Can we explain inflation persistence in a way that is consistent with the micro-evidence on nominal rigidity?

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Abstract

This paper adopts the Impulse-Response methodology to understand inflation persistence. It has often been argued that existing models of pricing fail to explain the persistence that we observe. We adopt a common general framework which allows for an explicit modelling of the distribution of contract lengths and for different types of price setting. We also evaluate how far the theories are consistent with recent evidence on price and wage rigidity. We find that allowing for a distribution of durations can take us a long way to solving the puzzle of inflation persistence, but not all the way yet.

Keywords: DSGE models, inflation, persistence, price-setting.

JEL: E17, E3.

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1 Introduction.

In this paper we explore how far existing theories of wage and price setting are consistent with two empirical features: first the macroeconomic persistence we observe in inflation, second the microeconomic data on nominal rigidity of wages and prices. There has been a considerable focus on the macroeconomic aspects of modelling inflation persistence\(^1\). However, more recently there is now a considerable amount of microdata available on the behaviour of prices in the Eurozone and the U.S., which allows us to evaluate existing theories of pricing\(^2\). Very broadly, we can divide the wage and price setting models into four categories, which differ in how they model wage-price behaviour\(^3\):

1. The wage-price is set in nominal terms for a fixed and known period (e.g. Taylor (1980), Fuhrer and Moore (1995))

2. The wage-price is set in nominal terms for a random duration (e.g. Calvo (1983))

3. There is a fixed or uncertain contract length, and the firm/union sets the wage-price for each period at the beginning of the contract (e.g. Fischer (1977), Mankiw and Reis (2002)).

4. The initial wage-price is set, but throughout the contract length the nominal wage-price is updated according to recent inflation (Indexation): (e.g. Woodford(2003, p. 213-218), Christiano et al. (2005), Smets and Wouters (2003)).

The theories in category 3 and 4 were developed in a large part to model inflation dynamics better and put right the shortcomings in the simple forms


\(^3\)This list includes all of the main models that have unchanging distributions of durations; it excludes state-dependent pricing models, such as Dotsey, King and Wolman (1999), which do not.
of theories 1 and 2 (Taylor and Calvo) which have for some time been known to be inadequate in this respect (e.g. Fuhrer and Moore (1995), Christiano et al. (2005)). There is a crucial difference between the first two categories and the second two: in 1 and 2 the nominal wage-price set will last more than one period (usually a period corresponds to a quarter); in 3 and 4, the wage-price will in general be different in every period. Whilst the sticky information (Mankiw and Reis (2002)) and Calvo-with-indexing (Christiano et al. (2005), Smets and Wouters (2003)) are able to model inflation persistence well, it is at the cost of having prices and/or wages changing every quarter, which contradicts the empirical micro evidence. If we explicitly allow for sectoral heterogeneity with the ranges of contract lengths suggested by the data, we move a long way towards explaining inflation persistence whilst being consistent with the micro evidence on nominal rigidity. We consider two distributions of contract lengths: the Calvo distribution and one derived from US data (Bils and Klenow (2004)).

In section 2 we present the empirical evidence on inflation and nominal rigidity. In section 3 we outline a generic macroeconomic framework which allows us to explore the different models and generate impulse-response functions (IRFs) for inflation for different models within a common environment. We also discuss the calibration of the common framework. In section 4 we evaluate the IRFs of the different wage-price models and in section 5 we discuss how these respond to a key parameter. In section 6 we conclude.

2 The Evidence on Inflation Persistence and Nominal Rigidity.

Empirical studies show that monetary policy shocks have persistent effects on inflation. A common way to illustrate this fact is to regress inflation on its own lags and then sum the coefficients on lagged inflation. If the sum of the coefficients is high, then a shock will lead to a changed level of inflation for an extended period. For the U.S., Clark (2006) finds that the sum of the autoregression ($AR$) coefficients for the aggregate inflation series is about 0.9. Batini (2002) finds that for the Euro zone 1970-2002 the coefficients sum to around 0.7, varying across countries. Whilst some studies (e.g. Cogley and Sargent (2001), Levin and Piger (2004), Taylor (2000)) show that the coefficient is reduced if you allow for structural breaks and regime switches,
few would argue for coefficients near to zero.

Second, there is the evidence of vector autoregressions (VARs) which introduce another dimension: the shape and timing of the response of inflation to monetary policy. It is widely agreed that inflation exhibits a delayed response to monetary policy. That is, the maximum effect of a policy occurs sometime after the policy: there is a hump-shaped response. Views about the timing of the peak differ. The traditional view was put forward by Friedman: monetary policy has "long and variable lags"; the impact on inflation could peak as long as eight quarters or even more. Certainly, this is the view taken by the Bank of England: when setting monetary policy, the Monetary Policy Committee (MPC) looks eight quarters ahead. The European Central Bank (ECB) takes the view that the maximum impact is six quarters. Different researchers have estimated the response to be anywhere from four quarters (e.g. Smets and Wouters (2003)) to twelve quarters (e.g. Nelson (1998), Batini and Nelson (2001), Batini (2002)).

We can summarize these observations by three stylized facts or features:

**Feature 1** The biggest effect is not on impact (hump shape)

**Feature 2:** The biggest effect is (a) after 4Q, (b) after 8Q, or (c) after 12Q (timing of hump)

**Feature 3:** After 20 Q, the effect on inflation is (a) 1%, or (b) 5% of the maximum (persistence).

In the case of Feature 2, we take three different values for the timing, corresponding to the moderate view (8Q), the preemptive view (12Q) and the rapid view (4Q). Likewise, for Feature 3, we have two thresholds.

Turning to the microdata on nominal rigidity, there has recently been a huge increase in what we know about pricing as a result of economists gaining access to the data collected by national statistical offices for the purpose of constructing price indices. Most significant here are the Inflation Persistence Network (IPN) in the Eurozone which covers all of the major Euro economies and the work of Bils and Klenow (2004) in the U.S.

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4 In fact, Pivetta and Reis (2007) show that the evidence for a reduced coefficient in cases of structural breaks and regime switches is not statistically significant.

5 Oddly enough, though, the Bank of England’s own model (BEQM) has the peak impact at six quarters.
The BK data set give us the proportion of prices that change (on average) per month in each of the goods and services covering 70% of the U.S. CPI for the period 1995-7. We can interpret this as a sector-specific Calvo reset probability and then generate the corresponding distribution of durations (by duration we mean a "price spell", period in which the nominal price remains unchanged). To do this we use Dixon and Kara (2006a) which generates the distribution of durations across firms. Having the distribution across each type of good or service, we can then aggregate across sectors using the CPI weights to give us the corresponding aggregate distribution of durations in the U.S. Figure 1 plots this distribution in terms of quarters:

\[ \text{Figure 1: BK - Distribution of durations across firms.} \]

The mean contract length is 4.4 quarters. Perhaps the most striking aspect of this distribution is its skewness: there is a very high share of short-term durations, the share of 1 and 2 quarters is about 50%, but also a tail of very long durations. The European data is similar in broad outline.

PMD1: Nominal prices and wages remain unchanged for about 4Q on average.

PMD2: There is a highly skewed distribution of durations, with a high proportion of flexible prices but a tail of long durations.

Having reviewed the stylised facts we can ask a fundamental question. Is there a model which is generated from a theory consistent Dynamic Stochastic General Equilibrium (DSGE) framework with reasonable calibrations which is consistent both with the stylised facts about inflation persistence, and also consistent with the micro data? One key feature to note is the combination of Feature 2 and PMD1: if we take the consensus view of an 8Q hump, then we need a theory that can yield a hump despite having a mean contract length of around 4Q.

3 The Model.

We present a framework which is able to encompass all four wage-price setting frameworks based on the idea of an economy consisting of many sectors.

\[^6\]The distribution across firms is equivalent to the cross section across prices set.
differentiated by how long a contract (whether of Type 1-4) lasts. Following Dixon and Kara (2007), we present the log-linearized equilibrium conditions of a DSGE model in which there can be potentially many sectors, each with a different contract length\(^7\). When each sector has a Taylor-style (Type 1) contract we have a Generalized Taylor Economy (\(GTE\)). When each sector has a Fischer-style (Type 3) contract we have a Generalized Fischer economy (\(GFE\)). We will also want to allow for indexation (Type-4). The exposition here aims to outline the basic building blocks of the model. We first describe the structure of the contracts in the economy, the wage-setting process under different models and monetary policy. We then describe the behavior of wages and prices in a log-linear model which encompasses most approaches. For convenience, we present the model in terms of wage-setting, but the framework is also consistent with price-setting (as we discuss below).

3.1 The Structure of Contracts.

In this section we outline an economy in which there are potentially many sectors differentiated by the duration of contracts. There are \(N\) sectors\(^8\), \(i = 1...N\), with sector shares \(\alpha_i\) summing to unity \((\sum_{i=1}^{N} \alpha_i = 1)\). Contracts in sector \(i\) last for \(i\) periods. There is a unit interval of firms \(f \in [0, 1]\) and a matched unit interval of firm-specific household-unions (one per firm). The sector share \(\alpha_i\) is the measure of firms in sector \(i\). Within each sector \(i\) there are \(i\) equally sized cohorts of unions and firms: each period one cohort comes to the end of its contract and starts a new one. A standard Taylor model is represented by an economy in which one sector (usually \(i = 2\) or 4) has a share of unity, the rest zero. In the \(GTE\), in each sector \(i\) there is a Taylor contract; in the \(GFE\), a Fischer-style contract.

The simple Calvo model is different from the \(GTE\) because the wage setters do not know how long the contract will last: each period a fraction \(\omega\) of firms/households chosen randomly start a new contract. However, the Calvo process can be described in deterministic terms at the aggregate level because the firm-level randomness washes out. As shown in Dixon and Kara (2006a), the distribution of contract lengths across firms is given by \(\alpha_i = \omega^2 i (1 - \omega)^{i-1} : i = 1...\infty\), with mean contract length \(T = 2\omega^{-1} - 1\). The

\(^7\)The detailed derivations can be found in the appendix of an earlier draft of this paper, ECB working paper 672, *Understanding Inflation Persistence*.

\(^8\)\(N\) can be infinite.
Calvo model with indexation has the same structure of contract lengths, but there is indexation throughout the contract life in response to past inflation. The Mankiw-Reis sticky-information (SI) model is a special case of the GFE with the Calvo distribution of contract lengths.

3.2 The Macroeconomy.

Here we present a common framework in the form of a log-linearized macroeconomic model which reflects the generic form that is derived from DSGE models with nominal rigidities in prices and/or wages. It is assumed that there is a final aggregate output produced from intermediate goods under constant returns which is available for consumption. The sectoral output level $y_{it}$ can be expressed as a function of the sectoral price $p_{it}$ relative to the aggregate price level $p_t$ and aggregate output $y_t$ where the coefficient $\theta$ is the elasticity of demand (this is the log-linearization of a CES production function relating intermediate outputs to aggregate output):

$$ y_{it} = \theta(p_t - p_{it}) + y_t $$

(1)

In the intermediate good sector, labour is the only input and returns are constant, so that prices are a markup over wages, the markup being determined by the elasticity of demand $\theta$. In log deviation form, sectoral price levels are given by the average wage set in the sector, and the wage is averaged over the $i$ cohorts in sector $i$:

$$ p_{it} = w_{it} = \frac{1}{i} \sum_{j=1}^{i} w_{ijt} $$

(2)

The log-linearized aggregate price index in the economy is the average of all sectoral prices:

$$ p_t = \sum_{i=1}^{N} \alpha_i p_{it} $$

(3)

The inflation rate is given by $\pi_t = p_t - p_{t-1}$.

We close the model with the demand side, which is given by a simple quantity theory relation:

$$ y_t = m_t - p_t $$

(4)
The money supply follows the following process,

\[ m_t = m_{t-1} + \ln(\mu_t), \quad \ln(\mu_t) = v \ln(\mu_{t-1}) + \xi_t \]  

(5)

where \(0 < v < 1\) and \(\xi_t\) is a white noise process with zero mean and a finite variance.

### 3.3 Wage-Setting Rules.

Before defining the optimal wage setting rules under different models, let us define the optimal wage that would occur if wages were perfectly flexible (we call this "the optimal flex wage"). The log-linearized version of the optimal flex wage in each sector\(^9\) is given by

\[ w_t^* = p_t + \gamma y_t \]  

(6)

with the coefficient on output \(\gamma\) being:

\[ \gamma = \frac{\eta_{LL} + \frac{\eta_{cc}}{1 + \theta \eta_{LL}}}{1} \]  

(7)

Where \(\eta_{cc} = \frac{-U_{cc} C}{U_c}\) is the parameter governing risk aversion, \(\eta_{LL} = \frac{-V_{LL} H}{V_L}\) is the inverse of the labor elasticity, \(\theta\) is the sectoral elasticity \((1)\).

We can represent the alternative wage-setting behaviour in terms of a two general equations: one for the reset wage in sector \(i\) \((x_{it})\), one for the average wage in sector \(i\) \((w_{it})\). For the GTE, these are\(^{10}\):

\[ x_{it} = \sum_{j=1}^{i} \lambda_{ij} E_t w_{t-j-1}^* - a \sum_{j=1}^{i} \sum_{k=j}^{i} \lambda_{ij+k} \pi_{t+j-1} \]  

(8)

\[ w_{it} = \sum_{j=1}^{i} \lambda_{ij} \left( x_{it-j-1} + a \sum_{k=0}^{j-1} \pi_{t+k-1} \right) \]  

(9)

where \(\lambda_{ij} = \frac{1}{i}\) and \(0 < a < 1\) measures the degree of indexation to the past inflation rate. Without indexation \((a = 0\)) the reset wage \((8)\) in sector \(i\)

\(^9\)Note that the optimal flex wage in each sector is the same. This is because it is based on the demand relation \((1)\) which has the same two aggregate variables \(\{p_t, y_t\}\) for each sector. Also, we make the common approximation for quarterly data that \(\beta = 1\).

\(^{10}\)The detailed derivations of the equations are presented in a technical appendix, which is available in the working paper version.
is simply the average (expected) optimal wage over the contract length (the nominal wage is constant over the contract length). Note that the reset wages will, in general, differ across sectors, since they take the average over a different time horizon. With indexation, the initial wage at the start of the contract is adjusted to take into account of future indexation over the lifetime of the contract. The average wage in sector $i$ (9) is related to the past reset wages and how far they have been indexed.

The two equations (8 and 9) can also represent the simple Calvo economy. To obtain the simple Calvo economy from (8), all reset wages at time $t$ are the same ($x_{it} = x_t$), the summation is made with $i = \infty$ and $\lambda_{ij} = \omega(1 - \omega)^{j-1} : j = 1...\infty$. and there is no indexation $a = 0$. Assuming $0 < a \leq 1$ extends these model to the case in which the wages are indexed to past inflation.

The standard equation for the average wage is obtained by setting $w_{it} = w_t$, and setting the summation as $i = \infty$ in (9).

In a GFE, the trajectory of wages is set at the outset of the contract. Suppose an $i$-period contract starts at time $t$; then the sequence of wages chosen from $t$ to $t + i - 1$ is $\{E_tw_t^s\}_{s=0}^{i-1}$. Hence, the average wage in sector $i$ at time $t$ is

$$w_{it} = \sum_{j=1}^{i} \lambda_{ij}E_{t-j+1}w_t^s$$

which is the average of the best guesses of each cohort for the optimal flex wage to be holding at $t$ and embodies "sticky information" idea in Fischer contracts: part of current wages are based on old information. In the GFE, since cohorts are of equal size within sector $i$, $\lambda_{ij} = \frac{1}{i}$. The Mankiw-Reis sticky-information (SI) model has $\lambda_{ij} = \omega(1 - \omega)^{j-1} : j = 1...\infty$.

### 3.4 The Choice of Parameters.

Following the literature, we set $\eta_{CC} = 1$ and $\eta_{LL} = 4.5$. The parameter $\theta$ determines the steady state markup $\left(\frac{\theta}{\theta-1}\right)$. Studies by Eichenbaum and Fisher (2004) and Kimball (1995) suggest a value of $\theta = 11$. Chari, Kehoe and McGrattan (2000) use a value of $\theta = 10$. Coenen et al. (2007) consider a range of values, from 5 to 20. In light of these studies, we set $\theta = 12$. Given the calibrated values of $\eta_{CC}$, $\eta_{LL}$ and $\theta$, we get a value of $\gamma = 0.1$, which is the value adopted by Mankiw and Reis (2002) and which we take as our initial reference value. With price setting, Chari et al. (2000) argue that the correct calibrated level of $\gamma$ is much larger, being 1.2. However,
Edge (2002) argues that these authors’ finding can be misleading, as their conclusion relies heavily on the assumption that all firms use identical inputs: she demonstrates that if CKM were to assume a firm specific labour market, then they would have obtained a similar value of $\gamma$ whether price or wage setting is assumed\textsuperscript{11}. One further rationale for lower values of $\gamma$ is given by Kimball (1995), who assumes that firms are reluctant to increase their price above the average if demand for their own good decreases sharply. Several studies appeared recently in which the Kimball (1995) aggregator is assumed (e.g. Altig et al. (2004), Coenen et al. (2007), Smets, Wouters and de Walque (2006)). Coenen et al. (2007) argue that there is a reasonable calibration of $\gamma$ based on firm-specific input that is consistent with a value of $\gamma = 0.027$. Non-microfounded econometric estimates of $\gamma$ tend to be smaller: Taylor (1980) estimates $\hat{\gamma} = 0.05$, Coenen et al. (2007) estimate $\hat{\gamma} = 0.003 - 0.027$ and Fuhrer and Moore (1995) estimate $\hat{\gamma} = 0.005\textsuperscript{12}$. Our reference set for $\gamma$ is thus \{0.1, 0.05, 0.027, 0.01, 0.005\}. When it comes to the serial correlation of money growth $\nu$, we follow Christiano et al. (2005) and set $\nu = 0.5\textsuperscript{13}$.

4 The Impulse Response Functions for Inflation.

We first look at the Calvo simple Taylor ($ST$) models. As is well known, neither of these fare well in modeling the stylised features. The policy we are simulating is a one off 1% shock in $\xi$ at $t = 0$. In this section, all reported simulations adopt benchmark values $\gamma = 0.1$ and $\nu = 0.5$: the latter implies a long-run cumulative effect of 2% on inflation from the $AR(1)$ process (5).


The Calvo (1983) pricing model has a single parameter: the reset probability or hazard rate, $\omega$, which gives the non-duration-dependent probability that a firm/union will have the option to reset its wage in any period. Figure

\textsuperscript{11}See also Ascarì (2003) and Woodford (2003).

\textsuperscript{12}See Roberts (2005) for a survey and an attempt to reconcile the differences in published estimates.

\textsuperscript{13}Note that some other studies suggest even higher values. Chari et al. (2000) estimate a value of $\nu = 0.57$ and Huang, Liu and Phaneuf (2004) use a value of $\nu = 0.68$. 

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2a illustrates the IRF of inflation to a one percent innovation in the money supply.

*Figure 2 Calvo & Simple Taylor*

As the figure shows, the Calvo model cannot deliver a hump shaped inflation response; the maximum is always in the first period (unless one imposes *ex-ante* pricing). As is well known (e.g., Woodford (2003), chapter 3), the purely forward-looking nature of the Calvo model is the main reason for this result. On the other hand, Feature 3(b) is satisfied for both values of \( \omega \). The failure of the Calvo model to generate the observed responses of inflation suggests that there might be a backward-looking element missing. The intertemporal backward-looking and forward-looking effects in the Taylor model are emphasized by Taylor (1980). Figure 2b displays the impulse response function of inflation in Taylor’s staggered contract model for contract lengths \( T = 2, 4, 6 \) and 8. The maximum inflation response in Taylor’s model is indeed delayed for a few quarters and it reaches its peak \( T - 1 \) quarters after the first period in which the shock occurs\(^{14}\). There is a hump shape of sorts, but a rather jagged one. Hence Features 1 and 2 can be met. However, the simple Taylor contract will only generate a hump at around two-years if the contract lasts for that length of time \( (T = 8) \) which is in direct conflict with the microdata PMD1. Furthermore, if we turn to Feature 3, inflation dies away rapidly \( T \) periods after the shock. In particular, for \( T = 4 \), the effects of the shock are almost gone after 15 periods; this certainly fails to meet even the weak criterion.

To summarise: Calvo can give us persistence (Feature 3) but no hump; Taylor gives us a hump at the length of the Taylor Contract, but fails Feature 3. The precise value of \( \gamma \) does not alter these conclusions much: Calvo always peaks on impact, Taylor \( T - 1 \) periods after the shock whatever the value of \( \gamma \), which influences persistence Feature 3. Taylor is inconsistent with the micro-data: even a simple Taylor with a mean of 4Q or 5Q will not display the distribution of lengths suggested by empirical studies. Calvo can be more consistent with PMD2, although the mean contract length requires a reset probability of \( \omega = 0.4 \).

\(^{14}\)Every cohort setting their wage from the period of the shock sets the wage knowing the innovation in the money supply. The last cohort to set its wage without this information is the one that set its wage the period before the innovation. Its contract ends \( T - 1 \) periods after the shock; inflation peaks when this cohort resets its wage.
4.2 Solution 1: Indexation in the Calvo Model.

There has been much empirical work done on the New Keynesian Phillips curve. As is well known, it does not do well in explaining the data (see for example Gali and Gertler (1999)). One model that does much better empirically is the hybrid Phillips curve, which takes the form

\[ \pi_t = (1 - \phi)E_t \pi_{t+1} + \phi \pi_{t-1} + b_t \] (11)

where \( \phi \in [0, 1] \) and \( \phi = 0 \) gives the New Keynesian Phillips curve. This has given rise to attempts to construct a theoretical model that can yield (11). The currently popular theoretical justification is to add indexation to the Calvo model (see for example Christiano et al. (2005), Smets and Wouters (2003), Woodford (2003))\(^{15}\): at the beginning of the contract the nominal wage is set, and for the contract duration this is updated by the previous period’s inflation (Christiano et al. (2005) call this "lagged inflation indexation"). This hardwires lagged inflation into current inflation. Woodford (2003) shows how this gives rise to a HPC with \( \phi = 0.5 \).

In Figure 3, we display the response of inflation to a monetary shock for \( \omega = 0.25 \) and \( \omega = 0.4 \) (average contract lengths of 7Q and 4Q respectively). As the figure illustrates, introducing backward-looking indexation can affect the impact of the shock on inflation and leads to a hump shaped response. The model can satisfy Feature 1 and provide a hump peaking at 5 – 6Q. However, even when \( \omega = 0.25 \), the model fails to generate enough persistence to satisfy Feature 3. Clearly, if we are thinking of prices, the notion of full indexation is inconsistent with the micro-data PMD2 and PMD1: a model with full or even partial indexation implies that every firm adjusts its price every period.

Recent work by Ireland (2007) (see also Cogley and Sbordone (2005)) argues that large and persistent movements in inflation could not happen unless consistent with monetary policy and reflecting implicit shifts in the inflation target. Ireland (2007) considers this possibility by using a Calvo model that allows for continual movements in the central bank’s inflation target. The difference between the Ireland model and the indexed-Calvo model is that the nominal wage-price is updated according to the central bank’s inflation target, rather than the previous period’s inflation rate. Ireland compares

\(^{15}\)Another justification is the older Fuhrer and Moore model, which we do not include here, but is covered in an older version of the paper (ECB working paper 672).
the empirical performance of his model with that of the Calvo model with full indexation. He finds that the new model performs better empirically than the indexed-Calvo model and, therefore, concludes that shifts in the central bank’s inflation target can substitute for backward-looking term in the Phillips curve in explaining inflation. However, updating prices with the implicit inflation target still falls foul of the microdata\textsuperscript{16}.

\subsection*{4.3 Solution 2: Distributions of Fischer Contract Lengths.}

In this section we consider a Generalized Fischer Economy (\textit{GF E}): an economy with many sectors, each with a Fischer contract where the wage-setter chooses a trajectory of wages, one for each period for the whole length of the contract Fischer (1977). The wages are thus conditional on the information the agent has when it sets the wages, so that as the contract gets older the information will be increasingly out of date\textsuperscript{17}.

There are two general points that need to be understood when interpreting the Fischer contracts. First, the \textit{IR} functions are generated by a single innovation in the initial period. The initial shock is perpetuated because we assume that money follows an \textit{AR}(1) process. However, any new contract that starts after the initial shock will be fully informed. Once all contracts have been renewed after the shock, the economy will behave as if there is full information and flexible wages/prices. The second point is that \textit{the length of the contract has no influence on the wages chosen for any specific period covered by the contract}. This is because a separate wage can be chosen for each period within the contract. Therefore, it makes no difference to the wage chosen for period 2 of the contract whether the contract will last for 2

\textsuperscript{16}Kiley (2005) adopts a slightly different approach to indexation. Kiley (2005) follows Gali and Gertler(1999) to assume a model with two different types of wage-setters: a proportion, \((1 - a)\), are Calvo wage-setters of the orthodox kind and the rest are "rule of thumb" agents who update using lagged inflation; however, lagged inflation is a moving average over the last \(b\) periods. Woodford (2003) is a special case when \(b = 1\) and \(a = 0.5\).

Our adaptation of Kiley’s approach has 4 parameters: \(\{a, b, \gamma, \omega\}\). We take the value of \(\gamma = 0.1\) and consider the two cases considered by Kiley for \(\{a, b\} = \{0.24, 1\}\) and \(\{0.17, 4\}\) with two values of \(\omega = 0.25\) and \(\omega = 0.4\). We find that the model’s performance is not significantly different from that of Woodford’s model. More specifically, in this model there is a hump, but it peaks well before 8\(Q\). In fact, it does not even reach 4\(Q\) even when \(\omega = 0.25\) with an average contract length of 7\(Q\).

\textsuperscript{17}An alternative interpretation is that the firm sets its wage or price optimally each period, but that it only updates its information infrequently.
periods or 10 periods: the period 2 wage will be its best guess at what the optimal wage is going to be in that period.

Mankiw and Reis’s Sticky Information model (SI) is a GFE where the distribution of contract lengths is Calvo with their choice of $\omega = 0.25$, resulting in an average length of 7 quarters. The parameter $\omega$ is presented as a "re-plan" probability\textsuperscript{18}: just as in the Calvo model, when the trajectory of wages is chosen at the outset of the contract, the wage-setter does not know how long it will last but has a subjective distribution over the lifetime. However, as we have noted, the length of the contract has no influence on the wage-setting behavior. Hence the SI model as presented by Mankiw and Reis is exactly the same as a Calvo-GFE: an economy where there is a Calvo distribution of contract lengths but in which each wage setter knows exactly how long the contract will run for. With Fischer contracts, the Calvo reset probability is only important in generating the distribution of durations: nothing else.

In Figure 4 we depict the IR functions for the SI model with $\omega = 0.25$. The SI model has a smooth hump, peaking at the 8th quarter, and inflation dies away slowly so that Feature 3(b) is satisfied. The reason for this shape is the distribution of contract lengths and in particular the longer contracts that let inflation persist. Hence, introducing heterogeneity into the Fischer model moves the model in the direction of explaining all three facts. With a Fischer contract, the price or wage setter tries to predict the optimal flex price or wage. Since this depends on the general price level, the trajectory of prices builds in anticipated inflation. The monetary policy IR has a hump shape because most firms have to wait to replan their price-plans once the new policy is in effect. Thus, for those yet to revise their plans, the pre-shock inflationary expectations are driving their prices. The Calvo distribution ensures that the hump is smooth and peaks at the required time.

As in the case of Calvo with indexation, inflation is "built in" in a way that is not consistent with the micro data: prices change every period and the calibrated value of $\omega = 0.25$ implies a mean contract length across firms much longer than 4 quarters. However, since the "contract" here refers to a planning horizon, there is no clear-cut microeconomic evidence for its appropriate calibration. However long or short the "contract", prices change every period which violates both PMD1 and PMD2.

\textsuperscript{18}The reset probability can also be interpreted as arising due to "optimal inattention", as in Reis (2006).
4.4 Solution 3: Distributions of Taylor Contract Lengths.

In this section, we now return to simple Taylor contracts, but with a distribution of contract lengths, considering two special GTEs. These differ in the share of weights across different durations, \( \alpha_i \). The Calvo-GTE, in which the share of each duration across firms is the same as generated by the Calvo model\(^{19} \): for \( \omega = 0.25 \), which has a mean contract length of 7Q and a modal lengths of 3 and 4Q. Second we use the distribution of duration data using the Bils and Klenow (2004) as shown in Figure 1.

The inflation impulse-responses for these two distributions of contract lengths are depicted Figure 5.

---

\(^{19}\) For computational purposes, we truncate the distribution at \( i = 20 \) and put all of the mass of the contracts \( i \geq 20 \) onto \( i = 20 \).
If we use exactly the same distribution of contract lengths as in the *SI* and Calvo with indexation, the Calvo-
*GTE* with \( \omega = 0.25 \), we can see that we can have a model that is more consistent with the microdata PMD2 and which gives a hump at 4Q and so is consistent with the rapid view Feature 3(a). However, it has the drawback of a mean contract length that is almost twice as long as PMD1.

5 Role of the Key Parameter \( \gamma \).

We have thus far considered the ability of the different models with calibrated parameters values to match the key features. We now examine how the changes in the key parameters influence the models with respect to macroeconomic Features 1-3\(^{20}\). The parameter \( \gamma \) is important as it determines the inflationary pressure on wages and prices that results from an increase in output. A low value of \( \gamma \) means that this inflationary pressure works through more slowly so that the reaction of inflation to output growth becomes slower. Table 1 shows how Features 1-3 fare for each of the models at the different reference levels of \( \gamma \) discussed in section 3.4: 0.1, 0.05, 0.027, 0.01, and 0.005. Where there are weak and strong criteria (Features 2 and 3), the more ticks indicate that the stronger criterion being met. Table 2 gives the exact timing of peak inflation.

\begin{table}[h]
\centering
\caption{Features 1-3 as \( \gamma \) varies.}
\begin{tabular}{ll}
\hline
Feature & Value \\
\hline
F1 & 0.1, 0.05, 0.027, 0.01, 0.005 \\
F2 & 0.1, 0.05, 0.027, 0.01, 0.005 \\
F3 & 0.1, 0.05, 0.027, 0.01, 0.005 \\
\hline
\end{tabular}
\end{table}

Let us first take the case of the models with the Calvo distribution of contract lengths in which prices change every quarter: *SI, IC*. With the standard Calibration of \( \omega = 0.25 \), both models satisfy F1 and strong F3 for all values of \( \gamma \). The key issue is the timing of peak inflation (F2), given in Table 2, which gets more delayed as \( \gamma \) falls. Sticky information meets the moderate view at \( \gamma = 0.1 \), and gets longer as \( \gamma \) gets smaller. Indexed-Calvo meets the moderate view somewhere between \( \gamma = 0.1 \) and \( \gamma = 0.027 \). These models both perform well at the macro-level, but at the cost of violating the microdata. If we impose the Calibration \( \omega = 0.4 \), then we see that matters are different. For the *IC* and *SI*, F3 is not satisfied in even its weak form when \( \gamma \geq 0.027 \). The moderate view of the peak is attained only when \( \gamma < 0.027 \).

\(^{20}\)Clearly, \( \gamma \) has no effect on or relevance for microeconomic features PMD1 and PMD2.
Turning to the Calvo–GTE, we see that with \( \omega = 0.25 \), F1 and F3 are satisfied for all \( \gamma \). The peak response meets the rapid criterion for \( \gamma = 0.1 \) and the moderate when \( \gamma = 0.027 \). This model has a distribution of contract durations, but the mean is too long. If we impose PMD1 and set \( \omega = 0.4 \), then the resulting Calvo distribution is much closer to the microdata on both counts. For \( \gamma \leq 0.027 \), the rapid peak and also the strong view of F3 are both satisfied. Thus, the Calvo–GTE is the only model with the Calvo distribution that is consistent with the microdata and also can satisfy the macro features F1-3. However, the peak response will be too rapid for many macroeconomists.

Lastly, we can look at the BK–GTE, which has the actual empirical distribution of contract lengths which by construction satisfies PMD1 and PMD2. For all values of \( \gamma \), F1 and F3 are satisfied. What of the peak inflation? Well, for "calibrated" \( \gamma = 0.027 \), the peak is at 3Q. This "almost" satisfies the rapid view (recall that we can follow Woodford (2003, p. 207-213) and introduce pre-set pricing to add an extra quarter lag into the pricing decision, taking the peak response form 3 to 4Q). What is more interesting is what happens when \( \gamma = 0.01 \). Even though the BK–GTE has an average contract length of 4.4Q, it peaks at 7Q. This would both satisfy the moderate view of peak inflation and be consistent with the microdata. However, as yet this can only be attained at a value of \( \gamma \) below the lowest "calibrated" value currently proposed.

When there is a distribution of contract lengths, a decrease in \( \gamma \) will tend to delay the maximum impact if there is already a hump shape and will move the models with a distribution significantly towards explaining all three features. In fact, this finding, to a large extent, explains the result obtained by Coenen et al. (2007). There, it is argued that a model with Taylor style contracts that allows for a distribution of contract lengths and assumes a Kimball(1995) aggregator along with the assumption of firm-specific input, which helps to lower \( \gamma \) ( \( \gamma = 0.027 \)), fits the German data and the US data very well without needing the assumption of backward-looking indexation.

6 Conclusions.

It has been long known that the new Keynesian dynamic wage and price setting models have problems in generating the sort of impulse-response functions generated by empirical VARS, and that they do not capture what policy
makers feel are the features of the response of inflation to monetary policy reflected in our stylized "features". This problem has lead to two main responses in the literature: the introduction of indexation into the Calvo model, and the adoption of Fischer contracts and a Calvo distribution of Contract lengths (Sticky Information). Both of these theories are inconsistent with the micro data on prices: not only do all prices change each period, but also with standard calibrations the average duration of contracts across firms is nearly twice as long as the Bils-Klenow data set suggests, and is almost as long as the "moderate" view of the hump at 8Q.

We have explored an alternative approach which is to keep to simple contracts which specify a given nominal wage or price for a specific length of time, but explicitly model the distribution off contract lengths. We do this for two types of distribution: the Bils-Klenow distribution and the Calvo distribution. We find that we can obtain a hump-shaped response with plenty of persistence. With the calibration suggested by Coenen et al. (2007) the Calvo–GTE is the only model that can satisfy all three inflation-persistence features, with the hump occurring at 4Q (the rapid view) and is consistent with the microdata PMD1-2. However, we have found the interesting phenomenon that for lower values of the \( \gamma \) parameter the hump in inflation can peak at 7Q or more with the empirical Bils-Klenow distribution, which is much longer than the mean contract length. We find that this is the only theory that is consistent with the micro-data on prices and can potentially explain inflation persistence.
References


21
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<th></th>
<th>$\gamma = 0.1$</th>
<th>$\gamma = 0.05$</th>
<th>$\gamma = 0.027$</th>
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Table 1: Features 1-3 as $\gamma$ varies
Table 2: The peak response of inflation (in quarters)

<table>
<thead>
<tr>
<th>$\gamma$</th>
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<th>0.027</th>
<th>0.01</th>
<th>0.005</th>
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<td>SI; $\omega = 0.40$</td>
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<tr>
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<td>6</td>
<td>8</td>
<td>10</td>
<td>13</td>
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</tbody>
</table>

Figure 1: $BK$ – Distribution
Figure 2: (a) Inflation response in the Calvo economy. (b) Inflation response in the simple Taylor economy.
Figure 3: Inflation response in the indexed-Calvo.

Figure 4: Inflation response in the SI.
Figure 5: (a) Inflation response in the Calvo-$GTE$ and the Calvo economy. (b) Inflation response in the BK-$GTE$. 