Wald statistic (for volatilities) \(99.4\%\)

Note: Values reported in table 4.10 are magnified by 1000 times as their original values.

In fact, the poor explanatory power of model three is not only detected by the ‘directed’ Wald, but also captured by the ‘full’ Wald when the model’s overall fitness

\textsuperscript{18} Note that the test results in this case are rather similar to their counterparts suggested for model two.
is considered: note that the ‘full’ Wald statistic reported in table 4.11 is 99.9%, which is another way of saying it is hardly possible for the model to have generated data that simultaneously fit the dynamics and volatilities observed in reality.

<table>
<thead>
<tr>
<th>The concerned model properties</th>
<th>‘Full’ Wald statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamics + Volatilities</td>
<td>99.9%</td>
</tr>
</tbody>
</table>

Thus model three, where a Taylor rule with ‘interest rate smoothing’ is in operation, cannot be considered to be a good proxy for the real-world economy.

5. Reconsidering the Prevailing Monetary Policy Rule in the Light of the Test Results

5.1 The best-fitting monetary policy rule in the US

While the performances of the three hypothetical NK models are evaluated in the last section, recall that these models only differ in the ways in which monetary policies are set. Hence, by ranking the models in terms of their ‘closeness’ to the real world, one will in effect be considering whether the observed data are more likely to have been generated with the optimal timeless policy or the original Taylor rule, or with a
Taylor rule where the interest rate is ‘smoothed’\textsuperscript{19}. For ranking the models’ performances, the test results revealed in section 4.3 are summarised as follows:

<table>
<thead>
<tr>
<th>NK models</th>
<th>‘Directed’ Wald statistics (for dynamics)</th>
<th>‘Directed’ Wald statistics (for volatilities)</th>
<th>‘Full’ Wald statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model one</td>
<td>98.2%</td>
<td>10.4%</td>
<td>96.5%</td>
</tr>
<tr>
<td>Model two</td>
<td>100%</td>
<td>91.5%</td>
<td>100%</td>
</tr>
<tr>
<td>Model three</td>
<td>99.9%</td>
<td>99.4%</td>
<td>99.9%</td>
</tr>
</tbody>
</table>

Given the test results reproduced in table 5.1, comparison by columns immediately shows the first model, which is combined with the optimal timeless rule, is generally superior to its rivals in fitting US data, as it consistently yields the lowest Wald statistics. More importantly, this model is the only one capable of explaining the dynamics and volatility of the data not only separately but also jointly. By contrast, in the cases where Taylor rules are incorporated into exactly the same economic environments, model two is only able to capture the scale of the economy’s volatility, whereas model three is completely rejected by the data in all dimensions.

5.2 Taylor rules as statistical relationships

The above suggests that the widespread success reported in single-equation regressions of Taylor rules on US data could simply represent some sort of statistical relationship emerging from the model with the optimal timeless policy. To examine this possibility, we treat the optimal timeless rule model again as the true model, the

\textsuperscript{19}That is, the ‘true’ monetary policy rule is identified as a part of the ‘true’ model in a relative sense.
null hypothesis and ask whether the existence of empirical Taylor rules would be consistent with it.

Suppose an arbitrarily specified Taylor-type regression is estimated to infer the potential ‘Taylor rule’ of the US economy. For simplicity, let the Taylor-type regression take the form:

$$\tilde{\pi}_t = \gamma_\pi \pi_t + \gamma_x x_t + \rho \tilde{\pi}_{t-1} + \xi_t$$

[5.1]

where variables have their usual meanings. Equation [5.1] can be estimated either using OLS if we assume the basic requirements for an OLS estimator are fulfilled, or via the IV approach to allow for possible correlations between the explanatory variables and the error term. The OLS and IV estimates based on the US data from 1982Q2 to 2007Q4 are summarised in table 5.2 below:

<table>
<thead>
<tr>
<th></th>
<th>$\gamma_\pi$</th>
<th>$\gamma_x$</th>
<th>$\rho$</th>
<th>Adjusted $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS estimates</td>
<td>0.0453</td>
<td>0.0922</td>
<td>0.8233</td>
<td>0.92</td>
</tr>
<tr>
<td>IV estimates</td>
<td>0.0376</td>
<td>0.1003</td>
<td>0.8017</td>
<td>0.90</td>
</tr>
</tbody>
</table>

Now, use the technique of ‘indirect inference’ to test if the observed ‘Taylor rule’ can be explained by model one based on the data simulated for the same periods.21. The test results are revealed as follows:

---

20 For the IV approach, here we take the lagged inflation and lagged output gap as instruments for their corresponding current values, respectively.
21 Note: a) While one may expect the estimates of $\gamma_\pi$ reported in table 5.2 be greater than one such that the ‘Taylor principle’ would be found, note that most existing literature has treated the interest rate series that is I(1) as a stationary series (See Carare and Tchaidze (2005), pp.17, footnote 17), whereas stationarity is obtained here by de-trending the data; Indeed, the ‘Taylor principle’ could be retrieved if the
According to table 5.3, although the real-data-based estimates of the ‘Taylor rule’ coefficients are not all individually captured by the model-implied 95% bounds, they are indeed explained as a set by the joint distribution of their simulation-based counterparts at the 99% confidence level, since the ‘directed’ Wald statistics are reported as 97.1% and 97.8% in panel A and panel B, respectively, indicating that it is statistically possible for model one to imply the ‘Taylor rules’ observed from both OLS and IV estimations as shown in table 5.2.
These results illustrate the identification problem with which we began this paper: a Taylor-type relation that has a good fit to the data may well be generated by a model where there is no *structural* Taylor rule at all\(^2\). Hence, any estimated or calibrated Taylor rule, no matter how well it might predict the actual movements of the nominal interest rate, is not by itself evidence that monetary policy follows this rule.

Note that table 5.4 below also summarises the Wald statistics when the optimal timeless rule model is used to explain several popular variants of the Taylor rule estimated with OLS. According to the reported Wald statistics, the real-data-based estimates of these Taylor rules are all well captured by the model. The model is thus *robust* in generating essentially the whole range of Taylor rules that have been estimated on US data.

<table>
<thead>
<tr>
<th>Taylor-type regressions</th>
<th>Adjusted (R^2)</th>
<th>‘Directed’ Wald statistic (for Taylor rule coefficients)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(	ilde{I}_t = \gamma_a \pi_t + \gamma_s x_t + \xi_t)</td>
<td>0.89</td>
<td>92.9%</td>
</tr>
<tr>
<td>(\xi_t = \rho_2 \xi_{t-1} + \varepsilon_t)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(	ilde{I}<em>t = \gamma_a \pi</em>{t-1} + \gamma_s x_{t-1} + \xi_t)</td>
<td>0.40</td>
<td>87.0%</td>
</tr>
<tr>
<td>(\tilde{I}<em>t = \rho</em>{\tilde{I}<em>{t-1}} + \gamma_a \pi</em>{t-1} + \gamma_s x_{t-1} + \xi_t)</td>
<td>0.90</td>
<td>97.9%</td>
</tr>
</tbody>
</table>

5.3 The ‘interest rate smoothing’ illusion: an implication

Another issue on which the test results in this paper and the analysis from the previous subsection sheds light is related to ‘interest rate smoothing’. In an early

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\(^2\) Note that the adjusted \(R^2\)’s reported in table 5.2 are as high as 0.92 for the OLS estimates and 0.90 for the IV estimates.
paper Clarida, Gali and Gertler (1999) claimed that a ‘puzzle’ regarding the central banks’ behaviour was yet to be solved, as the timeless rule generally derived from a standard NK model as optimal policy response to changes of macro variables would imply once-and-for-all adjustments of the nominal interest rate, whereas empirical ‘evidence’ from typical Taylor-type regressions estimated with the real data usually displayed a high degree of ‘interest rate smoothing’, in which case the sluggishness of interest rate variations could not be rationalized in terms of optimal behaviours.

While various authors explain such a discrepancy either at a theoretical level (e.g., Rotemberg and Woodford (1997, 1998), Woodford (1999, 2003a, 2003b)) or at an empirical level (e.g., Sack and Wieland (2000), Rudebusch (2002)), the tests in this paper support the optimal timeless rule but reject the Taylor rule with ‘interest rate smoothing’- implying the Fed has been responding to economic changes optimally without deliberately smoothing the interest rate. It is the persistence in the shocks themselves that induced the appearance of inertia in interest rate setting. Furthermore we show that one would find regressions of ‘interest-smoothing Taylor rules’ successfully fit the data even though this was being produced by the optimal timeless rule model. Hence we would argue that these optimal responses by policymakers have been incorrectly interpreted as ‘policy inertia’ due to these misleading regressions.

6. Conclusion

In this paper we have attempted to identify the principles governing US monetary policy since the early 1980s. The ‘Taylor rule’ is widely regarded as a good
description of these principles. Yet there is an identification problem plaguing efforts
to estimate it: other relationships implied by the DSGE model in which it is embedded
could imply a relationship that mimicked a Taylor rule. To get around this problem
we have set up three models, each with the same New Keynesian structure but
differing only in their monetary rules. The three different rules are an optimal timeless
rule, a standard Taylor rule and another with ‘interest rate smoothing’. We show,
using statistical inference based on the method of indirect inference, that only the
optimal timeless rule can replicate both the dynamics and the volatilities of the data.
We also show that if the optimal timeless rule model was operating it would have
produced data in which regressions of an interest-rate-smoothed Taylor rule would
have been found. In short, the policy of the Fed in this period appears to have been
approximately optimal and the fact that its behaviour looks like a Taylor rule with
interest-rate smoothing is a statistical artefact.

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