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Vo Phuong Mai Le and Patrick Minford

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Cardiff Business School
Aberconway Building
Colum Drive
Cardiff CF10 3EU
United Kingdom
t: +44 (0)29 2087 4000
f: +44 (0)29 2087 4419
business.cardiff.ac.uk

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Enquiries: EconWP@cardiff.ac.uk

CALVO CONTRACTS — OPTIMAL INDEXATION IN GENERAL EQUILIBRIUM*

Vo Phuong Mai Le[a]†, Patrick Minford[b]

[a]Cardiff University, [b]Cardiff University and CEPR

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Abstract

Calvo contracts, which are the basis of the current generation of New Keynesian models, widely include indexation to general inflation. We argue that the indexing formula should be expected inflation rather than lagged inflation. This is likely to optimise the welfare of the representative agent in a general equilibrium model of the New Keynesian type. The economy behaves under rational indexation is similar to a New Classical model, with shocks producing an immediate fluctuation in both prices and output followed by a fairly rapid return to steady state. The monetary policy that targets price level brings a greater economic stability.

Keywords: Calvo contracts, general equilibrium, rational indexation.

JEL Codes: F41, F42, E42

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†Corresponding Author: LeVP@Cardiff.ac.uk

1 Introduction

In this paper we consider the behaviour of a DSGE model with Calvo contracts in which there is some form of indexation. Specifically, we ask what would be the optimal form of any such indexation.

This question is interesting because within the context of the New Keynesian model, which is widely used by central banks for the analysis of monetary policy, recent work (eg Christiano et al 2005, Smets and Wouters, 2003, 2007) has maintained that an indexation mechanism is both theoretically attractive and empirically helpful. They have assumed that indexation would be to lagged prices or wages. However, other forms of indexation are possible; in particular a more natural and potentially effective mechanism would be forward-looking, indexation to the rational expectation of prices or wages. Thus we could think of agents raising wages and prices in line with what they currently anticipate (on the basis of available information) future inflation to be. As noted by Minford and Peel (2003, 2006) this has the effect of making the Calvo model behave much more like a New Classical model, with a high degree of flexibility in the general level of prices and wages.

Some New Keynesian authors have dismissed this idea on the grounds that the data firmly reject such a model. However, in a recent paper Meenagh et al (2008) have found that the Smets and Wouters (2003) DSGE model of the EU could fit the data only under the assumption that 94% of the labour and product markets were competitive. Also, Le and Minford (2008) find that the Smets and Wouters (2007) model of the US only fit the data given 90% of the labour markets and 80% of the product markets are perfectly competitive. Thus, these economies are modelled on New Classical assumptions. Hence it cannot be right to reject this sort of indexation on purely empirical grounds. On the contrary it might well be that it could rescue New Keynesian models from empirical failure.

2 The basis for indexation

The theoretical basis for nominal rigidity set out by Calvo (1983) has been widely adopted in recent work of the so-called New Keynesian type — also known as the New Neo-Keynesian Synthesis — for example Clarida, Gali and Gertler (1999) and Woodford (2003). In the Calvo contract nominal rigidity can last indefinitely in the sense that there is a limited chance for wage- or price-setters to change their setting in any period. Hence once a price or wage is 'out of line' with its equilibrium there is a chance it will

continue for ever. This has seemed an attractive set-up for modellers who wanted a basis for nominal rigidity with substantial potential real effects[1].

More recently, it has been recognised (Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2003, 2007 for example) that the uncompromising nominal rigidity in Calvo (1983) ought to be modified to allow for some indexing process whereby general inflation is passed through by wage/price-setters. The argument has been that the chances of changing price identified in the Calvo model relate to the changing of a relative price, for example on the grounds that some micro menu cost threshold is stochastically disturbed by some micro event. If then there is some general ongoing inflation this would be passed on by all including those who would not, on micro grounds, wish to change their relative price. Thus the 'menus' outside the restaurants are all updated for general inflation; some individual menus are then raised more or less than that according to micro shocks. In the literature, two specific ways have been widely pursued for doing this: indexing to 'core' inflation (the inflation trend, to be somehow determined) and alternatively to lagged inflation[2]. We can perhaps think of indexation as one element in the business of 'nominal protection', that is protecting price- and wage-setters against movements in the general price level. Another element is the use of expectations of inflation over the lifetime of the contract in the initial setting of wages and prices; notice that the basic Calvo model has this element but only for those who are able to change their prices. For wages the same arguments would indicate that wages should additionally be updated automatically for general rises in real wages, so that a general wage index would be used for them. In what follows we assume that indexation is of price inflation for prices and wage inflation for wages.[3]

Christiano et al. (2005) have noted how adding lagged indexation allows their model, with some other

[1] Nevertheless recent work has exposed a variety of puzzles arising from this set-up. On the one hand there are the empirical difficulties of the original theory noted for example by Mankiw (2001), Mankiw and Reis (2002), Ball (1994), Fuhrer and Moore (1995), Bakhshi et al (2003), Rudd and Whelan (2003) and Eichenbaum and Fisher (2003). On the other hand, a number of articles have pointed to the time-inconsistency problems posed for policy. The essence of these problems lies in that indefiniteness of duration for rigidity; once prices or wages have got out of line there is a strong incentive not to worsen matters by causing yet more prices or wages to get out of line, even if commitments have been made to stabilising prices or inflation along a particular initially-optimal path. A partial list of work that has addressed these issues would include: Goodfriend and King (2001), Khan, King and Wolman (2002), Svensson and Woodford (2003), McCallum (2003), Collard and Dellas (2003), and Woodford (2003).

[2] Further examples are Casares (2002), Ascari (2003); additionally Calvo, Celasun and Kumhof (2003, 2001) and Cespedes, Kumhof and Parrado (2003) have recently suggested further forms of indexing based on rules of thumb based on learning. All these schemes violate the strict natural rate hypothesis (that no monetary policy should be capable of permanently changing output and employment) whose absence from the original Calvo set-up was noted by McCallum (1998) — see also Minford and Peel (2002) for examples of how monetary policy can 'manipulate' real outcomes.

[3] By indexation we mean some automatic contract formula moving all prices (or wages) according to some publicly available information of the general price level. We rule out any other contingent contract clauses, simply because these are ruled out by the Calvo mechanism. Thus 'indexation' is not to be confused with the optimal price paths, conditional on available information, for example, of Mankiw and Reis (2003), where the sole constraint is the periodic arrival of new information; these paths will no doubt contain a planned reaction to expected future inflation among other things affecting the optimal path. But under Calvo contracts such flexibility is not allowed.

modifications of purely rational behaviour, to generate the hump-shaped impulse responses they find in the data. This empirical success has become a further defence of Calvo contracts with lagged indexation. However, recent works by Meenagh et al (2008) and Le and Minford (2008) have further found that impulse responses in the data can also be generated by models with very little nominal rigidity, such as the one with indexation to rationally expected inflation we also consider below; furthermore, when the shocks to the VAR are identified by using the model being tested, then many of these responses are not even hump-shaped.

How should indexation be best carried out however? This issue has not been addressed in the Calvo model though there has been a long series of papers on it in a variety of other macroeconomic models going back to Gray (1976) and Fischer (1977) (see also Barro's critique, 1977). They, building on earlier work, suggested labour contracts could act as insurance for workers (e.g. Azariadis, 1975, and Baily, 1974; and see Malcomson, 1999, for a review of the contract literature) and argued that wages would not be fully indexed because it was not feasible to draw up a fully-contingent contract, expressed throughout in real terms; hence the indexation parameter would stand in as a contingent response to shocks that is partial. More recently, Minford, Nowell and Webb (2003) revived this approach within a model of overlapping contracts; here indexation to lagged inflation operated side by side with expected inflation at the start of the contract.

These papers have found, unsurprisingly, that the degree of optimal nominal protection will vary with the characteristics of the monetary regime. At the one extreme where the regime is volatile and creates persistent shocks protection is high; at the other extreme where there is no monetary volatility at all protection is likely to be low and its exact nature also depends on how real shocks behave. One important aspect explored in Minford, Nowell and Webb (2003) was the effect of switching from a generic inflation targeting rule to a generic price level targeting rule; they found that because price level targeting increased price level certainty it reduced the need for indexation as a way of stabilising consumption. The model here is different but similar considerations apply; so we have also explored the effect of such a monetary rule switch here.

In assessing optimality these papers have usually considered a cooperative equilibrium strategy. This is a natural choice since agents could well be deterred from changing prices or wages automatically in response to some general measure of inflation unless they were sure others (especially in their own

industry) were doing the same. In this paper we also use this cooperative approach. Minford and Peel (2006, Appendix) showed how, in a partial equilibrium context, where agents were acting individually taking others' choices as given, it was optimal for a price-setter to index to lagged expected inflation rather than to lagged inflation; the reason was that, given expectations about the future state of the economy, the lagged expectation, being the optimal forecast from the lagged information available, allows the agent's expected plan for real outcomes to be achieved on average whereas the alternative creates a bias. It can be argued however that in general equilibrium a cooperatively-adopted rule of indexation would not necessarily give the same ranking; for example the forecast bias created might generate a useful offset to the distortion created by price/wage reactions to real shocks.

We do not consider in detail how this cooperative approach could be achieved in practice. Various mechanisms are possible. Trade unions or employers' organisations could organise it for example; or there could be a sharing of information across agents in which coordination was suggested and agreed by discussion; or it could spread from private maximisation in partial equilibrium as above to general acceptance.

In what follows we first sketch out in an overview how in a standard New Keynesian model the addition of indexing affects outcomes. In the next section we work analytically with a much simplified version of this model. We then proceed to show a wider set of results via numerical simulations on a calibrated version of the full model. Finally we consider the robustness of the results to changes in our assumptions, especially to the monetary rule.

3 The Model With Indexation — an overview

The model used is a nonlinear NNS model (Canzoneri et al., 2004), characterized by optimizing agents, monopolistic competition, nominal inertia and capital accumulation. It is very closely related to the models of Erceg, Henderson and Levin (2000) and Collard and Dellas (2003). Staggered price setting leads to a dispersion in firms' prices that creates an inefficient variation in output levels across firms, and staggered wage setting leads to dispersion in the distribution of employment across households.

We study various versions of this model. In the simplest we abstract from capital and assume flexible wages — this version is tractable analytically and we examine it in a loglinearised form. Our first version assumes lagged indexation. In it price-setters know that prices will rise by the lagged index anyway and

so their problem is to forecast real marginal costs and also the difference between the general price level and the index — this latter will turn real marginal costs plus the index into actual nominal marginal costs, which is what they aim to match. It is this version that we now proceed to analyse.

In this model the price-setter's forecasting problem relates to a real variable, real marginal costs, and a nominal error — the extent that the index fails to equal actual prices. Of course perfect indexation would simply be actual prices and so this error would disappear. When the index is lagged prices, the error is simply the current rate of inflation. Then the price-setter's problem is to forecast real marginal costs and current and future inflation. This is the standard problem solved by agents in Christiano et al (2005) and Smets and Wouters (2003, 2007). It has a familiar solution, given that both real marginal costs and inflation are persistent variables.

However in our second version where the index is the price level expected last period, this error is purely random. This means that the price is set solely in respect of expected real marginal costs, since the expected price error is zero. However now consider the actual price level; it is equal to the whole history of prices set by price-setters, all uprated by the price level expected last period. If we take rational expectations of this, it implies a restriction on expected prices being set by price-setters this period: the indexing process itself will uprate prices by any predictable move that price-setters would otherwise make, a move that is related to the previous pricing error. Price-setters must therefore be expected last period to offset this lagged error this period. We are more used to these restrictions in a plain New Classical model, where the expected output gap is forced to generate no 'inflationary pressure'; similarly here expected marginal costs must be such as to generate no 'inflationary pressure' or else infinite inflation would be produced.

In addition to this expected price-setting we must add the unexpected element in prices set, the pricing 'error', which will be related to the shock in productivity (real marginal costs) observed by price-setters in this period. Thus the previous pricing error will also be related to the lagged shock in productivity. It follows that prices set are equal to the expected part, related to the lagged shock in productivity, plus the effect of the current productivity shock.

Such are the restrictions placed by rational expectations when the indexing process itself is rational: the indexing process is aiming off for all systematic effects in price-setting. .

In effect this is forcing set-prices to be a moving average process of order one in a real variable, the

productivity shock. The actual price surprise is a function of the set-price surprise. But to find actual prices, the sum of the set-price and the indexing component, we have to solve for expected prices, the indexing component: this comes from the Taylor Rule and the expected behaviour of output (expected productivity again). Thus it is that we get a New Keynesian model with some resemblance to New Classical. The set-price (related to real marginal costs and so to the output gap) has moving average behaviour — much like the output gap in a New Classical model may have. Meanwhile price behaviour depends on the behaviour of monetary policy.

The account we have given relates specifically to the simplified model with no capital and wage flexibility. But the basic patterns are the same in more complex models, which we simulate numerically. Notice that there is still not complete price/wage flexibility because current shocks disturb both prices and output and create lagged effects. However plainly this New Keynesian model has far more price/wage flexibility than either the New Keynesian model with lagged indexation or that with no indexation at all.

4 A simplified model with exogenous capital and a competitive labour market

We first examine what private sector indexation behaviour would be optimal assuming that monetary policy followed a standard Taylor Rule. Our model contained three shocks: a money shock to the Taylor Rule, a productivity shock and a government spending shock. However the last turned out to be of no importance and thus in practice we considered only the first two — one nominal and one real.

As the full model is complex and nonlinear, we investigate a much simplified version in which capital is exogenous and made a non-tradeable endowment resource, while the labour market is assumed to be competitive. All the equations are loglinearised.

(1) Each firm is a price-setter, which forms the expectation of its price for period t based on micro information of period t and macro information of period $(t-1)$. Each firm f minimises its total cost

subject to its production function:

$$\min\ TC_t(f) = W_t L_t(f)$$

$$s.t\ Y_t(f) = Z_t \bar{K}^{\nu} L_t(f)^{1-\nu}$$

where $\bar{K}$ is exogenous and $\log Z_t = \rho_1 \log Z_{t-1} + z_t$. The labour demand function is derived from the above production function $L_t(f) = \left(\frac{\bar{Y}_t(f)}{Z_t \bar{K}^{\nu}}\right)^{\frac{1}{1-\nu}}$. The firm's cost minimising problem implies that the nominal marginal cost is

$$MC_t = \frac{1}{1-\nu} W_t L_t(f)^{\nu} \frac{1}{Z_t \bar{K}^{\nu}}$$

and thus, loglinearised real marginal cost is

$$\log mc_t = \log W_t - \log P_t + \nu \log L_t(f) - \log Z_t \tag{1}$$

In regards to households, each of them maximises the life-time expected welfare subject to his budget constraint and labour demand, but without the capital accumulation constraint in this simple set up. However, besides the assumption of fully complete contingent claims that make the households homogeneous in their consumption decisions, the competitive labour market means they are homogeneous in labour supply also. The welfare maximisation implies every household supplies N_t^{χ} units of labour: $N_t^{\chi} = \frac{W_t}{P_t C_t}$ and its log-linear form is given by

$$\chi \log N_t = \log W_t - \log P_t - \log C_t \tag{2}$$

The competitive labour market also means that in equilibrium the supply of and demand for labour must be equal so that the equation (1) becomes

$$\log mc_t = \log W_t - \log P_t + \nu \log N_t - \log Z_t$$

(2) The production function is given as:

$$\log Y_t = \log Z_t + (1-\nu) \log N_t \tag{3}$$

(3) Ignoring government spending, the market clearing condition gives

$$\log Y_t = \log C_t \tag{4}$$

(4) We use a simple Taylor Rule, without lags and with the real interest rate assumed to be set in response to inflation and the output gap, with a monetary shock:

$$r_t = \tau \pi_t + \sigma(\log Y_t - \log Z_t) + \log M_t \tag{5}$$

Adding in the Euler equation $\log C_t = \log C_{t+1} - r_t$ and allowing for market clearing gives us an Aggregate Demand curve:

$$\log Y_t = \frac{1}{1 - \sigma^* B^{-1}} \left\{ -\tau \sigma^* \pi_t + \sigma \sigma^* \log Z_t - \sigma^* \log M_t \right\}$$

where $\log Z_t = \rho_1 \log Z_{t-1} + z_t$; $\log M_t = \rho_2 \log M_{t-1} + \mu_t$; z_t and μ_t are *i.i.d.*; B^{-1} is the forward operator instructing one to lead the variable but keeping the expectations data-set constant; and $\sigma^* = \frac{1}{1+\sigma}$.

(5) The reset price level loglinearised around its equilibrium is:

$$\log P_t^* = \frac{(1 - \alpha\beta) E_t \left(\log mc_t + \log P_t - \log \tilde{P}_t \right)}{1 - \alpha\beta B^{-1}} \tag{6}$$

This is rewritten in terms of the determinants of real marginal cost (from equations (1) , (2) , (3) and (4) as:

$$\log P_t^* = \frac{(1 - \alpha\beta)}{1 - \alpha\beta B^{-1}} E_t \left(\frac{1 + \chi}{1 - \nu} (\log Y_t - \log Z_t) + \log P_t - \log \tilde{P}_t \right) \tag{7}$$

Notice that though the firm at t knows its own marginal cost and its own price, in order to forecast the future paths of variables required to set its price it needs to know Y_t, P_t and $\tilde{P}_t$ which are macro variables; of these it only knows $\tilde{P}_t$. It does of course know Z_t, its own productivity level.

(6) The log-linearised form of the aggregate price equation is given as

$$\log P_t - \log \tilde{P}_t = \alpha \left(\log P_{t-1} - \log \tilde{P}_{t-1} \right) + (1 - \alpha) \log P_t^* \tag{8}$$

(7) The loglinearised price dispersion, $\log DP_t$, is derived using a conventional second order Taylor expansion around $P^*_{t-i} = 1$:

$$\log DP_t = \frac{1}{2}\Sigma^{\infty}_{i=0}\phi_p\alpha^i(1-\alpha)\left[1-\alpha^i(1-\alpha)\right]\left(\log P^*_{t-i}\right)^2 - \Sigma^{\infty}_{j=0}\Sigma^{\infty}_{i=0}\phi_p\alpha^{i+j}(1-\alpha)^2\log P^*_{t-i}\log P^*_{t-j} \tag{9}$$

4.1 Solving the model under rational indexation

We now solve the model in turn under rational (this section) and lagged indexation (next section). so that we may compare the two welfare expressions. Our notation is as follows: $E^{t-1}x_t = E(x_t|\Phi_{t-1}); E_t x_t = E(x_t|\Phi_{t-1},\phi_t); x^{UE}_t = x_t - E^{t-1}x_t$, where Φ_{t-1} is the full information set from period $t-1$ and ϕ_t is the limited information set available (to the agent forming expectations) for period t.

Assume under rational indexation that

$$\log \tilde{P}_t = E\left(\log P_t|\Phi_{t-1}\right) \equiv E^{t-1}\log P_t$$

Applying this assumption to equation (8) , we get:

$$E^{t-1}\log P^*_t = -\frac{\alpha}{1-\alpha}\left(\log P_{t-1} - E^{t-2}\log P_{t-1}\right) \tag{10}$$

and

$$E^{t-1}\log P^*_{t+i} = E^{t-1}E^t\log P^*_{t+i} = 0;\ \forall i \geqslant 1$$

The reset price is therefore:

$$\log P^*_t = E^{t-1}\log P^*_t + \log P^{*UE}_t = \log P^{*UE}_t - \frac{\alpha}{1-\alpha}\log P^{UE}_{t-1}, \tag{11}$$

where from equation (8)

$$logP^{UE}_t = (1-\alpha)\,logP^{*UE}_t \tag{12}$$

and from equation (7) and the assumption that firms have knowledge of their own micro information (productivity, prices and costs) in period t as well as the macro information of period $(t-1)$:

$$\begin{aligned}\log P_t^{*UE} &= \log P_t^* - E^{t-1}\log P_t^* \\ &= (1-\alpha\beta)\,\Sigma_{i=0}\,(\alpha\beta)^i\left(\frac{1+\chi}{1-\nu}\right)\left[E^{t-1}\log Y_{t+i} - E^t\log Z_{t+i}\right] \\ &- (1-\alpha\beta)\,\Sigma_{i=0}\,(\alpha\beta)^i\left(\frac{1+\chi}{1-\nu}\right)\left[E^{t-1}\log Y_{t+i} - E^{t-1}\log Z_{t+i}\right] \\ &= (1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\frac{1}{1-\alpha\beta\rho_1}\,(-z_t) \qquad (13)\end{aligned}$$

Hence, given equations (11), (12) and (13) the reset price is rewritten as

$$\log P_t^* = \log P_t^{*UE} - \alpha\log P_{t-1}^{*UE} = \chi'\frac{(1-\alpha L)}{(1-\alpha)}\,(-z_t)\,, \qquad (14)$$

where $\chi' = (1-\alpha)\,(1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\frac{1}{1-\alpha\beta\rho_1}$.

Thus we have found as noted in the overview that rational expectations has placed restrictions on the behaviour of the reset price. Because of the relation between expected reset prices and expected future developments in the economy, we obtain further restrictions on the latter (just as in a New Classical model expected output is forced to converge on output potential) :

$$\begin{aligned}(a)\quad E^{t-1}\log P_{t+j}^* &= (1-\alpha\beta)\,\Sigma_{i=0}\,(\alpha\beta)^i\left(\frac{1+\chi}{1-\nu}\right)\left[E^{t-1}\log Y_{t+i} - E^{t-1}\log Z_{t+i}\right]_{+j} = 0\ \forall j \geqslant 1 \\ &\Leftrightarrow E^{t-1}\log Y_{t+i} = E^{t-1}\log Z_{t+i},\ \forall i \geqslant 1\end{aligned}$$

$$(b)\qquad E^{t-1}\log P_t^* = (1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\left[E^{t-1}\log Y_t - E^{t-1}\log Z_t\right] = -\frac{\alpha}{1-\alpha}\log P_{t-1}^{UE}$$

Now we look at the output side of the model, where under rational expectations output consists of the expected output and the surprise change in output. First, using the Aggregate Demand curve we obtain for surprise output:

$$\log Y_t^{UE} = -\tau\sigma^*\log P_t^{UE} + \sigma\sigma^* z_t - \sigma^*\mu_t \qquad (15)$$

and from the restrictions on expected reset prices above:

$$E^{t-1}\log Y_t = E^{t-1}\log Z_t - \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{(1-\alpha\beta)}\left(\frac{1-\nu}{1+\chi}\right)\log P_{t-1}^{UE}.$$

The latter equation is in turn written as follows

$$E^{t-1}\log Y_t = E^{t-1}\log Z_t + v'z_{t-1}, \tag{16}$$

where $v' = \left(\frac{\alpha}{1-\alpha}\right)\frac{1}{(1-\alpha\beta)}\left(\frac{1-\nu}{1+\chi}\right)$ and we have used the expressions above for surprise prices and reset prices.

Using equations (12) and (13), the unexpected component of output can be written

$$\log Y_t^{UE} = \sigma^*(\sigma + \tau\chi')z_t - \sigma^*\mu_t \tag{17}$$

The Aggregate Output therefore is just a sum of its expected and unexpected components — equations (16) and (17)

$$\log Y_t = \rho_1 \log Z_{t-1} + v'z_{t-1} + \sigma^*(\sigma + \tau\chi')z_t - \sigma^*\mu_t \tag{18}$$

and

$$\log Y_t - \log Z_t = v'z_{t-1} + \sigma^*(\tau\chi' - 1)z_t - \sigma^*\mu_t \tag{19}$$

4.1.1 Welfare under rational indexation

Under the flexible price and wage assumption, the welfare level would be

$$u_t^{FLEX} = \log Z_t \tag{20}$$

However, in the economy of price rigidity and competitive labour market, the welfare function is

$$u_t = \log C_t - \bar{N}^{\chi+1}\log N_t \tag{21}$$

where $\log C_t = \log Y_t = \log Z_t + (1-\nu)\log N_t - \log DP_t$ and $\log N_t = \frac{1}{1-\nu}\left(\log Y_t - \log Z_t\right)$.

We evaluate expected welfare in terms of its deviation from the flex-price optimum:

$$E\left(u_t - u_t^{FLEX}\right) = E\left[\left(\frac{1-\nu-\bar{N}^{\chi+1}}{1-\nu}\right)\left\{\begin{array}{c} v'z_{t-1} + \sigma^*(\tau\chi'-1)z_t \\ -\sigma^*\mu_t \end{array}\right\}\right] - E\log DP_t \tag{22}$$

Notice that the unconditional mean of the first element in this expected welfare term is 0. So effectively we only consider the second term, where it is known respectively from equations (9) and (14) that

$$\begin{aligned} E\left(u_t - u_t^{FLEX}\right) &= -E\log DP_t \\ &= -\frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i\left(1-\alpha\right)\left[1-\alpha^i\left(1-\alpha\right)\right]var\left(\log P_{t-i}^*\right) \\ &+ \Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}\left(1-\alpha\right)^2 Cov\left(\log P_{t-i}^*, \log P_{t-j}^*\right) \end{aligned}$$

and

$$\log P_t^* = -\chi'' z_t + \alpha\chi'' z_{t-1},$$

where $\chi'' = \frac{\chi'}{1-\alpha}$. The expected welfare is (Appendix 9.1.1):

$$E\left(u_t - u_t^{FLEX}\right) = -\phi_p\alpha\chi''^2 var(z) \tag{23}$$

4.2 Solving the model with lagged indexation

The solution for lagged indexation follows more familar lines. We write the index as

$$\log\tilde{P}_t = E^{t-1}\log P_t + \left(k\log P_{t-1} - E^{t-1}\log P_t\right) \tag{24}$$

where if $k = 1$ then there is full lagged indexation $\log\tilde{P}_t = \log P_{t-1}$, and if $k = 0$ then there is a no indexation and $\log\tilde{P}_t = 0$. We will focus here exclusively on the case of full lagged indexation, $k = 1$.

Using this assumption and equation (8), the general price is

$$\log P_t = E^{t-1}\log P_t + v_{t-1} + \frac{(1-\alpha)\log P_t^*}{1-\alpha L},$$

where $v_{t-1} = \log P_{t-1} - E^{t-1}\log P_t = -E^{t-1}\pi_t$. This equation can also be written as

$$\left(\log P_t - E^{t-1}\log P_t\right) - v_{t-1} + \alpha v_{t-2} - \alpha\log P_{t-1}^{UE} = (1-\alpha)\log P_t^*$$

and taking the expectation E^{t-1} throughout and manipulating this equation, we get expected reset price

$$E^{t-1}\log P_t^* = \frac{-v_{t-1} + \alpha v_{t-2} - \alpha \log P_{t-1}^{UE}}{1-\alpha} \tag{25}$$

What we see is that the expected reset price now contains the the last two expected inflation rates; these terms are the bias in indexation away from its rational value.

We use equation (7) and its expectation

$$E^{t-1}\log P_t^* = \frac{(1-\alpha\beta)}{1-\alpha\beta B^{-1}} E^{t-1} E_t \left(\frac{1+\chi}{1-\nu} (\log Y_t - \log Z_t) + \log P_t^{UE} - v_{t-1} \right)$$

to obtain unanticipated reset price

$$\log P_t^{*UE} = (1-\alpha\beta)\left(\frac{1+\chi}{1-\nu}\right)\frac{1}{1-\alpha\beta\rho_1}(-z_t) \tag{26}$$

4.2.1 Solving for $\log P_t$ and $\log P_t^*$ under lagged indexation

Using the equations for inflation together with the Aggregate Demand curve above yields (Appendix 1 section 9.2.1):

$$\pi_t = \frac{1-\alpha}{1-\alpha L}\frac{1-\alpha\beta}{1-\alpha\beta B^{-1}} E_t \left[\chi^* \left(\frac{1}{1-\sigma^* B^{-1}} \right) \left\{ -\tau\sigma^*\pi_t - \sigma^*(1-B^{-1})\log Z_t - \sigma^* \log M_t \right\} + \pi_t \right] \tag{27}$$

where $\chi^* = \frac{1+\chi}{1-\nu}$. The solution for the general inflation rate has the Wold decomposition form $\pi_t = \Sigma_{i=0}\zeta_i \mu_{t-i} + \Sigma_{i=0}\xi_i z_{t-i}$; use undetermined coefficients to solve for π_t. Thus the solution for ν_t is by implication:

$$\nu_t = -E^t \pi_{t+1} = -\Sigma_{i=0}\zeta_{i+1}\mu_{t-i} - \Sigma_{i=0}\xi_{i+1} z_{t-i} \tag{28}$$

We once again find the solution for output's deviation from its flexprice value which now becomes:

$$\begin{aligned} \log Y_t - \log Z_t &= \left(\log Y_t^{UE} - z_t\right) + E^{t-1}(\log Y_t - \log Z_t) \\ &= (\sigma^*(\tau\chi' - 1)z_t - \sigma^*\mu_t) + \\ &\left\{ \frac{1}{(1-\alpha\beta)\chi^*} \right\} \begin{pmatrix} \alpha\chi' z_{t-1} - \frac{\alpha(1+\beta)}{1-\alpha} v_{t-1} + \\ \frac{\alpha}{1+\alpha} v_{t-2} + \frac{\alpha\beta}{1-\alpha} E^{t-1} v_t \end{pmatrix} \end{aligned} \tag{29}$$

4.2.2 Welfare under lagged indexation

Expected welfare is

$$E\left(u_t - u_t^{FLEX}\right) = E\left[\frac{1-\nu-\bar{N}^{\chi+1}}{1-\nu}\left(\begin{array}{c}(\sigma^*(\tau\chi'-1)z_t - \sigma^*\mu_t) + \\ \left\{\frac{1}{(1-\alpha\beta)\chi^*}\right\}\left(\begin{array}{c} a\chi' z_{t-1} - \frac{\alpha(1+\beta)}{1-\alpha}v_{t-1} + \\ \frac{\alpha}{1+\alpha}v_{t-2} + \frac{\alpha\beta}{1-\alpha}E^{t-1}v_t \end{array}\right)\end{array}\right)\right] - E\log DP_t \quad (30)$$

But again, we only have to consider the second term in this welfare expression, where

$$\begin{aligned} -E\log DP_t = &-\frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i(1-\alpha)\left[1-\alpha^i(1-\alpha)\right]var\left(\log P_{t-i}^*\right) \\ &+ \Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}(1-\alpha)^2 Cov\left(\log P_{t-i}^*, \log P_{t-j}^*\right) \end{aligned}$$

with

$$\log P_t^* = \frac{\chi'}{1-\alpha}(-z_t + \alpha z_{t-1}) - \left(\frac{\nu_{t-1} - \alpha\nu_{t-2}}{1-\alpha}\right) \quad \text{(Appendix 9.2.2)} \quad (31)$$

We have the following expressions: $\nu_{t-1} = \log P_{t-1} - E^{t-1}\log P_t = -E^{t-1}\pi_t$ is correlated with z_{t-1}; but $\nu_{t-2} = \log P_{t-2} - E^{t-2}\log P_{t-1}$ is uncorrelated with z_t and z_{t-1}. Assume that $-\frac{\nu_{t-1}-\alpha\nu_{t-2}}{1-\alpha} = \psi_0 z_{t-1} + q_{t-1}$, where $\psi_0 z_{t-1}$ combines all terms in z_{t-1}, and q_{t-1} is uncorrelated with z_{t-1} and z_t. Comparing this with the new reset price under rational indexation, this lagged indexation's renewed price function has the extra term $-\frac{\nu_{t-1}-\alpha\nu_{t-2}}{1-\alpha}$. For the task below, we temporarily take the expected welfare under rational expectation as a benchmark. To compare the expected welfare under lagged indexation to the benchmark, we need to investigate whether this extra term in renewed price improves or worsens the expected welfare level in respect to the benchmark.

It can be seen that there are two elements in this term. The first, $\psi_0 z_{t-1}$, is potentially helpfully correlated with the lagged term in the rational indexation solution for $\log P_t^*$; hence it could potentially reduce $E\log DP_t$. The second, q_{t-1}, adds noise to the solution and hence must increase $E\log DP_t$. We can find a closed form expression for the latter but the former requires numerical calculation. In Appendix 9.2.3 we look at whether this latter term can be signed unambiguously.

What we find is that for productivity shocks the effect of rational compared with lagged indexation is ambiguous; extra noise is introduced by the lagged index but some of it is correlated with the lagged productivity in a potentially helpful way. For monetary shocks rational indexation is unambiguously

superior because in this case these shocks have no effect on the reset price and therefore on welfare; under lagged indexation monetary shocks at $t-1$ and before all enter the current reset price setting.

4.3 Conclusions from the simplified model

What we find is that it is optimal within this simplified version of the model to index reset prices to the rational expectation of the price level in the face of monetary shocks; in the face of productivity shocks it is ambiguous. We thus find, in line with earlier work, that the type of indexation will depend importantly on the monetary regime. Thus, to put it rather crudely, provided monetary shocks are large enough, rational indexation will be optimal.

We also find that should rational indexation be chosen for this reason, then expected welfare is invariant to monetary policy; and the economy's Phillips Curve defaults to a 'New Classical' one where output depends on current and lagged real shocks but otherwise only on monetary surprises as in Lucas (1972); there is an echo here of Sargent and Wallace's (1975) famous irrelevance result three decades ago. The intuition behind this result is that rational indexation builds into prices the effect of any shocks known at time $t-1$. Whatever has happened at $t-1$ is, in the case of the productivity shock, built into the expected real reset price for next period; this fixes expected real marginal cost and hence expected real output. The expected price index is then calculated as the necessary price increase that will accommodate this and the expected level of interest rates. Unexpected monetary shocks have no effect on prices because they have been pre-set in this way. Thus only lagged money shocks affect prices while only unexpected money shocks affect output under rational indexation in this model under the assumption made up to now that agents have no information on economy-wide variables.[4]

5 Extending our results — Stochastic simulation results on the full model under micro private information

So far we have been considering optimal indexation using a simplified linearised model, with only prices being set (and with no capital). We now use numerical methods to investigate the full model and other

[4] Notice too that in a corrolary of this point, again echoing Sargent and Wallace (ibid.), price level determinacy cannot be produced by an interest-rate rule targeting inflation unless the lagged price level is given; yet the model cannot generate such a lagged price level under such a regime unless again the twice-lagged price level was fixed and so on ad infinitum. It is necessary when using such a rule to specify the lagged price level exogenously via an initial condition, presumably indicating that at some previous point a different rule was being pursued.

assumptions about monetary policy; we also consider the possibility of less than full indexation, as well as a mixture of rational and lagged indexation. To do this we can no longer use analytic methods because of the rise in complexity; thus we move to numerical methods under calibration. We use Dynare (Juillard, 2003) which employs a second order approximation of the model. Throughout the simulation, we use a discount factor β of 0.99. The Cobb-Douglas capital share parameter, $\nu = 0.25$, implies that the output-labour elasticity is 0.75. The wage and price markup rates are $\mu_w = \mu_p = 1.167$. The constant probability determining the degree of price stickiness is $\alpha = 0.67$; this implies that an average price contract duration is 3 periods, while the probability of wage resetting is assumed to be $1 - \omega = 0.25$ in every period, implying an average wage contract length of 4 periods. The Frisch elasticity of labour supply is 0.33. These are the calibration parameters used by Canzoneri et al. (2004) in their benchmark specification.

We proceed to consider the stochastic simulations for expected welfare in terms of deviations from the flex-optimum under both lagged and rational indexation when only micro information is assumed to be known at period t. The aim is to relate these results to those from the analytic model above. Table 1 shows that expected welfare is maximised by rational indexation, just as in the analytic model.

	Type of shock	$\mathbf{k}_1$	$\mathbf{kw}_1$	$\mathbf{k}_0$	$\mathbf{kw}_0$	E(**Welfare**)
Inflation target	Money/productivity	0	0	1	1	−0.0048
		0	0	.8	.8	−0.01311
		.2	.2	1	1	−0.01626
		1	1	1	1	−0.02669
Price target	Money/productivity	0	0	1	1	−0.0048
		0	0	.8	.8	−0.01032
		.2	.2	1	1	−0.01120
		1	1	1	1	−0.01994

k_0 and kw_0 are the extent of price and wage indexation respectively (1=100% indexation; k_1 and kw_1 are the weight on rational indexation in price and wage equations respectively (1=100% weight on rational indexation)

Table 1: Expected welfare for different types of indexation assuming only current information is micro; interest rule with inflation and price level targets (stochastic simulation of monetary and productitivty shocks, each with standard error of 0.01)

We now find that monetary policy does, strictly speaking, have an effect on expected welfare under rational indexation. The reason for this is the introduction of wage-setting. Though prices are only affected by the productivity shock (because it alone is currently observed), the interest rate rule reacts to both inflation (or prices) and to the output gap while monetary shocks also affect the latter. This reaction alters output and so employment; with wages fixed this drives agents away from their flex-

price leisure choice, affecting their welfare. We also show in Table 2 how monetary policy choices affect expected welfare. The choice of whether to target inflation or prices is irrelevant since it is only the current price shock reaction that matters. Thus what matters in the interest rate rule is the size of the reactions to inflation or prices and to the output gap. Higher inflation or price coefficients worsen the effect of productivity shocks because they dampen price changes which means that real wages do not change as much as they should to match productivity change. Higher reactions to the output gap dampen movements in it and employment which move workers away from their flex-price choices. What we notice is that while there are effects here, they are not at all big, because the utility function does not have much curvature in leisure. In the Calvo model the big losses arise because of price and wage dispersion. Hence we can say that effectively the results are the same as in the analytic model: full rational indexation is optimal and in this case monetary policy is (effectively) impotent.

Full Rational indexing (always optimal) EWelfare:	**Productivity**	**Monetary**	**Total**
Inflation targeting	−0.00381	−0.00098	−0.0048
Price targeting	−0.00381	−0.00098	−0.0048
Stricter Inflation targeting	−0.00426	−0.00098	−0.00524
Stricter Price targeting	−0.00426	−0.00098	−0.00524
Inflation targeting (higher weight on output gap)	−0.00376	−0.00092	−0.00468
Price targeting (higher weight on output gap)	−0.00376	−0.00092	−0.00468

Table 2: Expected welfare for different types of indexation assuming only current information is micro; interest rule with inflation and price level targets (stochastic simulation of monetary and productitivity shocks, each with standard error of 0.01).

5.1 Conclusions on case of micro current information only

What we have found in this case of micro current infromation is that full rational indexation is the dominant strategy for private agents. This has strong implications for monetary policy. First, it is irrelevant whether the interest rate rule targets inflation or prices since only the shock to prices or inflation matters and it is the same under both rules. Second, the coefficients of the rule make no difference at all to expected welfare in the analytic model (because current prices respond to current productivity shocks only) and in the full model they make virtually no difference (since they only enter through the effect on employment whose effect on welfare is minor).

6 Full model results when agents observe full current information

We now turn to the case where full current information is available to private agents. This is the default assumption made in New Keynesian models. We also use the default assumption about information used by them and others who use this approach: namely that agents know all current information, macro and micro, when they form their expectations at t; the indexation formula still uses the lagged information because of lags in application. Hence our initial simulations use the model exactly as in this literature except for the addition of the indexation formula, so that we can gauge the effect solely of adding in this one element.

As we noted earlier the justification for this full current information assumption presumably lies in the overlap between the length of time in which prices and wages are not changed at all — a quarter — and the production of current macro information by statistics offices and the private sector itself. In the course of three months price and wage setters may well be fairly well informed about what is going on in that quarter so that the assumption of full knowledge may be a close approximation. At any rate we now explore the implications of this assumption within the full model.

We thus carry out the stochastic simulation under the assumption of full information being available in period t (Table 3) As in Table. 1 the pair $(k1, kw1)$ show the weights on lagged and rational indexation in indexation formulas for prices and wages respectively, while $(k0, kw0)$ shows whether prices and wages are partially or fully indexed. Our stochastic simulations are done for 100 sets of 40 overlapping shocks — with both productivity and monetary shocks. Like Minford and Nowell (2003), we treat each period outcome as a stochastic experiment of equal likelihood. We ignore the discount rate in calculation of the expected welfare. Firstly, in each set, in the first period, it runs for the first shock and records the welfare of this period. The first period values are then used as the base values for the next period simulation and so on. Then.we have 4000 observations which we average to get expected welfare. This process repeats for each $(k0, kw0, k1, kw1)$, where values of these parameters all belong to the interval of $[0, 1]$ and they move with a step of 0.2. Finally, we compare all the expected welfare values to find the weighting scheme that gives the maximum expected welfare.

Table 3 reports the results of stochastic simulations on the full model, showing the optimal indexation

scheme under our two shocks to productivity and money.

	k1	kw1	k0	kw0	Best expected welfare
Inflation targeting	0	0	1	1	−0.00572
Price targeting	0	0	0.8	1	−0.00096
Stricter price target	0	0	0.8	1	−0.00034
Shocks assumed to be both monetary and productivity each with standard error of 0.01.					

Table 3: Optimal index weighting scheme under inflation and price level targeting with full current information assumption

We note that:

(1) lagged indexation does not have any weight in the optimal indexation scheme.

(2) monetary policy is effective on welfare; as we move from inflation to price targeting and then to stricter price targeting expected welfare improves.

(3) the extent of price indexation also responds endogenously to this change in monetary policy: it drops somewhat. While full rational indexation is best under inflation targeting, price indexation drops to only 80% (though still on the rational index) as price level targeting is introduced.

Let us consider these points in turn.

(1) To understand why lagged indexation does not enter the optimal indexation scheme, we refer back above to where we showed that lagged indexation created an additional correlation between lagged price surprises and lagged prices: this tends to raise the variability of accumulated reset prices on balance. Hence the optimal indexation scheme only has rational indexation in it. The explanation can be briefly described as follows: the reset price under lagged indexation is given as the reset price under rational indexation plus an extra term

$$\log P_t^* \text{ (lagged)} = \log P_t^* \text{ (rational)} - \left(\frac{\nu_{t-1} - \alpha \nu_{t-2}}{1-\alpha} \right) \tag{32}$$

where $v_{t-1} = \log P_{t-1} - E_{t-1} \log P_t$. The reset price under the rational expectation is only a function of the current and lagged productivity shocks. The term $-\left(\frac{\nu_{t-1}-\alpha\nu_{t-2}}{1-\alpha}\right)$ in the reset price under lagged indexation contains all past productivity and monetary shocks. The difference between expected welfare under rational indexation and lagged indexation is divided into two parts. The first part consists of all the terms (q) that are not related to lagged productivity shock (z_{t-1}), such as all the productivity shocks that occur before or at period $(t-2)$ and all monetary shocks. These shocks are the ones that do not enter the reset price expression under rational indexation. The ex-

pected welfare of this part is $-\phi_p \frac{\alpha}{1+\alpha}\left(\frac{1-\rho}{1-\alpha\rho}\right) var\left(q\right)$ [5] which must worsen the expected welfare under lagged indexation compared with rational indexation. The second part of the expected welfare consists only of terms in z_{t-1} which are hence correlated with welfare under rational indexation. This part is $-\phi_p \frac{\alpha}{1+\alpha}\left(\chi''^2 + \left(\psi_0 + \alpha\chi''\right)^2 + \left(1-\alpha\right)\chi''\left(\psi_0 + \alpha\chi''\right)\right) var(z)$, which turns out for the calibrated values of the model to improve expected welfare compared with rational indexation. In aggregate the comparison between the resulted expected welfare levels under lagged and rational indexation depends on the magnitudes of the two parts above. The simulation results suggest that the first part dominates the second, so that lagged indexation lowers expected welfare by introducing additional lagged shocks into the reset price.

(2) Monetary policy is now effective on expected welfare because full information causes current monetary and productivity shocks to affect both reset prices and wages and the interest rate rule modifies these effects through its reaction coefficients. We can demonstrate this conclusion in the analytic model with its assumptions of no labour market, no capital accumulation, and only price setting. reset price. This model's solution for the reset price under full rational indexation is:

$$\log P_t^* = 1.5134 z_t - 1.0141 z_{t-1} - 1.4095 \mu_t + 0.7659 \mu_{t-1} \tag{33}$$

Here the monetary shock and its lagged value join the productivity shock and its lagged value in affecting the reset price and so expected welfare.

Moving from an inflation to a price-level target has the effect of increasing the response of real interest rates to price shocks and so dampening these. The reason is that price-level targeting effectively raises the real interest rate, r_t, response to a price shock because $r_t = R_t - E_t\pi_{t+1}$, which is $r_t = rule - (E_t P_{t+1} - P_t)$. Under inflation targeting, the last term in the real interest rate equation is small or zero so the real interest, r_t, is just the rule; but under price-level targeting, $E_t P_{t+1}$ is small or zero, so the rule becomes $rule + P_t$. Therefore, a price-level target stops prices moving as much as they do under an inflation target. This reduces the variance of the reset price and also that of both of the real wage and of employment which additionally enter the welfare function in the full model. We can see the effect of the reduced reset price variance from simulations of the analytic model under full current information

[5] $q_{t-i} = \rho^{i-j} q_{t-j}$

and full rational indexation — Table 4 below

Inflation target			Price target		
Productivity	Monetary	Total	Productivity	Monetary	Total
−0.001363	−0.001026	−0.002389	−0.00087	−0.000498	−0.00137

Table 4: Expected welfare for rational indexation under interest rule with inflation and price level targets, assuming full current information (stochastic simulation of monetary and productitivty shocks, each with standard error of 0.01)

(3) We turn last to why indexation is sensitive to monetary policy. We can understand this in terms of the alteration less than full rational indexation creates in the reset price, p_t^*. The reset price equation under full rational indexation can be written as a function of unexpected current and lagged price changes:

$$(1-\alpha)\log P_t^* = \log P_t^{UE} - \alpha \log P_{t-1}^{UE}, \tag{34}$$

If we now deviate from full rational indexation by reducing the indexation to $(1-k)$ this gives us instead

$$(1-\alpha)\log P_t^* = \log P_t^{UE} + kE_{t-1}\log P_t - \alpha \log P_{t-1}^{UE} - k\alpha E_{t-2}\log P_{t-1}, \tag{35}$$

We can see that this creates a potential correlation between $E_{t-1}\log P_t$and $\log P_{t-1}^{UE}$. Suppose there is a shock to the price level, then under price-level targeting there is a commitment to remove some or all of this shock from next period's price level; thus write $E_{t-1}\log P_t = \rho(1-\beta)\log P_{t-1}^{UE}$ where ρ is the model-generated persistence in prices and β is the extent of its removal by the price-targeting rule (thus we can think of ρ as the persistence that would occur as β tends to zero, ie the price-level target operates very slowly). It follows that the variance of $\log P_t^*$ which enters expected welfare will equal $(\frac{1}{1-\alpha})^2[Var\log P^{UE}](1+[k\alpha\rho(1-\beta)]^2+\{\alpha^2+[k\rho(1-\beta)]^2-2k\alpha\rho(1-\beta)\})$. The difference of this from the variance at $k=0$ is $(\frac{1}{1-\alpha})^2[Var\log P^{UE}]([k\alpha\rho(1-\beta)]^2+[k\rho(1-\beta)]^2-2k\alpha\rho(1-\beta))$. For this difference to be negative for positive $\{k,\rho(1-\beta)\}$ we require that $2\alpha > \rho(1-\beta)(1+k\alpha^2)$. Price-level targeting generates a value of β close to unity, hence reducing the right hand side to close to zero — the stricter the closer to zero. However, inflation-targeting tends to induce a positive serial correlation, ρ_π, between rates of inflation; thus the serial correlation between price levels is $1+\rho_\pi = \rho(1-\beta)$ when β is calculated from the inflation-targeting rule.

So what we find is that price-level targeting produces a reason to bias indexation away from 100% in

order to induce a helpful correlation offsetting the persistence. Effectively it is partly doing the job of indexation by brining prices back onto target.

7 Implications of rational indexation for the effects of monetary policy

What are the impulse responses to a monetary shock under the three monetary regimes we have identified? The charts show them in turn for inflation targeting, price targeting and stricter price targeting, the latter two with endogenously slightly lower than full (rational) indexation. What we see is that they have none of the supposed hallmark properties of New Keynesian models: there is little persistence, no 'hump shape' in either inflation or output, but rather there is a brief moving shock oscillation followed by virtually no residual effect at all. Price level targeting increases stability, the stricter the greater the increase. As for the productivity shock (Figure 2) we see a rather similar effect to that of a monetary shock superimposed on the steady declining effect of declining productivity on output and consumption. There is plainly no nominal rigidity to speak of in these effects; there is solely an effect of the Calvo mechanism causing relative prices to move in response to both shocks because only a minority of price and wage setters are able to change their relative price currently in response to a current shock.

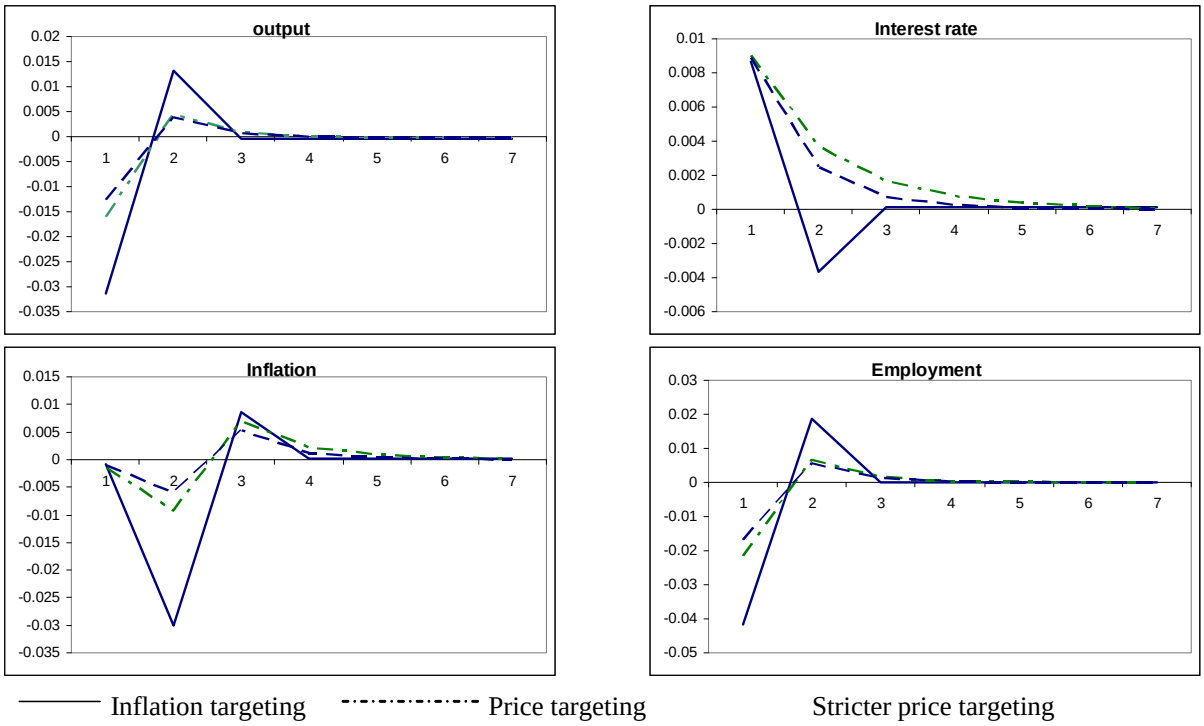


Figure 1: Dynamic paths after an unexpected 0.01 rise in interest rate under different monetary regimes

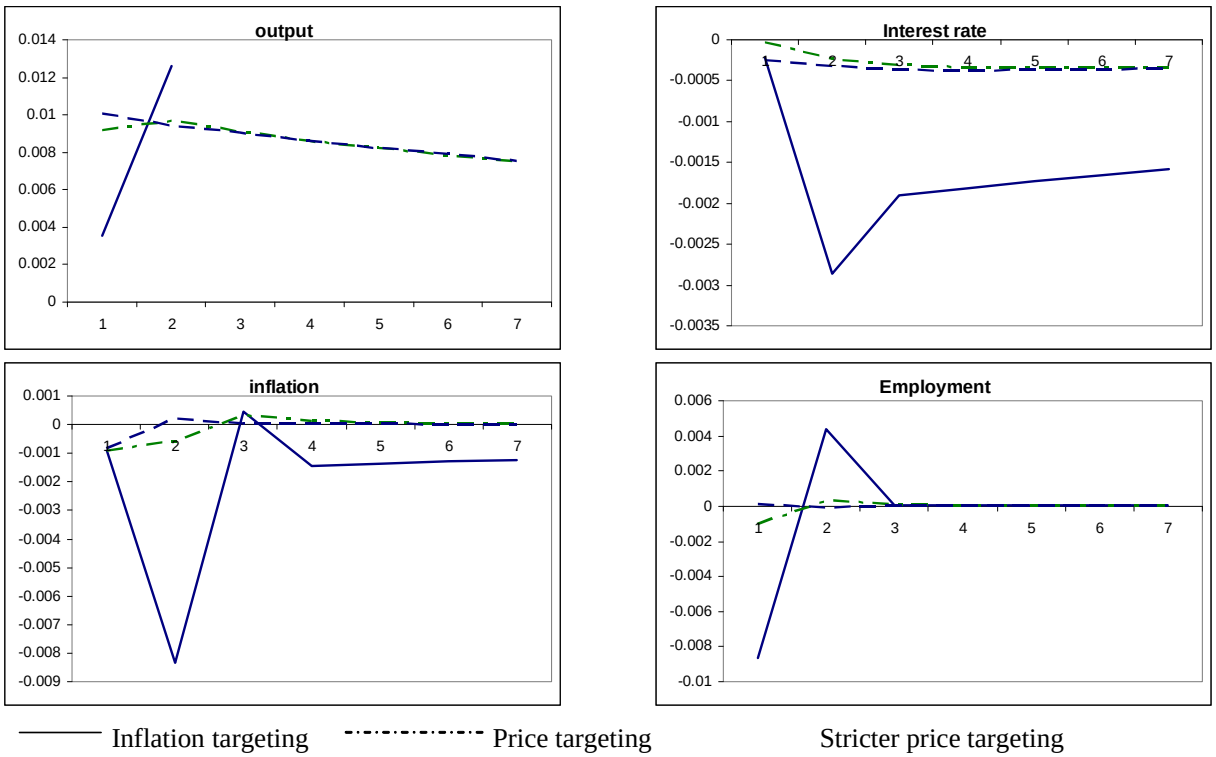


Figure 2: Dynamic paths after an unexpected 0.01 rise in productivity under different monetary regimes

8 General Conclusions

This paper set out to discover what sort of indexation arrangements would be made cooperatively by price- and wage-setting agents operating under Calvo contracts, assuming they were restricted to some response to lagged public price or wage information. We investigated two sorts of indexing formula: one is simply to the lagged price or wage, the other is to the lagged expectation of the current price or wage. We found, under a wide variety of assumptions about the structure of the model, about the extent of current macro information, and about the nature of monetary targeting, that either full or close to full indexation to the rational expectation of prices (for price-setters) and of wages (for wage-setters) was optimal — 'rational' indexation thus dominates.

The implications of this optimising choice for the behaviour of the economy and for the choice of monetary targeting rule are of some interest also. We found that the economy behaves under rational indexation somewhat similarly to a New Classical model, with shocks producing an immediate fluctuation in both prices and output followed by a fairly rapid return to steady state. As for monetary policy, the more this follows a price level target and the more strictly that it does so, the greater the economy's stability (and hence welfare).

It might be argued that these results are of little empirical relevance on the grounds that empirical impulse responses to shocks are 'hump-shaped' and that models with simple lagged indexation fit the data well. However, we have cited recent evidence showing both that empirical impulse responses are not necessarily hump-shaped and that models with little rigidity actually fit the data better than the Calvo model with lagged indexation. Thus we would argue that these results are of considerable relevance to those wishing to use models of the Calvo type but allowing for some indexation mechanism, as well as for those wishing to use models with limited nominal rigidity.

In effect we would like to suggest that rational agents facing micro-based limits on price- and wage-setting can circumvent them to a large degree by the appropriate choice of indexing mechanism, thus approximating fairly closely to the operation of flexible markets with lagged macro information. Thus there are incentives for them to cooperate in setting up such a mechanism. This set-up may also be empirically close to the data.

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9 Appendix

9.1 Solving the model under rational indexation

9.1.1 Welfare: equation (23)

$$\begin{aligned} E\left(u_t - u_t^{FLEX}\right) &= -E\ln DP_t \\ &= -\frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i\left(1-\alpha\right)\left[1-\alpha^i\left(1-\alpha\right)\right]var\left(\log P_{t-i}^*\right) \\ &+ \Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}\left(1-\alpha\right)^2 Cov\left(\log P_{t-i}^*, \log P_{t-j}^*\right) \\ &= (1)+(2) \end{aligned}$$

and

$$\log P_t^* = -\chi'' z_t + \alpha\chi'' z_{t-1}$$

Therefore, $var\left(\log P_{t-i}^*\right) = \left(1+\alpha^2\right)\chi''^2 var(z)$ and (1) is

$$\begin{aligned} (1) &= -\frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i\left(1-\alpha\right)\left[1-\alpha^i\left(1-\alpha\right)\right]\left(1+\alpha^2\right)\chi''^2 var(z) \\ &= -\frac{1}{2}\phi_p\left(1-\alpha\right)\left(1+\alpha^2\right)\chi''^2 var(z)\Sigma_{i=0}^{\infty}\alpha^i\left[1-\alpha^i\left(1-\alpha\right)\right] \\ &= -\frac{1}{2}\phi_p\left(1-\alpha\right)\left(1+\alpha^2\right)\chi''^2 var(z)\frac{2\alpha}{1-\alpha^2} \\ &= -\phi_p\chi''^2\frac{\alpha\left(1+\alpha^2\right)}{1+\alpha}var(z) \end{aligned}$$

and (2) is

$$\begin{aligned} (2) &= \phi_p\left(1-\alpha\right)^2\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\alpha^{i+j}E\left(\log P_{t-i}^*\log P_{t-j}^*\right) \\ &= \phi_p\left(1-\alpha\right)^2\begin{bmatrix} i=0;\ j=1,2... \\ \alpha\left(-\alpha\chi''^2\right)var(z) \\ i=1;\ j=2,3... \\ \alpha^3\left(-\alpha\chi''^2\right)var(z) \\ \vdots \end{bmatrix} \\ &= -\alpha\chi''^2\phi_p\left(1-\alpha\right)^2\frac{\alpha}{1-\alpha^2}var(z) \end{aligned}$$

Thus,

$$E\left(u_t - u_t^{FLEX}\right) = -\phi_p \alpha \chi''^2 var(z) \tag{36}$$

9.2 Solving the model under lagged indexation

9.2.1 Inflation: equation (27)

Under lagged indexation we start by deriving the equation (27) as in the text. Manipulate the equations for inflation and the Aggregate Demand curve respectively:

$$\pi_t = \log P_t - \log \tilde{P}_t = \frac{(1-\alpha)\log P_t^*}{1-\alpha L}$$

and

$$\log Y_t = \frac{1}{1-\sigma^* B^{-1}}\left\{-\tau\sigma^*\pi_t + \sigma\sigma^* \log Z_t - \sigma^* \log M_t\right\}$$

to get

$$(1-\alpha L)\,\pi_t = \frac{(1-\alpha)(1-\alpha\beta)}{1-\alpha\beta B^{-1}} E^t\left(\chi^*\left(\log Y_t - \log Z_t\right) + \pi_t\right)$$

and then

$$\pi_t = \frac{1-\alpha}{1-\alpha L}\frac{1-\alpha\beta}{1-\alpha\beta B^{-1}} E_t\left[\chi^*\left(\frac{1}{1-\sigma^* B^{-1}}\right)\left\{-\tau\sigma^*\pi_t - \sigma^*(1-B^{-1)})\log Z_t - \sigma^*\log M_t\right\} + \pi_t\right]$$

Due to the assumption that in period t producers know both macro information from period $(t-1)$ and micro information in period t, the above equation is rewritten as:

$$\begin{aligned}(1-\alpha L)\,\pi_t = {} & (1-\alpha)(1-\alpha\beta)\chi^*\sigma^* \frac{E_t}{(1-\alpha\beta B^{-1})(1-\sigma^* B^{-1})}(1-B^{-1})(-\log Z_t) \\ & - (1-\alpha)(1-\alpha\beta)\chi^*\sigma^* \frac{E^{t-1}\log M_t}{(1-\alpha\beta B^{-1})(1-\sigma^* B^{-1})} \\ & - \frac{(1-\alpha)(1-\alpha\beta)(\chi^*\tau\sigma^*)}{(1-\alpha\beta B^{-1})(1-\sigma^* B^{-1})} E^{t-1}\pi_t + \frac{(1-\alpha)(1-\alpha\beta)}{(1-\alpha\beta B^{-1})} E^{t-1}\pi_t\end{aligned} \tag{37}$$

Equation (37) can be written as:

$$(1-\alpha L)\,\pi_t - \frac{(1-\alpha)\,(1-\alpha\beta)}{(1-\sigma^* B^{-1})}E^{t-1}\pi_t + \frac{(1-\alpha)\,(1-\alpha\beta)\,(\chi^*\tau\sigma^*)}{(1-\alpha\beta B^{-1})\,(1-\sigma^* B^{-1})}E^{t-1}\pi_t$$
$$= (1-\alpha)\,(1-\alpha\beta)\,\chi^*\sigma^*\frac{E_t(1-B^{-1})\,(-\log Z_t)}{(1-\alpha\beta B^{-1})\,(1-\sigma^* B^{-1})} - (1-\alpha)\,(1-\alpha\beta)\,\chi^*\sigma^*\frac{E^{t-1}\log M_t}{(1-\alpha\beta B^{-1})\,(1-\sigma^* B^{-1})}$$

The LHS of this equation is rearranged into:

$$\begin{bmatrix} (1-\alpha L)\left(1-\alpha\beta E^{t-1}B^{-1}\right) \\ (1-\sigma^* E^{t-1}B^{-1})\pi_t \\ -(1-\alpha)\,(1-\alpha\beta)\,(1-\sigma^* E^{t-1}B^{-1})E^{t-1}\pi_t \\ +(1-\alpha)\,(1-\alpha\beta)\,\chi^*\tau\sigma^* E^{t-1}\pi_t \end{bmatrix} = \begin{bmatrix} 1-\alpha L - \begin{pmatrix} \sigma^*+\alpha\beta- \\ (1-\alpha)\,(1-\alpha\beta)\,\sigma^* \end{pmatrix} E^{t-1}B^{-1} \\ +\alpha\beta\sigma^* E^{t-1}B^{-2} + \alpha\left(\sigma^*+\alpha\beta\right)E^{t-2} \\ -\alpha^2\beta\sigma^* E^{t-2}B^{-1} \\ +(1-\alpha)\,(1-\alpha\beta)\,\chi^*\tau\sigma^* E^{t-1} \end{bmatrix}\pi_t$$
$$= \begin{pmatrix} 1-0.67L+1.42E^{t-1}B^{-1} \\ +0.56E^{t-1}B^{-2}+1.01E^{t-2} \\ -0.38E^{t-2}B^{-1}+0.88E^{t-1} \end{pmatrix}\pi_t$$

while the RHS is reduced to :

$$(1-\alpha)\,(1-\alpha\beta)\,\chi^*\sigma^*\begin{pmatrix} \frac{E_t(1-B^{-1})(-\log Z_t)}{(1-\alpha\beta E^t B^{-1})(1-\sigma^* E^t B^{-1})}\left(1-\alpha\beta E^{t-1}B^{-1}\right)(1-\sigma^* E^{t-1}B^{-1}) \\ -E^{t-1}\log M_t \end{pmatrix}$$
$$= (1-\alpha)\,(1-\alpha\beta)\,\chi^*\sigma^*\begin{pmatrix} \frac{(\rho_1-1)}{(1-\alpha\beta\rho_1)(1-\sigma^*\rho_1)}\left(1-\alpha\beta E^{t-1}B^{-1}\right)(1-\sigma^* E^{t-1}B^{-1})\log Z_t \\ -\rho_2\log M_{t-1} \end{pmatrix}$$
$$= \frac{(1-\alpha)\,(1-\alpha\beta)\,\chi^*\sigma^*}{(1-\alpha\beta\rho_1)\,(1-\sigma^*\rho_1)}\begin{bmatrix} (\rho_1-1)\begin{pmatrix} 1-\alpha\beta E^{t-1}B^{-1}- \\ \sigma^* E^{t-1}B^{-1}+\alpha\beta E^{t-1}B^{-2} \end{pmatrix}\log Z_t - \\ -\rho_2\,(1-\alpha\beta\rho_1)\,(1-\sigma^*\rho_1)\log M_{t-1} \end{bmatrix}$$
$$= \chi^{**}\begin{bmatrix} (\rho_1-1)\left(1-\alpha\beta E^{t-1}B^{-1}-\sigma^* E^{t-1}B^{-1}+\alpha\beta E^{t-1}B^{-2}\right)\log Z_t - \\ -\rho_2\,(1-\alpha\beta\rho_1)\,(1-\sigma^*\rho_1)\log M_{t-1} \end{bmatrix} \tag{38}$$

We are only interested in z_{t-i}, therefore the RHS can be simplified to:

$$\chi^{**}(\rho_1-1)\left(1-\alpha\beta E^{t-1}B^{-1}-\sigma^* E^{t-1}B^{-1}+\alpha\beta E^{t-1}B^{-2}\right)\log Z_t$$
$$=\chi^{**}(\rho_1-1)\left(\log Z_t-(\sigma^*+\alpha\beta)\rho_1^2\log Z_{t-1}+\alpha\beta\rho_1^3\log Z_{t-1}\right)$$
$$=\chi^{**}(\rho_1-1)\left(\frac{z_t}{1-\rho_1 L}-\rho_1^2(\sigma^*+\alpha\beta-\alpha\beta\rho_1)\frac{z_{t-1}}{1-\rho_1 L}\right)$$

Multiply both RHS and LHS by $(1-\rho_1 L)$,

$$\begin{aligned}
&\pi_t-(\rho_1+\alpha)\pi_{t-1}+(1-\alpha)(1-\alpha\beta)(\tau\sigma^*\chi^*-1)\left(E^{t-1}\pi_t-\rho_1 E^{t-2}\pi_{t-1}\right)\\
&+((1-\alpha)(1-\alpha\beta)\sigma^*-(\sigma^*+\alpha\beta))\left(E^{t-1}\pi_{t+1}-\rho_1 E^{t-2}\pi_t\right)-\alpha^2\beta\sigma^*\left(E^{t-2}\pi_{t+1}-\rho_1 E^{t-3}\pi_t\right)\\
&+\alpha(\sigma^*+\alpha\beta)\left(E^{t-2}\pi_t-\rho_1 E^{t-3}\pi_{t-1}\right)+\rho_1\alpha\pi_{t-2}+\alpha\beta\sigma^*\left(E^{t-1}\pi_{t+2}-\alpha E^{t-2}\pi_{t+1}\right)\\
&=\chi^{**}(\rho_1-1)\left(z_t-\rho_1^2(\sigma^*+\alpha\beta-\alpha\beta\sigma^*\rho_1)z_{t-1}\right)
\end{aligned}\tag{39}$$

Given the assumption that $\pi_t=\Sigma_{i=0}^{\infty}\xi^i z_{t-i}$, we collect terms:

(z_t)

$$\xi_0=\chi^{**}(\rho_1-1)\tag{40}$$

(z_{t-1})

$$\begin{aligned}
&\xi_1((1-\alpha)(1-\alpha\beta)(\tau\sigma^*\chi^*-1)+1)-(\rho_1+\alpha)\xi_0+\alpha\beta\sigma_3^*\xi+((1-\alpha)(1-\alpha\beta)\sigma^*-(\sigma^*+\alpha\beta))\xi_2\\
&=-\chi^{**}(\rho_1-1)\rho_1^2(\sigma^*+\alpha\beta-\alpha\beta\sigma^*\rho_1)
\end{aligned}\tag{41}$$

(z_{t-2})

$$\begin{aligned}
0=&\xi_2(1+(1-\alpha)(1-\alpha\beta)(\tau\sigma^*\chi^*-1)(1-\rho_1)+\alpha(\sigma^*+\alpha\beta))-\begin{pmatrix}(\rho_1+\alpha)+\rho_1(1-\alpha)\\(1-\alpha\beta)(\tau\sigma^*\chi^*-1)\end{pmatrix}\xi_1\\
&+\xi_3((1-\alpha)(1-\alpha\beta)\sigma^*-(\sigma^*+\alpha\beta)-\alpha\beta\sigma^*(\alpha+\rho_1))+\alpha\beta\sigma_4^*\xi_4+\rho_1\alpha\xi_0
\end{aligned}\tag{42}$$

$(z_{t-i},\ i \geq 3)$

$$0 = \rho_1 \alpha \xi_1 - \begin{pmatrix} (\rho_1 + \alpha) + \rho_1 (1 - \alpha) \\ (1 - \alpha\beta)(\tau \sigma^* \chi^* - 1) \end{pmatrix} \xi_2 + \alpha\beta\sigma^* \xi_5 + \begin{pmatrix} 1 + (1-\alpha)(1-\alpha\beta)(\tau\sigma^*\chi^* - 1) - \\ \rho_1 \left((1-\alpha)(1-\alpha\beta)\sigma^* - (\sigma^* + \alpha\beta)\right) \\ +\alpha(\sigma^* + \alpha\beta) + \rho_1 \alpha^2 \beta \sigma^* \end{pmatrix} \xi_3$$

$$+ \left((1-\alpha)(1-\alpha\beta)\sigma^* - (\sigma^* + \alpha\beta) - \alpha\beta\sigma^*(\alpha + \rho_1) - \rho_1 \alpha(\sigma^* + \alpha\beta)\right) \xi_4 \tag{43}$$

The last term can be generalised as a 4th order difference equation:

$$\xi_{i+2} - 0.763\xi_{i+1} + 3.43\xi_i - 4.32\xi_{i-1} + 1.1125\xi_{i-2} = 0 \quad \text{for} \quad i \geq 3, \tag{44}$$

which has characteristic roots of $[x = -0.226 \pm 1.904i; x = 0.86093 \text{ and } x = 0.35153]$. Given the first two roots are very small, the equation can be reduced to

$$(1 - 0.351513L)(1 - 0.86093L)\xi_i = 0 \tag{45}$$

$$\text{or } \xi_i - 1.2125\xi_{i-1} + 0.3026\xi_{i-2} = 0 \quad \text{for } i \geq 3$$

Therefore, from equation(40), (41), (42) and (45), we can build the system of equations and solve for ξ_0 and ξ_1.

$$-0.464 = \xi_0$$

$$0.393 = 1.88\xi_1 - 1.6\xi_0 + 0.564\xi_3 + 1.42\xi_2$$

$$0 = 1.5713\xi_2 - 2.34\xi_1 - 0.518\xi_3 + 0.564\xi_4 + 0.623\xi_0$$

$$0 = \xi_3 - 1.2125\xi_2 + 0.3\xi_1$$

$$0 = \xi_4 - 1.2125\xi_3 + 0.3\xi_2$$

The solution is $\xi_0 = -0.464$ and $\xi_1 = -0.151$

9.2.2 Reset Price: equation (31)

Substituting equation (12)

$$\log P_t^{UE} = (1-\alpha)\log P_t^{*UE}$$

into equation (25)

$$E^{t-1}\log P_t^* = \frac{-v_{t-1} + \alpha v_{t-2} - \alpha \log P_{t-1}^{UE}}{1-\alpha}$$

we get

$$E^{t-1}\log P_t^* = \frac{-v_{t-1} + \alpha v_{t-2} - \alpha(1-\alpha)\log P_{t-1}^{*UE}}{1-\alpha} \tag{46}$$

Under the rational expectation and equation (26), the new reset price is

$$\begin{aligned}
\log P_t^* &= E^{t-1}\log P_t^* + \log P_t^{*UE} = \\
&= \frac{-v_{t-1} + \alpha v_{t-2}}{1-\alpha} - \alpha \log P_{t-1}^{*UE} + \log P_t^{*UE} \\
&= \frac{-v_{t-1} + \alpha v_{t-2}}{1-\alpha} - \alpha\left(\frac{1-\alpha\beta}{1-\alpha\beta\rho_1}\right)\left(\frac{1+\chi}{1-\nu}\right)(-z_{t-1}) + \left(\frac{1-\alpha\beta}{1-\alpha\beta\rho_1}\right)\left(\frac{1+\chi}{1-\nu}\right)(-z_t) \\
&= \frac{-v_{t-1} + \alpha v_{t-2}}{1-\alpha} + \frac{\chi'}{1-\alpha}(-z_t + \alpha z_{t-1})
\end{aligned} \tag{47}$$

9.2.3 Welfare

The first one involves all the elements with q_{t-1} in the expression for $E\log DP_t$.

$$\begin{aligned}
E\ln DP_t(q_{t-1}) &= \frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i(1-\alpha)\left[1-\alpha^i(1-\alpha)\right]var\left(\log P_{t-i}^*\right) \\
&\quad - \Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}(1-\alpha)^2 Cov\left(\log P_{t-i}^*, \log P_{t-j}^*\right) \\
&= A(q_{t-1}) + B(q_{t-1})
\end{aligned}$$

Here, we only include in the variance of $\log P_t^*$ the terms in q_{t-1} where q_t follows some autocorrelation process $q_{t-i} = \rho_{i-j}q_{t-j}$, so that $var\left(\log P_t^*[q_{t-1}]\right) = var(q)$. Therefore

$$A(q_{t-1}) = \phi_p\frac{\alpha}{1+\alpha}var(q) \tag{48}$$

Consider $B(q_{t-1})$ now. First we assume that $\rho_{i-j}=1$; then

$$B(q_{t-1})=-\phi_p(1-\alpha)^2\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\alpha^{i+j}\rho_{i-j}var(q)\begin{bmatrix} i=0;\ j=1,2... \\ \alpha+\alpha^2+... \\ i=1;\ j=2,3... \\ \alpha^3+\alpha^4+... \\ : \end{bmatrix}=-\frac{\alpha\phi_p var(q)}{1+\alpha} \tag{49}$$

Summing up equations (48) and (49) gives

$$E\ln DP_t(q_{t-1})=0 \tag{50}$$

However, if $\rho_{i-j}<1$ for any $i-j$, then this term must be negative. Thus for example suppose that $\rho_{i-j}=\rho^{i-j}$ so that q is a first-order autocorrelation process, then

$$B(q_{t-1})=-\phi_p(1-\alpha)^2\Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\alpha^{i+j}\rho^{i-j}var(q)\begin{bmatrix} i=0;\ j=1,2... \\ \alpha\rho+\alpha^2\rho^2+... \\ i=1;\ j=2,3... \\ \alpha^3\rho+\alpha^4\rho^2+... \\ : \end{bmatrix}=-\frac{\alpha\rho(1-\alpha)\phi_p var(q)}{(1+\alpha)(1-\alpha\rho)} \tag{51}$$

As a result of equations (48) and (51), the expected price dispersion is

$$\begin{aligned} E\ln DP_t(q_{t-1}) &= \phi_p\frac{\alpha}{1+\alpha}(var(q))-\frac{\alpha\rho(1-\alpha)}{(1+\alpha)(1-\alpha\rho)}\phi_p var(q) \\ &= \phi_p\frac{\alpha}{1+\alpha}\left(\frac{1-\rho}{1-\alpha\rho}\right)var(q) \end{aligned} \tag{52}$$

So comparing equations (50) and (52), we find that this term q_{t-1} must raise $E\log DP_t$, this in turn reduces the expected welfare. Note that q contains all terms in money shocks and all terms in productivity shocks from $(t-2)$ backwards.

The second part involves all other terms that are not q_{t-1},that is the term $\psi_0 z_{t-1}$. Thus it analyses

the effect of this term $\psi_0 z_{t-1}$ on expected welfare. So looking at this part of $\log P_t^*$:

$$\log P_t^*(\psi_0 z_{t-1}) = \frac{\chi'}{1-\alpha}\left(\alpha z_{t-1} - z_t\right) + \psi_0 z_{t-1},$$

we find

$$\begin{aligned} E\ln DP_t(\psi_0 z_{t-1}) &= \frac{1}{2}\Sigma_{i=0}^{\infty}\phi_p\alpha^i\left(1-\alpha\right)\left[1-\alpha^i\left(1-\alpha\right)\right]var\left(\log P_{t-i}^*\right) \\ &\quad - \Sigma_{j=0}^{\infty}\Sigma_{i=0}^{\infty}\phi_p\alpha^{i+j}\left(1-\alpha\right)^2 Cov\left(\log P_{t-i}^*, \log P_{t-j}^*\right) \\ &= A(\psi_0 z_{t-1}) + B(\psi_0 z_{t-1}). \end{aligned}$$

Given

$$A(\psi_0 z_{t-1}) = \phi_p\frac{\alpha}{1+\alpha}\left(\chi''^2 + \left(\psi_0 + \alpha\chi''\right)^2\right)var(z)$$

and

$$B(\psi_0 z_{t-1}) = \phi_p\frac{\alpha\left(1-\alpha\right)}{1+\alpha}\chi''\left(\psi_0 + \alpha\chi''\right)var(z)$$

this part of expected price dispersion is

$$E\ln DP_t\left(\psi_0 z_{t-1}\right) = \phi_p\frac{\alpha}{1+\alpha}\left(\chi''^2 + \left(\psi_0 + \alpha\chi''\right)^2 + \left(1-\alpha\right)\chi''\left(\psi_0 + \alpha\chi''\right)\right)var(z) \tag{53}$$

Since equation (53) is derived using the part of equation (31) , which partly consists equation for $\log P_t^*$ under the rational expectation indexing, we compare equation (53) and the expected price dispersion under rational indexing (equation(23)):

$$E\ln DP_t = \phi_p\frac{\alpha}{1+\alpha}\left(\chi''^2 + \left(\alpha\chi''\right)^2 + \left(1-\alpha\right)\alpha\chi''^2\right)var(z) \tag{54}$$

It follows that the difference between $E\ln DP_t$ under lagged and rational indexation due to the term in $\psi_0 z_{t-1}$ is: $\phi_p\frac{\alpha}{1+\alpha}\left(\left(\psi_0\right)^2\left[1+\frac{(1+\alpha)\chi''}{\psi_0}\right]\right)var(z)$ from which it can be seen that for this term to worsen welfare under lagged indexation requires that $\frac{(1+\alpha)\chi''}{\psi_0}$ should be positive (or if negative should be greater than -1 which can effectively be ruled out). Now by construction $\psi_0 = \frac{-1}{1-\alpha}(-\xi_1)$ so the (relevant sufficient) condition for the term to worsen welfare is that $\xi_1 > 0$.

The coefficient ξ_1 comes from the path of prices in the period after the shock; this cannot be solved out analytically as it involves solving for all the ξ_i in the equation from Appendix 9.2.1 (involving among other things finding the two stable roots of a fourth order difference equation). Here we find it numerically using the calibration of Canzoneri et al. (2004); the value of ξ_1 turns out to be -0.151,which implies that the term in $\psi_0 z_{t-1}$ improves welfare.

The reason for this is that rational indexation causes reset prices to follow a first order moving average in which the lagged shock in prices when there is a productivity shock is corrected. The pattern is jagged and therefore costly in the first two quarters. Under lagged indexation the reset price follows a moving average in the path of inflation which responds smoothly to a productivity shock; the path is therefore smoother in the first two quarters.

What we have found overall therefore is that for productivity shocks the effect of rational compared with lagged indexation is ambiguous; extra noise is introduced by the lagged index but some of it is correlated with the lagged productivity in a potentially helpful way. For monetary shocks rational indexation is unambiguously superior because in this case these shocks have no effect on the reset price and therefore on welfare; under lagged indexation monetary shocks at $t-1$ and before all enter the current reset price setting.