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Speed Limit Policies versus Inflation Targeting: A Free Lunch?

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Abstract

Inflation targeting is currently popular with central banks. Is this popularity justified? I investigate this question by comparing a speed limit policy and inflation targeting with a Lucas-type Phillips curve capturing output gap persistence. If the output gap is at least moderately persistent, a speed limit policy can: (1) partly eliminate the state-contingent inflation bias, and (2) reduce inflation variability at no output gap variability cost.

Keywords: inflation targeting; speed limit policy; inflation bias; discretion; stabilisation.

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1. Introduction

When monetary policy is discretionary, inflation targeting is typically sub-optimal. Often, alternative policies will exist which impose lower costs on society. One such alternative is the speed limit policy which replaces the output gap with its first difference in the loss function of the central bank (Walsh, 2003a). Using a New Keynesian Phillips curve, Walsh showed a speed limit policy will outperform inflation targeting (unless inflation is predominantly backward-looking) because it imparts inertia like the optimal commitment policy when expectations are forward-looking (Woodford, 2003). This 'speed limit' result has been further investigated by Yetman (2006) who found it was robust to relaxations in standard assumptions regarding credibility and expectations formation.

The second factor in favour of a speed limit policy is measurement error. As Walsh (2003b) points out, revisions in the first difference of the US output gap are historically much lower than revisions in its level.[1] A speed limit policy therefore offers the prospect of reduced scope for real-time policy error. Interestingly, Walsh (2003a) notes that Federal Open Market Committee (FOMC) press releases suggest that the Fed already thinks about policy in speed limit terms, while both Peel et al. (2004) and Paez-Farrell (2007) find empirical support for a speed limit form of the Taylor rule in the US. Though there are identification problems with Taylor rule estimations generally (e.g. Gillman, Le and Minford, 2007[2]), measurement error issues do appear to have been given greater prominence in the US than in the UK or mainland Europe.[3]

In this paper, I follow Walsh (2003a) and Yetman (2006) by ignoring any measurement error benefits in order to focus on pure stabilisation issues. However, in contrast to these papers, I take a neoclassical Phillips curve as my starting point. The contribution of the paper is to show that benefits of speed limit policies are not confined to the New Keynesian Phillips curve specification. Specifically, a speed

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[1] For the period 1970-2002, full-sample revisions in the output gap were sometimes in excess of 3% of potential output. By comparison, revisions in the first difference of the output gap did not exceed 1.5% of potential output for the same period.

[2] These authors show that in an endogenous growth model with cash and credit in advance, a central bank following a Friedman rule for the money supply appears, via the Fischer equation for nominal interest rates, to be following a speed limit form of the Taylor rule.

[3] See, for example, Orphanides (2001).

policy and inflation targeting are compared with a neoclassical Lucas-type Phillips curve capturing output gap persistence. By appropriate choice of the relative weight on real economy stabilisation, a speed limit policy: (1) partly eliminates the state-contingent inflation bias of discretionary policy, and (2) reduces inflation variability for any given level of output gap variability. Both these results require the output gap to be 'at least moderately persistent'.[4]

2. The model

The model follows Svensson (1997, 1999) and Dittmar, Gavin and Kydland (1999). It comprises three equations:

$$L^{CB} = E_t \sum_{i=0}^{\infty} b^i L_{t+i} \qquad (0 < b < 1) \qquad \text{(1a)}$$

$$x_t = r x_{t-1} + a(p_t - p_t^e) + e_t \qquad (0 < r < 1) \qquad \text{(1b)}$$

$$p_t^e = E_{t-1} p_t \qquad \text{(1c)}$$

(1a) is the intertemporal loss function delegated to the central bank by society. The central bank's goal is thus to minimize the discounted sum of its expected future (period) losses. The period loss function L_t varies depending on the delegated policy:

$$L_t = \begin{cases} p_t^{\,2} + l^{IT} x_t^2 & \text{under inflation targeting} \qquad \text{(2a)} \\ p_t^{\,2} + l^{SL}(x_t - x_{t-1})^2 & \text{under a speed limit policy} \qquad \text{(2b)} \end{cases}$$

where p_t is inflation and x_t is the output gap, defined as the log of the ratio of output to its flex-price steady state value. The parameters l^{IT}, $l^{SL} > 0$ are the relative weights on real economy stabilisation delegated to the central bank under each policy.

(1b) is the economy's Lucas-type Phillips curve, which shows that the output gap persists and that inflation will influence the output gap if and only if actual and expected inflation differ (a so-called 'inflation surprise' term) . a is a positive

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[4] The term 'at least moderately persistent' is taken from Svensson (1999) who uses it to refer to a first-order autocorrelation in the output gap greater than or equal to $\frac{1}{2}$.

parameter representing the slope of Phillips curve and e_t is an i.i.d. supply shock with mean zero and variance s^2.

(1c) states that inflation expectations p_t^e are formed rationally conditional upon period t-1's information set. Using this rational expectations assumption in (1b) gives us a Phillips curve which is neoclassical in the sense that it satisfies the natural rate hypothesis.

3. Solving the model

Set-up the Lagrangian:

$$V = E_t \left\{ \sum_{i=0}^{\infty} b^i (L_{t+i} - m_{t+i}(x_{t+i} - r x_{t+i-1} - a(p_{t+i} - E_{t+i-1} p_{t+i}) - e_{t+i}) \right\} \qquad (3)$$

where L_t takes the form of (2a) or (2b) - which depends on the policy considered - and m_t is a Lagrange multiplier. The solution method follows Dittmar, Gavin and Kydland (1999). It consists of four steps:

1. Minimise with respect to p_t and x_t, and eliminate the resulting Lagrange multiplier to get a single first-order condition;

2. Posit a linear decision rule for inflation of the form:[5]

$$p_t = A_1 x_{t-1} + A_2 e_t \qquad (4)$$

and use this to find x_t as a function of x_{t-1} and e_t;

3. Express the first-order condition in terms of only constants, x_{t-1}, e_t and the undetermined coefficients A_1 and A_2;

4. Collect terms to identify A_1 and A_2, and hence the solutions for p_t and x_t.

[5] I have used the fact that there will be no average inflation bias, under either policy, to set the constant equal to the inflation target of zero.

3. 1 Inflation targeting

Following this method gives the solutions derived by Dittmar, Gavin and Kydland (1999):

$$p_t^{IT} = A_1^{IT} x_{t-1}^{IT} + A_2^{IT} e_t \quad (5a)$$

$$x_t^{IT} = r x_{t-1}^{IT} + \frac{1 - b r^2}{1 + a^2 l^{IT} - b r^2} e_t \quad (5b)$$

where

$$A_1^{IT} = -\frac{a\, l^{IT} r}{1 - b r^2} < 0$$

$$A_2^{IT} = -\frac{a\, l^{IT}}{1 + a^2 l^{IT} - b r^2} < 0$$

3.2 Speed limit policy

The first-order conditions with respect to x_t and p_t are:

$$x_t : 2\, l^{SL} \Delta x_t - 2\, l^{SL} b E_t \Delta x_{t+1} - m_t + b r E_t m_{t+1} = 0 \quad (6a)$$

$$p_t : 2 p_t + a m_t = 0 \quad (6b)$$

Combining these to eliminate the multiplier produces the first-order condition:

$$\frac{1}{a} p_t + l^{SL} \Delta x_t - l^{SL} b E_t \Delta x_{t+1} - \frac{b r}{a} E_t p_{t+1} = 0 \quad (7)$$

And via (4) and (1b):

$$\Delta x_t = (r - 1) x_{t-1} + (1 + a A_2^{SL}) e_t \quad (8a)$$

$$E_t \Delta x_{t+1} = (r - 1) x_t = (r - 1)(r x_{t-1} + (1 + a A_2^{SL}) e_t) \quad (8b)$$

$$E_t p_{t+1} = A_1^{SL} (r x_{t-1} + (1 + a A_2^{SL}) e_t) \quad (8c)$$

Using (8a)-(8c), (7) can be written in terms of x_{t-1} and e_t, allowing us to identify coefficients.

The solutions that emerge, after some simple but tedious algebra, are:

$$p_t^{SL} = A_1^{SL} x_{t-1}^{SL} + A_2^{SL} e_t \qquad (9a)$$

$$x_t^{SL} = r x_{t-1}^{SL} + \frac{1 - b r^2}{1 + a^2 l^{SL}(1 + b(1 - 2r)) - b r^2} e_t \qquad (9b)$$

where

$$A_1^{SL} = \frac{a\, l^{SL}(1 - r)(1 - b r)}{1 - b r^2} > 0$$

$$A_2^{SL} = -\frac{a\, l^{SL}(1 + b(1 - 2r))}{1 + a^2 l^{SL}(1 + b(1 - 2r)) - b r^2} < 0$$

4. The state contingent inflation bias

Consider the term $A_1^{IT} x_{t-1}^{IT}$ in (5a). Svensson (1997) calls this a 'state-contingent inflation bias' of discretionary policy. This bias arises because with persistence a non-zero output gap in the previous period is expected to translate into a non-zero output gap in the current period, thus conflicting with the inflation targeting central bank's output gap target of zero. In response, the central bank attempts to close such deviations by either 'climbing' or 'sliding down' the Phillips curve.

With rational expectations all such attempts are futile; they produce no effect on the output gap (because they are incorporated into expected inflation) but do increase inflation variability. It is this inefficiency in the discretionary inflation targeting solution which a speed limit policy can partly eliminate if output gap is at least moderately persistent (i.e. $r \geq \frac{1}{2}$).

In order to see this, consider the term $A_1^{SL} x_{t-1}^{SL}$ in (9a) which I shall call the state-contingent inflation bias of a speed limit policy. First, notice that the inflation bias term changes in sign under a speed limit policy because the central bank now expects persistence to keep the output gap too low relative to target (the previous period's output gap) when the previous period's output gap is positive, and vice versa when it is negative. Second, for a delegated weight $l^{SL} = \frac{l^{IT}}{1 + b(1 - 2r)}$ in the speed limit central bank's loss function, the state-contingent inflation bias is lower *in absolute* *terms* under a speed limit policy so long as the output gap is at least moderately persistent.[6] This observation leads to Proposition 1.

Proposition 1

If $r \geq \frac{1}{2}$ and $l^{SL} = \frac{l^{IT}}{1+b(1-2r)} \geq l^{IT}$ *, the state-contingent inflation bias is lower in absolute value under a speed limit policy:* $\left|A_1^{SL} x_{t-1}^{SL}\right| < \left|A_1^{IT} x_{t-1}^{IT}\right|$.

Proof. When $l^{SL} = \frac{l^{IT}}{1+b(1-2r)}$ the output gaps, via (5b) and (9b), are equal under both policies: $x_t^{SL} = x_t^{IT}$. It follows that the inflation bias term will be lower in absolute value under a speed limit policy if $A_1^{SL} < \left|A_1^{IT}\right|$.

By (5a) we have $\left|A_1^{IT}\right| = \frac{a\, l^{IT} r}{1-br^2}$, and with $l^{SL} = \frac{l^{IT}}{1+b(1-2r)}$:

$$A_1^{SL} = \frac{a\, l^{IT}(1-r)(1-br)}{(1+b(1-2r))(1-br^2)} \qquad (10)$$

Thus $A_1^{SL} < \left|A_1^{IT}\right|$ if:

$$2r(1+b(1-1.5r)) > 1$$

This inequality is satisfied by $r \geq \frac{1}{2}$ regardless of the value of b. Q.E.D.

This result is easily explained: persistence in the output gap 'aids' the speed limit central bank in achieving its output gap target- the previous period's output gap - thus giving it a lower incentive to climb, or slide down, the Phillips curve in the face of supply shocks.

[6] This finding indicates that the speed limit central bank has a lower incentive to spring inflationary (deflationary) surprises in an attempt to climb (slide down) the Phillips curve.

5. Inflation and output gap variability

Now consider the variances in inflation and the output gap resulting under each policy, as shown by equations (A1) to (A4) of Appendix A. It will be shown that Proposition 1 implies that, even if l^{IT} is optimally chosen, delegating a weight $l^{SL} = \dfrac{l^{IT}}{1+b(1-2r)}$ to the speed limit central bank delivers reduced inflation variability for any given level of output gap variability.

Proposition 2

If $r \geq \dfrac{1}{2}$ *and* $l^{SL} = \dfrac{l^{IT}}{1+b(1-2r)} \geq l^{IT}$, *the speed limit policy gives an identical output gap variance, but a lower inflation variance.*

Proof. By Proposition 1 the output gaps (and thus their variances) are equal, and substituting this l^{SL} in (9a) and comparing to (5a) shows that the inflation responses to a supply shock are also identical.

That is:

$$x_t^{SL} = x_t^{IT} \qquad (11a)$$

$$A_2^{SL} = A_2^{IT} \qquad (11b)$$

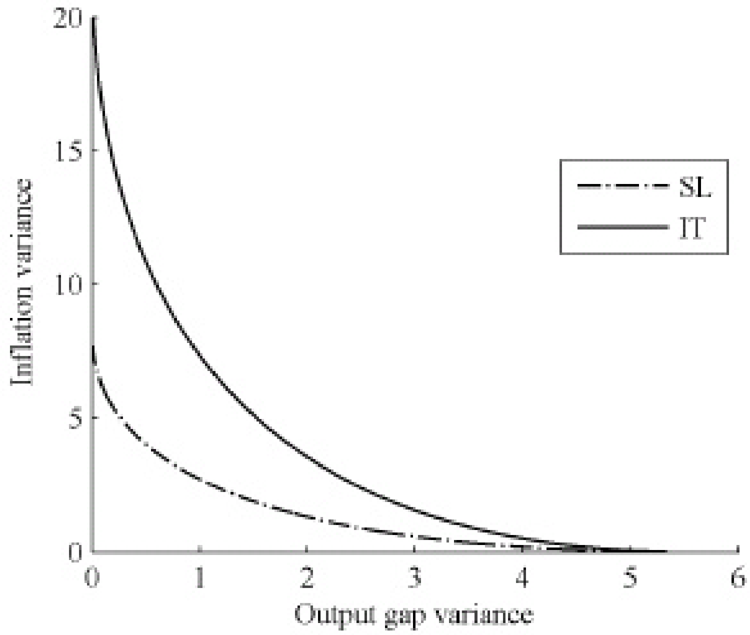
Using (11a) and (11b) in the inflation variance solutions (A2) and (A4) of Appendix A, there is a lower inflation variance under a speed limit policy, i.e. $\text{var}(p_t^{SL}) < \text{var}(p_t^{IT})$, if and only if $A_1^{SL} < \left|A_1^{IT}\right|$.

By Proposition 1, this inequality is satisfied by $r \geq \dfrac{1}{2}$. Q.E.D.

The intuition behind this result is straightforward. The lower incentive to inflate (for a given l) gives the speed limit central bank ‘leeway’ to increase its weight on real economy stabilisation above the inflation targeting central bank, and thus match the output gap variance it achieves, while still leaving the inflation bias, which contributes to inflation variability, lower.

We can see this result illustrated in Figure 1, which plots a sample inflation-output gap variance trade-off. The reduction in inflation variance for a given output gap variance, grows as the output gap variance is reduced because, in order to achieve this reduction, the central bank must be delegated an increasingly high relative weight on real economy stabilisation. A higher relative weight on real economy stabilisation increases the incentive for inflation bias, and this exactly what the speed limit policy partly eliminates. A speed policy is thus of particular benefit to a society which places a high weight on output gap stability.

Figure 1
The inflation-output gap variance trade-off ($b = 0.99$, $s^2 = 2$, $r = 0.8$ and $a = 0.5$)[7]



[7] Dittmar, Gavin and Kydland (1999) estimate the degree of persistence in the output gap in the G-10 countries to be around 0.8 (though it is notably higher in the US), and use $a = 0.5$ in their simulations. A copy of the MATLAB program used to construct Figure 1 is available from the author on request.

6. **Conclusion**

Previously, the monetary policy literature had shown speed limit policies to have desirable stabilisation properties only in forward-looking New Keynesian models. However, in this paper we have seen that a neoclassical model reaches a similar conclusion with the proviso that there is at least moderate persistence in the output gap. The stabilisation benefits of speed limit policies are therefore greater than has been commonly acknowledged.

Appendix A– Variance solutions

Inflation targeting

$$\mathrm{var}(x_t^{IT}) = \frac{(1-br^2)^2}{(1-r^2)(1+a^2 l^{IT} - br^2)^2} s^2 \qquad \text{(A1)}$$

$$\mathrm{var}(p_t^{IT}) = (A_1^{IT})^2 \mathrm{var}(x_t^{IT}) + (A_2^{IT})^2 s^2 \qquad \text{(A2)}$$

$$= \frac{a^2 l^{IT\,2}}{(1-r^2)(1-br^2+a^2 l^{IT})^2} s^2$$

Speed limit policy

$$\mathrm{var}(x_t^{SL}) = \frac{(1-br^2)^2}{(1-r^2)(1-br^2+a^2 l^{SL}[1+b(1-2r)])^2} s^2 \qquad \text{(A3)}$$

$$\mathrm{var}(p_t^{SL}) = (A_1^{SL})^2 \mathrm{var}(x_t^{SL}) + (A_2^{SL})^2 s^2 \qquad \text{(A4)}$$

$$= \frac{a^2 l^{SL\,2} s^2}{(1-r^2)(1-br^2+a^2 l^{SL}[1+b(1-2r)])^2} [(1-r)^2(1-br)^2 + (1-r^2)(1+b(1-2r))^2]$$

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